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Analysis of linear quadratic control loops with decentralized event-triggered sensing: rate and performance guarantees

M. Hadi Balaghi I., Duarte J. Antunes

Abstract— We consider a control loop with several distributed sensor agents. Each sensor measures a subset of the states and broadcasts data to a remote controller and the other sensors according to a threshold-based event-triggered policy. This policy specifies that each sensor transmits data when the norm of the error between the actual measurement and an estimate of this measurement exceeds a threshold. The controller incorporates a state estimator relying on the data received from the sensors and enforces a linear feedback control policy. Considering an average quadratic performance index, we provide: (i) upper bounds on the performance of such a decentralized event-triggered policy with respect to the optimal policy when all the agents transmit at maximum rate; and (ii) lower bounds on the average inter-transmission time for the overall networked control system. Moreover, we discuss the conservativeness of the bounds considering some simple cases. The usefulness of the results is highlighted via a numerical example.

I. INTRODUCTION

With the advent of the internet of things [1] and cyber-physical systems, more and more control systems are distributed with sensors, actuators and controllers communicating over networks. Moreover, interconnectivity is enabling new control applications which are expected to grow at a fast pace. However, a single control system typically operates periodically and requires a fast rate. Therefore, in settings where many control systems are connected to the same (local area) network, one may predict that fast periodic control will not be possible and communication management for control loops will play a key role.

One of the promising directions to tackle this challenge is to abandon the (fast) periodic control paradigm in favor of the event-triggered control paradigm. Conceptually, there is no reason for a control loop to be periodic, other than simplicity of analysis, e.g., the tools of classical control (loop shaping, optimal control) apply directly to periodic control. In fact, if transmissions in a closed-loop are expensive, triggering these only when necessary, e.g., when event occur, seems to be more reasonable. This is the idea behind event-triggered control (ETC) [2]–[8].

Although the literature on ETC is by now quite vast, most papers tackle a single control loop (see, e.g., [2]–[8]). However, the need to manage communication via ETC is more arguable when several agents in one or more control

loops transmit their information to the other agents. In this context, we can either consider settings with several control loops and where the control agents (sensors, actuators and sensors) are independent from the control agents of other loops (and must only share a communication network) - see [9]–[11]-, or settings where the control agents pertain to the same (large but possibly sparse) system and are interconnected - see, e.g., [12]–[17]. In the present work we will be mostly interested in the latter setting.

Most of the works in this setting consider deterministic process models [14]–[17], and there are also a few works considering stochastic process models as we will be considering in this paper [12], [13]. In [12] a system consisting of several interconnected stochastic linear time invariant sub-systems is considered. Assuming that the graph modeling the interconnections between the sub-systems is acyclic, [12] provides a distributed scheduling policy to prioritize the data to be transmitted over the network and establishes f -ergodicity of the overall error propagating in the network. In [13] an upper bound on the average quadratic cost for a decentralized event-triggered output feedback system with wireless communication is provided. In the context of deterministic models, many researchers have considered the multi-agent average consensus by using decentralized event-triggering schemes [15], [17]. For example, [15] proposed a Lyapunov-based decentralized triggering policy and proved that, in this case, events do not exhibit Zeno behaviour and consensus is achieved exponentially fast. There are also some deterministic decentralized event-triggered schedulers for nonlinear systems which guarantee $\mathcal{L}_p$ - gain performance and a lower bound on the inter-event times [14].

In this work we consider a control loop with several distributed sensor agents. The sensor agents measure a subset of the states and can broadcast data to the controller and the other sensors according to a transmission policy and the controllers can enforce a given control policy. Control performance of the system is measured by an average quadratic cost, as in standard LQG control. The goal is to find transmission and control policies to minimize both this average quadratic cost and the average transmission rate (or maximize the average inter-sampling time). This is a multi-objective decentralized decision making problem for which optimal solutions are hard to find due to its decentralized nature [18], [19]. Therefore, rather than attempting to solve this problem optimally, we impose sensor and controller policies, building upon analogous policies for the case where there is a single sensor agent, and analyze the performance and transmission rate of the overall system.

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In this context, we assume that the sensors transmit according to a threshold-based policy defined as follows. Transmissions occur when the norm of the error between the actual measurement and an estimate of this measurement exceeds a threshold. Moreover, the controller incorporates a state estimator based on the data received from the sensors and a linear feedback control policy. We then provide upper bounds on the performance of such a decentralized event-triggered policy with respect to the optimal policy when all the agents transmit at maximum rate and lower bounds on the average inter-transmission time of the overall closed-loop control system. We discuss the conservativeness of the bounds considering some simple cases. The usefulness of the results is highlighted with a numerical example.

The remainder of the paper is organized as follows. Section II provides the problem formulation. Section III starts by considering an event-triggered control loop with only one sensor and then it presents the main results for the decentralized sensor setting. Section IV presents simulation results and Section V provides concluding remarks. The proofs of the results are given in the appendix.

II. PROBLEM FORMULATION

Consider a discrete-time linear system

$$x_{t+1} = Ax_t + Bu_t + w_t, \quad t \in \mathbb{Z}_{\geq 0} \quad (1)$$

where $x_t \in \mathbb{R}^n$ is the state, and $u_t \in \mathbb{R}^m$ is the control input at time step $t \in \mathbb{Z}_{\geq 0}$, and $\{w_t \in \mathbb{R}^n | t \in \mathbb{Z}_{\geq 0}\}$ is a sequence of independent and identically distributed random disturbances with zero mean and covariance $W = \mathbb{E}[w_t w_t^\top]$ for every t . The pair (A, B) is assumed to be controllable.

Performance is measured by a quadratic average cost

$$J = \limsup_{T \rightarrow \infty} \frac{1}{T} \mathbb{E} \left[\sum_{t=0}^{T-1} x_t^\top Q x_t + u_t^\top R u_t \right] \quad (2)$$

where Q and R are positive-definite matrices and R is assumed to be diagonal for convenience.

Suppose that p sensors sample a subset of state components and that all the state components are measured by the sensors. Without loss of generality assume that $x = [(x^1)^\top \ (x^2)^\top \ \dots \ (x^p)^\top]^\top$ where $x^i \in \mathbb{R}^{n_i}$ are the state components measured by sensor $i \in \{1, 2, \dots, p\}$, $B = [B_1^\top \ B_2^\top \ \dots \ B_p^\top]^\top$ and $n = \sum_{i=1}^p n_i$. We define a vector $\sigma_t = (\sigma_t^1, \sigma_t^2, \dots, \sigma_t^p)$ where

$$\sigma_t^i = \begin{cases} 1 & \text{if sensor } i \text{ transmits at time } t \\ 0 & \text{otherwise} \end{cases}$$

If all the sensors would sample and broadcast their state measurements at every time step ($\sigma_t^i = 1$) for every i and t , the optimal control law would be $u_t = Kx_t$ where the control gains are given by

$$K = -(B^\top P B + R)^{-1} B^\top P A, \\ P = A^\top P A + Q - K^\top (B^\top P B + R) K.$$

and P is the unique positive-definite solution to this (Riccati) equation; the control gains can be written according to

the state partition as $K = [K_1 \ K_2 \ \dots \ K_p]$ where $K_i \in \mathbb{R}^{m \times n_i}$. This all-time transmission and control policy would lead to the average cost performance $\text{tr}(PW)$. Let the average transmission rate of each sensor be $\rho_i = \limsup_{T \rightarrow \infty} \frac{1}{T} \mathbb{E}[\sum_{t=0}^T \sigma_t^i]$. Note that the all-time transmission policy would result in an average transmission rate equal to one for all the sensors. Moreover, let

$$\bar{\tau}_i = 1/\rho_i \quad (3)$$

denote the average inter-transmission time of each sensor.

To account for the fact that the sensors do not transmit at every time step we assume that the control policy is instead given by

$$u_t = K \hat{x}_{t|t} \quad (4)$$

where $\hat{x}_{t|t}$ is a state estimate obtained from previous data provided by the sensors. It can be partitioned as the state vector $\hat{x} = [(\hat{x}^1)^\top, (\hat{x}^2)^\top, \dots, (\hat{x}^p)^\top]^\top$ and described by

$$\hat{x}_{t|t-1} = A \hat{x}_{t-1|t-1} + B u_{t-1} \\ \hat{x}_{t|t}^i = \begin{cases} x_t^i, & \text{if } \sigma_t^i = 1 \\ \hat{x}_{t|t-1}^i, & \text{otherwise.} \end{cases} \quad (5)$$

Given a transmission policy for each sensor such that the error $e_t := x_t - \hat{x}_{t|t}$ has the bounded covariance it can be shown from standard optimal control arguments [20, Ch. 8] that the performance is given by

$$J = \text{tr}(PW) + \limsup_{T \rightarrow \infty} \frac{1}{T} \mathbb{E} \left[\sum_{t=0}^{T-1} e_t^\top Y e_t \right] \quad (6)$$

where $Y := K^\top (B^\top P B + R) K$.

This motivates the use of a transmission policy resulting in a small error e_t . To this effect, we define policies where each sensor transmits to the network if a weighted norm of the error between the sensor measurement and the estimate seen by the controller $\bar{e}_t^i := x_t^i - \hat{x}_{t|t-1}^i$ is larger than a threshold, i.e.,

$$\sigma_t^i = \begin{cases} 1, & \text{if } \|\bar{e}_t^i\|_{Z_i} > \gamma_i \\ 0, & \text{otherwise.} \end{cases} \quad (7)$$

where $\|\bar{e}_t^i\|_{Z_i}^2 := (\bar{e}_t^i)^\top Z_i \bar{e}_t^i$ and the Z_i are positive definite matrices. In fact, if $\|\bar{e}_t^i\|_{Z_i} > \gamma_i$ at time t then $e_t^i = 0$, otherwise $e_t^i = \bar{e}_t^i$ and therefore $\|\bar{e}_t^i\|_{Z_i} \leq \gamma_i$ for every $t \in \mathbb{Z}_{\geq 0}$. Note that for the sensors to compute this rule they must also run the same estimator as the controller and therefore also listen to what is sent via the network.

Our goal in this paper is to analyze this decentralized networked control system taking into account that the positive definite matrices Z_i and scalars γ_i are tuning knobs. Note that these tuning knobs should be chosen to minimize the performance cost and maximize the average inter-transmission time of the overall closed-loop (or minimize the average transmission rate). While providing exact analytical expressions for the performance and for the average inter-transmission time is still hard, we aim at providing lower bounds for the average inter-transmission times (as large as possible) and upper bounds for the performance cost

(as small as possible). These bounds provide important indications on how to pick the tuning knobs Z_i and γ_i such that an appropriate trade-off performance vs average inter-transmission time can be achieved.

III. MAIN RESULTS

While we are mainly interested in the case of multiple agents/sensors, we start by providing in Section III-A bounds on performance and average inter-transmission time for the case of a single agent/sensor or centralized transmission. These bounds will build our intuition on how to provide bounds to the more challenging general case of multiple agents/sensors, which is presented in Section III-B. To smoothen the transition between the two sections, we provide a simple example at the end of Section III-A discussing the conservativeness of the bounds.

A. Single agent - centralized transmissions

We start by providing an upper bound on performance. For a given symmetric positive semi-definite matrix M let $\bar{\lambda}(M)$ indicate the largest eigenvalue.

Theorem 1: Consider the linear plant (1) with a single sensor $p = 1$ providing full state feedback $x_t^1 = x_t$ when $\sigma_t^1 = 1$ and controlled by (4), (5), (7) for $p = 1$. Then the performance index (2) is upper bounded by

$$J \leq \text{tr}(PW) + \beta$$

where $\beta = \gamma_1^2 \bar{\lambda}(S)$ and $S = (Z_1^{-1/2})^\top Y Z_1^{-1/2}$.

The proof of this result can be found in the appendix and it uses (6) along with the fact that the error e_t^i is bounded ($\|e_t^i\|_{Z_i} \leq \gamma_i$) at every time step. However, it does not exploit the fact that this error resets to zero when there is a transmission and the probability of being small in the time steps after a transmission is also larger than that for times far from the last transmission time. Therefore, as discussed in Example 1, this bound is in general conservative.

Note that the choice $Z_1 = Y$ would lead to the simple expression for the performance bound constant $\beta = \gamma_1^2$. However, this choice might not lead to a large average inter-transmission time.

We provide next a lower bound on the average inter-transmission time.

Theorem 2: Consider the linear plant (1) with a single sensor $p = 1$ providing full state feedback $x_t^1 = x_t$ when $\sigma_t^1 = 1$ and controlled by (4), (5), (7) for $p = 1$. Then the average inter-transmission time (3) is lower bounded ($\bar{\tau} \geq \xi$) by

$$\xi = \begin{cases} \theta, & \text{if } \alpha = 1 \\ -\log(1 + \frac{\theta}{\eta}) \frac{1}{\log(\alpha)}, & \text{otherwise.} \end{cases}$$

where $\theta = \gamma_1^2 / \text{tr}(Z_1 W)$, $\eta = \alpha / (1 - \alpha)$, and $0 < \alpha \leq 1$ is the maximum value such that for $Z_1 > 0$

$$\alpha A^\top Z_1 A - Z_1 \leq 0. \quad (8)$$

□

In Example 1 below we show that the bound provided in this theorem can be tight.

Based on these two theorems, Z_1 , γ_1 (and α) should be picked taking into account the following remarks:

- There is a clear trade-off in picking γ_1 : a small γ_1 leads to a smaller guaranteed performance (small β) but to a small guaranteed average inter-transmission time (small ξ).
- As the value of α increases, the minimum average inter-transmission time also increases. For stable and critically stable systems we can pick $\alpha = 1$ but for unstable systems we must have $\alpha < 1$.

Example 1: Suppose that $A = B = Q = R = 1$, in which case $P = 2$ and $Y = 1$, and $\{w_t | t \in \mathbb{Z}_{\geq 0}\}$ are uniformly distributed in the interval $[-\delta, \delta]$, $\delta = 1/10$ and $W = 1/300$. Without loss of generality we can assume that $Z_1 = 1$ and γ_1 is a tuning parameter. Then, for $t \in [0, \tau - 1]$ where τ is the first transmission time $\tau = \min\{s \in \mathbb{Z}_{>0} \mid |\bar{e}_s| > \gamma_1\}$. we have $e_{t+1} = e_t + w_t$. From Theorem 1 we conclude that

$$J \leq \text{tr}(PW) + \gamma_1^2 \quad (9)$$

Using a similar argument as in the proof of Theorem 2 we conclude that, for $V(\underline{e}) = \underline{e}^2$ and $\underline{e}_t = \bar{e}_t$ if $t > 0$ and $\underline{e}_0 = 0$,

$$\mathbb{E}[\underline{e}_\tau^2] = \mathbb{E}\left[\sum_{t=0}^{\tau-1} \mathbb{E}[V(\underline{e}_{t+1}) | \underline{e}_t] - V(\underline{e}_t)\right] = \mathbb{E}[\tau]W = \bar{\tau}W$$

Note that $\gamma_1^2 < \underline{e}_\tau^2 \leq \gamma_1^2 + \delta$ from which we conclude that $\gamma_1^2/W < \bar{\tau} \leq (\gamma_1^2 + \delta)/W$. If we assume that γ_1^2 is considerably larger than δ we see that the average inter-transmission time in this special case is very close to the minimum inter-transmission time presented in Theorem 1 when $\alpha = 1$, meaning that this bound can be tight in certain cases. However, the bound on performance (9) is not tight because in the proof of Theorem 1 the error e_t in expression (6) is bounded for every time step, even though right after a transmission the error e_t is small. As a result one can expect a gap between the bound and the actual performance. These remarks can be confirmed by comparing the bounds with the actual average inter-transmission time and performance obtained via Monte-carlo (MC) simulations. This is shown in the two most plots on the left hand side of Figure 1 as a function of the parameter γ_1^2 . Note that the gap between the bound and actual performance is quite large. This prevents concluding, based on the bounds, that the event-triggered policy actually achieves a better trade-off between performance and average inter-transmission time than periodic control as shown in the plot on the right-hand side (in particular it is consistent in the sense of [21]), which can actually be concluded based on MC simulations

B. Multi agents - decentralized transmissions

Considering now the general case with $p > 1$ sensors consider the following description of (1) based on a partition induced by the states measured by the sensors

$$x_{t+1}^i = A_{ii}x_t^i + \sum_{j \in \mathbb{S}_i} A_{ij}x_t^j + B_i u_t + w_t^i \quad (10)$$

Fig. 1: Performance and average inter-transmission time (AITT) plots for a simple example

where $\mathbb{S}_i$ is the set of all subsystems which influence the i th subsystem and $W_i = \mathbb{E}[w_t^i(w_t^i)^\top]$. The following theorem present an upper bound on the performance index. For an arbitrary matrix S let $\bar{\sigma}(S)$ indicate its largest singular value.

Theorem 3: Consider a discrete-time linear plant (1) consisting of p different coupled subsystems (10) with sensors providing x_t^i when $\sigma_t^i = 1$, suppose that the plant is controlled by (4), (5), (7). Then the performance index (2) is upper bounded by

$$J \leq \text{tr}(PW) + \sum_{i=1}^p \sum_{j=1}^p \beta_{ij}$$

where $\beta_{ij} = \bar{\sigma}(S_{ij})\gamma_i\gamma_j$, for $i, j \in \{1, \dots, p\}$, where $S_{ij} = (Z_i^{-1/2})^\top Y_i^\top Y_j Z_j^{-1/2}$ and $Y_i = (B^\top P B + R)^{1/2} K_i$. $\square$

The conservativeness of the performance bounds is asserted via examples in Section IV.

The next theorem provides lower bounds on the average inter-transmission time.

Theorem 4: Consider a discrete-time linear plant (1) consisting of p different coupled sub-systems (10) with sensors providing x_t^i when $\sigma_t^i = 1$ and suppose that the plant is controlled by (4), (5), (7). Then, the average inter-transmission time of each sub-system is lower bounded ($\bar{\tau}_i \geq \xi_i$) by

$$\xi_i = \begin{cases} \theta_i, & \text{if } \alpha_i = 1 \\ -\log(1 + \frac{\theta_i}{\eta_i}) \frac{1}{\log(\alpha_i)}, & \text{otherwise.} \end{cases}$$

where $\theta_i = \gamma_i^2 / (\text{tr}(Z_i W_i) + \bar{\epsilon}_i)$, $\eta_i = \alpha_i / (1 - \alpha_i)$ and

$$\bar{\epsilon}_i = \sum_{j \in \mathbb{S}_i} 2\Psi_{ij}^i + \sum_{j \in \mathbb{S}_i} \sum_{l \in \mathbb{S}_i} \Psi_{jl}^i,$$

$$\Psi_{jl}^i = \bar{\sigma}(G_{jl}^i)\gamma_i\gamma_j, \quad G_{jl}^i = (Z_j^{-1/2})^\top A_{ij}^\top Z_i A_{il} Z_l^{-1/2},$$

in which $0 < \alpha_i \leq 1$ is the maximum value such that $Z_i > 0$ and $\alpha_i A_{ii}^\top Z_i A_{ii} - Z_i \leq 0$. $\square$

Comparing Theorems 4 and 2, it is evident that the difference between the proposed bounds for the centralized and decentralized transmission policies is in the existence of the variables $\bar{\epsilon}_i$. These variables account for the coupling effects of other sub-systems. This plays the role of a disturbance in the proof of Theorem 4 and decreases the proposed lower bound of the average inter-transmission time.

From this discussion we see that the bounds depend on the topology of matrix A . In particular the average inter-transmission time for a given subsystem depends on the cardinality of $\mathbb{S}_i$. To illustrate this, consider a topology of three subsystems depicted in Fig. (2) where each S_i represents a subsystem with one state with the following A matrix.

Fig. 2: Nested system with three subsystems

$$A = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Based on Theorem (4), for the first subsystem $\bar{\epsilon}_1 = 0$ and we can use the results obtained for the single-agent transmission in Theorem 2 to find the minimum inter-transmission time for this subsystem. For the second subsystem

$$\bar{\epsilon}_2 = Z_2 \left(2|a_{22}a_{21}| \frac{\gamma_2\gamma_1}{\sqrt{Z_2 Z_1}} + a_{21}^2 \frac{\gamma_1^2}{Z_1} \right)$$

and for the third one

$$\begin{aligned} \bar{\epsilon}_3 = & Z_3 \left(2|a_{33}a_{31}| \frac{\gamma_3\gamma_1}{\sqrt{Z_3 Z_1}} + 2|a_{33}a_{32}| \frac{\gamma_3\gamma_2}{\sqrt{Z_3 Z_2}} \right. \\ & \left. + 2|a_{31}a_{32}| \frac{\gamma_1\gamma_2}{\sqrt{Z_1 Z_2}} + a_{31}^2 \frac{\gamma_1^2}{Z_1} + a_{32}^2 \frac{\gamma_2^2}{Z_2} \right) \end{aligned}$$

which shows as the amount of interconnections or coupling terms (a_{ij}) increase, the values of $\bar{\epsilon}$ increase which result in a smaller bound for the average inter-transmission time in Theorem 4. It can also be concluded that if subsystems are fully decoupled or the coupling terms are very small ($A_{ij} \approx 0$, $\forall i \neq j$), then for the lower bound of the average inter-transmission time we will arrive at the same result as in Theorem 2.

IV. SIMULATIONS

In this section we consider a system consisting of three states which depend on each other. The parameters of the system and of the cost function are

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.1 & 0.1 \\ 0.08 & -0.1 & 1.1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad W = 0.1I,$$

and $Q = I$, $R = 0.1I$. We assume the states are measured and transmitted according to two different methods explained next.

A. Centralized transmission

In this method, the entire state vector is measured and transmitted by one sensor. The controller of this system is designed based on (4) and (5). In the scheduler design, we set $\text{trace}(Z_1) = 15$ and $\alpha = 0.8196$ is the largest positive number so that (8) is feasible and

$$Z_1 = \begin{bmatrix} 5.4994 & -1.8814 & 1.8932 \\ -1.8814 & 4.7502 & 0.0006 \\ 1.8932 & 0.0006 & 4.7504 \end{bmatrix}.$$

Fig. 3: Performance trade off vs transmission rate, Solid lines: MC simulations, Dashed lines: Proposed bounds. a: single sensor b: ρ_1 c: ρ_2 d: ρ_3

We find bounds for the performance index and transmission rate when $\gamma_1 \in [\sqrt{2}, 10]$ based on Theorems 1 and 2. In Fig. (3, a) we compare this result with MC simulations. It is seen that the real performance of system is properly upper bounded by our proposed guaranteed bounds for the performance index and the transmission rate.

B. Decentralized transmission

In this method, each scalar component of the state is measured by one of three different sensors and transmitted based on (7). The controller remains the same as in the centralized method, but the parameters of the schedulers (α_i, Z_i, γ_i) are selected independently. Assume $\text{trace}(Z_i) = 5, \forall i \in \{1, 2, 3\}$. Based on the matrix A , the first state is marginally stable and the others are unstable and $\alpha_1 = 0.9999, \alpha_2 = \alpha_3 = 0.8266$ are the maximum values which will make $\alpha_i A_{ii}^T Z_i A_{ii} - Z_i \leq 0$ feasible and $Z_i = 5, \forall i \in \{1, 2, 3\}$. In this example we assume $\gamma_1 = \gamma_2 = \gamma_3 \in [\sqrt{2}, 10]$. Fig. (3,b-c) shows the performance index versus transmission rate of all three sensors and compares the MC simulation results with the proposed bounds. Guaranteed bounds in the decentralized case are much more conservative because of the coupling effect ($\bar{e}_i$) that appears explicitly in the transmission rate bound. It is obvious by Fig. (3, b-d) as the coupling effects increase, the upper bound for the transmission rate increases and become more conservative. In this case S_3 depends on the other two subsystems and S_2 depends on S_3 . This will cause an increase in the upper bound of their transmission rates.

V. CONCLUSION

In this paper we analyzed a deterministic threshold based policy for both centralized and decentralized data transmissions. First, an upper bound on the performance index and a lower bound on the average inter-transmission time in the centralized case were determined. The upper bound for the transmission rate depends on the covariance of the disturbance and on the plant's parameters. As the system

becomes unstable, the value of α decreases and the upper bound for the transmission rate increases. An increase in the covariance of the disturbance also increase the transmission rate. In the following, we focus on decentralized transmission strategy and find the upper bounds of the performance index and the transmission rate for each agent. The coupling in the dynamic of subsystems will increase the upper bound of the transmission rate. As this coupling becomes smaller, the bound of transmission rate approaches the one obtained in the centralized case.

APPENDIX

A. Proof of theorem 1

We start by proving that β is the optimal value of the problem

$$\max e^T Y e \quad \text{s.t.} \quad e^T Z_1 e \leq \gamma_1^2.$$

In fact, by defining $r = Z_1^{1/2} e$ the optimization problem can be written as

$$\max r^T S r \quad \text{s.t.} \quad r^T r \leq \gamma_1^2$$

where $S = (Z_1^{-1/2})^T Y Z_1^{-1/2}$. Therefore, $r^T S r \leq \bar{\lambda}(S) r^T r \leq \bar{\lambda}(S) \gamma_1^2 = \beta$ with equality when r is the eigenvector of S associated with $\bar{\lambda}(S)$. Then, for every $t \in \mathbb{N}$, $e_t^T Y e_t \leq \beta$ and the result follows from the expression for the performance index (6).

B. Proof of theorem 2

By construction of the triggering policy (7), the intervals between two consecutive transmissions times are independent and identically distributed. Then $\bar{\tau} = \mathbb{E}[\tau]$ where τ is the first transmission time $\tau = \min\{s \in \mathbb{Z}_{>0} \mid \bar{e}_s^T Z_1 \bar{e}_s > \gamma_1^2\}$. Define the process $\underline{e}_t = \bar{e}_t$ if $t > 0$ and $\underline{e}_0 = 0$ in the interval $\{0, 1, \dots, \tau\}$. Then $\underline{e}_{t+1} = A \underline{e}_t + w_t$. Let $V(\underline{e}_t, t) := \alpha^t \underline{e}_t^T Z_1 \underline{e}_t$. Then

$$\begin{aligned} & \mathbb{E}[V(\underline{e}_{t+1}, t+1) | \underline{e}_t] - V(\underline{e}_t, t) \\ &= \alpha^t \underline{e}_t^T (\alpha A^T Z_1 A - Z_1) \underline{e}_t + \alpha^{t+1} \text{tr}(Z_1 W) \leq \alpha^{t+1} \text{tr}(Z_1 W), \end{aligned}$$

where we used (8) and

$$\begin{aligned} & \mathbb{E}\left[\sum_{t=0}^{\tau-1} \mathbb{E}[V(\underline{e}_{t+1}, t+1) | \underline{e}_t] - V(\underline{e}_t, t)\right] \leq \\ & \mathbb{E}\left[\sum_{t=0}^{\tau-1} \alpha^{t+1} \text{tr}(Z_1 W)\right] = \mathbb{E}[\eta(1 - \alpha^\tau) \text{tr}(Z_1 W)] \leq \\ & \begin{cases} \mathbb{E}[\tau] \text{tr}(Z_1 W), & \text{if } \alpha = 1, \\ \eta(1 - \alpha^{\mathbb{E}[\tau]}) \text{tr}(Z_1 W), & \text{otherwise.} \end{cases} \end{aligned}$$

and the last inequality follows from the Jenssen's inequality for concave functions. Moreover,

$$\begin{aligned} & \mathbb{E}\left[\sum_{t=0}^{\tau-1} \mathbb{E}[V(\underline{e}_{t+1}, t+1) | \underline{e}_t] - V(\underline{e}_t, t)\right] = \mathbb{E}[V(\underline{e}_\tau, \tau)] - \\ & \mathbb{E}[V(\underline{e}_0, 0)] > \gamma_1^2 \mathbb{E}[\alpha^\tau] \geq \begin{cases} \gamma_1^2, & \text{if } \alpha = 1, \\ \gamma_1^2 \alpha^{\mathbb{E}[\tau]}, & \text{otherwise.} \end{cases} \end{aligned}$$

the last inequality follows also from the Jenssen's inequality for convex functions. Using the two inequalities, we obtain $\gamma_1^2 < \mathbb{E}[\tau]\text{tr}(Z_1 W)$ if $\alpha = 1$ and $\gamma_1^2 \alpha^{\mathbb{E}[\tau]} < \eta(1 - \alpha^{\mathbb{E}[\tau]})\text{tr}(Z_1 W)$ otherwise, from which we conclude the result.

C. Proof of theorem 3

Note that

$$e^\top Y e = \|Y_1 e_1 + \dots + Y_p e_p\|^2 = \sum_{i=1}^p \sum_{j=1}^p e_i^\top Y_i^\top Y_j e_j$$

where $Y_i = (B^\top P B + R)^{\frac{1}{2}} K_i$, $i \in \{1, \dots, p\}$. Consider the following optimization problem

$$\beta_{ij} = \max e_i^\top Y_i^\top Y_j e_j \quad \text{s.t.} \quad e_i^\top Z_i e_i \leq \gamma_i^2, \quad \& \quad e_j^\top Z_j e_j \leq \gamma_j^2.$$

Letting $e_i = Z_i^{-1/2} r_i$ and $e_j = Z_j^{-1/2} r_j$, the problem becomes

$$\beta_{ij} = \max r_i^\top S_{ij} r_j \quad \text{s.t.} \quad r_i^\top r_i \leq \gamma_i^2, \quad \& \quad r_j^\top r_j \leq \gamma_j^2,$$

where $S_{ij} = (Z_i^{-1/2})^\top Y_i^\top Y_j Z_j^{-1/2}$. We can conclude $\beta_{ij} = \bar{\sigma}(S_{ij})\gamma_i\gamma_j$ and the proof is complete based on (6) and following a similar reasoning to Theorem 1.

D. Proof of theorem 4

For each sub-system consider the time interval $\{0, 1, \dots, \tau_i\}$ where τ_i is the first transmission time of sensor i . By defining $V_i(\underline{e}_t^i, t) := \alpha_i^t (\underline{e}_t^i)^\top Z_i \underline{e}_t^i$, for each sensor, following the same procedure as in Theorem 2 and using the drift criteria for V_i during the transmission interval, we conclude that

$$\text{tr}(Z_i W_i) + \epsilon_i > \frac{\gamma_i^2 \mathbb{E}[\alpha_i^{\tau_i}]}{\mathbb{E}[\sum_{t=0}^{\tau_i-1} \alpha_i^{t+1}]}$$

where

$$\begin{aligned} \epsilon_i = & -\frac{1}{\mathbb{E}[\sum_{t=0}^{\tau_i-1} \alpha_i^{t+1}]} \mathbb{E}[\sum_{t=0}^{\tau_i-1} \alpha_i^t \epsilon_i(\underline{e}_t^i)^\top \underline{e}_t^i] + \\ & \frac{1}{\mathbb{E}[\sum_{t=0}^{\tau_i-1} \alpha_i^{t+1}]} \mathbb{E}[\sum_{t=0}^{\tau_i-1} \alpha_i^{t+1} \sum_{j \in \mathbb{S}_i} 2(\underline{e}_t^i)^\top A_{ii}^\top Z_i A_{ij} \underline{e}_t^j] + \\ & \frac{1}{\mathbb{E}[\sum_{t=0}^{\tau_i-1} \alpha_i^{t+1}]} \mathbb{E}[\sum_{t=0}^{\tau_i-1} \alpha_i^{t+1} \sum_{j \in \mathbb{S}_i} \sum_{l \in \mathbb{S}_i} (\underline{e}_t^j)^\top A_{ij}^\top Z_i A_{il} \underline{e}_t^l]. \end{aligned}$$

In order to find the minimum value for $\mathbb{E}[\tau_i]$, we calculate the maximum value of ϵ_i . The maximum of the first term of ϵ_i is zero. For finding the maximum of the other two terms we need to find the solution of the following optimization problem

$$\Psi_{jl}^i = \max \underline{e}_j^\top A_{ij}^\top Z_i A_{il} \underline{e}_l \quad \text{s.t.} \quad \underline{e}_j^\top Z_j \underline{e}_j \leq \gamma_j^2, \quad \& \quad \underline{e}_l^\top Z_l \underline{e}_l \leq \gamma_l^2.$$

This problem is solved as in Theorem 3 and $\Psi_{jl}^i = \bar{\sigma}(G_{jl}^i)\gamma_j\gamma_l$ where $G_{jl}^i = (Z_j^{-1/2})^\top A_{ij}^\top Z_i A_{il} Z_l^{-1/2}$. Then

$\bar{\epsilon}_i = \sum_{j \in \mathbb{S}_i} 2\Psi_{ij}^i + \sum_{j \in \mathbb{S}_i} \sum_{l \in \mathbb{S}_i} \Psi_{jl}^i$ and we conclude the following equation from which the result follows.

$$\text{tr}(Z_i W_i) + \bar{\epsilon}_i > \frac{\gamma_i^2 \mathbb{E}[\alpha_i^{\tau_i}]}{\mathbb{E}[\sum_{t=0}^{\tau_i-1} \alpha_i^{t+1}]} \geq \begin{cases} \frac{\gamma_i^2}{\mathbb{E}[\tau_i]}, & \text{if } \alpha_i = 1 \\ \frac{\gamma_i^2 \alpha_i^{\mathbb{E}[\tau_i]}}{\eta_i(1 - \alpha_i^{\mathbb{E}[\tau_i]})}, & \text{otherwise.} \end{cases}$$

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