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Magnetic artificial cilia for microfluidic propulsion

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1 Introduction

1.1 Lab-on-a-chip

Lab-on-a-chip (LOC) is a technology that aims at performing analyses of biological fluids (such as blood, saliva and urine), conventionally performed in a clinical lab, on a small chip. The analyses range from simple tests on biological samples to sophisticated DNA and cell analysis. The primary reason for the development of LOC is that the reduction in the sample size of the analyte enhances control and accuracy of bio-chemical reactions. The added advantage of the small size of the device is that smaller amounts of analyte and reagents are needed to perform the reactions, and the device becomes portable. As the facilities needed to perform the bio-chemical analysis are integrated in a small device, it was aptly named a micro-total analysis

system (micro-TAS) [71].

The important tasks performed in a micro-TAS are the treatment of an analyte with suitable reagents, subsequent bio-chemical reactions, separation of the molecules resulting from the reactions and the detection of these molecules. One example of such a bio-chemical reaction is a polymerase chain reaction (PCR), which is performed to amplify the concentration of specific DNA strands by several orders of magnitude. In this process, the analyte (containing a few DNA strands) is subjected to cyclic heating and cooling. When a PCR is miniaturised, the thermal response time becomes low leading to a drastic reduction in the PCR cycle time [55]. In addition, miniaturisation can lead to very rapid separation of bio-molecules using methods such as electrophoresis and chromatography [70, 36].

The micro-TAS can be encapsulated into a hand-held device which can be used for point-of-care (POC) testing, where the clinical diagnosis can be performed at the location of the patient, even by an untrained person. Instruments to perform simple analyses such as measuring glucose levels, hemoglobin levels and lithium levels have been commercialised. The importance of POC testing is especially applicable to patients with Type I diabetes, for which self monitoring of blood glucose is considered as an integral part of their treatment [54]. POC devices can also reduce the mortality rates in critical care units of hospitals [77] and are of immense help in disaster-affected areas, where it is difficult to perform regular clinical tests [56].

As the surface to volume ratio is high in a micro-TAS, physical phenomena associated with surfaces (e.g. surface tension and electrokinetics) gain importance. In addition, as the length scales involved are small (typically less than a millimetre), the viscous forces in the fluid dominate over the inertial forces leading to laminar flow profiles in a typical micro-TAS [83]. While some of these are beneficial, the presence of others are detrimental. For instance, the fabrication of devices to perform individual operations on a micro-TAS cannot always be done by simply downscaling conventional methodologies. Mixing of an analyte with another fluid is difficult to achieve in a microfluidic device due to the laminar nature of flow at these length scales. Another challenge is the pumping of fluids through the microchannels and testing chambers on a lab-on-a-chip. In some applications, a local control of the flow is also necessary, which calls for a localised pumping system that can be embedded into a microchannel.

The fluid propulsion in microfluidic systems is performed using three different approaches: (i) mechanical methods – such as external syringe pumps, peristaltic pumps [74, 64, 30, 86, 58, 31] and membrane pumps, (ii) using the electrokinetic properties of the fluids – such as in electro-osmotic pumps [97, 67] and magneto-hydrodynamic pumps [63, 37, 42, 93] – and (iii) acoustic methods [60, 73, 96, 95]. These fluid propulsion mechanisms have been developed only in recent years. On the other hand, nature has been using remarkable fluid propulsion mechanisms at micron length scales for the locomotion and fluid transport, which are primarily based on mechanical actuators that beat back and forth. In this work, we use principles inspired by natural systems to design a fluid propulsion system that can operate inside microchannels and controlled by external magnetic fields.

1.2 Micron-scale fluid manipulation in nature

Micron-scale fluid manipulation occurs in nature for two main reasons: locomotion and fluid transport. These are often (but not always*) performed using hair-like motile appendages known as cilia and flagella [72, 18]. The cilia can beat in two different ways. Firstly, the cilia on the external surfaces of organisms such as opalina beat in an asymmetric manner with a distinct effective and recovery stroke (see Fig. 1 (a)-(c)). During the effective stroke the cilia are straight and push large amounts of fluid, whereas during the recovery stroke they stay closer to the cell surface and pull back only a small amount of fluid. The net fluid propelled is in the direction of the effective stroke (see Fig. 1(c)). The hydrodynamic interaction causes adjacent cilia to beat out-of-phase leading to a wave-like motion which is commonly referred to as metachronal waves (see Fig. 1 (b)). Secondly, there is another category of cilia, called nodal cilia, that are present on node cells of embryos and revolve with a whirling motion about an axis that is non-orthogonal to the surface on which they are attached (see Fig. 1 (d)-(e)). These cilia create a large fluid flow when they are away from the surface and a low fluid flow in the opposite direction when they are close to the surface. As a result, a net flow is created in the direction of the upward stroke. A flagellum that is attached to a cell propagates waves

*The locomotion of a Cyanobacterium takes place by propagating waves of lateral displacement along its surface [23].

Figure 1: (a) Micrograph of cilia on *Opalina* [87]. (b) Metachronal waves are formed due to out-of-phase beating of the cilia. (c) Schematic diagram showing the movement of a cilium. (d) Micrograph of cilia on node cells (<http://www.physics.ubc.ca/steve/research/A-TopProt.html>). (e) Schematic representation of the motion of a nodal cilium. (f) and (g) Snapshots of a flagellum attached to a sperm cell [94]. The numbers refer to the sequences in time. The flagellum creates a wave of transverse displacement for fluid propulsion, the cilia in the case of *Opalina* beat with a distinct effective and recovery stroke and in the case of nodal cilia they describe a cone. Reproduced with permission from [43].

of transverse displacement along its length to exert a force on the fluid in the direction of the wave, which causes the flagellum and cell to move in the opposite direction (see Fig. 1 (f)-(g)).

Flagella are primarily known for locomotion [65, 14], whereas the cilia perform different functions like locomotion and fluid transport [72, 14, 26]. In some cases, the cilia are non-motile and perform sensing functions [26, 68]. The flagella are usually attached to a cell body, whereas the cilia occur in groups over the surface of microorganisms (for example on the surface of a *Paramecium*) or tissues of organs. The cilia are associated with micro- as well as macroorganisms. Microorganisms (such as *Paramecia*, *Opalina* and *Centophores*) have cilia on their outer surface for locomotion. In organisms such as *Lophophorates*, the cilia create a water current that brings suspended food particles near the organism's mouth [85]. The nodal cilia create a fluid flow that initiates the left-right asymmetry during embryonic development [34, 40]. The cilia on the surfaces of ventricular system of the brain play an important role in the transport of the cerebro-spinal fluid [78, 40]. Cilia are also present in the inner-lining of the respiratory tract and propel mucus out of the lungs [40].

1.3 Hydrodynamics at small length scales

At the small length scales of cilia and flagella (ranging from several microns upto hundreds of microns), viscous forces dominate over the inertial forces leading to low values in Reynolds number[†] ($Re \ll 1$) – the ratio of fluid inertial forces to the viscous forces. To get a feel for the viscous forces, consider a microorganism swimming at a steady velocity. The distance (normalised to its body length) this organism continues to move after its propulsion mechanism has stopped scales with $Re^\ddagger$ [62]. Since $Re \ll 1$, the organism cannot coast after it has stopped its propulsion mechanism. This is because the viscous forces are so large that they dissipate the kinetic energy of the organism instantaneously.

There are two consequences of high viscous forces. Firstly, a certain class of cyclic actuator motion – called reciprocal motion, in which the forward motion is exactly the same as the backward motion – cannot lead to a net fluid propulsion. An example of reciprocal motion is the back and forth oscillation of a rigid rod about one of its ends. During a reciprocal motion of the actuator the flow created during the forward motion of the actuator will be cancelled by the flow of the backward motion, creating zero net fluid transport. Secondly, the fluid behaviour is rate independent, i.e., even if the forward reciprocal motion of the actuator takes place faster than its backward motion, there will be no net fluid flow. The flow created by the fast forward motion will be exactly cancelled by the slow reverse motion of the actuator. Rigorous mathematical proofs of these two properties can be found elsewhere [16, 62].

As the cilia and flagella are able to create a fluid flow, their motion must be ‘non-reciprocal’. The cilia motion is non-reciprocal because of its distinct effective and recovery stroke. In the case of flagella, the wave of transverse displacements causes the slope of any segment before and after it has reached a maximum displacement to differ by a sign. This makes the flagella motion to be non-reciprocal. The non-reciprocal motion can also occur at larger length scales. For instance, a Cyanobacterium swims through a fluid by propagating waves of lateral displacement on its surface. Individual points on the surface oscillate about a mean position in a reciprocal fashion, but the directional wave of lateral displacements makes the surface motion non-reciprocal [84].

1.4 Artificial cilia in the literature

The early attempts to synthesise artificial cilia were made by Evans et al. [24] and den Toonder et al. [20]. In [24], rod-like magnetic cilia were fabricated using a template-based approach. A uniform magnetic field was applied to these cilia such that they prescribe a cone whose axis is non-orthogonal to the substrate, much like the nodal cilia shown in Fig. 1(d) and (e). It was shown that this motion of the cilia creates fluid transport above the cilia. Fluid transported by similar magnetic rod-like cilia was reported by Vilfan et al. [91]. However, in this case the cilia were fabricated by assembling super-magnetic beads using optical traps or by the use of dipole interaction between the beads when a magnetic field is applied. The cilia fabricated by den Toonder et al. [20] were curled plate-like structures aimed at mixing applications in lab-on-a-chip devices. These cilia were made of polyimide thin films of 100 micron length and coated with a thin layer of chromium. When an electric field is applied between the cilia and the substrate, the cilia snaps to the substrate and on removal of the electric field, it returns to the initial position. The cilia created flow although there was no significant asymmetry in the motion. It was later found by Baltussen et al. [4] that the main cause of the fluid flow is the inertial forces of the fluid. Plate like cilia made of a composite of a polymer filled with magnetic particles were reported by Fahrni et al. [25], Belardi et al. [7] and Hussong et al. [39]. The cilia fabricated by Fahrni et al. were actuated using an uniform rotating magnetic field generated by a quadrupole magnet, while the cilia reported by Hussong et al. were actuated using a strong permanent magnet that creates a strong and non-uniform fields. Timonen et al. [88] reported very slender magnetic cilia (aspect ratio larger than 100) made from magnetic particles glued together using a polymer solution. When properly actuated, these cilia were able to successfully mix liquid droplets of glycerol. Photo-actuation of plate-like artificial cilia made using light-responsive liquid crystal networks was demonstrated by van Oosten et al. [89]. Recently, a cost-effective in situ fabrication technique for artificial cilia was demonstrated by Wang et al. [92]. These cilia are constructed by self-assembly of micron sized magnetic beads and encapsulated with

[†]For cilia of length $10\ \mu\text{m}$ that beat at a frequency of 20 Hz in water, the Reynolds number is 2×10^{-3} .

[‡]Assuming the mass density of the organism is the same as the fluid.

soft polymer coatings.

2 Problem definition and modelling approaches

In the following we briefly discuss the problem at hand and the methods used for its solution. We consider a microchannel, filled with a Newtonian fluid, consisting of an array of magnetic artificial cilia, see Fig. 2. When a uniform external magnetic field $\mathbf{B}_0$ is applied, a magnetic body couple N_z (in units of couple per unit volume) acts on the cilia (see Fig. 2(b)). The magnitude of the magnetic couple depends on the magnitude of the applied magnetic field $|\mathbf{B}_0|$, the magnetic properties of the cilia and the relative orientation of the applied magnetic field and the artificial cilia. The magnetic body couple causes the motion of the cilia, which push the fluid. Due to the viscous and inertial resistance of the fluid, the cilia in turn experience a drag force, which have a tangential and normal components f_x and f_y , respectively (see Fig. 2(b)).

2.1 Governing equations

We now list the governing equations and boundary conditions that are relevant for the boundary value problem shown in Fig. 2. The magnetic field in the cilia are obtained by solving the static Maxwell's equations:

$$\begin{aligned}\nabla \cdot \mathbf{B} &= 0, \\ \nabla \times \mathbf{H} &= 0,\end{aligned}\tag{1}$$

where $\mathbf{B}$ is the magnetic flux density and $\mathbf{H}$ is the magnetic field. Inside a magnetic body they are related by the constitutive relation $\mathbf{B} = \mu_0(\boldsymbol{\chi} + \mathbf{I})\mathbf{H}$, where μ_0 is the magnetic permeability in vacuum, $\boldsymbol{\chi}$ is the magnetic susceptibility tensor and $\mathbf{I}$ is the identity tensor. Note that in vacuum, $\mathbf{B}$ and $\mathbf{H}$ differ only by the factor μ_0 .

The deformation of the cilia is modelled through the equations of elasto-dynamics:

$$\nabla_0 \cdot (\boldsymbol{\sigma} \cdot \mathbf{F}^T) = \rho \frac{d^2 \mathbf{u}}{dt^2},\tag{3}$$

where $\boldsymbol{\sigma}$ is the second Piola-Kirchhoff stress tensor, $\mathbf{u}$ is the displacement of the material particles, ∇_0 represents derivatives taken with respect to undeformed configuration and ρ is the mass density of cilia. The second Piola-Kirchhoff stress $\boldsymbol{\sigma}$ is assumed to be related to the Green-Lagrange strain tensor $\boldsymbol{\epsilon}$, through the constitutive relation $\boldsymbol{\sigma} = \mathbf{C}\boldsymbol{\epsilon}$, where $\mathbf{C}$ is the fourth order modulus tensor for an isotropic, linear elastic material.

The fluid is assumed to be Newtonian and incompressible. The physical behaviour of the fluid is governed by the Navier-Stokes equation:

$$-\nabla p + 2\mu \nabla \cdot \mathbf{D} = \rho_f \frac{d\mathbf{w}}{dt},\tag{4}$$

$$\nabla \cdot \mathbf{w} = 0,\tag{5}$$

where p is the pressure in the fluid, $\mathbf{D}$ is the rate of deformation tensor, $\mathbf{w}$ is the velocity of the fluid and μ is the viscosity of the fluid.

2.2 Modelling approaches

Theoretical and numerical studies have been undertaken by biologists and fluid mechanicians to understand the flow created by an array of cilia (see e.g., the reviews [14, 12, 82]). These methods have been developed with the aim of understanding the fluid transport created by natural cilia. Modelling approaches to understand the cilia-driven flow include the envelope model [14, 8, 10], the sublayer model [11, 33, 32, 66, 27, 81],

Figure 2: (a) Schematic of a micro-channel that comprises of an array of magnetic artificial cilia, which are driven by a uniform external magnetic field $\mathbf{B}_0$. (b) Schematic showing the axial forces (f_x), transverse forces (f_y) and magnetic couples (N_z) acting on a cilium.

fluid structure interaction models using the lattice-Boltzmann approach [53, 2, 28], fictitious domain method [45, 44, 46, 4, 52] and immersed boundary method [19].

In the limit of low Reynolds numbers, a net fluid transport by artificial cilia is ensured for non-reciprocal motion of the cilia only. A measure of the non-reciprocal motion is the area swept by the cilia tip during each beat cycle. In problems such as these, it is sufficient to model the interaction of the magnetic field and the cilia motion; the effect of the fluid is taken into account by a drag force on the cilia that is proportional to its velocity [45]. This is the simplest possible method of taking the effect of fluid forces into account. Such an approach is usually referred to as resistive force theory. A drawback of this method is that the interaction of the cilia with itself, its neighbours and the boundary on which it is attached cannot be captured.

The envelope model [13, 8, 10] considers that the cilia are assumed to be very densely spaced so that the fluid experiences an impenetrable oscillating surface consisting of the tips of the cilia, while the ciliated region below the surface is completely ignored. Such a surface is realised only when the tips of the cilia are spaced very close together, which has only been observed in the case of symplectic metachrony. Recently, Hussong et al. [38] introduced a modified version of the envelope model in which the ciliated region below the tip surface is assumed to be porous so that fluid can enter and leave this region.

The sublayer model was developed after Blake introduced the Green's function for a Stokeslet near a no-slip boundary [9]. In this method [11], the cilia are represented by a distribution of Stokeslets with appropriate mirror images to satisfy the no-slip condition on the surface to which the cilia are attached. However, the forces exerted on the fluid by the cilia were obtained from the resistive force theory. An alternative way of calculating the drag forces on the cilia is based on slender body theory. In this method, first, the velocity of a point on the cilia is written as a linear function of forces acting on various parts of the cilia. Second, this velocity is evaluated at all the points on the cilia. This results in a system of equations relating the forces acting at various points on the cilia and the resulting velocity. Now by specifying the velocity at these locations, the corresponding forces can be calculated. This approach was adopted by Smith et al. [81].

In most of the work done using the envelope model and slender body theory the fluid flow was analysed based on the experimentally deduced motion of the cilia. Only in the last decade the coupled fluid structure interaction has been captured using numerical models based on the sublayer model [27, 22], Lattice Boltzmann method [53, 2] and fictitious domain method [52, 45, 44, 46, 4].

Since the sublayer method is based on the availability of the Green's function, it can be used to analyse the fluid transported at low Reynolds number; the effect of inertial effects of the fluid, especially at high ciliary beat frequencies, cannot be studied. In the fictitious domain method, the fluid is modelled as an Eulerian

domain and the cilia are modelled as an overlapping Lagrangian domain. The coupling between the cilia and the fluid, i.e., the no-slip condition between the cilia and fluid, is established through Lagrange multipliers. Two drawbacks of the fictitious domain method have been reported. First, when the fluid and solid domains are overlapping, the conservation laws are applied to both the fluid and solid domains. Second, the accuracy of gradient quantities (e.g., velocity gradient) in the fluid domain just outside the solid is very sensitive to the discretization. To alleviate these issues, an extended finite element method based on a discontinuous Galerkin approach has been recently developed [1].

3 Magnetically actuated artificial cilia: a computational approach

In this section we will review the two-dimensional numerical solid-fluid magneto-elastic framework used to study the fluid-propulsion generated by a two-dimensional array of magnetically-driven artificial cilia [47, 46, 44, 48, 45]. To accurately capture the deformation of the artificial cilia and the resulting fluid transport, we have to (i) calculate the magnetic forces on the cilia due to the applied magnetic field, (ii) solve for the motion of the cilia under the action of the magnetic and fluid forces and to (iii) solve the equations of fluid dynamics to obtain the fluid velocity field due to the actuation of the cilia.

3.1 Solid dynamics model

Here, we start with a weak form of the (strong) solid dynamics equations given in the previous section – called the virtual work equation [69]. The principle of virtual work is used as a condition to establish equilibrium: when a consistent virtual displacement field is applied on a body, the body will be in equilibrium when the virtual work done by the internal forces δW_{int} equals the virtual work done by the external force δW_{ext} ,

$$\delta W_{\text{int}} = \delta W_{\text{ext}}.$$

We assume the cilia to be two-dimensional beams (plates) so that, the only non-zero component of the second Piola-Kirchoff stress tensor is the stress component along the axis. The internal virtual work can then be written as

$$\delta W_{\text{int}} = \int_{V_0} (\sigma \delta \epsilon + \rho (\ddot{u} \delta u + \ddot{v} \delta v)) dV_0,$$

where $\delta(\cdot)$ represents the variation of a quantity, σ is the component of the second Piola-Kirchoff stress component in the axial direction, ϵ is the Green-Lagrange strain component in the axial direction, ρ is the density of the cilia, u is the displacement in the x direction, v is the displacement in the y direction, V_0 represents volume in the reference configuration and a $\ddot{(\cdot)}$ implies a second derivative with respect to time.

To proceed further we assume that plane sections of the cilia remain plane and normal to the neutral axis and that the artificial cilia can be modelled as an assemblage of Euler-Bernoulli beam elements. The beam element has two nodes at the ends and has three degrees of freedom at each node: u the axial displacement, v the transverse displacement and $\phi = \partial v / \partial x$ the rotation, where x is the axial coordinate along the beam. The axial displacement along the beam (or film) is interpolated linearly and the transverse displacement is interpolated cubically,

$$u = \mathbf{N}_u \mathbf{p}, \quad v = \mathbf{N}_v \mathbf{p},$$

with $\mathbf{N}_u$ and $\mathbf{N}_v$ being the standard interpolation matrices [17] and $\mathbf{p} = \{u_1 \ v_1 \ \phi_1 l_0 \ u_2 \ v_2 \ \phi_2 l_0\}^T$, where the subscripts 1 and 2 refer to the node numbers and l_0 is a reference length. The nonlinear axial strain ϵ in the beam is

$$\epsilon = \frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial v}{\partial x} \right)^2 - y \frac{\partial^2 v}{\partial x^2} \equiv \bar{\epsilon} - y \chi.$$

By substituting the strains and defining $\int \sigma dA = P$ and $-\int \sigma y dA = M$ ($A = bh$ is the area of the cross section, h is the thickness and b is the out-of-plane width of the film), the internal virtual work at time $t + \Delta t$

can be written as the sum of an elastic and an inertial part

$$\delta W_{\text{int}}^{t+\Delta t} = \int_{x_0} (P^{t+\Delta t} \delta \bar{\epsilon}^{t+\Delta t} + M^{t+\Delta t} \delta \chi + \rho A (\ddot{u}^{t+\Delta t} \delta u + \ddot{v}^{t+\Delta t} \delta v)) dx.$$

We now expand the elastic part of the internal work linearly in time by substituting $(Q^{t+\Delta t} = Q^t + \Delta Q)$ for the field parameters in the above equation which gives

$$\begin{aligned} \delta W_{\text{int}}^{t+\Delta t} &= \int_{x_0} [(P^t \delta \bar{\epsilon}^t + M^t \delta \chi) + (\Delta P \delta \bar{\epsilon}^t + \Delta M \delta \chi) \\ &\quad + P^t \Delta \bar{\epsilon} + \rho A (\ddot{u}^{t+\Delta t} \delta u + \ddot{v}^{t+\Delta t} \delta v)] dx, \end{aligned}$$

in which terms of order higher than one are neglected. The following notations are introduced for convenience: $\partial u / \partial x = \mathbf{N}'_{\mathbf{u}} \mathbf{p} = \mathbf{B}_{\mathbf{u}} \mathbf{p}$, $\partial v / \partial x = \mathbf{N}'_{\mathbf{v}} \mathbf{p} = \mathbf{B}_{\mathbf{v}} \mathbf{p}$, $\partial^2 v / \partial x^2 = \mathbf{N}''_{\mathbf{v}} \mathbf{p} = \mathbf{C}_{\mathbf{v}} \mathbf{p}$, where the notation $(\cdot)'$ is used to denote $d(\cdot)/dx$. The constitutive relations are $\Delta P = EA \Delta \bar{\epsilon}$, $\Delta M = EI \Delta \chi$, with $E = \bar{E}/(1 - \nu^2)$ being the effective elastic modulus, ν being the Poisson's ratio and I being the second moment of area defined as $I = bh^3/12$. By choosing the domain of integration to be the current configuration (i.e., using an updated Lagrangian framework), the total displacements are zero, $\mathbf{p} = 0$ and we get

$$\delta W_{\text{int}}^{t+\Delta t} = \delta \mathbf{p}^T \mathbf{f}_{\text{int}}^t + \delta \mathbf{p}^T \mathbf{K} \Delta \mathbf{p} + \delta \mathbf{p}^T \mathbf{M} \ddot{\mathbf{p}}^{t+\Delta t}, \quad (6)$$

where

$$\mathbf{f}_{\text{int}}^t = \int [P^t \mathbf{B}_{\mathbf{u}}^T + M^t \mathbf{C}_{\mathbf{v}}^T] dx$$

is the nodal internal force vector,

$$\mathbf{K} = \int EA \mathbf{B}_{\mathbf{u}}^T \mathbf{B}_{\mathbf{u}} dx + \int EI \mathbf{C}_{\mathbf{v}}^T \mathbf{C}_{\mathbf{v}} dx + \int P^t \mathbf{B}_{\mathbf{v}}^T \mathbf{B}_{\mathbf{v}} dx$$

is the stiffness matrix, the first two terms of which represent the material stiffness and the third term represents the geometric stiffness, and

$$\mathbf{M} = \int \rho A (\mathbf{N}_{\mathbf{u}}^T \mathbf{N}_{\mathbf{u}} + \mathbf{N}_{\mathbf{v}}^T \mathbf{N}_{\mathbf{v}}) dx$$

is the mass matrix. The corresponding external virtual work is

$$\begin{aligned} \delta W_{\text{ext}}^{t+\Delta t} &= \int \left(f_x^{t+\Delta t} \delta u + f_y^{t+\Delta t} \delta v + N_z^{t+\Delta t} \frac{\partial \delta v}{\partial x} \right) A dx + \int (t_x^{t+\Delta t} \delta u + t_y^{t+\Delta t} \delta v) b dx \\ &= \delta \mathbf{p}^T \int \left[(f_x^{t+\Delta t} \mathbf{N}_{\mathbf{u}}^T + f_y^{t+\Delta t} \mathbf{N}_{\mathbf{v}}^T + N_z^{t+\Delta t} \mathbf{B}_{\mathbf{v}}^T) A \right. \\ &\quad \left. + b (t_x^{t+\Delta t} \mathbf{N}_{\mathbf{u}}^T + t_y^{t+\Delta t} \mathbf{N}_{\mathbf{v}}^T) \right] dx \\ &= \delta \mathbf{p}^T \mathbf{f}_{\text{ext}}^{t+\Delta t}, \end{aligned} \quad (7)$$

where f_x and f_y are the body forces in axial and transverse directions (for example, magnetic body forces), N_z is the body couple in the out-of-plane direction (for example, magnetic body couple) and t_x and t_y are the surface tractions (for example, fluid drag). By equating the internal and external virtual work and noting that the resulting equation holds for arbitrary $\delta \mathbf{p}$ we get

$$\mathbf{K} \Delta \mathbf{p} + \mathbf{M} \ddot{\mathbf{p}}^{t+\Delta t} = \mathbf{f}_{\text{ext}}^{t+\Delta t} - \mathbf{f}_{\text{int}}^t. \quad (8)$$

The motion of the film with time is obtained by solving Eqn. 8 using Newmark's algorithm with appropriate initial and boundary conditions.

3.2 Fluid dynamics model

The principle of virtual work in the rate form for the fluid problem is [5]

$$\int_V \sigma_{ij} \delta D_{ij} dV + \rho^f \int_V \frac{dw_i}{dt} \delta w_i dV + \int_V \delta p \frac{\partial w_i}{\partial x_i} dV = 0, \quad (9)$$

where the first and the second terms represent the work due to the internal stresses and inertia forces in the fluid, respectively, while the third term imposes the incompressibility condition. In Eqn. 9, σ_{ij} represents the components of the stress tensor in the fluid, D_{ij} represents the components of the deformation rate tensor in the fluid, ρ^f is the fluid density, w_i represents the components of the fluid velocity, $d/dt = \partial/\partial t + w_i \partial/\partial x_i$ represents the total derivative, p is the pressure and $dV = bdx dy$. The constitutive relation for the fluid is $\sigma_{ij} = -p\delta_{ij} + 2\mu D_{ij}$, where μ is the fluid viscosity. It is to be noted that x represents a spatial point in the fluid domain (Eulerian point of view), while it represents the position occupied by a material point on the beam in the solid domain (Lagrangian point of view). The following shape functions are used to interpolate velocity and pressure: $\mathbf{w} = \boldsymbol{\phi}^T \mathbf{U}$, $p = \boldsymbol{\psi}^T \mathbf{P}$

$$\begin{aligned} \begin{Bmatrix} w_1 \\ w_2 \end{Bmatrix} &= \boldsymbol{\phi}^T \mathbf{U}, \\ p &= \boldsymbol{\psi}^T \mathbf{P}, \end{aligned} \quad (10)$$

where, $\boldsymbol{\phi}$ and $\boldsymbol{\psi}$ are the shape functions used to interpolate the nodal velocities and pressure, respectively, and $\mathbf{U}$ and $\mathbf{P}$ are the nodal velocity and pressure vectors, respectively. Using Eqn. 10 the various terms in the virtual work equation are evaluated

$$\int_V \sigma_{ij} \delta D_{ij} dV = \int_V (-p\delta_{ij} + 2\mu D_{ij}) \delta D_{ij} = \delta \mathbf{U}^T (\mathbf{K}^{UU} \mathbf{U} + \mathbf{K}^{UP} \mathbf{P}), \quad (11)$$

$$\int_V \delta p \frac{\partial w_i}{\partial x_i} dV = \int_V \delta p_I \psi_I \frac{\partial \phi_{Ji}}{\partial x_i} U_J dV = \delta p_I \int_V \psi_I \frac{\partial \phi_{Ji}}{\partial x_i} dV U_J = \delta \mathbf{P}^T (\mathbf{K}^{UP})^T \mathbf{U}. \quad (12)$$

Assuming that the solution at time t is known, the convective term is linearized with respect to the known solution at time t and the time derivative of velocity is discretised using an implicit Euler scheme. Neglecting the higher order terms we get (see appendix A):

$$\begin{aligned} \int_V \frac{dw_i^{t+\Delta t}}{dt} \delta w_i dV &= \rho^f \int_V \left(\delta w_i \frac{\partial w_i^{t+\Delta t}}{\partial t} + \delta w_i \frac{\partial w_i^{t+\Delta t}}{\partial x_j} w_j^{t+\Delta t} \right) dV \\ &= \delta \mathbf{U}^T \hat{\mathbf{M}} \mathbf{U} + \delta \mathbf{U}^T \mathbf{K}^1 \mathbf{U} + \delta \mathbf{U}^T \mathbf{K}^2 \mathbf{U} - \delta \mathbf{U}^T \mathbf{F}^f. \end{aligned}$$

Substituting the above expressions in Eqn. 9 yields its discretised form

$$\delta \mathbf{U}^T \mathbf{K}^{UP} \mathbf{P} + \delta \mathbf{U}^T \hat{\mathbf{K}}_f \mathbf{U} - \delta \mathbf{U}^T \mathbf{F}^f + \delta \mathbf{P}^T (\mathbf{K}^{UP})^T \mathbf{U} = 0, \quad (13)$$

where $\hat{\mathbf{K}}_f = \mathbf{K}^{UU} + \hat{\mathbf{M}} + \mathbf{K}^1 + \mathbf{K}^2$.

3.3 Fluid-structure interaction

We now couple the Lagrangian beam element formulation of section 3.1 to the Eulerian finite element fluid model of section 3.2 using the fictitious domain method. The solid beam is considered as an internal boundary to the fluid domain. The solid and fluid models are coupled using the constraint that the velocity (or displacement) of the solid is equal to the velocity (or displacement) of the fluid. This coupling is established using the method of Lagrange multipliers [59]. In what follows, we apply the constraint to the fluid dynamics model, then add the virtual work done by the constraint force to the solid dynamics model and finally couple the equations so that the solid and the fluid equations of motion can be solved implicitly.

We first add the variation of the Lagrange multiplier times the constraint to the fluid dynamics model,

$$\begin{aligned} \delta \mathbf{U}^T \mathbf{K}^{UP} \mathbf{P} + \delta \mathbf{U}^T \hat{\mathbf{K}}_f \mathbf{U} - \delta \mathbf{U}^T \mathbf{F}^f + \delta \mathbf{P}^T (\mathbf{K}^{UP})^T \mathbf{U} \\ + \delta (\lambda_i^J (w_i^{Jf} - \dot{p}_i^J)) = 0, \end{aligned} \quad (14)$$

where, w_i^{Jf} and $\dot{p}_i^J$ represent the components of the fluid and solid beam velocity, respectively, in the i^{th} direction at the location where the J^{th} node of the solid beam is present. Discretising the constraint equation yields (see appendix A),

$$\begin{aligned} \delta \mathbf{U}^T \mathbf{K}^{UP} \mathbf{P} + \delta \mathbf{U}^T \hat{\mathbf{K}}_f \mathbf{U} - \delta \mathbf{U}^T \mathbf{F}^f + \delta \mathbf{P}^T (\mathbf{K}^{UP})^T \mathbf{U} + \delta \mathbf{U}^T \phi^J \boldsymbol{\lambda}^J \\ + \delta \boldsymbol{\lambda}^{JT} (\phi^{JT} \mathbf{U} - \mathbf{A} \dot{\mathbf{p}}^J) = 0, \end{aligned}$$

where $\mathbf{A}$ is a matrix that eliminates the rotational degrees of freedom from $\dot{\mathbf{p}}$. It can be noted that the term $\delta \mathbf{U}^T \phi^J \boldsymbol{\lambda}^J$ in the above equation represents the virtual work done by the Lagrange multiplier, i.e., the Lagrange multiplier acts as a force due to the constraint. Hence, the components of the Lagrange multiplier are to be interpreted as fluid drag forces. Invoking the arbitrary nature of the virtual fields and performing the standard finite element assembly yields the following set of equations

$$\hat{\mathbf{K}}_f \mathbf{U} + \mathbf{K}^{UP} \mathbf{P} + \boldsymbol{\Phi} \boldsymbol{\lambda} = \mathbf{F}^f, \quad (\mathbf{K}^{UP})^T \mathbf{U} = 0, \quad \boldsymbol{\Phi}^T \mathbf{U} - \mathbf{A} \dot{\mathbf{p}} = 0. \quad (15)$$

The fluid drag forces are now considered as nodal forces, whose virtual work is an additional contribution to Eqn. 8. This leads to

$$\delta \mathbf{p}^T \left(\mathbf{K} \Delta \mathbf{p} + \mathbf{M} \ddot{\mathbf{p}}^{t+\Delta t} - \mathbf{F}_{\text{ext}}^{t+\Delta t} + \mathbf{F}_{\text{int}}^t \right) - \boldsymbol{\lambda}^T \mathbf{A} \delta \mathbf{p} = 0, \quad (16)$$

where

$$\boldsymbol{\lambda} = \{ \lambda^1, \lambda^2 \}^T = \{ \lambda_1^1, \lambda_1^2, \lambda_2^1, \lambda_2^2 \}^T$$

is the Lagrange multiplier vector, λ_i^J is the Lagrange multiplier at the J^{th} node in the i^{th} direction. In general, the node of the solid beam is present inside a fluid element and does not coincide with a fluid node, so that the constraint is satisfied in an approximate sense. As the virtual quantity $\delta \mathbf{p}$ is arbitrary, we can write

$$\mathbf{K} \Delta \mathbf{p} + \mathbf{M} \ddot{\mathbf{p}}^{t+\Delta t} - \mathbf{F}_{\text{ext}}^{t+\Delta t} + \mathbf{F}_{\text{int}}^t - \mathbf{A}^T \boldsymbol{\lambda} = 0. \quad (17)$$

The motion of the cilia with time is obtained by solving Eqn. 17 with appropriate initial and boundary conditions. Newmark's integration scheme is used to integrate Eqn. 17 in time, based on which Eqn. 17 can be re-written to solve for the nodal velocities as

$$\hat{\mathbf{K}} \dot{\mathbf{p}}^{t+\Delta t} - \mathbf{A}^T \boldsymbol{\lambda} = \mathbf{F}_s^{t+\Delta t}. \quad (18)$$

The exact form of $\hat{\mathbf{K}}$ and $\mathbf{F}_s^{t+\Delta t}$ is provided in [49]. Equation 18 contains the discretised equations of motion for one beam element. After performing the standard finite element assembly procedure [17] we get the discretised equations of motion for the whole cilia (dropping the superscript $(t + \Delta t)$):

$$\hat{\mathbf{K}}_s \dot{\mathbf{p}} - \mathbf{A}^T \boldsymbol{\lambda} = \mathbf{F}_s. \quad (19)$$

Combining the equations of motion for the solid (Eqn. 19) and fluid (Eqn. 15) results in

$$\begin{bmatrix} \hat{\mathbf{K}}_f & \mathbf{K}^{UP} & \mathbf{0} & \boldsymbol{\Phi} \\ (\mathbf{K}^{UP})^T & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \hat{\mathbf{K}}_s & -\mathbf{A}^T \\ \boldsymbol{\Phi}^T & \mathbf{0} & -\mathbf{A} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{U} \\ \mathbf{P} \\ \dot{\mathbf{p}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{F}^f \\ \mathbf{0} \\ \mathbf{F}_s \\ \mathbf{0} \end{bmatrix}. \quad (20)$$

This set of equations is solved to obtain the velocities at the solid and fluid nodal points, the pressure in the fluid and the Lagrange multipliers at the solid nodal points. It is to be noted that in Eqn. 20 the velocity of the cilia and the fluid are simultaneously solved for every time increment. This approach is commonly referred to as the monolithic approach.

3.4 Magnetostatics

Maxwell's equations for the magnetostatic problem with no free currents are [41]

$$\nabla \cdot \mathbf{B} = 0 \quad (21)$$

$$\nabla \times \mathbf{H} = 0, \quad (22)$$

with the constitutive relation

$$\mathbf{B} = \mu_0(\mathbf{M} + \mathbf{H}), \quad (23)$$

where $\mathbf{B}$ is the magnetic flux density (or magnetic induction), $\mathbf{H}$ is the magnetic field, $\mathbf{M}$ is the magnetization which includes the remnant magnetization, and μ_0 is the permeability of free space. Substituting Eqn. 23 into Eqn. 21 yields

$$\nabla \cdot \mathbf{H} = -\nabla \cdot \mathbf{M}. \quad (24)$$

As $\nabla \times \mathbf{H} = 0$, a scalar potential ϕ exists, such that $\mathbf{H} = -\nabla\phi$. Substituting this in Eqn. 24 yields a Poisson equation for ϕ , $\nabla^2\phi = -\nabla \cdot \mathbf{M}$. By taking into consideration the effect of discontinuity in the medium, the general solution of the Poisson equation can be found [41], resulting in

$$\phi(\mathbf{x}) = -\frac{1}{4\pi} \oint \frac{\mathbf{n}' \cdot \mathbf{M}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} dS' + \frac{1}{4\pi} \int \frac{\nabla' \cdot \mathbf{M}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} dV'. \quad (25)$$

where $\mathbf{n}'$ is the outward normal to the surface of V . The magnetic field $\mathbf{H}(x)$ can be found from the gradient of $\phi(x)$.

We now discretise the cilia into a chain of rectangular segments, within which the magnetization is assumed to be uniform. As a result, $\nabla' \cdot \mathbf{M} = 0$ and the volume integral vanishes. The field is now only due to the jump of magnetization across the surface of each segment and is given by the surface integral in Eqn. 25. The magnetic field in coordinates local to the segment i (denoted by $\hat{\cdot}$) due to its four surfaces can be calculated at any position $(\hat{x}, \hat{y})$ by evaluating the surface integral in Eqn. 25, resulting in

$$\hat{\mathbf{H}}_i = \mathbf{G}_i \hat{\mathbf{M}}_i, \quad (26)$$

where $\hat{\mathbf{M}} = [\hat{M}_x \ \hat{M}_y]^T$, $\hat{\mathbf{H}} = [\hat{H}_x \ \hat{H}_y]^T$ and $\mathbf{G}_i$ can be obtained from Eqn. 25. Note that in this section Einstein's summation convention is not applied. The field due to segment i with respect to the global coordinates is

$$\mathbf{H}_i = \mathbf{R}_i \hat{\mathbf{H}}_i, \quad (27)$$

where

$$\mathbf{R}_i = \begin{bmatrix} \cos \theta_i & -\sin \theta_i \\ \sin \theta_i & \cos \theta_i \end{bmatrix}, \quad (28)$$

with θ_i the orientation of segment i with respect to the global coordinates. The field at any element j because of the magnetization of all the segments throughout the cilia is

$$\mathbf{H}_j = \mathbf{H}_0 + \sum_{i=1}^N \mathbf{R}_i \mathbf{G}_{ij} \hat{\mathbf{M}}_i = \mathbf{H}_0 + \mathbf{H}_{\text{self}}, \quad (29)$$

where $\mathbf{G}_{ij}$ properly accounts for the relative positioning of segments i and j and the geometry of segment i , $\mathbf{H}_0$ is the externally applied magnetic field, far away from the cilia, and N is the total number of segments. Equation 29 clearly shows that the magnetic field $\mathbf{H}$ in the cilia is the sum of the external field $\mathbf{H}_0$ and the demagnetizing field $\mathbf{H}_{\text{self}}$. By rotating $\mathbf{H}_j$ back to the local coordinates we get

$$\hat{\mathbf{H}}_j = \mathbf{R}_j^T \mathbf{H}_0 + \sum_{i=1}^N \mathbf{R}_j^T \mathbf{R}_i \mathbf{G}_{ij} \hat{\mathbf{M}}_i. \quad (30)$$

For the situation of a permanently magnetized cilia with magnetization $\hat{\mathbf{M}}_i, i = 1, \dots, N$, Eqn. 30 gives the magnetic field in all the segments. However, in case of a super-paramagnetic cilia, the magnetization $\hat{\mathbf{M}}_j$ is not known a-priori, but depends on the local magnetic field through

$$\begin{aligned}\hat{\mathbf{M}}_j &= \hat{\chi} \hat{\mathbf{H}}_j \\ &= \hat{\chi} \mathbf{R}_j^T \mathbf{H}_0 + \sum_{i=1}^N \hat{\chi} \mathbf{R}_j^T \mathbf{R}_i \mathbf{G}_{ij} \hat{\mathbf{M}}_i,\end{aligned}\quad (31)$$

with

$$\chi = \begin{bmatrix} \hat{\chi}_x & \hat{\chi}_{xy} \\ \hat{\chi}_{xy} & \hat{\chi}_y \end{bmatrix}.$$

There are N similar pairs of equations. In total these are $2 \times N$ equations for the $2 \times N$ unknown magnetizations. This set of equations is solved to get the magnetization with respect to the local coordinate frame. From the magnetization, the field can be found from Eqn. 30 and the magnetic flux density can be found by using Eqn. 23. The advantage of the proposed method is that we need not model the medium around the magnetic cilia to determine the magnetic field in the cilia. Once the magnetization is calculated by solving Eqn. 31, the magnetic body couple and body force per unit volume can be found from $\mathbf{N} = \mathbf{M} \times \mathbf{B}_0$ and $\mathbf{f} = \mathbf{M} \cdot \nabla \mathbf{B}_0$ (with $\mathbf{B}_0 = \mu_0 \mathbf{H}_0$), and given as input to Eqn. 7.

4 Dimensional analysis

In this section we use the principle of virtual work to identify the dimensionless parameters that govern the deformation behaviour of the artificial cilia. Considering only the transverse deformations and magnetic body torques we have,

$$\int EI \frac{\partial^2 v}{\partial x^2} \frac{\partial^2 \delta v}{\partial x^2} dx + \int \rho A \frac{\partial^2 v}{\partial t^2} \delta v dx - \int N_z \frac{\partial \delta v}{\partial x} A dx - \int \Lambda \delta v b dx = 0,$$

where, the first term represents the virtual elastic work done by the internal moments, the second term represents the virtual work done by the inertial forces of the beam, the third term represents the virtual work done by the magnetic couple and the last term represents the work done by the fluid drag forces. In the above equation, Λ is the traction due to fluid drag on the cilia in the transverse direction and has units of force per unit area. We introduce the dimensionless variables V , T and X , such that $v = VL$, $x = XL$ and $t = Tt_{\text{ref}}$, where L is a characteristic length (taken to be the length of the cilia) and t_{ref} is a characteristic time. Substitution yields

$$\int \left(\frac{Ebh^3}{12L^2} \frac{\partial^2 V}{\partial X^2} \frac{\partial^2 \delta V}{\partial X^2} + \frac{\rho b h L^2}{t_{\text{ref}}^2} \delta V \frac{\partial^2 V}{\partial T^2} \right) dX - \int \left(N_z h b \frac{\partial \delta V}{\partial X} - \Lambda L \delta V b \right) dX = 0, \quad (32)$$

from which the elastic ($Ebh^3/12L^2$), the inertial ($\rho b h L^2/t_{\text{ref}}^2$), the magnetic ($N_z h b$) and the viscous ($\Lambda L \delta V b$) terms can be easily identified. By normalising with the elastic term, we get

$$\int \left(\left(\frac{\partial^2 V}{\partial X^2} \frac{\partial^2 \delta V}{\partial X^2} \right) + I_n \left(\frac{\partial^2 V}{\partial T^2} \delta V \right) \right) dX - \int \left(M_n \left(\frac{\partial \delta V}{\partial X} \right) + F_n \delta V \right) dX = 0. \quad (33)$$

Here, the three governing dimensionless numbers are defined as the inertia number $I_n = 12(\rho/E)(L/t_{\text{ref}})^2(L/h)^2$ (the ratio of inertial to elastic force), the magnetic number $M_n = 12(N_z/E)(L/h)^2$ (the ratio of magnetic to elastic force) and the fluid number $F_n = 12(\Lambda/E)(L/h)^3$ (the ratio of fluid to elastic force). From dimensional considerations Λ should scale with μ/t_{ref} , leading to $F_n = 12(\mu/Et_{\text{ref}})(L/h)^3$. To identify the dimensionless parameters that govern the fluid flow, we start from the Navier-Stokes equations for the fluid

$$-\nabla \cdot \boldsymbol{\sigma} + \rho \frac{d\mathbf{w}}{dt} = 0 \quad \text{in } A, \quad (34)$$

$$-\|\boldsymbol{\sigma}\| \cdot \mathbf{n} + \mathbf{\Lambda} = 0 \quad \text{on } \Gamma_s. \quad (35)$$

The ciliary motion creates a flow that has a dominant velocity in the channel direction, i.e. x -direction. Therefore, we consider only the x -component of Eqn. 34. Using the constitutive relation for the fluid gives:

$$\rho^f \left(\frac{\partial w_x}{\partial t} + w_x \frac{\partial w_x}{\partial x} \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 w_x}{\partial x^2} + \frac{\partial^2 w_x}{\partial y^2} \right). \quad (36)$$

The relevant length scales can be identified by looking at the mechanism of fluid flow inside the channel (Fig. 3). The cilia of length L are placed periodically at a distance a in a channel of height H . The fluid in between the cilia is directly driven by them, and the momentum diffuses from this region upwards into the channel. The relevant length scales are the cilia spacing a in the x -direction, L and $H - L$ in the y -direction, and the relevant time scale is t_{ref} . Because the velocity of the fluid after a units will be the same (in the case of uniformly beating cilia), the net velocity and pressure gradients in the x -direction vanish over a distance a . Now, introducing these length and time scales in the above equation leads to:

$$\text{Re}_f \left(\frac{\partial \bar{w}_x}{\partial \bar{t}} \right) \approx \left(\frac{\partial^2 \bar{w}_x}{\partial \bar{y}^2} \right) \quad \text{for } 0 < y < L, \quad (37)$$

$$\text{Re}_H \left(\frac{\partial \bar{w}_x}{\partial \bar{t}} \right) \approx \left(\frac{\partial^2 \bar{w}_x}{\partial \bar{y}^2} \right) \quad \text{for } L < y < H, \quad (38)$$

where the terms in brackets are nondimensional, $\text{Re}_H = \rho^f (H - L)^2 / \mu t_{\text{ref}} = t_{\text{diff}} / t_{\text{ref}}$ is the diffusion Reynolds number and $\text{Re}_f = \rho^f L^2 / \mu t_{\text{ref}}$ is the flapping Reynolds number. The diffusion Reynolds number signifies how long it takes for the momentum to diffuse into the fluid (t_{diff}) compared to t_{ref} , whereas the flapping Reynolds number Re_f quantifies how large the inertia forces are compared to the viscous forces[§]. The question one might ask is, which of these Reynolds numbers is important? With the help of a simple analytical model it has been shown that Re_H has two effects [49]; first, it determines the duration of the transient period and secondly, it reduces the fluctuating component of the fluid transported. However, the mean propulsion velocity created by the cilia in the steady state is independent of Re_H , leaving the flapping Reynolds number Re_f to be the main parameter that governs the fluid transported. In the following, we take $\text{Re} \equiv \text{Re}_f = \rho^f L^2 / t_{\text{ref}}$.

5 How to create non-reciprocal motion at low Reynolds numbers?

A number of concepts has been proposed in the literature to create a non-reciprocal asymmetric motion of artificial cilia. Evans et al. [24] proposed to use rod-like magnetic cilia that are driven by a three-dimensional magnetic field to mimic the motion of nodal cilia. Similar concepts have been developed by Shields et al. [80], Vilfan et al. [91] and Wang et al. [92]. One of the first designs for two-dimensional actuation was proposed by Kim and Netz [53] comprised of slender rod-like filaments that are hinged on a substrate and driven by applied moments at the hinges. Gauger et al. [27] proposed a mechanism for a planar actuation of rod-like magnetic cilia. The rod-like magnetic cilia were actuated by a magnetic field that oscillates back and forth. They demonstrated that the cilia exhibit a non-reciprocal motion when the angular velocity of the forward actuation is much larger than the reverse actuation. A useful measure of the non-reciprocal motion is the area swept by the tip of the cilia during a beat cycle, relative to the area of a semi-circle with radius equal to cilia length. In articles of Kim and Netz and Gauger and coworkers, the magnitude of normalised swept area was small. New actuation schemes for artificial cilia based on magnetic buckling and rotating magnetic fields were investigated by Khaderi et al. [45] that led to considerably larger swept areas. These actuation schemes are discussed below.

Khaderi et al. [45] used the numerical model reviewed in section 3 to study a periodic arrangement of cilia in a microfluidic channel of height $5L$, with the cilia spaced $5L$ apart in the limit of low Reynolds numbers, where L is the length of the cilia. A square unit-cell is identified consisting of one cilium. No-slip boundary conditions are applied at the top and bottom boundaries of the channel and periodic boundary conditions at the left and right ends of the unit-cell (see Fig. 3). The fluid has a viscosity of water $\mu = 1$ mPas. The cilia

[§]The fluid velocity near the cilia, the velocity gradient and the viscous energy dissipated per unit time scale with L/t_{ref} , $1/t_{\text{ref}}$ and μ/t_{ref}^2 , respectively. The inertia forces scale with $\rho L/t_{\text{ref}}^2$, hence the kinetic energy input per unit time to the system scales with $\rho L^2/t_{\text{ref}}^3$. Their ratio gives the flapping Reynolds number Re_f .

Figure 3: Schematic side view of the microfluidic channel used in the simulations. The unit-cell used is shown using dashed lines. Reproduced with permission from [49].

have a thickness $h = 2 \mu\text{m}$, an elastic modulus of 1 MPa and a density $\rho = 1600 \text{ kg/m}^3$, unless mentioned otherwise. In the following, the Eulerian fluid mesh is not shown for clarity.

Partly magnetic cilia with cracks. Early studies on the mechanical properties of a cilium, when no information on the microstructure was available, showed that the natural cilium has a larger stiffness during the effective stroke compared to the recovery stroke [29]. Because of the large stiffness, the cilium does not deform during the effective stroke, while due to the low stiffness the drag forces cause the cilium to deform considerably so that it stays closer to the cell boundary during the recovery stroke. To use this concept the magnetic cilia need to possess a large bending stiffness in the effective stroke while pushing the fluid and to possess a low stiffness during the recovery stroke. This can be achieved by introducing cracks on one side of the cilia and by making only a portion of the cilia magnetic. The cilium, which is straight initially, is attached at the left end and has cracks of size $1 \mu\text{m}$ at the bottom. As only a part of the cilium is magnetic, it behaves like a flexible oar (as also mentioned in [76]). Only 20% of the cilium near the fixed end is magnetic. The assumed remnant magnetization is 15 kA/m , with the magnetization vector pointing from the fixed end to the free end. The applied magnetic field ($B_0 = 75 \text{ mT}$) initially points in the positive x -direction, after which it is rotated by 180° in the counterclockwise direction in 20 ms and then rotated back to the initial position in the next 10 ms, thereafter this cycle is repeated. The movement of a cilium under the action of the applied magnetic field is shown in Fig. 4. When the external magnetic field is applied, for the first 20 ms, the magnetic couples act on the magnetized portion of the cilia in a counterclockwise manner. The drag forces are acting on the top part of the cilia, which close the cracks and make the cilia stiff. The cilium thus remains nearly straight and rotates about the fixed end to perform the effective stroke (see instances 1, 2 and 3 in Fig. 4). When the applied field rotates back to the initial position during the next 10 ms, the drag forces act on the bottom part of the cilia which open the cracks making the cilia floppy and bend it (see instances 4 and 5 in Fig. 4). Such an interaction of magnetic couples, elastic forces and drag forces leads to an asymmetric motion, as can be seen from Fig. 4.

Buckling of a straight magnetic cilium In this configuration, a straight horizontal magnetic cilium with a perturbation is used to achieve an asymmetric motion. The cilium considered here has a uniform remanent magnetization of 15 kA/m with the direction of magnetization pointing from the fixed end of the cilium to the free end. When an external magnetic field is applied in the direction opposite to the cilium's magnetization, there will be no resulting magnetic couple when the cilium is straight, hence it will not move. However, if the cilium is not initially straight but given an initial perturbation, due to, for instance, manufacturing imperfections, then it will buckle under the influence of the external magnetic field. By assuming a uniform magnetization in the cilium and neglecting drag forces, the critical magnetic field that will cause buckling can be calculated [43]. Figure 5 shows the asymmetric motion due to the buckling of a magnetic cilium. The length of the cilium is 100 microns. The external magnetic field is linearly increased from zero to a maximum of B_0 (25 mT) in the direction opposite to the magnetization in the cilium in 15 ms. Then it is rotated by 180° degrees in the clockwise direction in the next 15 ms, after which it is reduced to zero in the next 1 ms and the cycle is repeated. At instant 1 in Fig. 5 when the cilium is nearly horizontal the magnitude of the counterclockwise magnetic couple acting on the cilium is low. Under the influence of this magnetic couple, the cilium buckles to instances 2 and 3 and performs the recovery stroke. When the applied field is rotated by 180° in the clockwise direction, the cilium follows the applied field to perform the effective stroke and comes back to the initial position (instances 4 and 5). The effective fluid propulsion

Figure 4: Cilia with cracks with only a part (20% of the length) near the fixed end magnetized. Instances 1, 2, 3, 4 and 5 refer to a time of 5, 11, 20, 27 and 30 ms, respectively. The arrows show the direction of the applied field. Reproduced with permission from [43].

takes place when the cilium returns to the initial position.

Curled permanently magnetic cilium The effect of the buckling-induced recovery stroke can be enhanced by choosing the initial geometry of the cilium to be a quarter of a circle with radius $100\ \mu\text{m}$, see instant 1 in Fig. 6(a). The direction of the magnetization is along the cilium with the magnetization vector pointing from the fixed end to the free end. The remanent magnetization of the cilium is taken to be $M_r = 15\ \text{kA/m}$. A uniform external field of magnitude $B_0 = 13.3\ \text{mT}$ is applied at 225° to the x axis from $t = 0\ \text{ms}$ to $t = 1\ \text{ms}$ and then linearly reduced to zero in the next $0.2\ \text{ms}$. The results of the non-reciprocating motion of the cilium in the fluid during magnetic actuation are shown in Fig. 6(a). When the external field is applied, clockwise torques (N_z is the magnetic body torque) are acting on the portion near the fixed end of the cilium while near the free end counter-clockwise torques develop (see instance 1 in Fig. 6(b)). Under the influence of such a system of moments, the cilium undergoes a buckling-kind of instability. This can be nicely seen from instances 1 and 2 in Figs. 6(a) and 6(b). During this stage the position of zero torque is almost fixed, while the torques at the free end increase. This causes the cilium to snap through to configurations 3 and 4 during which the zero-torque position travels to the fixed end. Clearly, the initially opposing directions of the internal magnetization and the applied magnetic field are essential in generating an instability that causes a large bending deformation during application of the field. Then, the applied field is reduced to zero and the cilium returns to the initial position through instance 5 in Fig. 6(a). Note that the propulsive action in the effective stroke (red) takes place during the elastic recovery of the cilium, while the cilium stays low in the recovery stroke due to the buckling-enforced snap-through.

Super-paramagnetic cilium For a PM cilium the torques are maximum when the local magnetic field is perpendicular to the (remanent) magnetization. For a super-paramagnetic (SPM) cilium, however, the magnetization is induced by the field itself, posing different requirements on the applied magnetic fields in order to deform the cilium. A straight, magnetically anisotropic SPM cilium (having susceptibilities 4.6 and 0.8 in the tangential and normal directions, respectively [90]), is subjected to a magnetic field with magnitude $B_0 = 31.5\ \text{mT}$ that is rotated from 0° to 180° in $t = 10\ \text{ms}$ and then kept constant during the rest of the cycle. The cilium has a length $L = 100\ \mu\text{m}$. Its cross-section is tapered, with the thickness varying linearly along its length, having $h = 2\ \mu\text{m}$ at the left (attached) end and $h = 1\ \mu\text{m}$ at the right end. Figure 7(a) shows that in the effective stroke the portion of the beam near the free end is nearly straight. This is due to the fact that in this region the cilium can easily follow the applied field so that the field and magnetization are almost parallel, causing the magnetic torque to be low in this region of the cilium (instances 2, 3 and 4 in Fig. 7(b)). When the cilium has reached position 4, the magnetization in the cilium is such that the torques are oriented clockwise near the fixed end and anticlockwise near the free end, resulting in strong bending of the cilium. From Fig. 7(b) it can be seen that during the recovery stroke (shown using circles) the position

Figure 5: Asymmetric motion of a perturbed permanently magnetic cilium. The magnetization points from the fixed to free end. During instances 1 and 2, the cilium buckles and exhibits a recovery stroke and during instances 3, 4 and 5 it performs effective stroke. Instances 1, 2, 3, 4 and 5 correspond 2, 10, 12, 15 and 20 ms, respectively. Reproduced with permission from [43].

of zero torque propagates from the fixed end to the free end (from instance 4 to 5). Here the tapering is essential, causing the torque per unit length to be higher at the fixed end, allowing the cilium to recover to the initial position (1). The generated asymmetric motion is very similar to that of natural cilia [72]. It is to be noted that the cilium recovers in the presence of an applied magnetic field. This sensitive interplay between stored elastic energy and controlled applied field can be exploited to provide a large asymmetry in motion. In the rest of the review we focus our attention on the super-paramagnetic cilia.

6 Fluid transport by super-paramagnetic artificial cilia at low Reynolds numbers

In this section we review the fluid propulsion created by a two-dimensional array of tapered super-paramagnetic artificial cilia, described above, which are actuated by a rotating magnetic field, $B_x = B_0 \cos(\omega t)$, $B_y = B_0 \sin(\omega t)$, which is uniform in space, B_0 is the magnitude of the applied magnetic field, $\omega = 2\pi/t_{\text{ref}}$ is the angular frequency and t_{ref} is the time period of rotation of the magnetic field. A unit-cell, as shown in Fig. 3, containing one cilium is chosen to study the fluid flow. To elucidate the main features of fluid propulsion, the fluid, Reynolds, magnetic and inertia numbers are set to 0.015, 0.001, 10.89 and 0.0048, respectively. This indicates that the fluid forces are low compared to the elastic forces, but are large compared to the fluid inertia forces.

The fluid propelled is characterised by two parameters: the net volume of the fluid transported during a ciliary beat cycle and the effectiveness. The velocity field in the fluid, at any x position, integrated along the channel height gives the instantaneous flux through the channel. This flux when integrated in the direction of the effective and recovery strokes gives the positive (Q_p) and negative (Q_n) flows, respectively (see Fig. 3). Due to the asymmetric motion, the positive flow is larger than the negative flow, generating a net area flow per cycle ($Q_p - Q_n$) in the direction of the effective stroke. The effectiveness, defined as $(Q_p - Q_n)/(Q_p + Q_n)$, indicates which part of the totally displaced fluid is effectively converted into a net flow. An effectiveness of unity represents unidirectional flow.

The deformed geometry of a typical cilium at different time instants when a rotating magnetic field is applied is shown in Figs. 8(a)-(f). Let us now analyse the fluid flow due to the ciliary motion. When the cilia move, they set the fluid nearby into motion. The momentum diffuses to the rest of the fluid due to the viscous forces. Since the viscous forces are large compared to the inertia forces, this momentum diffuses

Figure 6: Buckling of a curled permanently magnetic (PM) cilium as a result of magnetic actuation, during the propulsion of fluid. (a) Snapshots of the cilium at 0 ms, 0.3 ms, 0.6 ms, 1.1 ms and 3 ms. (b) Normalised torque distribution along the cilium corresponding to the snapshots shown in (a). Reproduced with permission from [43].

instantaneously and the fluid velocity nicely follows the cilia velocity at all time instants. The cilia create a positive flux (to the left) during the effective stroke (see the instantaneous flux at the instances corresponding to Figs. 8(a) and 8(b) in Fig. 8(g)), and drag back a part of the fluid creating a negative flux (to the right) during the recovery stroke (see the instantaneous flux at the instances corresponding to Figs. 8(c) - 8(e) in Fig. 8(g)). The evolution of the flow during the beat cycle can be seen from the total flow created, shown using the solid line in Fig. 8(g). The flow created by the cilia increases during the effective stroke and decreases during the recovery. Due to the asymmetric motion of the cilia, the positive flow is larger than the negative flow; the cilia create a net flow of magnitude 0.2 (in units of $\pi L^2/2$) to the left by the end of the cycle ($t = t_{\text{beat}}$). In the process of creating this flow, the cilia push the fluid back and forth, which leads to a low value of the effectiveness (0.2). The position of the fluid particles (which formed a straight line at the beginning of the beat cycle) at the end of the beat cycle provides a Lagrangian perspective of the fluid transported. The recovery stroke takes place with a whip-like motion of the cilia in a short duration of time, which leads to a larger instantaneous fluid velocity and flux during the recovery stroke than the effective stroke (compare instances (4) and (5) with (1) and (2) in Fig. 8). As the Reynolds number is low, the kinetic energy input by the cilia is instantaneously dissipated due to the large viscous forces. Therefore, even if the cilia impart a large velocity to the fluid during the whip-like recovery stroke (see Figs. 8(d) and 8(e)), the velocity and flux drop to zero as soon as the cilia have returned to the initial position (see Fig. 8(f) and instant (6) in Fig. 8(g)). It is to be noted that due to the low viscous forces (compared to elastic forces) the cilia complete the beat cycle in $0.89t_{\text{beat}}$. The cilia remain idle in this ‘dead position’ until the start of the next cycle.

7 The effect of Reynolds number

In nature, the length of the cilia is on the order of $10 \mu\text{m}$, and they beat with a frequency of 10 Hz. As a result, the Reynolds number is $\approx 1 \times 10^{-3}$, and the effect of fluid inertia is negligible in comparison to the effect of viscous forces in the fluid. The fabricated artificial cilia are typically $100 \mu\text{m}$ long. In addition, the beat frequency can be externally tuned (10 Hz - 100 Hz). Hence, for artificial cilia, the influence of fluid inertia cannot be neglected. Consequently, the flow through the microfluidic channel depends on the inertia number I_n , fluid number F_n , magnetic number M_n and Reynolds number Re (see section 4). The effect of the fluid number, inertia number and magnetic number in the limit of low Reynolds number was studied in [45]. It has been shown that the inertia number does not play a major role and when the fluid number is increased (for a given magnetic number) the asymmetry (and therefore the flow) decreases. In the following we will review the effect of Re for given values of the magnetic and fluid numbers [49].

Figure 7: Motion of a super-paramagnetic (SPM) cilium in a rotating magnetic field, during the propulsion of fluid. (a) Snapshots of the cilium at 0 ms, 2.5 ms, 5.0 ms, 7.5 ms and 8.5 ms. (b) Normalised torque distribution along the cilium corresponding to the snapshots shown in (a). Here μ_0 is the permeability of vacuum, h is the thickness at position ζ and $h_{\zeta=0}$ is the thickness at the fixed end. Reproduced with permission from [43].

The area flow is plotted as a function of the Reynolds number (Re) for a fluid number (F_n) of 0.015 and a magnetic number of 10.89 in Fig. 9. As the Reynolds number is increased, the net flow changes direction so that the fluid propelled is in the direction of the recovery stroke (dashed box). On increasing the Reynolds number further, the fluid flow reverses direction again and flow occurs in the direction of the effective stroke. It is to be noted that the flow created at high Reynolds number is larger than that of the Stokes regime. To segregate the contributions to the net fluid flow, we also plot the positive and negative flow created in Fig. 9. When the Re is increased, the positive flow remains nearly constant, but the negative flow increases and causes the area flow to decrease. On a further increase of Re , the negative flow reaches a maximum, after which the positive and negative flow start decreasing. At high Re , the positive flow starts increasing again. We now investigate what mechanisms are involved that drive the flow direction reversal at moderate Reynolds number ($Re \approx 1$) and what causes a higher flow at high Re compared to the Stokes regime.

Before proceeding any further we would like to restate the importance of fluid inertia. At high Reynolds numbers, the viscous forces are relatively low. The energy input to the fluid during the effective and recovery stroke is not instantly dissipated, but is carried over to the next recovery and effective stroke, respectively (effect of Re). Also, at high Reynolds numbers, the diffusion from the cilia to the fluid takes place over a finite interval of time (effect of D_H). As a result, the fluid velocity field does not instantaneously follow the velocity of the cilia.

At Reynolds number $Re = 1$, the velocity diffusion time t_{diff} is finite (effect of Re_H , see section 4). The velocity of the fluid does not follow the cilia velocity instantaneously. This causes a negative velocity only near the bottom half of the channel during the recovery stroke (see Figs. 10(b)-10(d)), and a consequent reduction of the maximum flux transported at ($Re \approx 1$) (compare instant (3) in Fig. 10(e) with instant (5) in 8(g)). As the fluid number is low ($F_n = 0.015$), the cilia return to the initial position before $t = t_{beat}$ and remain idle in the dead position (Figs. 10(c)-10(d)) until the start of the next cycle. As the viscous forces are comparable to the inertia forces, the kinetic energy input to the fluid during the whip-like recovery stroke, is not dissipated instantly, but creates a large negative flow during the dead position (after the recovery is complete and before the start of the next cycle) and during the initial part of the effective stroke (Fig. 10(d) and 10(e)). This causes an increase of the negative flow during recovery stroke compared to the Stokes regime, despite the lower instantaneous flux. Consequently, during the first half of the subsequent effective stroke, the cilia have to spend energy in cancelling the negative flow, instead of creating a positive flow. Hence, the positive flow is lower than that of the Stokes regime. This leads to flow direction reversal at moderate Reynolds numbers.

Next, we analyse the flow behaviour at a Reynolds number $Re = 10$ (see Fig. 11). At $Re = 10$, the

Figure 8: (a)-(f) Contours of absolute velocity (normalised with L/t_{beat}) at different time instants. The direction of the velocity is given by the streamlines and the white arrows represent the magnetic field at the respective time instants. The circles represent fluid particles. The parameters used are $F_n = 0.015$, $\text{Re} = 0.001$ and $M_n = 10.89$. Four unit-cells are shown for clarity. (g) Instantaneous flux and accumulated flow as a function of time. The time instants corresponding to (a)-(f) are duly marked. Note that the cilia have completed one beat cycle while the magnetic field rotates by π . Reproduced with permission from [49].

Figure 9: Flow at the end of one cycle as a function of Reynolds number (Re) for $F_n = 0.015$ and $M_n = 10.89$. Fluid propelled by the cilia in the direction of the effective and recovery stroke are termed as positive (Q_p) and negative (Q_n) flows (see also figure 3). Different regions of significance are marked I, II and III. Region I: The large kinetic energy input during the recovery stroke leads to a negative flow during the dead position. Region II: The negative flow decreases due to the delayed momentum diffusion, and the positive flow decreases because the effective stroke has to first cancel the negative flow. Region III: The positive flow increases because of the continuous velocity in the direction of the effective stroke. Reproduced with permission from [49].

viscous forces are so low compared to inertial forces that they do not dissipate the energy input to the fluid during the effective stroke. This can be seen from the continuous fluid velocity in the direction of the effective stroke near the top of the channel (see the contour plots of the absolute velocity in Fig. 11). This leads to an increase of the positive flow. It is to be noted that because of resistance offered by the fluid inertia forces to the cilia motion, they return to the initial position at $t \approx t_{\text{beat}}$, reducing the idle time in the dead position to zero (see Fig. 11(f)). The effect of the increased t_{diff} is to restrict the fluid velocity in the negative direction to a narrow region close to the cilia (see Figs. 11(e) and 11(f)). This reduces the maximum negative flux even further (compare instant (3) in Fig. 10(e) against instant (e) in 11(g)). In this situation (low maximum negative flux and the absence of the dead position) the high energy input during the recovery stroke cannot create a substantial flow, because the next effective stroke already starts and effectively blocks the negative flow. This leads to a low negative flow compared to $Re = 1$. The total flow created by the cilia at the end of the cycle is 0.32 in units of $\pi L^2/2$ (see Fig. 11(g)), which is higher than that of the Stokes regime (0.2). This can also be observed from the position of the fluid particles at the end of the beat cycle. The fluid particles have undergone a larger displacement (Fig. 11(f)) compared to the Stokes regime (Fig. 8(f)).

The observations of this section can be summarised as follows (see Fig. 9). The quantity of the negative flow depends on the competition between the inertia-induced negative flow and its obstruction by the subsequent effective stroke. At small Re , the negative flux is large but only occurs over a short duration of time (Fig. 8(g)). When Re increases, inertia in the fluid causes the flow to be more local to the cilia but it sustains over a longer period of time so that the total negative flow increases (see Fig. 9 and Fig. 10). The net effect is that the total flow decreases and becomes negative (i.e., in the recovery direction, see region I of Fig. 9). When Re further increases (i.e., region II of Fig. 9), the positive flow decreases because a part of the effective stroke is spent in overcoming the negative flow caused by the previous recovery stroke. The negative flow also decreases because of the localization of the negative velocity near the cilia during a recovery stroke. Finally, in the region III, inertia is so dominant that the flow becomes almost unidirectional with the total flux being positive during most of the cycle, only showing a small dip during recovery (see Fig. 11). In region III, the flow continues to increase with inertia.

To also include the effect of fluid number (F_n), the flow and effectiveness are calculated for a range of fluid number (F_n) and Reynolds number (Re) in Fig. 12. The contour plots suggest three regions of significance,

Figure 10: (a)-(e) Contours of absolute velocity (normalised with L/t_{beat}) at different time instants. The direction of the velocity is given by the streamlines and the white arrows represent the magnetic field at the respective time instances. The circles represent fluid particles. The parameters used are $F_n = 0.015$, $M_n = 10.89$ and $Re = 1$. Four unit-cells are shown for clarity. The legend for the contours is the same as that of Fig. 8. (e) Instantaneous flux and accumulated flow as a function of time. The time instances corresponding to (a)-(d) are duly marked. Note that the cilia have completed one beat cycle while the magnetic field rotates by π . Reproduced with permission from [49].

Figure 11: (a)-(f) Contours of absolute velocity (normalised with L/t_{beat}) at different time instants. The direction of the velocity is given by the streamlines and the white arrows represent the magnetic field at the respective time instants. The circles represent fluid particles. The parameters used are $F_n = 0.015$, $\text{Re} = 10$ and $M_n = 10.89$. Four unit-cells are shown for clarity. (g) Instantaneous flux and accumulated flow as a function of time. The time instants corresponding to (a)-(f) are duly marked. Note that the cilia have completed one beat cycle while the magnetic field rotates by π . Reproduced with permission from [49].

namely A, B and C. Region A represents the Stokes regime where the fluid viscous forces dominate over the inertia forces. In region A, the effectiveness of the fluid propulsion remains constant at ≈ 0.2 (see Fig. 12(b)). The fluid propelled decreases when the fluid number is increased; the increase of the fluid number increases the viscous forces, which decrease the area swept by the cilia tip and therefore the flow. The reader is referred to [45] for a more detailed discussion. In region B, the flow is negative and thus takes place in the direction of the recovery stroke. Now, if either the Reynolds number or the fluid number is increased, the fluid flows in the direction of the effective stroke. In region C, the fluid inertia forces are large and the effectiveness of fluid propulsion is high. In this regime, for low fluid numbers the fluid transported is large compared to the Stokes regime. However, when the fluid number is large, the area swept is lower (due to large viscous forces) and the flow created is comparable to that of the Stokes regime. Interestingly, the effectiveness of the fluid propulsion is large even in this region, thus creating an unidirectional flow. This suggests that we

Figure 12: Parametric area flow and effectiveness as a function of fluid number F_n and Reynolds number Re at a magnetic number of 10.89. Reproduced with permission from [49].

can use the inertia forces in the fluid to generate a unidirectional flow with periodically beating artificial cilia. In natural ciliary systems such a unidirectional fluid flow is achieved by the hydrodynamic interaction between individual cilia, which leads to a metachronal wave [79]. In artificial ciliary systems, however, the metachronal wave can be established by magnetically forcing the cilia to beat out-of-phase (using period unit-cells that include multiple cilia)[47]. The study performed in the current section also suggests that a given ciliary system can be made to switch the flow direction depending on the operating frequency (see the dashed arrow in Fig. 12(a), which shows the direction of increasing frequency or decreasing t_{beat}). A ciliary system in region B creates a flow in the direction of the recovery stroke. Now, if the frequency of magnetic actuation is increased, both the fluid number as well as the Reynolds number will increase, resulting in a strong increase of the effectiveness and large unidirectional flow which is maximal for the range of F_n and Re studied (see Fig. 12).

8 Effect of metachronal waves

In previous sections attention was focused on the flow created by an array of synchronously-beating cilia whose motion is planar and asymmetric, in the absence and presence of fluid inertia. In natural ciliary systems, however, there is a pronounced phase difference between neighbouring cilia. Such an out-of-phase behaviour gives rise to travelling waves on the surface of the cilia, which are often referred to as metachronal waves. When the direction of the metachronal wave is in the same direction as the effective stroke, it is referred to as symplectic metachrony; and when the direction of the metachronal wave is in the opposite direction as the effective stroke, it is referred to as antiplectic metachrony. The effect of the out-of-phase motion in magnetically driven artificial cilia has been studied numerically by Gauger et al. [27] and Khaderi

Figure 13: (a) Schematic representation of the problem analysed. We study an infinitely long microfluidic channel consisting of equal-sized cilia spaced a distance a apart. The variation of magnetic field in space is shown using blue arrows. Q_p and Q_n denote the flow in the direction of the effective and recovery stroke, respectively. (b) Typical asymmetric motion of a cilium. The dashed lines represents the trajectory of the tip of an individual cilium. Reproduced with permission from [47].

et al. [47]. These studies focussed on the following aspects. How does the generated flow in the presence of metachrony differ from the flow generated by cilia that beat in phase? How does the flow depend on the metachronal wave speed and its direction, and how does it depend on the cilia spacing?

Gauger and coworkers [27] performed simulations using a finite number of cilia at a fixed spacing. They reported that the flow created by the antiplectic (symplectic) metachrony is larger (smaller) than synchronously-beating cilia. In contrast, Khaderi and coworkers [47] analysed a periodic array of artificial cilia for various inter-cilia spacings and reported that the fluid transport is always enhanced in the presence of metachronal waves. The magnitude of fluid transported by antiplectic metachrony differs from that of symplectic metachrony only for cilia spacings less than twice the cilia length. In these cases the fluid flow generated by antiplectic metachrony is larger than symplectic metachrony. In the following we review the results of Khaderi et al. [47] in detail.

8.1 Externally imposed out-of-phase motion

Khaderi et al [47] studied the flow in an infinitely long channel of height H created by a two-dimensional array of magnetic artificial cilia, which are actuated using a rotating magnetic field which is uniform over each cilium, but with a phase difference between adjacent cilia. The external magnetic field experienced by the i^{th} cilium is

$$B_{xi} = B_0 \cos(\omega t - \phi_i), \quad B_{yi} = B_0 \sin(\omega t - \phi_i), \quad (39)$$

where B_0 is the magnitude of the applied magnetic field, the phase of the magnetic field $\phi_i = 2\pi(i-1)/n$, $\omega = 2\pi/t_{\text{ref}}$ is the angular frequency and t_{ref} is the time period of rotation of the magnetic field. The magnetic field experienced by the individual cilia during a particular instance in time is shown using the blue arrows in Fig. 13(a). The phase difference in the applied magnetic field between adjacent cilia is $\Delta\phi = 2\pi/n$. The chosen form of the phase ϕ_i makes the phase of the magnetic field at every n^{th} cilium identical. That is, the magnetic field is periodic after n repeats of cilia. Consequently, the applied magnetic field travels n cilia units in time t_{ref} , so that the phase velocity of the magnetic field is $n/t_{\text{ref}} = \omega/\Delta\phi$ (in cilia per second). The phase velocity is to the right (positive) and the magnetic field at each cilium position rotates counterclockwise with time. The typical asymmetric motion of a cilium is shown in Fig. 13(b). The cilia are tethered at one end to the surface, while the other end is free. The trajectory of the free end of a typical cilium is represented by the dashed lines in Fig. 13(b), with the arrows representing the direction of motion.

When $0 < \Delta\phi < \pi/2$ the metachronal wave velocity is positive, i.e. to the right in Fig. 13. Consequently, the metachronal wave velocity is opposite to the direction of the effective stroke, which is commonly addressed as antiplectic metachrony (AM). When $\pi/2 < \Delta\phi < \pi$, the metachronal wave velocity is in the same direction as the effective stroke and is referred to as symplectic metachrony (SM). The flow created was analysed as a function of the cilia spacing a (normalised with the length L) and the phase difference $\Delta\phi$ for the following set of parameters: $F_n = 0.15$, $M_n = 12.2$, $I_n = 4.8 \times 10^{-3}$ and $H/L = 2$.

8.2 Results and discussion

To obtain an understanding of fluid flow due to the out-of-phase motion of cilia, we analyse the case of antiplectic metachrony with a phase difference $\Delta\phi = 2\pi/n = 2\pi/12$. Since n is even, a unit-cell of width $6a$ consisting of 6 cilia is chosen, see Fig. 14. The contours represent the absolute velocity normalised with L/t_{beat} . The direction of the velocity field can be determined from the arrows on the streamlines. The white arrows represent the applied magnetic field for each cilium.

The snapshots shown in Figs. 14(a)-14(f) correspond to the time instances when the flux generated by the cilia is maximum. In Fig. 14(g) the instantaneous flux as a function of time t (right axis) and the flow (accumulated flux at time t , left axis) are plotted. The time instances corresponding to Figs. 14(a)-14(f) are marked in Fig. 14(g). The motion of the fluid particles near the third cilium under the influence of the velocity field caused by the ciliary motion is also shown. It can be observed from Fig. 14(g) that one beat cycle consists of six sub-beats, which correspond to the traveling of the magnetic couple from one cilium to the next. The traveling of the metachronal wave to the right can, for instance, be seen by looking at the cilia which exhibit the recovery stroke (i.e. cilium 1 in Fig. 14(a), cilium 2 in Fig. 14(b), etc). The negative flow created by the cilia during their recovery stroke is overcome by the flow due to the effective stroke of the rest of the cilia; this leads to a vortex formation near the cilia exhibiting their recovery stroke. As a result, the negative flow is completely obstructed for most of the time during the recovery stroke. It can be observed from Fig. 14(g) that no flux (right axis) is observed in the negative direction, and that the flow (left axis) continuously increases during each sub-beat. Moreover, the increase in the flow during each sub-beat is similar (see Fig. 14(g)). Thus, the total flow per beat cycle (left axis of Fig. 14(g)) is the sum of the flows generated during each sub-beat (i.e. flow per beat = $6 \times$ flow generated during one sub-beat). Therefore, it is sufficient to analyse the fluid flow during one sub-beat.

In the following, we analyse the fluid motion and the resulting flow during the second sub-beat. The velocity profiles at different instants of this sub-beat are shown in Figs. 15(a)-15(d). The corresponding flow and the flux generated are shown in Fig. 15(e). At $t_{\text{beat}}/6$, the third cilium starts its recovery stroke and the particles near the top boundary are driven by the positive flow created by cilia 4, 5 and 6 (see Fig. 15(a)). At this instant, as only one cilium is exhibiting a recovery stroke, the flux created by the cilia is maximum (see instant 'a' in Fig. 15(e)). In Fig. 15(b), the third cilium also has begun its recovery stroke and now the negative flow caused by both the second and third cilia is opposed by the effective stroke of the other cilia. The high velocity of the second cilium during its recovery stroke decreases the flux caused by the other cilia (see instant 'b' in Fig. 15(e)). When the third cilium is half-way through its recovery stroke (see Fig. 15(c)), the second cilium is about to finish its recovery, which generates a large velocity, due to the whip-like action [45,], to the right. Now, the position of the third cilium is such, that it opposes the negative flow caused by the second cilium. This leads to a strong vortex formation near the second and third cilia, with only a small flux in the direction of the recovery stroke (to the right). The small negative flux caused by the whip-like motion of the second cilium can be seen by the instant marked 'c' in Fig. 15(e), causing a momentary decrease in the flow. The vortex imparts a high velocity in the direction of the effective stroke to the particles away from the cilia. As the third cilium progresses further in its recovery stroke, the particles come under the influence of the flow due to the rest of cilia, which are now in different phases of their effective stroke (see Fig. 15(d)). Now, only the third cilium is in the recovery stroke; this again leads to a maximum value of the flux (similar to Fig. 15(a)). The key observation of Figs. 14 and 15 is that the negative flow created during the recovery stroke of the cilia creates a local vortex due to the positive flow created by other cilia. This shielding effect during the recovery stroke leads to a drastic increase in the net propulsion rate for cilia beating out-of-phase, compared to synchronously beating cilia.

The fluid propelled and the corresponding effectiveness are plotted for different values of $\Delta\phi$ and a/L in Fig. 16. The metachronal wave velocity is plotted as a function of $\Delta\phi$ and is shown using dashed lines in Fig. 16(a). As mentioned earlier, when the metachronal wave velocity is positive an antiplectic metachrony (AM) results, and when the metachronal wave velocity is negative we get a symplectic metachrony (SM). When all the cilia are moving synchronously ($\Delta\phi = 0$ or π), the flow (normalised by $\pi L^2/2$) will be approximately 0.22 for $a/L = 5$. As the cilia density is increased by decreasing a from $a/L = 5$ to $a/L = 1.67$, the viscous resistance per cilium decreases, which causes the normalised flow to increase to 0.25. When the cilia beat in-phase, the effectiveness of fluid propulsion is very low, see Fig. 16(b). The fluid propelled shows a substantial increase once the cilia start beating out-of-phase (Fig. 16(a)). When the cilia spacing

Figure 14: (a)-(f) Out-of-phase motion of cilia during a representative cycle for $\Delta\phi = \pi/6$ ($n = 12$) with the wave moving to the right (antiplectic metachrony) for $a/L = 1.67$. The contours represent the absolute velocity normalised with L/t_{beat} . The direction of the velocity is represented by streamlines. The white circles represent fluid particles. The applied magnetic field at each cilium is represented by the white arrows. (g) Instantaneous flux (right axis) and flow (or accumulated flux, left axis) as a function of time with the instants (a)-(g) duly marked. Reproduced with permission from [47].

Figure 15: (a)-(d) Snapshots for the out-of-phase motion of cilia between the time instances of Figs. 14(b) and 14(c) for $\Delta\phi = \pi/6$ ($n = 12$) with the wave moving to the right (antiplectic metachrony) for $a/L = 1.67$. The contours represent the absolute velocity normalised with L/t_{beat} . The direction of the velocity is represented by streamlines. The white circles represent fluid particles. The applied magnetic field at each cilium is represented by the white arrows. (e) Instantaneous flux (right axis) and flow (left axis) as a function of time with the instances (a)-(d) duly marked. Reproduced with permission from [47].

Figure 16: Flow and effectiveness as a function of the phase difference $\Delta\phi$ for different inter-cilium spacings a/L . AM and SM refer to antiplectic metachrony (the wave direction is opposite to the direction of the effective stroke) and symplectic metachrony (the wave direction and the effective stroke direction are the same), respectively. Reproduced with permission from [47].

Figure 17: (a) Positive (Q_p) and negative flow (Q_n) (see Fig. 13) created by the cilia corresponding to the results presented in Fig. 16. (b) Flux vs time (scaled with the time t_1 taken by the magnetic couple to travel from one cilium to the next) for $a/L = 1.67$ and different wave speeds. Reproduced with permission from [47].

is large ($a/L = 5$ and 2.5), the flow generated remains approximately constant for all metachronal wave speeds. The increase in flow by decreasing the cilia spacing from $a/L = 5$ to $a/L = 2.5$ is much larger when the cilia beat out-of-phase compared to the increase when the cilia beat in-phase. However, when the cilia spacing is low ($a/L = 1.67$), we see a larger increase in the fluid flow when there is an antiplectic metachrony (AM) compared to a symplectic metachrony (SM). Also, the effectiveness sharply increases from around 0.3 (i.e., 30% of the totally displaced fluid is converted into net flow) to 1 (fully unidirectional flow), see Fig. 16(b). To analyse these trends a bit further, we plot the positive and negative flow (Q_p and Q_n in Fig. 13) created during a beat cycle for different phase differences in Fig. 17(a). It can be seen that the cilia do not create a negative flow when they beat out-of-phase for all cilia spacings, resulting in a unidirectional flow (effectiveness = 1). This reduction in negative flow is due to the shielding of flow during the recovery stroke caused by the effective flow of other cilia. It can also be noted that the positive flow is also reduced compared to in-phase beating, but the reduction is considerably less than the reduction in negative flow. Thus, the net flow increases as soon as the cilia start to beat out-of-phase (see Fig. 16(a)). It can be seen from Fig. 17(a) that in the presence of metachronal waves when the cilia spacing is large ($a/L = 5$), the fluid transported during the effective stroke remains nearly the same for all values of the wave velocities. For small cilia spacing ($a/L = 1.67$), however, the positive flow is maximal for antiplectic metachrony, which leads to a larger net flow for antiplectic metachrony compared to symplectic metachrony.

To understand the difference in positive flow for opposite wave directions for small inter-cilium spacing ($a/L = 1.67$), we plot the flux as a function of time scaled with the time taken by the magnetic couple to travel from one cilium to the next t_1 , for two different metachronal wave velocities ($3/t_{\text{beat}}$ and $6/t_{\text{beat}}$ cilia per second), see Fig. 17(b). The corresponding phase differences are also shown in the legend. It can be seen that the flux in the case of antiplectic metachrony is larger than the flux created by the symplectic metachrony for the same wave speed. This difference in flux for opposite wave directions can be understood by analysing the velocity field corresponding to symplectic and antiplectic metachrony at time instances when the flux is maximum (see Fig. 18). Figure 18(a) and 18(b) correspond to different phase differences ($\Delta\phi = \pi/6$ and $\Delta\phi = 5\pi/6$, respectively) leading to a similar wave speed of $6/t_{\text{beat}}$ cilia per second. The fifth cilium is in the peak of its effective stroke for both AM and SM. In the case of symplectic metachrony, the positive flow created by the fifth cilium is obstructed by the close proximity of the fourth cilium, which has just started its effective stroke. As a result, we observe the formation of a vortex. In the case of antiplectic metachrony, however, the position of the fourth cilium is such that the positive flow created by the fifth cilium is not obstructed. This leads to larger fluid flow in the positive direction, so that the net flow created by an antiplectic metachrony is larger than that created by its symplectic counterpart.

Figure 18: Snapshots for antiplectic ($\Delta\phi = \pi/6$) and symplectic metachrony ($\Delta\phi = 5\pi/6$) for a wave speed of $6/t_{\text{beat}}$ cilia per second and cilia spacing $a/L = 1.67$ at $t = 0.1t_1$ of Fig. 17(b). The contours represent the absolute velocity normalised with L/t_{beat} (blue and red colours represent a normalised velocity of 0 and 2, respectively). The direction of the velocity is represented by streamlines. The applied magnetic field is shown by the white arrows. Reproduced with permission from [47].

9 Three dimensional model of magnetically driven artificial cilia

In the previous sections we reported on the results of two-dimensional simulations of ciliary motion and fluid flow. Such a two-dimensional analysis is valid for cilia widths that are larger than the cilia length and channel height. However, the cilia that have been fabricated are three-dimensional structures and have a finite width (typically one fifth of their length [39]). In these situations, the effect of the cilia width and the spacing between the cilia along the width direction play an important role in determining the fluid transported. Also, in experimental systems the magnetic field can be applied in three spatial directions. In order to model these effects a three-dimensional numerical model has been developed which can accurately describe the motion of the cilia, the velocity field in the fluid and the magnetic field in the cilia, for the limiting case of low Reynolds numbers. In the following we summarize the numerical model developed by Khaderi et al. [50]. The reader is referred to this reference for details.

9.1 Solid mechanics model

The cilia are modelled using shell elements based on the superposition of the bending and membrane stiffness in the local coordinate axis of the shell element. The membrane stiffness is based on constant strain triangles with drilling degrees of freedom [3], and the bending stiffness is based on the discrete Kirchhoff triangular elements proposed in [6]. We start with the principle of virtual work containing the relevant energies, linearize it, and finally adopt an updated Lagrangian framework to arrive at the final set of equations. The resulting stiffness matrix includes the geometric non-linearity, which accounts for large deformations, but small strains. The internal virtual work after linearization and discretization can be written in the global coordinates as

$$\delta W_{\text{int}}^{t+\Delta t} = \delta \mathbf{P}^T \mathbf{F}_{\text{int}}^t + \delta \mathbf{P}^T (\mathbf{K}_M + \mathbf{K}_G) \Delta \mathbf{P}, \quad (40)$$

where $\mathbf{P}$ is the global displacement vector, $\mathbf{F}_{\text{int}}^t$ is the internal force vector at time t , $\mathbf{K}_M$ and $\mathbf{K}_G$ are the material and geometric stiffness matrices and $\Delta \mathbf{P}$ represents the increment in the nodal displacements. Similarly, the external virtual work at time $t + \Delta t$ due to body forces and body moments can be written as

$$\delta W_{\text{ext}}^{t+\Delta t} = \delta \mathbf{P}^T \mathbf{F}_{\text{ext}}^{t+\Delta t}. \quad (41)$$

9.2 Fluid dynamic model

To model the fluid we use the boundary element method. The cilia, which are immersed in the fluid and fixed to a substrate, exert forces on the fluid. The velocity at a point in the fluid $\mathbf{r}_i$ due to a point force exerted on the fluid by the cilia at a position $\mathbf{r}_j$ can be obtained using the Green's function as,

$$\mathbf{u}^f(\mathbf{r}_i) = \mathbf{G}(\mathbf{r}_i - \mathbf{r}_j, H) \mathbf{f}(\mathbf{r}_j); \quad \mathbf{u}_i^f = \mathbf{G}_{ij} \mathbf{f}_j. \quad [\text{no summation over } j] \quad (42)$$

The Green's function $\mathbf{G}_{ij}$ for a point force $\mathbf{f}_j$ acting in a fluid near a no-slip boundary is given in [75]. We assume that this point force is distributed over the boundary of the cilia as a traction $\mathbf{t}_j^f$. Now, the velocity in the fluid due to the traction acting over an elemental area of the cilia dS can be written as $\mathbf{u}_i^f = \mathbf{G}_{ij} \mathbf{t}_j^f dS$. Integrating this expression over the cilia surface gives the velocity in the fluid due to the force exerted by the cilia on the fluid,

$$\mathbf{u}_i^f = \int \mathbf{G}_{ij} \mathbf{t}_j^f dS. \quad (43)$$

The cilia surface is discretized using triangular elements, with the tractions assumed to be varying linearly in the element,

$$\mathbf{t}_j^f = \mathbf{N} \left[\{t_1^x \ t_1^y \ t_1^z \dots\}_j^f \right]^T = \mathbf{N} \mathbf{T}_j^f,$$

where $\mathbf{N}$ is a matrix of shape functions and t_j^i is the traction at the j^{th} node in the i^{th} direction. We replace the integral in Eqn. 43 with a sum over the elements and use the above equation to get,

$$\mathbf{u}_i^f = \sum_j^{\text{nelm}} \int \mathbf{G}_{ij} \mathbf{t}_j^f dS_j = \sum_j^{\text{nelm}} \int \mathbf{G}_{ij} \mathbf{N} dS_j \mathbf{T}_j^f. \quad (44)$$

When i is chosen to be the nodes of the cilia mesh, the integration in Eqn. 44 can be performed to obtain a system of equations relating the velocity of the i^{th} node to all forces exerted by the surface of the cilia on the fluid. The integration procedure is adopted from [75]. Equation 44 is evaluated at all nodes on the cilia, and the obtained equations are assembled in a matrix $\mathbf{G}$, which relates the traction exerted by the cilia on the fluid to its velocity, $\mathbf{U}^f = \mathbf{G} \mathbf{T}^f$. Once the velocity of the surface is known, this relation can be inverted to obtain the nodal tractions : $\mathbf{T}^f = \mathbf{G}^{-1} \mathbf{U}^f$.

9.3 Solid-fluid coupling

The effect of fluid drag is incorporated as an external force, which provides an additional contribution to the stiffness matrix. The external virtual work at time $t + \Delta t$ on the j^{th} shell element due to the fluid drag is

$$\delta W_{\text{fluid}}^{t+\Delta t} = - \int \mathbf{t}_j^f \cdot \delta \mathbf{u} dS = - \int (\delta \mathbf{u} \cdot \mathbf{N}) dS \mathbf{T}_j^f \approx -\delta \mathbf{p} \int \mathbf{N}^T \mathbf{N} dS \mathbf{T}_j^f = -\delta \mathbf{p} \mathbf{M}^j \mathbf{T}_j^f, \quad (45)$$

where $\mathbf{M}^j = \int \mathbf{N}^T \mathbf{N} dS$, $\mathbf{u}$ is the displacement vector and $\mathbf{p}$ is the local nodal displacement vector. After performing the standard finite element assembly procedure we get,

$$\delta W_{\text{fluid}}^{t+\Delta t} = -\delta \mathbf{P} \mathbf{M} \mathbf{T}^f = -\delta \mathbf{P} \mathbf{M} \mathbf{G}^{-1} \mathbf{U}^f. \quad (46)$$

Using the no-slip boundary condition $\mathbf{U}^f = \mathbf{A} \Delta \mathbf{P} / \Delta t$, Eqn. 46 can be written as,

$$\delta W_{\text{fluid}}^{t+\Delta t} = -\delta \mathbf{P} \mathbf{M} \mathbf{G}^{-1} \mathbf{A} \Delta \mathbf{P} / \Delta t = -\delta \mathbf{P}^T \mathbf{K}_f \Delta \mathbf{P} \quad (47)$$

where $\mathbf{K}_f = \mathbf{M} \mathbf{G}^{-1} \mathbf{A} / \Delta t$ is the stiffness contribution due to the presence of the fluid and $\mathbf{A}$ is a matrix that eliminates the rotational degrees of freedom from the global displacement vector $\Delta \mathbf{P}$.

Equating the internal (Eqn. 40) and the external virtual work (the sum of Eqns. 47 and 41), and invoking the arbitrariness of the virtual displacements, we get the final equation of motion for the fluid-structure interaction problem:

$$(\mathbf{K}_M + \mathbf{K}_G + \mathbf{K}_f) \Delta \mathbf{P} = \mathbf{F}_{\text{ext}}^{t+\Delta t} - \mathbf{F}_{\text{int}}^t. \quad (48)$$

After incorporating the appropriate boundary conditions, Eqn. 48 is solved for the displacement increment $\Delta \mathbf{P}$.

9.4 Magnetostatic model

The cilia are magnetic films which respond to an external magnetic field. The magnetic property of the cilia is characterised by the magnetic susceptibility tensor χ . The magnetic body couple acting on the cilia $\mathbf{N}$ is obtained from the cross product of the magnetization and the magnetic field intensity $\mathbf{N} = \mathbf{M} \times \mathbf{B}_{\text{ext}}$, where $\mathbf{M}$ is the magnetization and $\mathbf{B}_{\text{ext}}$ is the external magnetic field. The magnetization $\mathbf{M}$ has to be found by solving the Maxwell's equations of electromagnetism. However, we adopt a simpler approach and make use of the fact that the cilia are slender enough to not perturb the external magnetic field significantly; the magnetic field just outside the cilia is equal to the applied magnetic field. The magnetic field inside the cilia can be determined from the electromagnetic boundary conditions: $B_z = B_z^{\text{ext}}$, $H_x = H_x^{\text{ext}}$, $H_y = H_y^{\text{ext}}$, where $\mathbf{H}_{\text{ext}} = \mathbf{B}_{\text{ext}}/\mu_0$, μ_0 is the permeability of free space and x , y and z refer to the local coordinate axes of the shell element. Using the first boundary condition we can find the field H_z inside the cilia as,

$$H_z = \frac{B_z^{\text{ext}} - \mu_0 (H_x^{\text{ext}} \chi_{xz} + H_y^{\text{ext}} \chi_{yz})}{\mu_0(1 + \chi_{zz})}. \quad (49)$$

Once the field in the cilia $\mathbf{H}$ is determined, the magnetization of the cilia ($\mathbf{M} = \chi \mathbf{H}$) and the magnetic body couple ($\mathbf{N}$) can be found. The only assumption made in this approach is that the magnetic field outside the cilia is the external magnetic field, i. e. we neglect the magnetic field caused by the magnetization of cilia.

9.5 Effect of the cilia width and spacing

The modelling capacity of the above magneto-mechanical model was demonstrated using following examples. In the cases shown, the length L of the cilia is 100 micron and the thickness h is 2 micron. The elastic modulus of the cilia E is 1 MPa, the Poisson's ratio $\nu = 0.0$ and the fluid viscosity μ is 1 mPas. The magnetic susceptibilities of the cilia are $\chi_{xx} = 4.6$ and $\chi_{yy} = \chi_{zz} = 0.8$ (all other components are taken to be zero). We apply a magnetic field of magnitude 20 mT, rotating about the $-y$ axis with a frequency of 50 Hz. As the cilia are super-paramagnetic, the cilia complete one beat cycle in $t_{\text{beat}}=10$ ms [47]. In the simulations, the fixed edges of the cilia are placed $0.1L$ above the no-slip boundary, to mimic the presence of the sacrificial layer used during the manufacturing process [20, 25].

9.5.1 Motion of a cilium with non-uniform width

The motion of one cilium whose width decreases linearly from the fixed end to the free end was first studied. To quantify the amount of non-uniformity (tapering) in the width, we define the geometric parameter $T = (b - b_1)/b$, where b and b_1 are the widths of cilium at the fixed and the free edges, respectively. The width of the cilium at the fixed edge is taken to be $L/10$. When T is zero the cilium has uniform width; whereas the cilium is triangular for a tapering T of unity. First, we find what value of T is required to generate an asymmetric motion. The trajectory of the tip of the cilium for different values of T is shown in Fig. 19. The area swept by the free end of the cilium increases when the tapering T is increased (see the left inset of Fig. 19).

The volume flow per cycle (in microlitres per minute) for different values of tapering is shown in the right inset of Fig. 19(a). When the tapering is increased, the area swept increases, whereas the area of the cilium which drives the flow decreases. Thus the created flow is due to the competition between the swept area and the area of the cilium that pushes the fluid. As a result, we see an initial increase of the flow, which reaches a maximum for a tapering T of 0.5, and then decreases. In the same inset, the fluid flow (normalised with $L^2(b_1 + b)/2$) is plotted as a function of the area swept (normalised with L^2) in the $x - z$ plane. It is to be noted that normalising the fluid flow with $(b_1 + b)/2$ gives the area flow per unit average width of the cilia, which on further normalisation with L^2 becomes the normalised area flow. Hence, similar to the two-dimensional cases presented in [45, 44], the flow scales linearly with the swept area.

To gain insight on the fluid propulsion, we look at the position of fluid particles[¶] which initially formed a

[¶]At every time instant, the displacement of the fluid particles is calculated using their velocity (Eqn. 44). The new position is found by adding the displacement to their current position.

Figure 19: Displacement of the cilium tip for different tapering in the width. The inset at the right shows the fluid flow (in microlitre per minute) for different values of tapering T , and the flow (normalised with $L^2(b_1 + b)/2$) as a function of the area swept (normalised with L^2). The inset at the left shows the area swept normalised with (L^2) as a function of T . Reproduced with permission from [50].

plane parallel to the $y - z$ plane for a cilium with a tapering T of 0.5, see Fig. 20. During the effective stroke, the cilium displaces the fluid particles in the negative x -direction (see Figs. 20(a) and (c)), after which the fluid particles are dragged back during the recovery stroke (see Figs. 20(d) and (f)). The displacement of the fluid particles can be observed by comparing the position of the particles in Figs. 20(a) and (f). The additional information, which we obtain from the three-dimensional model compared to the two-dimensional model, is the displacement of the fluid particles that are not present in the plane of beat.

9.5.2 Effect of the cilium width and spacing

We now examine the effect of the width and the cilium spacing on the flow generated by one row of cilia placed along the width. The geometry is shown in Fig. 21(a). We take n cilia of width b at the fixed edge whose tapering T is 0.5. The spacing between the cilia is p (the pitch), so that the total width occupied by the cilium row is np . For the simulations we choose $n = 4$ and calculate the flow created as a function of the spacing between the cilia $p' = p - b$. The spacing p' is varied from 0 (no spacing between the cilia at their fixed edge) to $0.9L$.

The horizontal lines in Fig. 21(b) represent four times the flow created by one cilium for different widths b . In the cases considered the increase of flow does not scale with the increase of the cilium width b . This suggests that the forces acting on the fluid do not scale linearly with the cilium width b . Such a behaviour is also present in the case of ellipsoids [35,].

The flow as a function of the spacing p' for various widths for $n = 4$ is also shown in Fig. 21(b). The flow created by the cilia is larger when they are further apart. The beating of a cilium imparts velocity to a small fluid region around it. When four cilia are used, the region influenced by the cilia strongly depends on their spacing. When the spacing between them is small these regions overlap so that they collectively influence only a small region of the fluid. The total fluid region that can be influenced by the cilia increases as the spacing is increased, reaching a maximum for spacings when the cilia do not hydrodynamically interact. In these cases, the flow converges to four times the flow caused by one cilium. This can also be rationalised from a force point-of-view. When the cilia are spaced closer, they can move the fluid with less effort; this reduces the forces acting on the fluid due to the cilium motion. Consequently, the flow generated is low. A similar behaviour can also be seen in the case of two spheres which are translating at a given velocity [35]. The force exerted by the spheres on the fluid is reduced when they are brought closer together.

Figure 20: Snapshots of the motion of the cilium, beating in the $x-z$ plane, at different instances of time for a representative cycle. Figures (a)-(c) and (d)-(f) represent the effective and recovery strokes, respectively. The evolving surface represents contours of the x coordinates of fluid particles which at time $t = 0$ were parallel to the $y-z$ plane. During the effective stroke, the cilium displaces the fluid particles in the negative x -direction, and during the recovery stroke, the fluid particles are dragged back. The displacement of the fluid particles can be observed by comparing the position of the particles in Figs. (a) and (f). Reproduced with permission from [50].

Figure 21: (a) Parameters used to study the effect of cilia spacing p' and width b for a given tapering. (b)-(c) Flow as a function of width and the pitch of the cilia spacing for cilia having a taper along the width. Reproduced with permission from [50].

Figure 22: Schematic diagram showing the arrangement of cilia, the direction of effective and recovery strokes along with different kinds of metachronal waves. Reproduced with permission from [50].

A practically-relevant question is: How much flow can be generated by the cilia per unit width? This question can be answered by normalising the flow in Fig. 21(b) with the width of the cilia row (np). It can be seen that when the spacing p' is very low compared to the length L , the narrow cilia produce more flow; when the spacing is comparable to the length, the broader cilia create a higher flow. Note that for a given width, many narrow cilia spaced close together create the largest flow. Interestingly, this is the option chosen by nature. The natural cilia are hair-like structures that are spaced very close together.

9.6 Effect of metachronal waves in the out-of-plane direction

We now analyse the flow when an array of cilia move out-of-phase in the direction of their beat motion (antiplectic and symplectic metachrony) and in the direction orthogonal to it (laeoelectric and diaplectic metachrony) [16]. In laeoelectric and diaplectic metachrony, the effective stroke is to the left and right of the direction of propagation of the metachronal wave (see Fig. 22). To perform the simulations we choose 5 rows^{||} of cilia, with each row containing 5 cilia (see Fig. 22). The cilia have a uniform width $b = 0.1L$ and a tapering in the thickness, such that the thickness of the cilia at the fixed end is 2 microns, which decreases linearly along the length to a thickness of 1 micron at the free end. The cilia spacings are $a = 1.1L$ (along the length) and $p = 0.2L$ (along the width). A rotating magnetic field with a magnitude of 20 mT is applied to every cilium at a frequency of 50 Hz. The phase difference in the magnetic field between adjacent cilia is varied from $\Delta\phi_x = -\pi/2$ to $\pi/2$ in the beat direction, and from 0 to $\Delta\phi_y = 2\pi/10$ in the direction normal to beat plane^{**}. A zero phase difference in any direction represents uniformly beating cilia in that direction, and a phase difference of $\pi/2$ represents the situation when adjacent cilia are in anti-phase (standing wave). As the metachronal wave can also travel in a direction normal to the cilia beat, we also analyse the flow in this direction. In the following, the flow in the plane of the ciliary beat is referred to as primary flow and the flow normal to this plane is called secondary flow. The primary flow is plotted as a function of the phase differences $\Delta\phi_x$ and $\Delta\phi_y$ in Fig. 23.

The flow is always larger for the cilia beating with antiplectic metachrony compared to synchronously beating cilia. In the case of symplectic metachrony, the cilia obstruct the flow caused by their neighbours during the effective stroke. As a result, for antiplectic metachrony the flow is larger and for symplectic metachrony the flow is smaller than synchronously beating cilia, although the magnitude of increase is larger (for antiplectic metachrony) than the decrease (for symplectic metachrony). The flow obstruction is maximum for $-0.2\pi < \Delta\phi_x < 0$, in such cases the flow created is less than that created by synchronously beating cilia. In these cases, however, when $\Delta\phi_y > 0$ the decrease is lower because of the relaxation of the obstruction of positive flow. The flow exhibits a fluctuating behaviour only for the cases enclosed in the

^{||} A row refers to the arrangement of cilia in the y -direction.

^{**} The metachrony normal to the beat plane will create symmetric waves about $\Delta\phi_y = 0$. Hence, simulations are performed only for $\Delta\phi_y > 0$.

Figure 23: Primary flow created by cilia due to metachrony along and normal to the beat direction, respectively. The primary flow is larger when the cilia beat out-of-phase compared to synchronously beating cilia except for $-2\pi/10 < \Delta\phi_x < 0$ and $0 < \Delta\phi_y < \pi/10$. The secondary flow is created due to the plate-like motion of the rows of cilia, which reaches a maximum when the cilia motion between rows is anti-phase and $\Delta\phi_y = \pi/10$. AM, SM and LM represent antiplectic, symplectic and laeoplectic metachrony, respectively. Reproduced with permission from [50].

white curve, outside this region (in the direction of the arrow) the flow is unidirectional.

9.7 Out-of-plane actuation of cilia

In nature, the cilia on a Paramecium beat in a plane normal to the surface during the effective stroke, and during the recovery stroke they beat in a plane parallel to the surface. To achieve such a motion, we apply a magnetic field so that the magnetic field vector can be oriented in three-dimensional space (see Fig. 24).

Figure 25 shows the motion of a SPM cilium in which the cilium performs the effective stroke in the $x-z$ plane and the recovery stroke in the $x-y$ plane near the no-slip boundary. This results in large flow during the effective stroke and a small flow during the recovery stroke. The effective stroke consists of a uniform bending of the cilium in the $x-z$ plane. During the recovery stroke, the cilium undergoes a significant amount of twisting, and comes back to the initial position (see Figs. 25(d) - (f)).

10 Realistic flow geometries

10.1 Flow geometries in lab-on-a-chip systems

Since the main application of artificial cilia is in lab-on-a-chip devices, it is important to note that the magnetic artificial cilia have to be designed for typical lab-on-a-chip channel geometries. Then, we should ask the question how the generated flux and pressure can be optimized. Two channel configurations that are typically encountered in lab-on-a-chip applications: (i) A closed-loop channel (see Fig. 26(a)), and (ii) an open-loop channel (see Fig. 26(b)). In the closed-loop channel, fluid is initially pumped in by an external device and then it is propelled around the channel. Closed-loop channels are used, for instance, to perform polymerase chain reaction (PCR) [93]. In an open-loop channel we have well-defined inlet and outlet points for the fluid to enter and leave the channel. The fluid is propelled by an array of artificial cilia inside the

Figure 24: The arrows represent the applied magnetic field vector at different instances in time for a cilium to exhibit the effective stroke in the $x - z$ plane and the recovery stroke in the $x - y$ plane. The numbers adjacent to the arrows show the time in milliseconds. The magnitude of the magnetic field at time instances 0, 5, 7.5 15 and 25 ms is 20 mT. Reproduced with permission from [50].

Figure 25: Snap shots of the motion of a SPM cilium due to a 3D magnetic field. The arrow shows the applied magnetic field. The cilium performs the effective stroke in the $x - z$ plane (see (a)-(c)) and the recovery stroke in the $x - y$ plane (see (d)-(f)). Significant twisting of the cilium during the recovery stroke can be observed from the instances shown in (d) and (e). Reproduced with permission from [50].

channel. For example, the open-loop channel can be used to supply the fluid to a closed-loop channel. For the closed-loop channel, we assume the radius of the loop to be much larger than the width of the channel and the spacing between the cilia. In that case, the analysis can be performed by using a periodic unit-cell which contains one cilium. However, in the case of the open-loop channel there is no periodicity due to the presence of the inlet and outlet, and the analysis has to be performed with the channel containing multiple cilia. The cilia span the entire width of the channels for an optimal performance (see Fig. 26), and the width of the channels is taken to be larger than the height. As a result, two-dimensional simulations are sufficient for both channel geometries.

Figure 26: Two possible applications of artificial cilia in microfluidics: (a) Closed-loop channel and (b) open-loop channel. In the closed-loop channel, the cilia can be used to propel the fluid inside a circular channel for a well-defined period of time. The closed-loop channel can, for example, be used to perform a polymerase chain reaction (PCR). In the open-loop channel, the cilia intake the fluid from one end of the channel and pump it to the other end. (c) Schematic representation of the typical motion of a magnetically-actuated artificial cilium during its beat cycle. The shaded region bounded by a dashed curve represents the area swept by the cilium. The direction of motion of the cilium is shown using the arrow on the dashed curve. The effective stroke is represented by the instances 1, 2 and 3, and the recovery stroke by the instances 4 and 5. Reproduced with permission from [46].

The performance of the artificial cilia is quantified by the flux and pressure they can generate. We describe the performance of the artificial cilia for a given set of parameters as a function of the channel height H and cilia spacing a , normalised by their length L . The following set of parameters are used: The length of the super-paramagnetic cilia (L) is 100 microns, they have linearly varying tapered cross-section with a thickness being $h = 2 \mu\text{m}$ at the fixed end and $1 \mu\text{m}$ at the free end, an elastic modulus $E = 1 \text{ MPa}$, and density $\rho = 1600 \text{ kg/m}^3$. The fluid viscosity $\mu = 1 \text{ mPa}$. The super-paramagnetic cilia are subjected to a magnetic field with magnitude $B_0 = 31.5 \text{ mT}$ that is rotated from 0° to 180° in $t = 10 \text{ ms}$ and then kept constant during the rest of the cycle. At 0° , the magnetic field is directed along the original length of the cilia (instance 1 in Fig. 26(c)). The anisotropic magnetic susceptibilities of the cilia are 4.6 along the length and 0.8 along the thickness [90].

10.1.1 Closed-loop channel

The unit-cell for the closed-loop channel has a width a and height H , containing one cilium of length L . The top and bottom surfaces of the unit-cell are no-slip boundaries, while the left and right boundaries are periodic in velocity. This results in a pressure distribution which is also periodic, so that no pressure gradient can be generated by the closed-loop channel.

Figure 27 shows the area flow per cycle normalised by $(\pi L^2/2)$, as a function of the cilia spacing a/L for various channel heights H/L . Also shown is the corresponding volume flow assuming an out-of-plane channel width of 1 mm. It can be seen that the flow decreases as the cilia spacing is increased. There can be

two reasons for the decrease in the flow. Firstly, a decrease in the area swept by the cilia tip decreases the area flux [45]. Secondly, when the cilia spacing is increased, the fluid drag forces imposed by top and bottom channel surfaces increase and thus the flux gets reduced. It can be seen from Fig. 27 that the swept area shows a minor increase for large a/L , indicating that only a limited amount of hydrodynamic interaction is operative when the cilia move apart. Hence, the reduction in flow (as we increase the spacing) is only due to increased fluid viscous forces when the cilia spacing becomes larger. It can also be observed from Fig. 27

Figure 27: Volume flow (left axis), area flow (left axis) and area swept (right axis) as a function of the cilia spacing (a/L) for different channel heights (H/L) for the closed-loop channel, L is the cilia length. The solid lines correspond to the left axes, and the dashed lines correspond to the right axis. The volume flow is calculated by taking the out-of-plane width of the channel to be 1 mm. The fluid flow increases when the cilia spacing is decreased, and when the channel height is increased. The area swept by the cilia is not substantially influenced by either the cilia spacing or the channel height. Reproduced with permission from [46].

that the fluid flow scales almost linearly with the height of the channel. When the height of the channel is substantially larger than the cilia length ($H > L$), the cilia can be considered to create a fluctuating Couette flow (shear flow) in the channel, with the top boundary being a no-slip boundary and the bottom boundary being displaced with a tangential velocity as imposed by the cilia. In such cases, the flow scales linearly with the height of the channel [57]. The typical volume flow rates that can be created with the current configuration is tens of microlitres per minute (for a frequency of 50 Hz), which is comparable to the flow generated by typical dynamic pumps [61].

10.1.2 Open-loop channel

10.1.3 Flux calculation

The open-loop microfluidic channel has well-defined inlets and outlets, between which the fluid is propelled by an array of artificial cilia, see Fig. 28(a). As mentioned earlier, the performance of any pumping device is characterised by the flux and pressure it can generate. Different experimental set-ups are usually adopted to probe the flux and pressure. The flux measurement for an open system is typically performed by maintaining constant fluid levels at the inlet and outlet when the pumping mechanism is switched on [42] (see Fig. 28(a), top). As a result, the actuating mechanism does not have to work against a pressure gradient, but only has to overcome the frictional resistance of the channel, as in the closed-loop channel of the previous section. The fluid that spills out at the outlet and that is replenished at the inlet equals the fluid transported by the artificial cilia.

To perform numerical simulations, we neglect the frictional loss in the regions L_1 and L_2 and model only the portion of the channel where the cilia are placed (see Fig. 28(a), bottom). By assuming the left and right boundaries of the channel to be traction free, we properly account for the constant fluid levels at the inlet and outlet. For a given cilia spacing, we increase the number of cilia and compute the area flow per cycle. By plotting the area flow as a function of cilia spacing a/L , we found that the open-loop results converge to the closed-loop results depicted in Fig. 27 when large number of cilia are used. From this it can be concluded that the dependence of the flux generated by an open-loop channel on the channel height and cilia spacing is the same as that of a closed-loop channel, in situations where sufficiently many cilia are present in the open-loop channel.

10.1.4 Pressure calculation

The pressure in the open-loop channels is experimentally measured by comparing the fluid heights between two capillary tubes at the inlet and outlet of the channel [42, 15] (see Fig. 28(b), top). When the pumping mechanism is switched on, the initially equal fluid heights ($h_1 = h_2$) will become different. After this transient phase, the system reaches a steady state in which h_1 and h_2 remain constant. This level difference $h_1 - h_2$ gives the pressure difference generated by the pumping system. In the numerical analysis, we neglect the transient phase during which the fluid levels are changing, but focus on the steady state. At steady state, the fluid will be in motion only near the artificial cilia. At a distance L_D away from the cilia the velocity will be zero (see Fig. 28(b), top). Within this region a circulating flow is established that generates a mean pressure of $\rho g h_1$ and $\rho g h_2$ at the left and right of this region, respectively. This gives a pressure difference $\Delta p = \rho g (h_1 - h_2)$, with ρ the density of the fluid and g the acceleration due to gravity.

Motivated by this, we choose our computational domain to comprise of a closed channel containing multiple cilia, with the distance between the cilia and the left and right channel boundaries being L_D (see Fig. 28(b), bottom). We now choose L_D to be large enough, so that the fluid at distances larger than L_D away from the cilia remains static. By performing simulations for different heights and L_D values, we found that the minimum L_D required is equal to the height of the channel (H). Figure 29 shows the pressure field for $L_D = H$, $H = 2L$, $a = 2L$ and for 4 cilia ($n = 4$) at a particular instant during the effective stroke. Since the effective stroke is to the left, the pressure builds up to the left, and the fluid circulates clockwise in the channel.

We now calculate the pressure gradient by integrating the pressure difference Δp (between the left and right boundaries) over a cycle and divide by the channel length na . Figure 30 shows the normalised pressure gradient $L \int_{\text{cycle}} \Delta p dt / n \mu a$ as a function of normalised cilia spacing (a/L) for different channel heights. It can be seen that as the cilia spacing and channel height decrease, the pressure gradient increases. The slope of the lines in Fig. 30 is -1, indicating that the pressure jump Δp generated by a cilium is independent of the cilia spacing a . As a result, the total pressure difference generated by an array of n cilia is simply $n \Delta p$. On the right axis of Fig. 30 the pressure head generated assuming a total channel length of 10 cm and fluid density of 1000 kg/m³ is shown. This pressure jump per cilium decreases with the height of the channel.

To investigate the height dependence we plot the pressure gradient as a function of height for $a = 2L$ in the inset of Fig. 30. The dashed line is a fit to the data in Log-Log scale. The slope of the fitted line is -2.4. This relatively strong dependence on the height of the channel may be related to two contributions - Firstly, the fluid can more freely flow backwards as the height H is increased, so the cilia have to exert less force. Secondly, the pressure on the boundaries is due to the force exerted by the cilia divided by the boundary area. Since the boundary area scales with H , the pressure is further decreased. It is to be noted that the maximum pressure generated by the cilia is 11 mm, which is lower than the pressure generated by typical dynamic pumps [61]. Therefore, the artificial cilia can find applications in microfluidic channels where the back pressure to be overcome is moderate.

Figure 28: Schematic pictures representing the approaches taken to calculate the flow (a) and pressure (b) for an open-loop channel. (a) Flow calculation: (Top) Experimental method: The cilia inside an open-loop channel intake the fluid from one end and pump it to the other end. The levels of the inlet and outlet are maintained at the same elevation. (Bottom) Computational model: Simplified model to calculate the flow through an open-loop channel. The pipings L_1 and L_2 are ignored for simplicity. (b) Pressure calculation: (Top) Experimental method: The channel on both the ends is attached with capillary tubes. When the cilia operate, there will be a difference in the levels of the liquid between the left and the right capillary tubes, which gives the pressure generated by the cilia. The fluid will be in motion only near the cilia, while far away from the cilia (beyond a distance L_D) the fluid will be stationary. (Bottom) Computational model: In the simulations, the cilia are made to operate in a closed channel with the distance between the left/right boundaries and the cilia is L_D . The difference between the pressure calculated on these two boundaries gives the pressure generated by the cilia. Reproduced with permission from [46].

Figure 29: Pressure contours with multiple cilia for $H/L = 2$, $a = 2L$, $n = 4$ and $L_D = H$. Because the channel is closed, the fluid simply circulates in the channel. As the cilia are moving to the left, we see a uniform positive pressure on the left boundary and a uniform negative pressure on the right boundary. The gradual build-up of the pressure due to multiple cilia can be seen. Reproduced with permission from [46].

Figure 30: Variation of the pressure gradient generated with cilia spacing a/L for different channel heights (left axis). Also shown (right axis) is the corresponding pressure head generated assuming a total channel length of 10 cm and fluid density of 1000 kg/m^3 . The pressure generated decreases when either the cilia spacing or the channel height increase. The inset shows the pressure gradient as a function of the height of the channel for $a = 2L$. The pressure generated decreasing drastically when the height of the channel is increased. Reproduced with permission from [46].

Figure 31: (a) Picture of the substrate and placed cartridge. The cilia area covers the whole channel section of $5 \times 10 \text{ mm}^2$. The transport direction of the cilia is indicated with a red arrow. (b) Schematic of the channel with three components: the ciliated substrate in the bottom, the middle cartridge and the channel cover with integrated fluid connections and the vacuum channel. (c) Schematic of the cilia array. All dimensions are in μm . The dimension of $80 \mu\text{m}$ refers to the length of cilia ($70 \mu\text{m}$) and the region fixed to the substrate ($10 \mu\text{m}$). Approximately 10000 individual rectangular-shaped structures are distributed over the channel bottom. Reproduced with permission from [51].

10.2 Experiments using plate-like magnetic artificial cilia

Based on the theoretical studies reviewed in previous sections, plate-like artificial cilia have been fabricated and integrated into a microfluidic channel [7]. The theoretical studies on the magnetically driven artificial cilia are based on a uniform distribution of the magnetic field. Realising such uniform magnetic fields experimentally involves an elaborate design of the actuation system [80, 25, 91]. Alternatively, a simple actuation system was proposed using a non-uniform magnetic field, created by a rotating permanent magnet [7, 39]. Fluid flow measurements performed show that the velocities generated by the cilia subject to such non-uniform magnetic field can reach values around $\approx 130 \mu\text{m/s}$ [39]. However, the underlying physical mechanisms that are responsible for these fluid velocities remain unclear. The dominant cause for the breaking of asymmetry in these experimental systems leading to flow was analysed by Khaderi et al. [51].

10.2.1 Experimental setup

An array of artificial cilia was fabricated using a composite of poly(n-butylacrylate) (PnBA) containing photoreactive side groups and super-paramagnetic magnetite nano-particles (with particle diameter $10\text{-}20 \text{ nm}$ and volume fraction $\approx 9\%$) [7]. These artificial cilia are flap-shaped with length $L = 70 \mu\text{m}$, width $b = 20 \mu\text{m}$ and thickness $h = 0.9 \mu\text{m}$, elastic polymer structures that are attached at one end to the substrate surface of a silicone wafer. The cilia are located $20 \mu\text{m}$ apart (along the width direction) while the streamwise distance (along the length direction) from cilia tip to cilia tip equals $100 \mu\text{m}$. To create a microfluidic chamber around the cilia, a channel is assembled on top of the cilia-covering substrate in a sandwich-like system (see Fig. 31). A cartridge made of poly(N,N-dimethylacrylamide) of $500 \mu\text{m}$ thickness is placed on the substrate surface. The cartridge has a rectangular void of 10 mm length and 5 mm width in its middle which defines the side walls of the fluid channel. The channel is closed by placing a glass cover on top of the cartridge. The glass cover has integrated inlet and outlet channel connections, therefore allowing to fill the channel after assembly with fluid. All three components are hold together by clamping. Additionally, a connection to a vacuum channel allows sealing the microfluidic channel by under pressure during usage. A rotating permanent magnet of dimensions $10 \text{ mm} \times 25 \text{ mm} \times 50 \text{ mm}$ that is placed below the channel is used to actuate the cilia, see Fig. 32. The magnetization of the magnet is determined by comparing the field caused by the magnet with the field created by a rectangular permanent magnet [49].

Phase-locked micro-PIV measurements were performed at 20 different phases throughout an actuation cycle to quantify the instantaneous velocity distributions of the flow in bottom parallel measurement planes. Measurements were performed in a stack of 20 planes with a spacing of $20 \mu\text{m}$ between each plane. To increase

Figure 32: Experimental set-up used to study the fluid transport due to artificial cilia (not to scale). Magnetic cilia of length $70 \mu\text{m}$, spaced $100 \mu\text{m}$ apart are used to create a flow in a closed channel of height 0.5 mm , width 5 mm and length 20 mm . The artificial cilia are driven using a permanent magnet having a remanent magnetization $\mathbf{M}_r$ that rotates about the z axis. Reproduced with permission from [51].

the reliability of the measurement results compared to instantaneous vector results, ensemble averages were computed for each vector field from a set of 100 image pairs. The measurement region spans an area of $400 \times 300 \mu\text{m}$, therefore covering a field of approximately 30 individual cilia. Measurements are performed at the channel symmetry plane such that sidewall effects in the measurement result can be neglected.

10.2.2 Computational model

Motivated by the experimental parameters, a periodic unit-cell of width 0.1 mm and height 0.5 mm containing one cilium is used for the simulations (see Fig. 32). The cilia have a length $L = 70 \mu\text{m}$, thickness $h = 2 \mu\text{m}$ and a width $b = 20 \mu\text{m}$. The fluid has a viscosity of $\mu = 1 \text{ mPa}\cdot\text{s}$ and a density of $\rho_f = 1000 \text{ kg/m}^3$. No-slip boundary conditions are applied to the top and bottom of the channel. The initial configuration of the cilia is not exactly known from the experiments. Hence, the initial configuration is chosen to be an arc of a circle of length L with the height of the segment equal to aR , where R is the radius of the circle. In the simulations, the fixed edges of the cilia are placed $h/2$ above the no-slip boundary, to mimic the presence of the sacrificial layer used during the manufacturing process[20]. The other input quantities of the numerical model are the elastic and magnetic properties, which were estimated to be 0.65 MPa and 1.27 MA/m .

The flow caused by the cilia is due to the competition between magnetic forces, elastic forces of the cilia, fluid viscous and inertia forces. Consequently, three physical dimensionless parameters can be identified [49]: (i) the fluid number $F_n = (\mu/12Et_{\text{beat}})(L/h)^3$ - the ratio of viscous to elastic forces, (ii) the Reynolds number $Re = \rho_f L^2/t_{\text{beat}}$ - the ratio of fluid inertial to viscous forces and (iii) the magnetic number $M_n = (\mu_0 H_0^2/12E)(L/h)^2$ - the ratio of magnetic to elastic forces. Here, t_{beat} is the time taken by the cilia to complete one beat cycle and is equal to half the time period of the rotating magnet, μ_0 is the magnetic permeability in vacuum and H_0 is the magnetic field caused by the permanent magnet. By assuming that the magnet can be approximated as a dipole, whose magnetic field decreases as $1/r^3$ with distance r , H_0 is taken to be of the form $|\mathbf{M}_r|(h_r/2s)^3$, where h_r is the thickness of the rotating magnet and s is the distance

between the magnet and the microfluidic channel (see Fig. 32). The relevant geometrical dimensionless parameters are H/L and a/L , where H is the height of the microfluidic channel and a is the cilia spacing along length. The dimensionless parameters, when the frequency of the rotating magnet is 10 Hz, are $F_n = 1.21 \times 10^{-3}$, $M_n = 0.281$, $Re = 0.098$, $H/L = 7.14$ and $a/L = 1.42$.

10.2.3 Results

To simulate the closed channel, a zero-flux condition is imposed on the periodic edges of the unit-cell (see Fig. 32). Since we simulate a single cilium in a periodic domain, this model implicitly neglects the phase lag between the cilia due to a possible non-uniform magnetic field as observed by Belardi et al.[7]. The data points in Fig. 33 show the time evolution of the instantaneous horizontal velocity^{††} at a plane 60 microns ($= 0.85L$) from the substrate for a frequency of 10 Hz. The initial configuration used is an arc of a circle with $a = 0.7$ and $\phi = -10^\circ$. As shown in Fig. 33, there is a good agreement between the numerical and experimental data.

Figure 33: Velocity at a plane 60 microns from the base during a half magnet rotation at 10 Hz. Reproduced with permission from [51].

Fluid transport by artificial cilia can be due to (1) spatial asymmetry, (2) temporal asymmetry and (3) orientational asymmetry. The relative contribution of temporal and orientational asymmetry is difficult to identify in the presence of inertia, and we therefore combine these two effects into an ‘inertia-induced’ contribution, in addition to the ‘spatially-induced’ contribution. In the following we analyse which of these contributions is dominant in causing the net fluid transport.

To ascertain if there is a spatial asymmetry, we have to observe the cilia motion from the side of the channel. As this is a difficult task experimentally due to restricted optical access, we resort to the simulations to analyse this. Figure 34 shows the deformed shape of the cilia for various time instances. The circular arrows at the two ends of the cilia represent the direction of the magnetic body couple acting on the respective ends. It can be seen that when the magnet turns by 180° , the cilia complete one beat cycle (t_{beat} is half the time period of the rotating magnet). To analyse the area swept by the cilia, we plot the trajectory of the tip of the cilia in Fig. 34, with the straight arrows showing the direction of motion. It can be clearly seen that the cilia exhibit a spatially asymmetric motion during a beat cycle with the rotation of cilia tip

^{††}The instantaneous velocity in the simulations was calculated as $\int u_x(y = 60 \mu\text{m}, t) dx/a$, where a is the spacing between the cilia and u_x is the velocity in the x direction.

Figure 34: Configuration of cilia at various time instances. The top figure shows the deformed geometry of the cilia and the dots show the trajectory of the tip of the cilia. The straight arrows, curved arrows and dots represent the direction of the cilia motion, the magnetic body couple and the trajectory of the tip, respectively. The bottom figure shows the orientation of the magnet and the distribution of magnetic field in a circle around the magnet. The box represents the micro-channel that contains the cilia. Reproduced with permission from [51].

in a direction opposite to that of the magnet during the effective stroke. During the effective stroke, a counter-clockwise/clockwise torque acts on the cilia near the fixed/free end. This straightens-up the cilia and enables the cilia to follow the magnetic field during the effective stroke to the right, see Figs. 34(a) and (b). As the cilia cannot continue to follow the magnetic field forever, due to the elastic forces and due to the presence of the channel walls, they become locked momentarily while the magnetic field keeps on rotating, see Fig. 34(c). As the magnetic field continues to rotate, the magnetic torque is reversed such that a counter-clockwise/clockwise torque act on the cilia near the free/fixed end, see Figs. 34(d) and (e). This makes the cilia more curved as they return to the initial position. We note that the effective stroke takes place during the first $\approx 111^\circ$ rotation of the magnet, leading to a positive velocity. As the magnet continues to rotate further, the recovery stroke takes place, which leads to a negative velocity. The mean velocity of the recovery stroke is about 1.5 times larger than that of effective stroke.

Figure 34 clearly shows that the cilia exhibit a spatially asymmetric beat cycle, which contributes towards a net fluid flow even in the absence of fluid inertia. To explore whether temporal or orientational asymmetry also contribute to the net flow, we now investigate the effect of inertial forces.

10.2.4 Effect of fluid inertia

To see if there is any effect of fluid inertia on the fluid transported we performed simulations in which the inertia terms in the Navier-Stokes equations were neglected. The results are plotted in Fig. 35. The spatial asymmetry, i.e. the area swept by the cilia was found to be identical in the presence and absence of fluid inertia. The results show that the fluid inertia plays an important role even at low frequencies of 10 Hz and a cilia length of 70 micron (corresponding to a Reynolds number $\rho L^2/\mu t_{\text{beat}} = 0.098$). The mean velocity profile shows that the maximum fluid velocity in the absence of inertia (due to the spatial asymmetry) is nearly half that of the flow in the presence of inertia at some elevations (see Fig. 35(a)). The instantaneous velocity at an elevation of 60 micron is compared for Stokes and inertial flow in Fig. 35(b). The effect of fluid inertia is to increase the positive velocity and decrease the negative velocity^{††}. The cause of this increased positive flow has been explained by comparing the velocity contours of the Stokes and inertial flows [51].

Figure 35: Effect of fluid inertia: (a) Mean velocity profile at 10 Hz. (b) Velocity at a plane 60 microns from the base during half a magnet rotation at 10 Hz. Reproduced with permission from [51].

11 Concluding remarks

In this article we have reviewed various theoretical results regarding fluid transport using magnetically actuated artificial cilia, such as the effect of fluid inertia, geometry of the channel, metachronal waves and the influence of three-dimensional motion of the cilia. There also has been significant effort from the experimental side towards the fabrication, actuation and flow characterization. Several studies have shown that the actuation of artificial cilia in microfluidic chips can be used to generate substantial fluid flow as well as effective fluid mixing (see e.g., [21] for an overview).

There are at least three issues yet to be addressed for a successful deployment of artificial cilia-induced fluid flow in commercial LOC systems.

- The fluid that will be analysed in a LOC is typically non-Newtonian (like saliva and blood). This feature of the analyte brings in additional time and length scales. The question now is- how much of the known results can be applied to non-Newtonian fluid transport. To answer this question new theoretical and experimental developments are needed.
- New methods have to be developed that allow reliable, large scale fabrication of the artificial cilia in micro-channels and their deployment in LOCs (see e.g., Wang et al. [92]).
- Simple ways have to be found in which the actuation mechanism of the artificial cilia can be integrated within a LOC, making use of electromagnets or moving magnets, for instance.

^{††}No significant differences were observed in the area swept and the cilia velocity between the results presented in Fig. 35.

In addition to their use for microfluidic flow and mixing, artificial cilia might also find application as anti-fouling surfaces, for the transportation of cells or other particles, as microsensors for flow and/or viscosity, and when functionalized chemically, for the sensing of chemical agents or biochemical targets (e.g. proteins)).

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A Discretisation of various terms used in section 3.

$$\begin{aligned}
\int_V \sigma_{ij} \delta D_{ij} dV &= \int_V (-p \delta_{ij} + 2\mu D_{ij}) \delta D_{ij} \\
&= \int_V \left(-p \delta_{ij} + \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right) \frac{\partial \delta u_i}{\partial x_j} dV, \\
&= \mu \int_V \delta U_I \frac{\partial \phi_{Ii}}{\partial x_j} \left(\frac{\partial \phi_{Ji}}{\partial x_j} + \frac{\partial \phi_{Jj}}{\partial x_i} \right) U_J dV - \int_V \delta U_I \frac{\partial \phi_{Ii}}{\partial x_j} \psi_J P_J dV, \\
&= \mu \delta U_I \int_V \frac{\partial \phi_{Ii}}{\partial x_j} \left(\frac{\partial \phi_{Ji}}{\partial x_j} + \frac{\partial \phi_{Jj}}{\partial x_i} \right) dV U_J - \delta U_I \int_V \frac{\phi_{Ii}}{\partial x_j} \psi_J dV P_J, \\
&= \delta U_I^T (K_{IJ}^{UU} U_J + K_{IJ}^{UP} P_J) = \delta \mathbf{U}^T (\mathbf{K}^{UU} \mathbf{U} + \mathbf{K}^{UP} \mathbf{P}).
\end{aligned}$$

$$\begin{aligned}
\int_V \frac{du_i^{t+\Delta t}}{dt} \delta u_i dV &= \rho \int_V \delta u_i \left(\frac{\partial u_i^{t+\Delta t}}{\partial t} + \frac{\partial u_i^{t+\Delta t}}{\partial x_j} u_j^{t+\Delta t} \right) dV \\
&= \rho \int_V \delta u_i \left(\frac{u_i^{t+\Delta t} - u_i^t}{\Delta t} + \frac{\partial(u_i^t + \Delta u_i)}{\partial x_j} (u_j^t + \Delta u_j) \right) dV \\
&= \rho \int_V \delta u_i \left(\frac{u_i^{t+\Delta t} - u_i^t}{\Delta t} + \frac{\partial u_i^t}{\partial x_j} \Delta u_j + \frac{\partial u_i^t}{\partial x_j} u_j^t + \frac{\partial \Delta u_i}{\partial x_j} u_j^t \right) dV \\
&= \rho \int_V \delta u_i \left(\frac{u_i^{t+\Delta t} - u_i^t}{\Delta t} - \frac{\partial u_i^t}{\partial x_j} u_j^t + \frac{\partial u_i^t}{\partial x_j} u_j^{t+\Delta t} + \frac{\partial u_i^{t+\Delta t}}{\partial x_j} u_j^t \right) dV \\
&= \rho \int_V \delta u_i \left(\frac{u_i^{t+\Delta t}}{\Delta t} - \frac{u_i^t}{\Delta t} - \frac{\partial u_i^t}{\partial x_j} u_j^t + \frac{\partial u_i^t}{\partial x_j} u_j^{t+\Delta t} + \frac{\partial u_i^{t+\Delta t}}{\partial x_j} u_j^t \right) dV \\
&= \rho \int_V \delta U_I \phi_{Ii} \left(U_J^{t+\Delta t} \frac{\phi_{Ji}}{\Delta t} - \left(\frac{u_i^t}{\Delta t} + \frac{\partial u_i^t}{\partial x_j} u_j^t \right) + \frac{\partial u_i^t}{\partial x_j} \phi_{Jj} U_J^{t+\Delta t} + \right. \\
&\quad \left. \frac{\partial \phi_{Ji}}{\partial x_j} U_J^{t+\Delta t} u_j^t \right) dV \\
&= \rho \delta U_I \int_V \frac{1}{\Delta t} \phi_{Ii} \phi_{Ji} dV U_J^{t+\Delta t} + \rho \delta U_I \int_V \frac{\partial u_i^t}{\partial x_j} \phi_{Ii} \phi_{Jj} dV U_J^{t+\Delta t} \\
&\quad + \rho \delta U_I \int_V u_j^t \phi_{Ii} \frac{\partial \phi_{Ji}}{\partial x_j} dV U_J^{t+\Delta t} - \rho \delta U_I \int_V \phi_{Ii} \left(\frac{u_i^t}{\Delta t} + \frac{\partial u_i^t}{\partial x_j} u_j^t \right) dV \\
&= \delta U_I \hat{M}_{IJ} U_J^{t+\Delta t} + \delta U_I K_{IJ}^1 U_J^{t+\Delta t} + \delta U_I K_{IJ}^2 U_J^{t+\Delta t} - \delta U_I F_I \\
&= \delta \mathbf{U}^T \hat{\mathbf{M}} \mathbf{U}^{t+\Delta t} + \delta \mathbf{U}^T \mathbf{K}^1 \mathbf{U}^{t+\Delta t} + \delta \mathbf{U}^T \mathbf{K}^2 \mathbf{U}^{t+\Delta t} - \delta \mathbf{U}^T \mathbf{F}, \\
&= \delta \mathbf{U}^T \hat{\mathbf{M}} \mathbf{U} + \delta \mathbf{U}^T \mathbf{K}^1 \mathbf{U} + \delta \mathbf{U}^T \mathbf{K}^2 \mathbf{U} - \delta \mathbf{U}^T \mathbf{F}.
\end{aligned}$$

$$\begin{aligned}
\delta(\lambda_i^J (u_i^{Jf} - \dot{p}_i^J)) &= \lambda_i^J \delta u_i^J + \delta \lambda_i^J (u_i^J - \dot{p}_i^J) \\
&= \lambda_i^J \phi_{Ii}^J \delta U_I + \delta \lambda_i^J (\phi_{Ii}^J U_I - \dot{p}_i^J) \\
&= \delta \mathbf{U}^T \phi^J \lambda^J + \delta \lambda^{JT} (\phi^{JT} \mathbf{U} - \mathbf{A}^J \dot{\mathbf{p}}^J).
\end{aligned}$$