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Early age mechanical behaviour of 3D printed concrete: Numerical modelling and experimental testing

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ABSTRACT

A numerical model was developed to analyse the mechanical behaviour of fresh, 3D printed concrete, in the range of 0 to 90 min after material deposition. The model was based on a time-dependent Mohr-Coulomb failure criterion and linear stress-strain behaviour up to failure. An experimental program, consisting of unconfined uniaxial compression tests and direct shear tests, was set-up and performed to obtain the required material properties. The material tests showed that the Young's modulus and cohesion linearly increase with fresh concrete age, as do the compressive and shear strength. The Poisson's ratio and angle of internal friction, on the other hand, remain constant. Subsequently, the model was validated by comparison to printing experiments. Modelling of the printed samples reproduced the experimental results qualitatively, but the quantitative agreement with the print experiments could be improved. However, the deviations can well be explained and the type of failure-deformation mode was predicted accurately.

1. Introduction

For some decades now, the construction industry has gradually moved towards a digitalization of processes. In the design phase, architects work in a digital environment, and engineers have adopted numerical tools for structural analyses. Data exchange using Building Information Models (BIM), which allow automated design evaluation, has become standard in many practices. Consequently, automation in the construction phase is becoming more common, as illustrated by e.g. advanced prefabricated and precast industries. More recently, explorations have started into new, additive manufacturing techniques for the construction industry. Many construction materials can be used with various 3D printing methods to manufacture objects ranging in scale from connection elements, to components, to complete buildings. Although 3D print techniques for e.g. ceramics, steel, polymers, foams, glass, or concrete are still in different stages of Technology Readiness Level, their potential in terms of design optimization and customization is becoming increasingly clear [1–5].

3D Concrete printing is one of the areas which is rapidly developing, as illustrated by the high frequency in which new projects are being presented by a growing number of private enterprises and research institutes worldwide [6–9]. These projects still have a case study character: they showcase the possibilities of what could be printed. However, a fundamental understanding of the particularities of the print process and its relation to the properties of the printed product has yet to be developed.

An obvious path of research for 3D concrete printing focuses on the final, hardened printed product, including the interface strength between layers, which is expected to deteriorate for larger interval times between layers [10]. Le et al. [11] discuss a reduction in bond strength between layers as the printing time gap increases, whereas Duballet et al. [12] report a toolpath dependent failure mode in printed specimens.

However, there is an additional, new phase that needs to be addressed: the mechanical behaviour during the 3D printing process. While printing, layers of concrete are deposited on top of each other without the presence of formwork to confine and stabilize the material. The fresh concrete should therefore be sufficiently strong, stiff, and stable to carry its self-weight and the weight of the layers above it, and limit deformations.

First experiences with 3D concrete printing by the authors [5,13] and other research institutes [14,15] have shown that both the printability (i.e. the structural integrity of the object during printing) and the post-print properties (e.g. interface strength) are highly dependent on print process parameters, such as time, temperature, kinematics etc. This has resulted in premature object failures during printing as well as variations in quality of the printed product.

The process-product dependency is also known from 3D printing processes used in other industries. For instance, Dunbar et al. [16] identify a number of build failures that can occur in laser powder bed fusion (LPBF) due to residual thermal stresses, including layer

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delamination, post-build distortions, and more. To be able to accurately predict print product quality (during and after printing) and avoid failures (in other words: obtain a robust print process), extensive finite element-based methods have been developed to model the printing process, including transient behaviour such as time dependencies.

For 3D concrete printing, however, such a virtual production tool has hitherto been non-existent. For low-cost/crude-quality applications, the necessity of such a tool could be debated. But when aiming to compete with existing high-quality manufacturing methods generally associated with prefabricated concrete, it is indispensable. Therefore, this study presents an experimentally validated modelling method of the 3D concrete printing process to assess the printability, based on perhaps the primary process-product dependency: the time-dependent development of mechanical properties of fresh concrete during printing.

Experimental results [17,18] and an analytical framework on fresh printed concrete [19] provide a starting point. However, these concern a purely strength-based failure criterion, whereas in-print failures are also often initiated by loss of stability, as observed in a range of (largely unpublished) printing experiments. Besides, layers may not be stacked exactly on top of each other, either on purpose (cantilevering objects), or due to imperfections/vibrations in the printing process. The printing process itself is not necessarily a constant one, as stops may occur or the printing strategy may change from layer to layer. Additionally, concerning multi material printing or filling the printed structure with a secondary material while printing, stresses may occur not exclusively in vertical direction. In these cases the critical layer is no longer the one with the highest loading, i.e. the initial layer, and an analytical strength-based criterion no longer holds. Any approach should thus be based on the development of both the mechanical properties (including stiffness), and the 3D geometry of the printed object over time.

Developing and validating a modelling method for 3D concrete printing presents four major challenges. First, a suitable material failure model has to be selected. Conventional experiments and analyses on fresh concrete generally focus on concrete in a dynamic state, in relation to e.g. pumpability or filling of formworks. These theories do not hold for fresh 3D printed concrete, which is in a static state after extrusion. Therefore a model was adopted for the state of the material directly after printing, i.e. in between a Bingham fluid and a proper solid. Secondly, specific experimental methods, based on geotechnical test methods of soils, had to be devised to determine the relevant material properties. Once the tests had been carried out, a FE-model was developed as a third step, using the commercially available Abaqus FE-code. It was used to model the material tests and evaluate the results, which proved to be in very close agreement with the experimental results. Finally, a 3D concrete print was modelled with the developed method and validated by comparing the results to actual prints of the same design. This involved development of custom optical measuring methods to track the object behaviour during printing and allow for comparison to the FE-results. Although the quantitative agreement could be improved, the qualitative similarity between the experimental and numerical results already is striking.

2. Modelling parameters for 3D printed concrete

3D concrete printing generally requires a low- to zero-slump material to maintain shape and position after deposition. In this research, a custom designed printable concrete mix was applied as described by [5], containing Portland cement (CEM I 52.5 R), siliceous aggregate with a maximum particle size of 1 mm, limestone filler, additives, rheology modifiers and a small amount of polypropylene (PP) fibers. Fig. 1 shows a close up of the concrete during printing.

The shape stability is also an essential property for other concrete construction processes in which the material is loaded in the dormant period, like slip forming [20,21]. The ‘green strength’ which allows fresh concrete to carry its own weight immediately after mixing or

Fig. 1. Close-up of concrete filament during printing: a low- to zero-slump material is required to maintain shape and position after deposition.

compacting, is attributed to a combined inter particle friction, and cohesion [22]. This mechanical behaviour is similar to that of soils, and as such a Mohr-Coulomb yield criterion is proposed by [23–26], and likewise adopted in the present study, albeit in an expanded form to include time dependent development of the material properties, as given in Eq. 1:

$$\tau_y = C(t) + \sigma_n \tan(\varphi(t)) \quad (1)$$

where C is the cohesion between particles bonded by cement, and φ is the angle of internal friction caused by the frictional resistance and interlocking between internal particles, both of which may be time dependent. The shear yield stress and acting normal stress are given by τ_y and σ_n , respectively.

Two competing time dependent processes during printing determine structural failure of a printed object: the increasing strength and stiffness caused by thixotropic build-up of the concrete [27,28] needs to keep up with the gradually increasing load as more layers are deposited on each other. The latter effect can be incorporated in the numerical model by stepwise load increments, whereas the former is accounted for by updating the yield criterion over time. Experimental findings presented by [29–31] have shown the thixotropic build-up causes the Mohr-Coulomb parameters to evolve significantly within the time frame of a typical printing process. Consequently, the parameters $C(t)$ and $\varphi(t)$ have been established experimentally in this study through direct shear tests on specimens of different age.

However, since print object failures are often stability-driven, the stress-strain relation before failure also was established to determine the stiffness, taken as the tangent Young's modulus $E(t)$, and Poisson ratio $\nu(t)$. These parameters are both needed to model the object response before yielding. As the direct shear test is unsuitable to obtain these due to the non-uniform stress distribution in the sample, an unconfined compression test was adopted. Both experiments are discussed in the next section.

A print object is considered failed when the yield stress is reached at any point in the object, as it will likely coincide with extensive deformations, progressive collapse, and cracking. Therefore, complex post-yield phenomena like viscous behaviour and dilatancy effects have not been established experimentally. Rather, a dilatancy angle ψ has been assumed merely in order to avoid abortion of the numerical analysis when the yield stress is (locally) exceeded, as discussed further in Section 3.3.3.

3. Experimental program

An experimental program is designed to define the Mohr-Coulomb parameters, and strength and stiffness development of fresh 3D printed concrete. Similar to [17,29,32,33] the authors adopted geotechnical

Table 1

Summary of experimental program, where the number of samples indicates the amount of tests for each variable.

	Unconfined uniaxial compression test (UUCT)	Direct shear test (DST)
Age [min]	0, 15, 30, 60, 90	0, 15, 30, 60, 90
Normal load [N]	–	1.5, 11.5, 21.5
Number of samples per set	6	5
Total number of samples	30	75

(soil) tests to apply for concrete in the dormant phase.

Contrary to tests on soil, it is essential that the tests on fresh concrete can be executed swiftly to eliminate the influence of thixotropic build-up within a single test and to be able to attribute the properties to a specific age. Additionally, the tests should be easily repeatable to assess the properties at different material ages. The preparation and testing procedures should be such that unintentional breakdown of the thixotropic build-up is minimized.

The unconfined uniaxial compression test and direct shear test fulfil these requirements and can be used to define the Mohr-Coulomb strength and stiffness parameters. Both tests have been performed at multiple fresh concrete ages of $t = 0, 15, 30, 60$ and 90 min. Here $t = 0$ is the earliest time possible taking into account compaction, demolding, placing of the specimens in the test setup and starting the test, which takes approximately 5 min. The time range of 0 to 90 min corresponds to the typical duration of a 3D printing process, and is well within the dormant period of the material, as the initial setting time is defined at approximately 2 h by standard Vicat measurements [34]. The experimental program is summarized in Table 1. In total, 30 compression tests

Fig. 2. 3DCP Setup, consisting of 4 axis gantry robot, numerical control unit, and concrete mixer and pump.

Fig. 3. Schematic of uniaxial unconfined compression test (left) and photograph of a sample (right). The white dots are used as strain measurement points.

and 75 shear tests have been performed.

The fresh concrete is extracted after mixing and pumping using the 3DCP 3D printer setup [5], which consists of an M-Tec Duomix 2000 mixer-pump with a linear displacement pump that feeds concrete through a Ø1 inch, 10 m length hose (Fig. 2). Consistent mixing speed and pumping frequency are maintained for every test. All samples were prepared and tested at room temperature $T = 22^\circ\text{C}$. The sample temperature at loading was slightly elevated to an average of $T = 24^\circ\text{C}$, which is attributed to the accumulated heat in the mixer-pump.

3.1. Uniaxial unconfined compression test

Uniaxial unconfined compression tests (UUCT) were performed on cylindrical samples. The specimen dimensions were designed according to the ASTM D2166 [35]. The cylinder diameter of $d = 70$ mm is large enough to eliminate size effects due to particle size and distribution, while the height is $h = 140$ mm, so that $h/d = 2$ to allow a diagonal shear failure plane to form. The material was extracted into steel cylindrical molds lined with a thin sheet of Teflon and compacted three times for 10 s on a 30 Hz vibration table to realize a homogeneous sample. Just before testing the sample was carefully demolded and the Teflon sheet removed.

The specimens were loaded in an INSTRON test rig equipped with a 5 kN load cell, see Fig. 3. The load is transferred into the sample through a steel plate with the same diameter. A double sheet of Teflon is placed on both sides of the sample, to realize free support conditions. From soil tests it is known that the rate of loading influences the strength and deformation characteristics, and should be chosen in relation to the material's application. Thus, the displacement controlled tests have been performed at a rate of 30 mm/min, chosen to mimic the loading rate during printing and allow the test to be performed fast enough to neglect effects of thixotropic build-up. The average test time up to 25% strain was equal to 1.2 min. In total 6 specimens were tested for each fresh concrete age, resulting in a total of 30 specimens (Table 1).

The machine load and displacement were recorded. In addition, the vertical and lateral deformation of the specimen was measured. Due to the low stiffness of the fresh material and the occurring large deformations, it appeared not possible to apply a physical measurement device without severely affecting the specimen. Non-contacting, optic measurement systems were used instead. Local deformations were measured at mid height of the sample, by means of two white dots in both vertical and lateral direction, as shown in Fig. 3. These dots were tracked and recorded by the optic measurement system. However, in some of the tests, a local shear plane formed in between the measurement dots, which rendered the deformation measurements unrepresentative for the behaviour of the specimen. Thus, additionally the global deformation was measured by taking high resolution (18 MP) photographs during the test, and post processing this output in National

Fig. 4. Schematic of direct shear test (left) and photograph of the setup without a normal load applied (right).

Instruments Vision Builder software. Results of both systems were compared and deemed similar and accurate.

3.2. Direct shear test

Direct shear tests (DST) were performed on cylindrical samples. The specimen dimensions were designed according to ASTM D3080 [36]. Two horizontal plates with a circular opening with diameter $d = 70$ mm were positioned on top of each other, such that the total specimen height $h = 40$ mm. The plates were filled in three steps with fresh concrete and compacted for 10 s on a 30 Hz vibration table to realize a homogeneous sample.

The specimens were loaded in a Schenck RM100 test rig equipped with a 100 kN load cell. A steel cable translated the vertical displacement of the load cell via a pulley to the top plate, depicted in Fig. 4. This resulted in a constant horizontal displacement rate of 15 mm/min, imposed on the top plate. Additionally, steel weights were used to apply external normal loads as a test variable to the top surface of the specimens. Three load values were applied: 0 N (no weight), 10 N and 20 N, so that the two unknowns $C(t)$ and $\phi(t)$ could be established. Together with the self-weight of the top half of concrete specimen, this resulted in normal loads of 1.5 N, 11.5 N, and 21.5 N, respectively, on the shear plane, which is well defined in the middle of the specimen. The average test time was equal to 1.2 min, which is considered as fast enough to neglect effects of thixotropic build-up during a single test. In total 5 specimens were tested for each age and normal load combination, resulting in a total of 75 specimens (Table 1).

To minimize the friction between the two halves of the shear box, a well-known issue reported e.g. by [37,38], two grooves were machine-milled in the top and bottom part of the plates. Four steel balls were placed in these grooves as rollers. Calibration of the test setup showed the friction in the shear box was thus reduced to 6% of the average failure load of the weakest (i.e. $t = 0$) samples, as approximately 0.8 N was required to move the top plate over the bottom one.

3.3. Results and discussions

3.3.1. Compression tests

Compression tests were performed up to 50% vertical strain, i.e. 70 mm displacement. Load-displacement results are shown in Fig. 6. Note that one result from both $t = 0$ and $t = 15$ min was excluded from further discussions as these test were improperly recorded due to issues with the measurement device.

From Fig. 6 it is clearly observed that the mechanical behaviour of fresh concrete changes in the early age of 0 to 90 min. The peak force increases for older specimens, and the post initial-path behaviour varies with specimen age. For every sample, the load initially increases approximately linearly as the vertical displacement increases. After this initial path, the increment of loading decreases as deformations grow. The younger specimens (i.e. $t = 0, 15$, and 30 min) show an increment in load with increasing deformations, up to a certain plateau. The older specimens (i.e. $t = 60$ and 90 min), on the other hand, show a load decrease after the initial peak, again to a certain limit. This difference in

behaviour is attributed to the occurring lateral deformations and failure behaviour. Due to the relatively low stiffness, the early age specimens significantly expand in lateral direction as the vertical deformation increases. A distinct failure plane is not formed and the specimens fail by ‘barreling’ (bulging), see Fig. 5. As a result, the cross section grows under vertical displacement, leading to an increase of force. The older specimens however, expand much less in lateral direction and have a more distinct failure plane formation (Fig. 5), which roughly coincides with the peak force, after which the loading decreases. This is a well-known phenomenon in soils, where the failure modes of barreling or by shear plane are closely related to the ductility and brittleness of the material [39–41].

The load-displacement data for each specimen was translated into stress-strain diagrams. Due to the large occurring deformations, especially at early ages, all stresses are calculated using an updated cross section. The horizontal displacement measurements from the optic measurement systems are used to update the cross section at every measurement point. The data is cut-off at 25% strain, as severe deformations and cracks were observed after this point and the measured deformations resulted in unrealistic values. For every specimen age the ultimate strength was achieved well before the cut-off value.

Arguably, a correction for self-weight could be included, in particular for early concrete ages ($t = 0$ min), for which the total self-weight of the specimen of 10.7 N corresponds to approximately 23% of the average failure load. At older ages ($t = 90$ min), this percentage reduces to 11%. However, the failure plane usually does not occur at the bottom edge of the sample, but in the center region of the cylinder. Thus, the full self-weight should not be included in the strength calculation. As the failure plane does not always occur at the same specimen height, each test should be corrected for a different self-weight, resulting into difficult comparisons. Moreover, not incorporating the self-weight is a safe approximation of the strength. Thus, the self-weight of each sample will not be added in the strength derivations.

Fig. 5. Typical failure observed in compression tests on fresh concrete. An early age sample of $t = 15$ min shows ‘barreling’ (left), while an old age specimen of $t = 90$ min shows a distinct shear failure plane (right).

Fig. 6. Force - displacement diagrams of compression tests for concrete age $t = 0$ to 90 min.

Fig. 7. Stress-strain diagram of compression tests for concrete age $t = 30$ min. The grey lines indicate the individual tests results, the solid black line represents the average stress-strain relation.

Fig. 8. Average stress-strain relations for each concrete age $t = 0$ to 90 min.

Typical stress-strain data is shown in Fig. 7 for a concrete age of 30 min. Using the 6 experiments for each age, an average stress-strain relation can be determined as indicated by the black solid line. Fig. 8 gives an overview of the average stress-strain relation for each concrete age, clearly showing a significant increase of both strength and stiffness in time. The unconfined compressive strength σ_y is defined for each test as the maximum occurring stress after area correction, and is summarized in Table 2 along with each sample's density, and Young's modulus measured at 5% strain. The 5% strain limit was within the linear range for each age. Moreover, it provides a practical limit in the printing process, as significant deformations would compromise the geometrical accuracy of the object.

The average compressive strength of the youngest specimens is equal to 6.37 kPa, which increases to an average 18.93 kPa at 90 min. These values are in the same order of magnitude as reported by other early age compressive studies on printable concrete [17,18]. A linear fit is found, as depicted in Fig. 9. The average Young's modulus starts at 0.074 MPa and develops linearly to 0.186 MPa (Fig. 9). When normalizing the strength and stiffness value to their maximum average value at $t = 90$ min, their rates of development can be compared and are deemed similar. This is indicated by the approximately 45° angle in Fig. 10, which is a linear fit through the normalized average strength and stiffness values. The measured densities are approximately constant and therefore excluded from further discussions.

3.3.2. Shear tests

Shear tests were performed up to 35 mm horizontal displacement. Typical load-displacement results are shown in Fig. 11 for a 1.5 N normal load (only self-weight). For every sample the load initially increases approximately linearly as the horizontal displacement increases. After this initial path, the increment of loading decreases as deformations grow. This post-initial path behaviour varied with specimen age. Similar to the compression tests, the material shows a more plastic behaviour at early age, which gradually becomes a more brittle

Table 2

Compressive strength, Young's modulus and density derived from the uniaxial unconfined compression test, with average values μ , standard deviation σ , and relative standard deviation RSD.

Concrete age [min]	Compressive strength σ_y [kPa]			Young's modulus E [MPa]			Density ρ [kg/m ³]		
	$\mu_{\sigma y}$	$\sigma_{\sigma y}$	RSD	μ_E	σ_E	RSD	μ_ρ	σ_ρ	RSD
0	6,37	1,07	17%	0,074	0,015	21%	2039	15,0	0,7%
15	7,71	1,42	18%	0,099	0,022	22%	2007	8,90	0,4%
30	10,05	1,31	13%	0,117	0,016	14%	2033	9,15	0,4%
60	15,52	2,29	15%	0,154	0,021	14%	2030	10,6	0,5%
90	18,93	3,92	21%	0,186	0,036	19%	1994	15,0	0,8%

Fig. 9. Compressive strength development (left) and Young's modulus development (right) up to 90 min derived from the compression tests. The grey marks indicate the individual test results, the dashed black line represents a linear fit with $R^2 = 0.99$, based on the average test results.

Fig. 10. Comparison between the strength and stiffness development up to 90 min. All values are normalized to their maximum average value at $t = 90$ min.

failure for older specimens. This is shown by a steady decrease of force after the initial peak for specimens $t = 0$, $t = 15$, and $t = 30$, while a more distinct drop in force is observed for the older ages of $t = 60$ and $t = 90$.

The load-displacement data for each specimen is translated into shear stress-shear displacement diagrams. The data is cut-off at 25 mm, as after this deformation the measured data were disturbed by the friction of the top half of concrete sliding over the bottom aluminum plate as observed by the oscillations in the force displacement diagrams. For every specimen age and normal load combination the ultimate strength was achieved well before the cut-off value.

Fig. 11. Force - displacement diagrams of shear tests for concrete age $t = 0$ to 90 min and normal load 1.5 N.

Fig. 12. Shear stress-displacement diagram of direct shear tests for concrete age $t = 30$ min and normal load 1.5 N. The grey lines indicate the individual tests results, the solid black line represents the average relation.

Fig. 13. Average stress-strain relations for each concrete age $t = 0$ to 90 min and normal load 1.5 N.

Table 3

Shear strength and density derived from the direct shear test, with average values μ , standard deviation σ , and relative standard deviation RSD.

Concrete age [min]	Normal load [N]	Shear strength τ_y [kPa]			Density ρ [kg/m ³]		
		μ_{τ_y}	σ_{τ_y}	RSD	μ_ρ	σ_ρ	RSD
0	1.5	3,11	0,48	15%	2033	40,0	2,0%
15	1.5	3,95	0,37	9%	2076	30,1	1,5%
30	1.5	5,08	0,30	6%	2064	41,6	2,0%
60	1.5	6,87	0,99	14%	2077	38,1	1,8%
90	1.5	8,92	1,20	13%	2066	34,6	1,7%
0	11.5	4,35	0,90	21%	2158	82,7	3,8%
15	11.5	4,66	0,94	20%	2059	51,3	2,5%
30	11.5	4,95	1,01	20%	2092	27,9	1,3%
60	11.5	7,13	0,90	13%	2102	35,2	1,7%
90	11.5	9,34	2,19	23%	2086	51,0	2,4%
0	21.5	5,30	0,83	16%	2076	56,2	2,7%
15	21.5	4,76	0,40	8%	2044	23,0	1,1%
30	21.5	6,32	1,13	18%	2099	29,0	1,4%
60	21.5	8,38	1,51	18%	2027	34,6	1,7%
90	21.5	12,43	2,74	22%	2044	37,8	1,8%

Due to the large occurring deformations all stresses were calculated using an updated cross section. The horizontal deformation is used to update the cross section A_{new} at every measurement point, by using Eq. (2) [42]:

$$A_{new} = A_0 \cdot \frac{2}{\pi} \left\{ \cos^{-1} \left(\frac{\Delta h}{D} \right) - \left(\frac{\Delta h}{D} \right) \sqrt{1 - \left(\frac{\Delta h}{D} \right)^2} \right\} \quad (2)$$

where A_0 is the initial sample area, D is the sample diameter, and Δh the horizontal shear displacement.

Typical shear stress-displacement data are shown in Fig. 12 for a concrete age of 30 min. Similar to the compression tests, using the 5 experiments for each age and normal load combination, an average shear stress-displacement relation can be determined as indicated by the black solid line. Fig. 13 gives an overview of the average stress-strain relation for each concrete age. The shear strength τ_y is defined for each test as the maximum occurring stress after area correction, and is summarized in Table 3 along with each sample's density.

The average shear strength at $t = 0$ is equal to 3.11 kPa and develops in a linear fashion towards 8.92 kPa at $t = 90$ min, as depicted in Fig. 14. This rate of strength development found in the DST is equal to the results of the UUCT test as shown in Fig. 15, where the normalized compressive strength is plotted versus the normalized shear strength. The linear fit makes a 45° angle with the abscissa and ordinate, which

Fig. 14. Shear strength development up to 90 min derived from the direct shear tests with normal load 1.5 N. The grey marks indicate the individual test results, the dashed black line represents a linear fit with $R^2 = 0.99$, based on the average test results.

indicates an equal rate of development.

As expected according to the Mohr-Coulomb theory, the addition of a vertical normal load increases the shear strength. Fig. 16 shows the relationship between the maximum shear stresses and the corresponding normal stresses for concrete age $t = 0$ to 90 min.

3.3.3. Derivation of model parameters

A linear development of strength and stiffness in time was derived for the first 90 mins of concrete age. This linear trend falls in line with common considerations of fresh concrete prior to setting [28,43].

Using the results of the direct shear tests, the Mohr-Coulomb parameters can be defined as a function of age for this material. The cohesion C corresponds to the intercept of the dashed lines with the ordinate in Fig. 16 and the slope is the angle of internal friction φ . The cohesion is equal to 3.05 kPa at $t = 0$ min and increases in a linear fashion to 8.27 kPa at $t = 90$ min, given by the function:

$$C(t) = 0.058 \cdot t + 3.05 \quad (3)$$

It should be noted that the experimental results inhibit considerable scatter, which may be attributed to the sensitivity of the 3DCP setup. Even though this still results in a clear linear time-dependency for the cohesion, such an obvious proportion between age and the angle of internal friction could not be established. Fig. 17 shows that the average values of φ for each age all lie within one standard deviation of each other. Therefore, a constant value of angle of internal friction $\varphi = 20^\circ$ was adopted. The constant nature of this value falls in line with experimental findings on fresh concrete [25] and is a conservative choice.

Substituting (3) in (1) results in the Mohr-Coulomb criterion as a function of concrete age:

$$\tau(t) = C(t) + \sigma_n \cdot \tan(\varphi(t)) = (0.058 \cdot t + 3.05) + \sigma_n \cdot \tan(20^\circ) \quad (4)$$

Using the results of the compression tests, the stiffness and deformation behaviour in time is derived. The Young's modulus as a function of concrete age was determined at 5% strain and equals:

$$E(t) = 0.0012 \cdot t + 0.078 \quad (5)$$

The Poisson's ratio was derived from the lateral deformations in the compression tests, again at 5% strain. This ratio appears to be approximately constant in the first 90 min and is equal to $\nu = 0.3$.

Concrete, like soil and rock, shows a non-associated, dilatant plastic behaviour. As dilation effects largely occur after the peak load has been reached (and the object is considered failed), they have not been

Fig. 15. Comparison between the compressive strength and shear strength up to 90 min. All values are normalized to their maximum average value at $t = 90$ min. The dashed line represents the linear fit and makes a 45° angle with the axes, indicating an equal rate of development.

Fig. 16. Shear stress versus normal stress for concrete age $t = 0$ to 90 min, derived from the direct shear tests. The black, dark grey, and light grey marks indicate respectively 1.5 N, 11.5 N, and 21.5 N vertical force applied. The dashed line represents a linear fit through the average values.

Fig. 17. Angle of Internal Friction for concrete age $t = 0$ to 90 min. The black marks represent the average values, and the standard deviation is indicated by error bars. They dashed grey line indicates a constant value equal to $\varphi = 20^\circ$.

measured in this study. For the numerical analysis, a dilatancy angle of $\psi = 13^\circ$ was assumed, equal to a typical value for hardened concrete [44]. Similar to the angle of internal friction, it is independent of fresh concrete age.

Five essential material parameters have now been defined to numerically model printed concrete of age $t = 0$ to 90 min namely the Young's modulus $E(t)$, Poisson's ratio ν , the Mohr-Coulomb parameters cohesion $C(t)$ and angle of internal friction φ , and the dilatancy angle ψ . Two parameters are linearly age dependent, the other three are constant within the considered time frame. They are used as input for numerical modelling as discussed in the next section.

4. Numerical model

Finite Element Method (FEM) analyses were carried out in two steps using the commercially available Abaqus FE-code. The first step

Fig. 18. Comparison between the experimental (grey) and numerical (black) stress-strain results of the compressions test, for concrete age $t = 0$ to 90 min.

concerns validation of the Mohr-Coulomb material model in Abaqus by simulating the compression and shear tests from the experimental program as described in Section 3. In the second step this material model is used to simulate the structural behaviour during an actual printing process and study the failure-deformation mode.

4.1. Compression test

A numerical model of the compression test was used to analyse the strength and deformation characteristics of the fresh concrete, based on the experimental results. An axisymmetric model was adopted, given the cylindrical shape of the specimens. The dimensions were taken equal to those of the experiments, i.e. $r = 35 \text{ mm}$ and $h = 140 \text{ mm}$. A regular mesh of axisymmetric linear 4-node continuum elements was used, which consisted of 10×40 elements in the radial r , and axial Z , directions respectively.

Due to the Teflon on Teflon contact used in the experiments, the

Fig. 19. Comparison between experimental and numerical results of the shear tests. The dashed lines represent the Mohr-Coulomb failure criterion for each concrete age, and the markers indicate the failure stresses derived from the FEM analyses.

boundary conditions at the specimen edges are taken as free in radial r direction. A deformation of 35 mm in negative z direction was imposed on the top edge of the cylinder, which corresponds to 25% vertical strain. The bottom edge was simply supported in z direction.

The age dependent material parameters as discussed in Section 3.3.3. were used. The analysis was performed for each age as tested in the experimental program. The resulting stress-strain diagrams in loading direction were compared to the experimental results, see Fig. 18. The numerical results (in black) are in good agreement with the experimental findings (in grey). As such, the numerical model is deemed suitable to predict the stiffness and deformation behaviour of fresh concrete.

4.2. Shear test

A numerical model of the direct shear test was used to evaluate the Mohr-Coulomb parameters as derived from the experimental results. A cylindrical 3D model was used, with dimensions equal to those of the experiments, i.e. $d = 70$ and $h = 40$. Because of symmetry only half of the specimen was modelled. The aluminum plates of the shear box apparatus containing the specimens were modelled as rigid surfaces, including the small gap of 0.5 mm between the two plates as used in the experimental setup. The interface between concrete and the box walls was modelled using Tie-constraints. A regular mesh of linear 8-node

continuum elements was applied. The Mohr-Coulomb failure criterion was used, and the material behaviour was assumed to be linear elastic up until the failure stress as found in the experimental program was reached.

The analysis was carried out in two steps. In the first step, the self-weight of the sample was activated and, when applicable, a normal load is applied on top of the specimen. In the second step, a horizontal displacement of 25 mm was imposed onto the top plate. The analysis was carried out for three external normal loads of 0, 10 and 20 N, which were applied as an evenly distributed pressure load. These three analyses were repeated for each concrete age as tested in the experimental program. The failure stress was then recorded and compared with the experimental results. The Mohr-Coulomb criterion as a function of time (Eq. 4) was plotted for the five different concrete ages as used in the experimental program, represented by the dashed grey lines in Fig. 19. The failure stress from the FEM analyses for three normal loads is indicated by the markers. The numerical results are in accordance with the experimental findings. Considering the agreement between experimental results on the one hand, and the material model and failure criterion on the other, the numerical method was considered suitable to analyse the printing process.

4.3. Print process

The printing process was modelled for a layered cylinder with a heart line radius of 250 mm, a thickness of 40 mm, and a layer height of 10 mm. These layer dimensions follow from the nozzle used in the 3DCP setup. By applying the model change option in ABAQUS, the layers were added stepwise on top of each other continuously during the analysis, until failure occurs. The time interval $t_{interval}$ between these layers was based on the chosen printing speed of $v_{print} = 5000$ mm/min, divided by the circumference of each layer: $t_{interval} = \pi d / v_{print} = 0.31$ min. The strength and stiffness properties of each layer are then calculated during the analysis based on their age in the printing process: e.g. after 10 layers, the initial layer will be 3.10 min old and use the corresponding properties of that age, while the fifth layer of the model will have the properties of 1.55 min old, and so on.

Axisymmetric linear 4-node continuum elements were applied. Each layer consists of 40 by 10 elements in width and height direction, respectively. The bottom layer is fixed, due to the high friction on the print bed in practice. Each layer has a gravity loading pointing downwards based on the average density of 2070 kg/m³ measured in the shear and compression tests. A Geometrical Non Linear analysis was performed to incorporate the influence of large deformations during the printing process. Default direct solver settings were used, with the exception of an unsymmetrical matrix storage due to the non-associated material behaviour. No initial imperfections were defined, as the

Fig. 20. Numerical results of an axisymmetric modelled cylinder, depicted for 5 to 40 layers, using the lower bound material properties. The plotted colors represent the radial deformations. On the right, the red color indicates where the yield stress is reached and plastic deformation occurs. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Fig. 21. FEM results showing deformed shape of cylinders for the lower bound (left), average (center) and upper bound (right) case.

Table 4
Summary of numerical results for the lower bound, average, and upper bound analyses.

Analysis	Number of layers	Max. radial deformation before failure [mm]	Z position of max. radial deformation [mm]
Lower bound	40	11,0	106,7
Average	46	13,89	115,8
Upper bound	53	13,05	116,5

failure-deformation mode can be initiated by restrained lateral deformations (expansion) close to the support.

Three different cases were analysed to study the impact of the relatively large scatter in material properties during the printing process. The average values derived from the experiments were used, along with an upper and lower bound of strength and stiffness parameters. The relative standard deviation of the experimental strength and stiffness values is approximately 17.5% of their corresponding average values. This value was therefore used as both reduction and increase of the average (AVG) values, i.e. $0.825 \cdot \text{AVG}$ and $1.175 \cdot \text{AVG}$, to obtain a lower and upper bound estimation of the failure-deformation mode.

Fig. 20 shows the deformation shape and horizontal deformations in the layers at different stages, i.e. 5 to 40 layers, for the analyses with lower bound material properties. It is observed that in all three analyses the failure-deformation mode is a combination of instability (cylindrical buckling) and material yielding. Generally, cylindrical buckling is initiated first by restrained radial deformations due to the support conditions. The radial buckling deformation increases as layers are added to the structure. Simultaneously, the stresses due to the self-weight of the layers increase and ultimately reach the limit as defined by the Mohr-Coulomb criterion in the lower parts of the cylinder. At this point, the material will start yielding and significantly deform. The regions in a plastic state just before failure are indicated by the red color in Fig. 20. These plastic deformations, along with the continuous increase of vertical loading, lead to a second order bending moment which grows until global failure is reached.

In Fig. 21 the deformed cylinders are plotted in different stages for all three analyses, and the results are summarized in Table 4. The 17.5% increase or decrease in material properties significantly influence the moment of failure. The cylinder with AVG values failed after 46 layers, while the lower bound values led to failure at 40 layers, and the higher bound at 53 layers. The type of failure however, remains

identical. No clear relation for the maximum radial deformation can be derived, which can be explained by two counteracting phenomena. A reduction of stiffness leads to higher deformations, while simultaneously, due to the reduction of strength, the cylinder fails at a lower number of layers. As such, the overall loading and corresponding deformation is less. The vertical position where the radial deformation is at a maximum is approximately equal for all three analyses.

5. Validation

The cylinder of the numerical program was printed using the same settings, i.e. layer dimensions of 40 by 10 mm, a radius of 250 mm, and printing speed of 5000 mm/min. To compare the failure-deformation mode of the numerical analyses with the printed cylinders, a non-contacting optic measurement system was used (Fig. 22). During the printing process, small circular markers were placed onto the fresh material on two sides of the cylinder. The object was exposed to red colored LED light, to increase the contrast between the markers and the fresh concrete. Two Basler Ace Ethernet camera's with an Edmund Optics fixed focal length lens were placed on both sides of the cylinder, and continuously took high resolution photographs of the current state

Fig. 22. Experimental setup to monitor deformations and failure during the printing process.

Fig. 23. Deformed shape of a cylinder during the printing process, visualized at 5 to 30 layers.

Fig. 24. Radial deformation plots of 5 cylinders during the printing process, visualized at 23 (left), 26 (center), and 29 (right) layers. The markers indicate the measurement points, the continuous lines are a smooth fit through these points.

of the printed object. These photos were then analysed real time in NI Vision software, where the circular trackers were used to compute the deformation during the printing process. This print experiment was carried out 5 times.

Typical deformation results are visualized in Fig. 23, for a printed cylinder after 5, 10, 15, 20, 25, and 30 layers. A cylindrical buckling deformation is observed for each printed cylinder, similar to the results of the FEM analyses. The five printed cylinders collapsed after 30, 25, 31, 27, and 31 layers respectively, resulting in an average cylinder height of 29 layers.

Using the Vision software, deformation plots can be generated similar to the numerical results of the previous section. Fig. 24 depicts the deformed shape of all 5 cylinders after 23, 26, and 29 layers. The markers indicate the measured values, while the continuous lines are a fit-line through these points that represent the geometry of the cylinder. Note that the step size of three layers follows from the spacing of markers in the cylinder, i.e. one in every three layers.

When comparing the numerical results to the printing experiments, it can be concluded that the FEM model qualitatively predicts the failure-deformation mode during the printing process. In practice

however, the cylinders collapse in an earlier stage and deform more at a lower load (i.e. fewer layers). The experimental results are summarized in Table 5. The average number of 29 layers reached in the experiments deviates 27.5% from the 40 layers according to the lower bound numerical analysis. The average experimental radial deformation just before failure, here measured at 29 layers, is equal to 15.3 mm, which corresponds to 139% of the lower bound numerical analysis. The

Table 5
Summary of failure-deformation results of five printed cylinders, with average values μ , standard deviation σ , relative standard deviation RSD.

		Max. radial deformation [mm]			Z position of max. radial deformation [mm]		
		Layer 23	Layer 26	Layer 29	Layer 23	Layer 26	Layer 29
μ	29	10,3	12,7	15,3	114,7	116,5	114,7
σ	2,4	2,2	2,7	2,2	4,5	2,8	2,6
RSD	8%	21%	21%	14%	4%	2%	2%

location (vertical height) at which the maximum radial buckling deformation is at a maximum, is approximately equal for both the experimental and numerical results, as all results are within a range of 10 mm, i.e. one layer, of each other.

Two main reasons are distinguished for the discrepancy between numerical and experimental results. Firstly, axially loaded shells are sensitive to geometrical imperfections that cause eccentricity in the application of vertical loading, and to material imperfections (non-homogeneity). The critical buckling load of a shell with imperfections can be several times lower than according to classical theory [45]. As imperfections have not yet been incorporated into the numerical model, it underestimates deformations and overestimates the moment of failure.

Secondly, the properties as derived in the experiments are for a material that has been extracted from the 3D printer, and compacted afterwards to realize a homogeneous sample. This, however, significantly results in improved material properties (increased strength and stiffness, decreased compressibility) in comparison to the actual printed concrete that is not compacted in the process.

6. Conclusions

In this study, a numerical model was developed to analyse the mechanical behaviour of early age 3D printed concrete, in the range of 0 to 90 min after material deposition. An experimental program, consisting of unconfined uniaxial compression tests (UUCT) and direct shear tests (DST), was set-up and performed to obtain the required material properties. Subsequently, the model was validated by comparison to printing experiments. A number of novel insights and conclusions are summarized below:

The numerical model is deemed suitable to analyse the structural behaviour of a concrete object during 3D printing, and qualitatively predict the corresponding failure-deformation mode. Quantitatively, however, the model overestimates the strength and stability of the object. This is likely due to overestimation of the material properties from testing because of compaction of the specimens, and due to disregarding the influence of geometrical and material imperfections induced by the printing process.

The Mohr-Coulomb theory as applied in the numerical model, i.e. extended with time dependent development of material properties, may be considered as a satisfactory failure criterion for fresh 3D printed concrete. In this study, the cohesion was found to be a linear function of time, and the angle of internal friction independent of age, within the time frame of a typical printing process.

Finally, it may be concluded that geotechnical (soil) tests are suitable to assess the properties of early age printed concrete. In this study, the Young's modulus, compressive strength and shear strength were found to be a linear function of time, and develop proportionally with fresh concrete age. The authors recommend to develop and apply tests which can assess the properties of printed concrete without compaction.

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