

A reduced-complexity linear precoding strategy for massive MIMO base stations

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A Reduced-Complexity Linear Precoding Strategy for Massive MIMO Base Stations

Amirashkan Farsaei

Electrical Engineering Department
Eindhoven University of Technology
Eindhoven, The Netherlands
Email: a.farsae@tue.nl

Frans Willems

and Alex Alvarado
Eindhoven University of Technology
Eindhoven, The Netherlands
Email: F.M.J.Willems@tue.nl
Email: A.Alvarado@tue.nl

Ulf Gustavsson

Ericsson
Gothenburg, Sweden
Email: ulf.gustavsson@ericsson.com

Abstract—Conjugate beamforming (CB) and zero-forcing (ZF) are well-known linear precoders, which have optimized hardware implementations. In this work, a linear precoder is proposed based on switching between CB and ZF for line-of-sight propagation environments. The proposed idea is to predict and use the precoder, which results in the highest sum-rate for a given channel. To this end, three regimes are introduced for the ratio of power per user at the base station (BS) over noise power at a user receiver. For low values of this ratio, CB and for high values, ZF result in the highest sum-rate. For moderate values, a precoding strategy is proposed to switch to the best precoder. The switching mechanism is based on an upper bound for the ZF sum-rate, which we introduced in this work. The proposed precoding strategy achieves a sum-rate higher than both CB and ZF. Simulation results for a massive MIMO system including a 100-antenna BS, show up to 8% improvement on the sum-rate compared to CB and ZF. In addition, the proposed precoder reduces computational complexity up to 16.5% compared to the ZF precoding.

I. INTRODUCTION

Linear precoders, e.g., conjugate beamforming (CB) and zero-forcing (ZF) are low-complexity precoders, which can achieve the capacity in a favorable propagation (FP) environment [1], [2]. Therefore, linear precoders are viable candidates for massive MIMO with FP [3]. Designing a linear precoder includes choosing a precoding matrix and a set of power allocation coefficients. By choosing either CB or ZF precoding matrices, one can find the power allocation coefficients by maximizing a utility function [4], which is subject to practical constraints, e.g., the total power available at the BS [5].

Throughput and fairness are two popular utility functions. However, it has been shown that harmonic mean provides a trade-off between maximizing these two, which is desirable in practice [6]. In this work, by modifying the utility function of harmonic mean, closed-form solutions for power allocation coefficients of CB and ZF are derived. This results in reducing the computational complexity compared to maximizing throughput and fairness, which require complex optimization algorithms. Consequently, a utility function based on the modified harmonic mean is viable for CB and ZF in terms of performance and computational complexity.

In a massive MIMO system with an M -antenna BS and K single-antenna users, by having the channel matrix, finding

the ZF precoding matrix has $O(MK^2)$ complexity due to having a matrix inversion and matrix multiplication. However, the CB precoding matrix is available without any processing, since it is equal to the complex conjugate of the channel matrix. Therefore, in terms of complexity, the CB precoding matrix is preferred in massive MIMO BSs. To the best of our knowledge, [7, Section 5.3] and [8] are the only works suggesting switching between CB and ZF to improve the sum-rate. However, no precise method is given, and there is no report on the gain of adaptively using CB and ZF in terms of computational complexity and sum-rate. In our paper, a systematic method is proposed for switching between CB and ZF.

In this paper, three regimes are defined for the ratio of power per user at the BS over noise power at a user receiver, i.e., low, moderate and high value regimes. We show that the CB precoding matrix for low value regime results in the highest sum-rate, whereas ZF results in the highest sum-rate for high value regime. Thus, CB and ZF can be used for low and high value regimes, respectively. For moderate value regime, both CB and ZF do not outperform the other. We propose a precoding strategy that lets the BS adaptively switch between CB and ZF to use the precoder with the highest sum-rate. By switching between CB and ZF, the computational complexity is also reduced compared to a BS that uses ZF. Therefore, the idea of switching between CB and ZF is beneficial in terms of sum-rate and computational complexity. This is the main idea behind the proposed precoding strategy.

The contributions of this work are as follows. First, a reduced-complexity linear precoder is proposed based on switching between CB and ZF. This precoder lets the BS predict and choose the precoder with the highest sum-rate in all regimes. The modified harmonic mean is used as the utility function to derive the power allocation coefficients for CB and ZF. Second, an upper bound is proposed for the ZF sum-rate, which is used in the proposed switching mechanism to compare the CB and ZF sum-rate. To show the effectiveness of the proposed precoding strategy, and following [3], single-cell LOS scenarios are considered for the simulations. The simulation results show up to 8% improvement on the sum-rate and up to

16.5% reduction in computational complexity.

The structure of this paper is as follows. In Section II, the principles of CB and ZF precoding are explained in detail. In Section III the proposed precoding strategy is introduced, and in Section IV the simulation results are presented. Finally, Section V concludes the paper.

The following notation is used throughout the paper. Bold lowercase and uppercase letters denote column vectors and matrices, respectively. Lowercase letters denote scalars. The symbols $|\cdot|$ and $\|\cdot\|$ denote the absolute value and l^2 -norm operators, respectively. The superscript $*$ denotes conjugate. The superscripts T and H denote un-conjugated transpose and conjugated transpose, respectively. The superscript $^{-1}$ stands for the inverse of a matrix. The symbol \mathbb{C} denotes complex numbers. A diagonal matrix with diagonal entries taken from the vector \mathbf{p} is denoted by $\text{diag}(\mathbf{p})$ and \mathbf{I}_K denotes the identity matrix of size $K \times K$. The complex inner product of two vectors \mathbf{a} and \mathbf{b} is denoted by $\langle \mathbf{a}, \mathbf{b} \rangle = \mathbf{a}^H \mathbf{b}$.

II. LINEAR PRECODERS

In this section, we introduce the system model under consideration. Then, the CB and ZF precoding are reviewed.

A. System Model

In this paper, we consider the downlink channel shown in Fig. 1, where an M -antenna BS transmits symbols to K single-antenna users. Let $\mathbf{s} = (s_1, \dots, s_K)^T$ be the vector of symbols to be transmitted to the users. These symbols are assumed to be uncorrelated, zero mean and unit variance. To compensate the channel effects and serve all the users, the BS uses a set of power allocation coefficients $\mathbf{p} = (\sigma_1, \dots, \sigma_K)^T$ and a precoding matrix $\mathbf{U} \in \mathbb{C}^{M \times K}$. The transmitted vector $\mathbf{x} \in \mathbb{C}^{M \times 1}$ can be expressed as:

$$\mathbf{x} = \mathbf{U} \text{diag}(\mathbf{p}) \mathbf{s}. \quad (1)$$

The precoding matrix is defined as $\mathbf{U} = (\mathbf{u}_1, \dots, \mathbf{u}_K)$, which satisfies $\|\mathbf{u}_i\| = 1$ for $i = 1, \dots, K$. The power allocation coefficients are positive real-values. We assume a total power constraint at the BS, i.e., $\|\mathbf{p}\|^2 \leq P_{\text{tot}}$, where P_{tot} is the maximum available power at the BS.

The BS transmits \mathbf{x} through the channel, which is then received by the users. The downlink channel is modeled as:

$$\mathbf{y} = \mathbf{H} \mathbf{x} + \mathbf{n}, \quad (2)$$

where $\mathbf{y} \in \mathbb{C}^{K \times 1}$ is the received signal vector of the users, $\mathbf{n} \in \mathbb{C}^{K \times 1}$ is the noise vector and $\mathbf{H} = (\mathbf{h}_1, \dots, \mathbf{h}_K)^T \in \mathbb{C}^{K \times M}$ is the downlink channel matrix. The noise components are modeled as independent, zero mean circularly-symmetric complex Gaussian random variables with covariance matrix of $\sigma_n^2 \mathbf{I}_K$. The \mathbf{h}_i is the channel vector from the BS antennas to the user i . The i th received signal is given by:

$$y_i = \mathbf{h}_i^T \mathbf{x} + n_i = \langle \mathbf{h}_i^*, \mathbf{x} \rangle + n_i, \quad (3)$$

which is shown in Fig. 1.

For notation simplicity, we use:

$$\mathbf{h}_i = c_i \tilde{\mathbf{h}}_i, \quad c_i = \|\mathbf{h}_i\|, \quad i = 1, \dots, K. \quad (4)$$

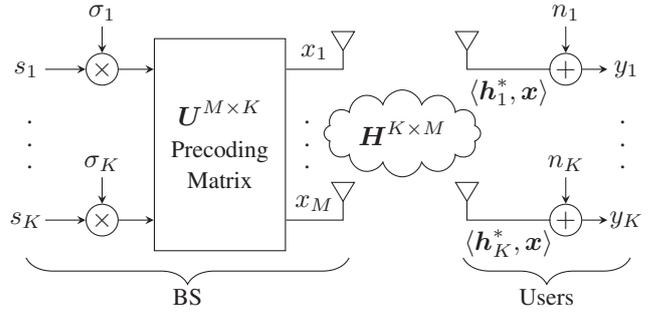


Fig. 1. General structure of downlink with an M -antenna BS serving K single-antenna users.

Thus, $\tilde{\mathbf{h}}_i$ is the unit norm channel vector for the user i . The received signal for the i th user can be further simplified (after some simple algebra) as:

$$y_i = c_i \sigma_i \langle \tilde{\mathbf{h}}_i^*, \mathbf{u}_i \rangle s_i + \sum_{j \neq i} c_j \sigma_j \langle \tilde{\mathbf{h}}_i^*, \mathbf{u}_j \rangle s_j + n_i, \quad (5)$$

where the first term is the desired signal, the second term is the interference from other users, and the last term is noise.

For a given channel, the signal to noise plus interference ratio (SINR) for the user i is defined based on (5) as:

$$\text{SINR}_i = \frac{c_i^2 \sigma_i^2 |\langle \tilde{\mathbf{h}}_i^*, \mathbf{u}_i \rangle|^2}{\sum_{j \neq i} c_j^2 \sigma_j^2 |\langle \tilde{\mathbf{h}}_i^*, \mathbf{u}_j \rangle|^2 + \sigma_n^2}. \quad (6)$$

We define the ratio of the radiated power per user at the BS over the noise power at a user receiver as:

$$\eta = \frac{P_{\text{tot}}/K}{\sigma_n^2}. \quad (7)$$

It is worth to mention that $K\eta$ is equivalent to ρ_d in [3]. The η has to compensate the propagation loss in order to satisfy throughput requirements. For a fixed precoding matrix, we use the power allocation coefficients, which maximize the harmonic mean [6] as:

$$\mathbf{p} = \underset{\mathbf{p}: \|\mathbf{p}\|^2 \leq P_{\text{tot}}}{\text{argmax}} \left(\sum_{i=1}^K \frac{1}{\text{SINR}_i} \right)^{-1}, \quad (8)$$

where SINR_i is given by (6). In this paper, the precoding matrix is assumed to be either CB or ZF, which are briefly reviewed in the following sections.

B. Conjugate Beamforming

The CB precoding matrix $\mathbf{U}^{\text{CB}} = (\mathbf{u}_1^{\text{CB}}, \dots, \mathbf{u}_K^{\text{CB}})$ maximizes the inner product of the user component in (5). The i th column of CB precoding matrix is obtained by:

$$\mathbf{u}_i^{\text{CB}} = \underset{\mathbf{u}_i: \|\mathbf{u}_i\|=1}{\text{argmax}} |\langle \tilde{\mathbf{h}}_i^*, \mathbf{u}_i \rangle|^2 = \tilde{\mathbf{h}}_i^*. \quad (9)$$

This leads to $\mathbf{U}^{\text{CB}} = (\tilde{\mathbf{h}}_1^*, \dots, \tilde{\mathbf{h}}_K^*)$. To derive the power allocation coefficients of CB, (8) can be solved by using water-filling [6], which is computationally expensive. Due to the

fact that the channel vectors are nearly orthogonal in massive MIMO systems [1], we propose to solve:

$$\mathbf{p}^{\text{CB}} = \underset{\mathbf{p}: \|\mathbf{p}\|^2 \leq P_{\text{tot}}}{\text{argmax}} \left(\sum_{i=1}^K \frac{\sigma_n^2}{c_i^2 \sigma_i^2} \right)^{-1}, \quad (10)$$

where $\frac{c_i^2 \sigma_i^2}{\sigma_n^2}$ is the received SNR of each user, which follows from the solution of (9). A closed-form solution for (10) is found by using the Lagrangian multiplier as:

$$\sigma_i^{\text{CB}} = \sqrt{\frac{P_{\text{tot}}}{\sum_{j=1}^K \frac{c_i}{c_j}}}. \quad (11)$$

This solution suggests that users with a high c_i should receive less power, whereas users with a low c_i should receive more power. Our proposed power allocation coefficients in (10) and (11) result in the following SINR for the i th user:

$$\text{SINR}_i^{\text{CB}} = \frac{1}{\sum_{j \neq i} \frac{c_i}{c_j} \rho_{ij}^2 + \frac{1}{K\eta} \sum_{j=1}^K \frac{1}{c_i c_j}}, \quad (12)$$

where $\rho_{ij} = |\langle \tilde{\mathbf{h}}_i^*, \tilde{\mathbf{h}}_j^* \rangle|$.

C. Zero-Forcing

The ZF precoding matrix removes the multi-user interference for all the users in (5), while it maximizes the desired signal. The i th column of ZF precoding matrix is found by:

$$\mathbf{u}_i^{\text{ZF}} = \underset{\substack{\mathbf{u}_i: \|\mathbf{u}_i\|=1, \\ \langle \tilde{\mathbf{h}}_j^*, \mathbf{u}_i \rangle = 0, \quad j \neq i}}{\text{argmax}} |\langle \tilde{\mathbf{h}}_i^*, \mathbf{u}_i \rangle|^2. \quad (13)$$

The resulting $\mathbf{U}^{\text{ZF}} = (\mathbf{u}_1^{\text{ZF}}, \dots, \mathbf{u}_K^{\text{ZF}})$ given by (13), can be found by normalizing the columns of pseudo-inverse of the channel [5] to have unit norm. Unlike CB, ZF removes the multi-user interference, and thus, (8) is solved by using the Lagrangian multiplier as:

$$\sigma_i^{\text{ZF}} = \sqrt{\frac{P_{\text{tot}}}{\sum_{j=1}^K \frac{c_i \gamma_i}{c_j \gamma_j}}}, \quad (14)$$

where $\gamma_i = |\langle \tilde{\mathbf{h}}_i^*, \mathbf{u}_i^{\text{ZF}} \rangle|$ and $\gamma_j = |\langle \tilde{\mathbf{h}}_j^*, \mathbf{u}_j^{\text{ZF}} \rangle|$. This results in the following SINR for the user i :

$$\text{SINR}_i^{\text{ZF}} = K\eta \frac{c_i \gamma_i}{\sum_{j=1}^K \frac{1}{c_j \gamma_j}}. \quad (15)$$

III. PROPOSED PRECODING STRATEGY

The proposed precoding strategy lets the BS choose the precoder that results in the highest sum-rate. The selection is based on η and a proposed upper bound for the ZF sum-rate. In this section, we first find two thresholds for η , which determine the best precoder for the low and high η regimes. Then, we derive an upper bound for the ZF sum-rate, which is later used in the switching mechanism of the proposed precoder.

A. Evaluation Of Precoders

The downlink sum-rate R of a linear precoder for a given channel matrix in bits/second/Hz, can be evaluated as [9, chapter 10], [10]:

$$R = \sum_{i=1}^K \log_2(1 + \text{SINR}_i), \quad (16)$$

where SINR_i is given by (6). We replace (12) and (15) in (16) to obtain the CB and ZF sum-rates as:

$$R^{\text{CB}} = \sum_{i=1}^K \log_2 \left(1 + \frac{1}{\sum_{j \neq i} \frac{c_i}{c_j} \rho_{ij}^2 + \frac{1}{K\eta} \sum_{j=1}^K \frac{1}{c_i c_j}} \right), \quad (17)$$

$$R^{\text{ZF}} = \sum_{i=1}^K \log_2 \left(1 + K\eta \frac{c_i \gamma_i}{\sum_{j=1}^K \frac{1}{c_j \gamma_j}} \right). \quad (18)$$

We compare the SINR_i formulas of these two precoders to find a condition in which CB results in a higher rate for all the users compared to ZF. This is stricter than only having a higher sum-rate. In fact, this condition guarantees that CB has a higher sum-rate. The same condition is found for ZF. The following theorem explains these conditions.

Theorem 1. For a given channel realization, when $\eta < \eta^{\text{CB}}$, CB, and when $\eta > \eta^{\text{ZF}}$, ZF result in a higher rate for all the users, where:

$$\begin{aligned} \eta^{\text{CB}} &= \min_{i=1, \dots, K} a_i, \\ \eta^{\text{ZF}} &= \max_{i=1, \dots, K} a_i, \end{aligned} \quad (19)$$

and a_i is given by:

$$a_i = \frac{1}{K c_i \gamma_i} \left(\frac{1}{\sum_{j \neq i} \frac{c_i}{c_j} \rho_{ij}^2} \right) \sum_{j=1}^K \frac{1}{c_j} \left(\frac{1}{\gamma_j} - \gamma_i \right). \quad (20)$$

Proof. See Appendix A. \square

The interpretation of Theorem 1 is that there are three regimes for η , i.e., low, moderate and high, which are graphically shown in Fig. 2. It is concluded from Theorem 1 that for low and high η regimes, CB and ZF result in the highest sum-rate, respectively. For moderate η regime, more analysis is required to find the precoder with the highest sum-rate. This analysis is presented in Section III-C.

The η^{CB} and η^{ZF} are functions of the channel and thus, are random variables. We study the probability density function (PDF) of η^{CB} and η^{ZF} for a large number of channel realizations with a fixed number of users. The PDF plots are used to find two thresholds T_{min} and T_{max} , which define the low and high η regimes, respectively. We define a point on the right tail of η^{CB} as T_{max} . Above T_{max} , the probability that CB results in a higher data rate for all the users is 0.5%. The choice of 0.5% and not 0% is to exclude rare scenarios. Similarly, we define a point on the left tail of η^{ZF} as T_{min} . More details are given in Section IV on how the low and high η regimes can be realized for a fixed or variable number of users.

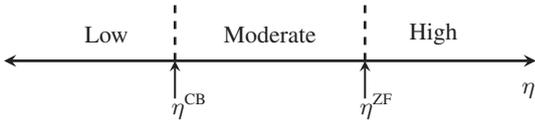


Fig. 2. The introduced regimes for η .

B. Correlation Between Channel Vectors

By having the correlation between the channel vectors, i.e., $\rho_{ij}, j \neq i, i, j = 1, \dots, K$, we can calculate the CB sum-rate based on (17). The following Lemma shows that the γ_i can be bounded, and thus, the ZF sum-rate can be bounded as well.

Lemma 2. *The value of γ_i is bounded by:*

$$\gamma_i \leq \sqrt{1 - \operatorname{argmax}_{j:j \neq i} \rho_{ij}^2} \quad i, j = 1, \dots, K. \quad (21)$$

Proof. See Appendix B. \square

Lemma 2 states that two users with large ρ_{ij} , have very small γ , which implies that the SINR^{ZF} of those users are very small (see (15)). The following Corollary gives an upper bound for the ZF sum-rate. Its proof follows from using the right-hand side of (21) in (18).

Corollary 3. *The ZF sum-rate is upper bounded by:*

$$R^{\text{ZF}} \leq \sum_{i=1}^K \log_2 \left(1 + K\eta \frac{c_i \gamma_i^U}{\sum_{j=1}^K \frac{1}{c_j \gamma_j^U}} \right), \quad (22)$$

$$\text{where } \gamma_i^U = \sqrt{1 - \operatorname{argmax}_{j:j \neq i} \rho_{ij}^2}.$$

The difference between the proposed bound and the ZF sum-rate is very small when the BS serves a low number of users. The accuracy of the proposed bound is studied in Section IV. The proposed upper bound is used in the proposed precoding strategy to find the best precoder in moderate η regime.

C. The Proposed Precoder

The proposed precoder is illustrated in Fig. 3. First, given the possible number of users, the PDF plots of η^{CB} and η^{ZF} are used to find T_{\min} and T_{\max} , or equivalently realize the low and high η regimes. In practice, this can be done either by running a periodic set of measurements or an offline set of measurements. In low η regime, CB and in high η regime, ZF are chosen as explained in Section III-A. In moderate η regime, a switching mechanism is proposed, which is based on the proposed upper bound for the ZF sum-rate. By measuring the channel, all the ρ_{ij} and c_i are found. These values are used to calculate the CB sum-rate based on (17) and the upper bound for the ZF sum-rate based on Theorem 2. We use the proposed bound rather than (18) to avoid a matrix inversion and multiplication. We use R_{\max}^{ZF} to denote the right-hand side of (22). Whenever R^{CB} exceeds R_{\max}^{ZF} , CB definitely results in the highest sum-rate. Thus, in these cases, the BS switches to CB. Otherwise, the BS uses ZF, due to the fact the proposed

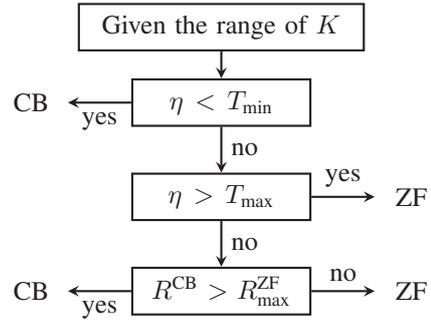


Fig. 3. Illustration of the proposed precoding strategy, given the possible range of K .

upper bound is quite close to the actual ZF sum-rate. In the next section, the results of applying the proposed precoding strategy for a massive MIMO system are presented.

IV. SIMULATIONS

In this section, two examples are given to show the effectiveness of the proposed precoder in moderate η regime. The first example shows how the proposed precoder performs for a fixed number of users. In the second example, the performance of the proposed precoder is evaluated when the number of users changes from $K = 5$ to $K = 25$ for a given η . A massive MIMO system including a linear array with 100 antennas with half-wavelength spacing at the BS is assumed that serves K single-antenna users in a single-cell LOS channel. The channel from the user u to the antenna v at the BS is modeled as $h_{uv} = \sqrt{\beta} e^{-jkR_{uv}}$, where k is the wave number, R_{uv} is the distance from the user u to the antenna v , and β is the average path loss given by the COST-WI model. The following assumptions are assumed:

- The carrier frequency = 1.9 GHz.
- The bandwidth = 20 MHz.
- The BS antenna gain = 0 dBi.
- The mobile antenna gain = 0 dBi.
- The mobile receiver noise figure = 9 dB.
- The users are uniformly distributed in a 120-degree sector, which are 20 m to 2 km far from the BS.
- The minimum distance between the users = 1 cm.

The PDF plots of η^{CB} and η^{ZF} are used to find T_{\min} and T_{\max} . In Fig. 4, the PDF plots are shown for $K = 5$ and $K = 25$. The moderate η regime for $K = 5$ is found as $(T_{\min}, T_{\max}) = (91 \text{ dB}, 111 \text{ dB})$ and for $K = 25$ is found as $(T_{\min}, T_{\max}) = (101 \text{ dB}, 114 \text{ dB})$.

In moderate η , the proposed upper bound is used to find the best precoder as explained in Section III-C. The proposed upper bound and actual ZF sum-rate are shown in Fig. 5 for a different number of users in a wide range of η . For a low number of users ($K = 5$), the difference between the proposed bound and actual ZF sum-rate are negligible. However, by increasing the number of users to $K = 20$ the difference is increased. The proposed upper bound can be used in scenarios with a small number of users relative to the number of BS antennas ($M \gg K$),

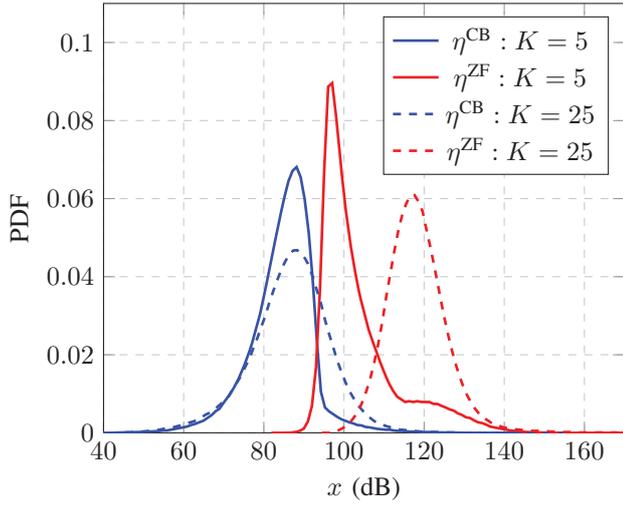


Fig. 4. The PDF plots of η^{CB} and η^{ZF} for 10^6 random realizations when $K = 5$ and $K = 25$.

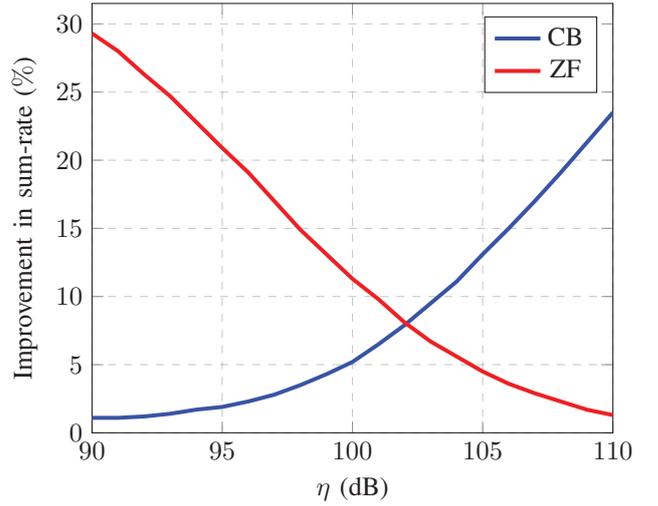


Fig. 6. The improvement of the proposed precoding strategy for $K = 20$ compared to CB and ZF.

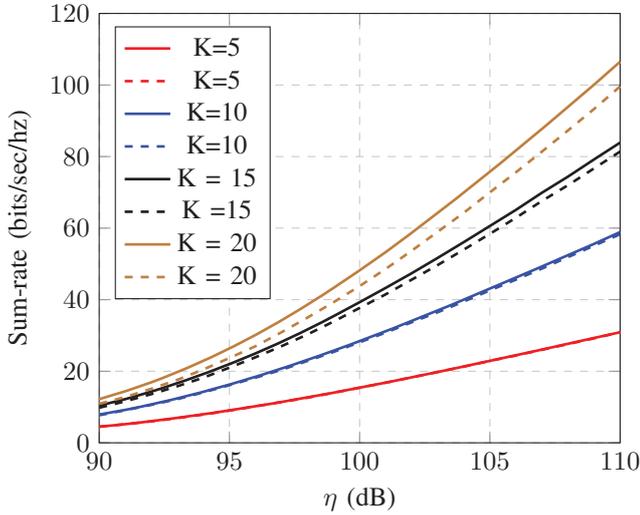


Fig. 5. The proposed upper bound compared to the ZF sum-rate of (18) for 10^5 random realizations, for a different number of users. The solid lines are the upper bound and the dashed lines are the achieved sum-rate.

which is the case for massive MIMO systems. The sum-rate improvement of the proposed precoder is presented with two examples in the next sections.

A. Example 1

In this example, the BS serves $K = 20$ users. The moderate η regime is realized by studying the PDF plots of η as $(T_{\min}, T_{\max}) = (98 \text{ dB}, 111.2 \text{ dB})$. Then, the proposed switching mechanism is used to find the best precoder. The effectiveness of the proposed precoder is shown in Fig. 6 for a wide range of η , which shows that the proposed precoder improves the sum-rate of CB and ZF. Specifically, the intersection point in Fig. 6 shows 7.5% improvement in sum-rate compared to both CB and ZF.

B. Example 2

In this example, the number of served users is changed from $K = 5$ to $K = 25$. The moderate η regime is found by considering the PDF plots of η^{CB} and η^{ZF} for all the values of $K = 5$ to $K = 25$. The common moderate η regime is found as $(T_{\min}, T_{\max}) = (101 \text{ dB}, 111 \text{ dB})$. The proposed precoder is used in this regime to use the best precoder. The results are shown in Fig. 7 for $\eta = 102.5 \text{ dB}$. The proposed precoder improves the sum-rate up to 8% compared to both CB and ZF (the intersection point). Moreover, the proposed precoder reduces the computational complexity up to 16.5% compared to the case, where a BS uses ZF all the time. This is due to the switching to CB. The proposed precoder has a complexity of $O(MK^2/2)$ when CB is chosen. However, it does not change the order of complexity when ZF is chosen due to the fact that its main processing is a part of the ZF precoder. Therefore, it reduces the total complexity compared to a ZF precoder. The complexity reduction while improving the sum-rate is of great importance for the massive MIMO systems, which is achieved by the proposed precoder.

V. CONCLUSION

In this paper, a linear precoder is proposed based on switching between CB and ZF. The proposed idea is to use the precoder with the highest sum-rate for a given channel realization. An upper bound for the ZF sum-rate is proposed, which is used in the proposed precoding strategy. The proposed precoder improves the CB and ZF sum-rate for moderate η regime, while it reduces the computational complexity. Simulation results show up to 8% improvement on the sum-rate and up to 16.5% reduction in the computational complexity. For future works, we consider applying the proposed idea for millimeter wave massive MIMO systems.

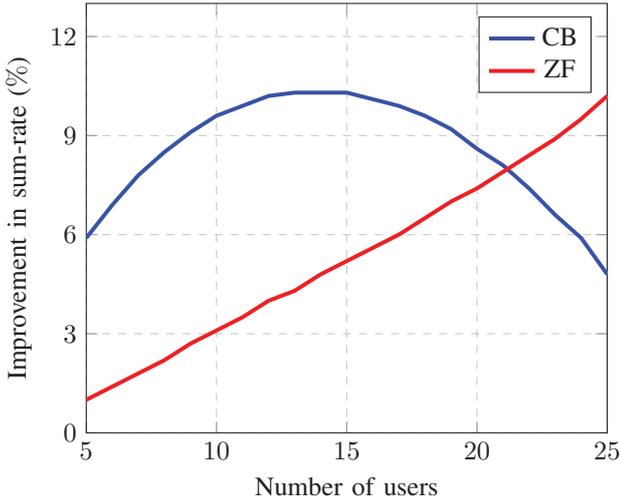


Fig. 7. The improvement of the proposed precoding strategy in $\eta = 102.5$ dB compared to CB and ZF when the number of users changes from 5 to 25.

VI. ACKNOWLEDGMENT

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APPENDIX A

PROOF OF THEOREM 1

To prove (19), we find the condition in which CB results in a higher rate for all the users. This clearly guarantees the CB sum-rate is higher than the ZF sum-rate. For i th user, the condition $\text{SINR}_i^{\text{ZF}} < \text{SINR}_i^{\text{CB}}$ is expressed as:

$$K\eta \frac{c_i \gamma_i}{\sum_{j=1}^K \frac{1}{c_j \gamma_j}} < \frac{1}{\sum_{j \neq i} \frac{c_i}{c_j} \rho_{ij}^2 + \frac{1}{K\eta} \sum_{j=1}^K \frac{1}{c_j \gamma_j}}. \quad (23)$$

Equation (23) is simplified as:

$$K\eta c_i \gamma_i \sum_{j \neq i} \frac{c_i}{c_j} \rho_{ij}^2 + \gamma_i \sum_{j=1}^K \frac{1}{c_j} < \sum_{j=1}^K \frac{1}{c_j \gamma_j}. \quad (24)$$

Finally, the following condition is found for each user:

$$\eta < \frac{1}{K c_i \gamma_i} \left(\frac{1}{\sum_{j \neq i} \frac{c_i}{c_j} \rho_{ij}^2} \right) \sum_{j=1}^K \frac{1}{c_j} \left(\frac{1}{\gamma_j} - \gamma_i \right). \quad (25)$$

By finding the minimum value of the right-hand side of (25) over all the users, η^{CB} is found. The η^{ZF} is found similarly by finding the maximum.

APPENDIX B

PROOF OF LEMMA 2

The orthogonal projection of $\tilde{\mathbf{h}}_i^*$ onto the subspace of other users' channel vectors, i.e., V , results in the vector \mathbf{a}_i . The γ_i is the distance from $\tilde{\mathbf{h}}_i^*$ to V , which is shown in Fig. 8. For a given vector, the orthogonal projection to a sub-space has the minimum distance, compared to the other non-orthogonal projections. Consequently, γ_i has the minimum length, or

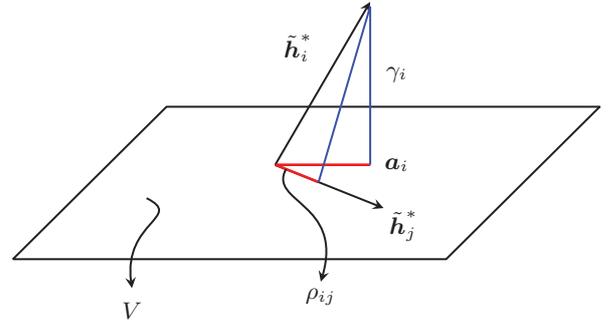


Fig. 8. The illustration of why \mathbf{a}_i has the maximum norm among all the other projections. The blue vectors are orthogonal lines from $\tilde{\mathbf{h}}_i^*$ to the directions of red vectors on V subspace.

equivalently \mathbf{a}_i has the largest length among the other projections of $\tilde{\mathbf{h}}_i^*$ on V . Thus, for any unit vector $\tilde{\mathbf{h}}_j^*$, $j \neq i$ in V , the following holds:

$$\|\mathbf{a}_i\|^2 \geq |\langle \tilde{\mathbf{h}}_i^*, \tilde{\mathbf{h}}_j^* \rangle|^2 \Rightarrow \|\mathbf{a}_i\|^2 \geq \rho_{ij}^2. \quad (26)$$

Thus, we can conclude that:

$$\|\mathbf{a}_i\|^2 \geq \operatorname{argmax}_{j:j \neq i} \rho_{ij}^2 \quad i, j = 1, \dots, K. \quad (27)$$

Therefore, considering $\|\mathbf{a}_i\|^2 = 1 - \gamma_i^2$ and (27), γ_i is bounded by:

$$\gamma_i \leq \sqrt{1 - \operatorname{argmax}_{j:j \neq i} \rho_{ij}^2} \quad i, j = 1, \dots, K, \quad (28)$$

which concludes the proof.

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