Representativeness of compressed flange behaviour for trapezoidal steel sheeting under combined web crippling and bending

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REPRESENTATIVENESS OF COMPRESSED FLANGE BEHAVIOUR FOR TRAPEZOIDAL STEEL SHEETING UNDER COMBINED WEB CRIPPLING AND BENDING

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Abstract. An existing theoretical model indicates that the behaviour of trapezoidal steel sheeting under combined web crippling and bending moment may be modelled by only the sheeting’s compressed flange. Therefore, this paper presents finite element models that simulate only the compressed flange of the sheeting, applying a prescribed displacement field along the flange longitudinal edges to simulate the loading of the bending moment. Seven different and increasingly complex types of these finite element models of a flange are compared with verified finite element models of sheeting for (a) first yield load; (b) ultimate load; (c) yield line patterns; and (d) out-of-plane deformation patterns. It is concluded that a bottom flange only, or a flange with a rounded bottom corner and load bearing plate do not predict (a) and (b) for sheeting correctly. A moving load bearing plate improves to some extent the prediction of (d) and in combination with an imposed curvature on the flange also (a). Future work should investigate the introduction of large, web-crippling deformation sized imperfections in the flange models, also for flanges representing second-generation sheeting.

1 INTRODUCTION

Trapezoidal steel sheeting is a popular product in the built environment. It is light, strong, durable, formable, and -if well designed- it may be aesthetically pleasing as well. Three generations exist: First-generation sheeting that has plain webs and flanges. Second-generation sheeting shows longitudinal stiffeners in webs, in flanges, or in both. Finally, third-generation sheeting is characterized by longitudinal and transverse stiffeners, the latter often in the webs. Regardless whether applied for the construction of claddings, roofs, or floors, sheeting is often applied as a multi-span system, for which then an intermediate support results in a local reaction force —possibly leading to web crippling, which is a reduction of the section height due to buckling and plastic yield lines— additional to bending due to distributed loading.

For safe application, experiments [1-12], numerical simulations [5,8,9,12-21], and theoretical models [6,7,10,19,22-31] have been developed, and many of these efforts have contributed to the state of current design standards. Both the European [32] and the American standards [33] provide as a standard approach: (a) the prediction of the web-crippling only strength, assuming a neglectable small bending moment, via curve-fitting of experimental results; (b) the prediction of the bending only strength using the effective-width method for the local buckling involved; (c) a standard formula for interaction strength of web crippling and bending, again based on curve-fitting of experiments. Alternatively, to predict the
bending only strength, the American standard allows for the application of the Direct Strength Method. Research is also underway to consider the Direct Strength Method for web crippling predictions, e.g. [34,35]. To overcome the empirical basis of the design standards, which lack insight in the failure modes and may yield significant different predictions, a theoretical model for first generation trapezoidal steel sheeting under combined web crippling and bending has been developed [8]. It considers a square plate that is representative for a square part of the sheeting’s compressed flange at one side of the load bearing plate. This plate is first given an imperfection shape and size equal to its deformation that is normally caused by the concentrated load. Then it uses Marguerre's equations to predict the failure of this plate due to normal stresses, which are normally caused by the bending moment. As the Marguerre's equations are complex, for these equations an alternative so-called two-strip model has been developed. The two-strip model requires finite element simulations of the square plate for its set-up and as such performs well [36]. However, it comes out that for commonly used second-generation sheeting, the required finite element simulations (for the two-strip model) need to model the complete compressed flange along the length of the section, instead of just a square plate, because otherwise boundary conditions cannot be applied correctly [37].

In conclusion, the theoretical model indicates that only a square part of the compressed flange is needed to predict the failure of first-generation sheeting successfully. Nevertheless, if this theoretical model is developed further for second-generation sheeting, it occurs that the required finite element models (for setting up the two-strip model) need to model the complete compressed flange. Several questions can now be asked: (a) Will the complete compressed flange predict failure of second-generation sheeting successfully, just like the square part for first-generation sheeting? If positive, this could lead to new accurate design rules for second-generation sheeting, which also provide insight in the failure behaviour. (b) If a square plate predicts failure of first-generation sheeting correctly, will it also predict correctly first yield, yield-line patterns, and out-of-plane deformations in the sheeting's compressed flange? This could improve the understanding of the already complex behaviour of first-generation sheeting under combined web crippling and bending moment. (c) Or do we need to model the complete compressed flange for this, even for first-generation sheeting? If the complete compressed flange would be needed, it is applicable for second-generation sheeting too, and it may also be used to study why a symmetric or asymmetric (along the length) failure mode can occur in sheeting. This is not possible using only a square plate of the compressed flange, for this is located at one side of the load-bearing plate and so is inevitably asymmetric from the beginning. This is the reason that in this paper question (b) is skipped and immediately question (c) will be addressed. Future work can then focus on question (a) subsequently.

Thus, the question to be answered by this paper is: Can a model of only the complete compressed flange predict first yield, failure, yield-line patterns, and out-of-plane deformations in the compressed flange of complete sheeting. Therefore, a systematic comparison between the behaviour of only compressed flanges and the behaviour of the accompanying trapezoidal sheeting is carried out. As the finite element method will be used throughout, and the finite element models of the flange are a part of the models for the full sheeting, first the general setup of the finite element models will be presented in section 2. Three fundamental failure modes exist for trapezoidal sheeting under web crippling and bending, and the finite element model is consequently validated and verified for each failure mode in section 3. The following section 4 presents in detail the several types of compressed flange models and their results, also in relation with the previously presented full models. Section 5 briefly considers some additional simulations of second-generation sheeting, to understand the behaviour of stiffened bottom flanges. Finally, section 6 concludes the work and suggests future research to be carried out.
2 FINITE ELEMENT MODEL PRINCIPLES

To investigate the behaviour of trapezoidal sheeting, which has normally a very wide cross-section and is applied continuously along multiple spans, experiments are commonly based on a sheet-section, and use a three-point bending test setup. A sheet-section is similar to a well-known hat-section, except that boundary conditions are applied such that it behaves equally to an imaginary hat-section part within the sheeting. In experiments, these boundary conditions are often realised by steel strips—placed at regular distances $l_s$ along the length—that connect the two half flanges, thus avoiding spreading of the webs. With respect to the three-point bending test setup, the experimental span length is determined to obtain realistic ratios between the concentrated load and the bending moment as occurring for the multi-span situations in practice. Here the finite element model for trapezoidal sheeting follows the same modelling approach, see figure 1. The sheet-section is supported at the left and right by support strips, and is loaded at midspan via a load bearing plate, which would be the intermediate support for the multi-span situation. Depending on the failure mode, only a quarter (grey areas) or a half sheet-section (grey + lightly hatched areas) is modelled, through boundary conditions at the longitudinal and (for a quarter model only) cross-sectional symmetry lines. The dotted vertical lines, equally distributed along the length and their number given by $l_s < 250$ mm, indicate the steel strips. The cross-sectional views present variables of some basic dimensions; the longitudinal view shows the mesh density regions.

The finite element model was developed based on experience gained in previous research, e.g. [8,37]. For the top flange elements throughout, and for the bottom flange and web for the coarse mesh, $22 \times 22$ mm four-node (each 6 DOF’s) double curved shell S4 [38] elements are present. The fine part of the mesh, the bottom flange and web, is meshed $6 \times 6$ mm. The rounded top corner has a single element along the circumference, being 8 mm long along the length. This is also the case for the rounded bottom corner for the coarse mesh part, whereas for the fine mesh 4 elements are used, being 3 mm in length direction. For the transition mesh, intermediate values and a free meshing algorithm are used. For the coarse and fine mesh, a regular mesh is generated. The load bearing plate is a single rigid solid with between the plate and sheet-section "surface-to-surface" contact with "finite sliding", with the tangential behaviour set to "frictionless"; normal behaviour set to "hard"; and the constraint enforcement method being the default "kinematic" [38]. Boundary conditions are for nodes along the cross-section: at the longitudinal symmetry line $UX=RY=RZ=0$ and at the cross-sectional

Figure 1: Finite element model setup.
symmetry line $UZ=RX=RY=0$; and for nodes on the top flange: at the dotted support line $UX=UY=RY=RZ=0$, at the dotted steel strip lines $UX=0$, and at the cross-sectional symmetry line, simulating the deformation strip, $UX=RZ=0$. Material properties are given in real strains and stresses, following coupon tests [8,39].

An experiment from the past (nr. 25 in [8]) has been used to study the model. The span length of this experiment equals 1200 mm, the plate thickness (without zinc layer) equals 0.68 mm, the yield stress is 355.9 N/mm², and the cross-sectional dimensions as shown in figure 1 equal: $b_{tf}=136.6$ mm, $b_w=100.1$ mm, $b_{bf}=40.5$ mm, $\theta_w=49.9^\circ$, and $r_{bf}=r_{bf}=2.6$ mm. As this experiment failed by the symmetric so-called yield-arc failure mode (to be discussed in the next section), a quarter finite element model was used. Furthermore, a static simulation was carried out. Figure 2 on the left shows load versus web crippling deformation, the latter being the reduction in cross-section height above the support at midspan. This for the experiment, the finite element model "FEM", and a previous finite element model in Ansys "FEM (old)" [8]. Similarity between the finite element models is excellent, and their deviations from the experiment are likely to be caused by the inevitable inaccuracies associated with experiments. Further comparisons of the load versus beam deflection, the support rotation versus web crippling deformation, and plastic behaviour confirmed this conclusion. Changing the number of elements along the circumference of the rounded bottom corner from 3 to 16 influenced the behaviour of the model significantly, as can be seen in figure 2 on the right which zooms in on the figure on the left with respect to the scale of the horizontal axis. A similar important change in behaviour was observed when the element aspect ratio for the bottom corner elements was varied from 1 to 5. Other variations, namely of the overall mesh density, the element type, the solution settings (precision, constraint enforcement method), and the presence of strips preventing spreading of the webs, did not lead to significantly different outcomes.

Other failure modes than the yield-arc mode may be asymmetric in length direction and then show local dynamic phenomena. Thus a half model and dynamic analyses are needed. To verify the model for these dynamic analyses, first an implicit dynamic simulation was carried out, in which dynamical equilibrium is solved to a certain accuracy. Mass-scaling and damping were not applied. For the load bearing plate displacement, an in-time linear increasing velocity was selected, as a linear increasing displacement induced undesirable dynamic effects in the load versus web crippling diagram, whereas a linear increasing acceleration showed an increasing ratio between the kinetic energy over the internal (strain) energy. Time for the end displacement (15 mm) was tuned: 0.01 seconds introduced too much.

Figure 2: Finite element model setup, t1:first yield, t2:some yielding of compressed flange, t3:yielding of almost complete flange above plate, t4:complete yield-line pattern.

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kinetic energy, more than 1% of the internal energy, however from 0.1 seconds on this
threshold was met and thus 0.1 seconds was selected. Results of the implicit dynamic
simulation with these settings were completely equal to the static simulations. Finally, an
additional verification was made by using an explicit dynamic simulation over the same time
and using the same loading. Oscillations occurred, but by filtering these out around 120 Hz
results were comparable to both the static and dynamic implicit simulations. As the dynamic
implicit simulations find equilibrium, are suitable for all possible failure modes, and have
been verified as presented in the section, this type will be used for most of the simulations to
follow.

3 SHEET-SECTION FINITE ELEMENT SIMULATIONS

For first-generation sheeting, with practical cross-sectional dimensions, and loaded by
practical ratios between web crippling load and bending moment, three failure modes may
occur: the symmetric yield-arc mode as shown in figure 3 on the left, the asymmetric yield-
eye mode on the right and a yield-arc to yield-eye transition mode that starts as yield-arc but
after some deformation transforms into a yield-eye mode (not shown). The yield-line pattern
of the yield-arc mode starts with an arc-like shaped yield line in the web, and hereafter several
yield lines in web and flange follow. The asymmetric yield-eye mode starts with two yield
lines in the flange, together shaped like an eye, thereafter followed by secondary lines in the
web and a single one in the flange. In the subsequent sections, for each failure mode the finite
element model of section 2 is adjusted to a dedicated experiment from the past to provide a
verified simulation for that specific mode. Different from experiment nr. 25 in section 2,
which was only selected to fail by the yield-arc mode and to be well documented, the
dedicated experiments in this section have been selected to cover different corner radii sizes,
bottom flange widths, among others. Furthermore, the here selected yield-arc (nr. 42) and
yield-eye (nr. 61) experiments were among the few that were simulated with two different
programs in respectively [8] and [37], which provides additional verification.

![Figure 3: Half finite element models. On the left a yield-arc mode, which starts with the inner arc in the web. On the right the yield-eye mode, which starts with the two most right yield lines in the bottom flange.](image)

3.1 Yield-arc failure mode

The verification of the finite element model as presented in section 2 indicated that an
element aspect ratio for the bottom corner of 2 yields acceptable results. Combined with the
required number of elements along the corner and overall mesh compatibility, this led to the
following mesh: For the top flange elements throughout, and for the bottom flange and web
elements for the coarse mesh, 20 × 20 mm elements are present. For the fine mesh, the bottom
flange and web have been meshed 2 × 2 mm. The rounded top corner has single elements
along the circumference, being 2 mm (fine part) respectively 10 mm (course part) along the
length. This is also the case for the rounded bottom corner, but for the fine part, 3 elements
along the corner are used, being 2 mm in length direction. A constant acceleration of the load bearing plate equal to 500 mm/s² for 0.316 s was used with an implicit dynamic solver. Experiment 42 [8] was modelled half, with properties $b_{tf}=109.8$ mm, $b_w=99.4$ mm, $b_{bf}=40.8$ mm, $\theta_w=50.9^\circ$, $r_{bf}=r_{tf}=3.1$ mm, $t=0.68$ mm, and a span length equal to 1802 mm. Not shown, but with respect to a) the ultimate load; b) load-web crippling behaviour; c) load-beam deflection; d) support rotation-web crippling behaviour; and e) yield line patterns and failure mode, the finite element model shows excellent agreement with the experiment and its finite element simulation at that time [8]. Also a static run with a quarter model, or an explicit dynamic run yield the same results. Only for a single and very large imperfection among a variety of imperfections —imperfections to be presented in the next section, where they have a crucial role— a yield-eye failure mode was found once.

3.2 Yield-eye failure mode

For the yield-eye failure mode, experiment 61 [8] was used, with $b_{tf}=141.0$ mm, $b_w=100.5$ mm, $b_{bf}=39.0$ mm, $\theta_w=44.2^\circ$, $r_{bf}=r_{tf}=5.8$ mm, $t=0.68$ mm, and a span length equal to 2400 mm. Due to these different dimensions compared to the model in the previous subsection, some element sizes in the fine meshed part differ as follows: The bottom flange and web have been meshed now $3 \times 3$ mm, and consequently the rounded top and bottom corner elements are now 3 mm in length direction too. The yield-eye failure mode is very sensitive to imperfections [37]. As such, different imperfections have been studied. Figure 4 shows thickness reduction imperfections, the first type being b3 on the right, for which the red shaded part has been given 0, 2.5, 5, and 10 % thickness reduction (type b2 is used in the next section). Furthermore, the first Eigen mode as imperfection has been tried too, with a maximum displacement scaled to 1/1000 or 1/250 times the span length.

![Figure 4: On the left imperfection type b2, for which the red part is given a reduced thickness. On the right imperfection type b3, which is similar but does not include the bottom corner.](image)

Again implicit dynamic simulations have been carried out, with a final speed equal to 175 mm/s. With no thickness imperfection or a very small imposed Eigen mode (1/1000), a yield-arc failure mode was found, but for all larger imperfections the yield-eye mode occurred. Larger thickness reductions reduces the ultimate load slightly, but again, with respect to the a) to e) comparisons as mentioned in the previous subsection, the finite element model agreed very well with the experiment and its finite element simulation at that time [8]. Kinetic energy was monitored and remained within safe limits compared to internal (strain) energy for regarding the simulation as being quasi-static. However, different from the yield-arc failure mode, the yield-eye mode occurred together with some lightly damped vibrations, clearly indicating that this mode has inherent dynamic properties.

3.3 Yield-arc to yield-eye transition mode

Experiment 56 [8] failed first symmetrically by the yield-arc failure mode. However, after further loading, asymmetric deformation occurred that resembled the yield-eye mode. This kind of failure has been defined as the yield-arc to yield-eye transition mode. The experiment had dimensions $b_{tf}=141.2$ mm, $b_w=100.8$ mm, $b_{bf}=93.7$ mm, $\theta_w=71.5^\circ$, $r_{bf}=r_{tf}=3.6$ mm, $t=0.68$
mm, and a span length equal to 2401 mm. As such, the finite element model for this transition mode could use the same mesh sizes as the model for the yield-eye mode. Simulations were carried out with imperfection type b3 with a variety of thickness reductions from 0 up to 35%. In general, small imperfections led to the yield-arc mode, large imperfections led to the yield-eye mode, and for 7 out of 14 cases a transition mode was found. Figure 5 shows that although load versus beam deflection is similar for all cases, load versus web-crippling behaviour is different. For very small imperfections the yield-arc mode occurred, which is symmetric and thus shows increasing values of web crippling deformation during the whole simulation. Very large imperfections led to the yield-eye mode. As this mode is asymmetric, the sheet-section bottom flange moves to an inclined position relative to the load bearing plate. At one side it loses contact, then transferring the load via the other side. Due to the inclination of the section, the distance between the load bearing plate surface and the top flange increases. As the original section height minus this distance is used for the web crippling deformation, the as such defined web crippling deformation reduces. Although this may be seen as a flaw in the experimental setup and in the definition of the web-crippling deformation, it actually provides a convenient measure that easily distinguishes between the several failure modes. For intermediate sized imperfections it can be seen that first the yield-arc mode occurred (no snap back), for larger sized imperfections this yield-arc mode was followed (earlier if imperfections were larger) by snap-back behaviour, as explained belonging to the yield-eye mode.

![Figure 5: Web crippling deformation snaps back immediately after the ultimate load for the yield-eye mode (35% reduction), during further deformation for the transition mode, or not at all for the yield-arc (5% reduction).](image)

Using alternatively an explicit dynamic solver, or imperfection type b2, led to slightly different values for the imperfection size that decided between a full yield-arc, full yield-eye, or transition mode. This showed again that the yield-eye and transition modes are very sensitive for imperfection types and sizes, and even different solving approaches.

### 3.3 Sheet-section models to be used for comparisons

The sheet-section finite element models, dedicated to the three failure modes, were able to simulate the associated experiments well. To compare the behaviour of their bottom flanges to the upcoming compressed flange finite element models, for each sheet-section model specific settings have been selected: For the yield-arc model imperfections are not used. For the yield-eye model imperfection b3 with 10% reduction of the thickness has been selected. And as the transition mode is very sensitive to imperfections, and compressed flange finite element models will simulate bottom flanges both with and without rounded corners, two simulations have been selected: a simulation with imperfection b2 with 13% reduction and another simulation with b3 with 25% reduction. This latter 25% reduction is unrealistically large, but represents a complex of imperfections in the real experiments and delivers a clear transition.
mechanism, as shown in figure 5 on the right. All sheet-section models are analysed with an implicit dynamic approach, with the program specific solver setting "quasi-static", which directs the solver to take large time increments, however also allows for considerable numerical dissipation if e.g. snap-back behaviour occurs [38].

4 COMPRESSED FLANGE FINITE ELEMENT SIMULATIONS

Referring to figure 1, which shows the setup of the sheet-section finite element models, a similar figure can be imagined for the compressed flange models, but now with only half the bottom flange (with or without the rounded bottom corner) shaded grey. Figure 13 in section 4.3 shows a part of a sheet-section model, which may help further understanding. For the longitudinal symmetry line normal symmetry conditions UX=RY=RZ=0 apply, whereas for the other longitudinal edge, being (a) the junction of the rounded bottom corner and web or (b) the junction between bottom flange and rounded bottom corner, boundary conditions have been used as shown in table 1: BC1 provides free longitudinal movement, however transverse movement is not possible. BC2 allows transverse movements but keeps the edge straight. BC3 allows for the edge to wave free in-plane. Transverse edges are completely free, as is the case if they are part of a sheet-section.

Table 1: Boundary conditions for compressed flange edges.

<table>
<thead>
<tr>
<th>BC</th>
<th>Visualisation</th>
<th>Longitudinal edges</th>
<th>Transverse edges</th>
</tr>
</thead>
<tbody>
<tr>
<td>BC1</td>
<td></td>
<td>UX=0</td>
<td>No BC's</td>
</tr>
<tr>
<td></td>
<td></td>
<td>UY=0</td>
<td></td>
</tr>
<tr>
<td>BC2</td>
<td></td>
<td>UX coupled</td>
<td>No BC's</td>
</tr>
<tr>
<td></td>
<td></td>
<td>UY=0</td>
<td></td>
</tr>
<tr>
<td>BC3</td>
<td></td>
<td>UY=0</td>
<td>No BC's</td>
</tr>
</tbody>
</table>

The bending moment in a sheet-section is caused by the concentrated load applied via the load bearing plate. Assuming linear behaviour of the geometry and material, the normal strain in the compressed rounded corner in z-direction due to this bending moment is given in figure 6. Integrating the strain yields the displacements u as shown at the bottom of figure 6 and by equations (1) to (3), with a being the load bearing plate width [40].

Figure 6: Strain and displacement functions due to bending along the length of a compressed bottom flange, boundary conditions not shown.
Using this setup, seven different compressed flange models have been developed and investigated. The first model is presented in the next section in detail, including a comparison with the sheet-models. The remaining six models are presented only for their differences with the first model.

4.1 Long plate without rounded bottom corner

The most basic model is a compressed bottom flange without rounded corner, and loaded by prescribed displacements according to the displacement function along the longitudinal edge as presented in the previous subsection. Note that this model can be used with three different boundary conditions (table 1), and for three experiments (42: yield-arc, 61: yield-eye, 56: transition). In general, for the imperfection the displacements of the first Eigen mode were imposed on the geometry with the maximal displacement scaled to 1/100 of the thickness. As an example, figure 7 shows the Eigen mode for the compressed flange with BC1 (table 1) and properties equal to the yield-arc experiment (42). Obviously other boundary conditions led to different first Eigen modes.

Explicit dynamic solutions were obtained, with the displacement quadratically increasing from \( u_{z=a/2} = 0 \) to -0.5 mm, due to a linearly increasing velocity from 0 to -6.67 mm/s\(^2\), at end time 0.15 s. Figure 8 on the left shows on the vertical axis the sum of the reaction forces in z-direction along the left half of the longitudinal edge, which was loaded by the prescribed displacements. On the horizontal axis the steering displacement \( u_{z=a/2} \) in equations (1) to (3) has been made equivalent to a representative load \( F_{rep} \) on a sheet-section as follows. The derivative of equation (2) yields strain. Using standard beam theory, this strain can be used to calculate the bending moment as given in equation (4) top left. Here \( E \) is Young's modulus, \( I_{x, cg} \) is the second moment of area about the neutral axis parallel to the \( x \)-axis, and \( y_{cg-bf} \) the distance between this neutral axis and the bottom flange.

\[
\begin{align*}
\frac{-L}{2} \leq z \leq -\frac{a}{2} : & \quad u(z) = \frac{2u_{z=a/2}}{a(L-a)} z^2 + \frac{2Lu_{z=a/2}}{a(L-a)} z + \frac{au_{z=a/2}}{2(L-a)} \\
\frac{-a}{2} < z < \frac{a}{2} : & \quad u(z) = \frac{2u_{z=a/2}}{a} z \\
\frac{a}{2} \leq z \leq \frac{L}{2} : & \quad u(z) = -\frac{2u_{z=a/2}}{a(L-a)} z^2 + \frac{2Lu_{z=a/2}}{a(L-a)} z - \frac{au_{z=a/2}}{2(L-a)} 
\end{align*}
\]

\[
M(z) = \frac{2u_{z=a/2}}{a} \frac{EI_{x, cg}}{y_{cg-bf}} \quad \Rightarrow \quad F_{rep} = \frac{8u_{z=a/2}EI_{x, cg}}{(L-a)ay_{cg-bf}} \quad (4)
\]
For a three-point bending test, the bending moment at $z=0$ and the concentrated load $F$ are related as shown in the equation bottom left, leading to the concentrated load $F_{\text{rep}}$ on a sheet-section as function of the steering displacement $u_{z=a/2}$, as indicated in the right hand side of the equation. Note that this load is only fully representative in the case of linear behaviour, both geometrically and materially.

First visible yield in the contour plots of figure 9 occurs at point "ty" in figure 8, somewhat after the first Eigen value for buckling, the latter indicated by the vertical dotted line marked $F_{\text{buckle}}$. First yield is quickly followed by the ultimate load at "tu". This ultimate load is related to a representative load on the horizontal axis equal to 4041N, which is 1.73 times larger than the ultimate load of the sheet-section (2339 N). Finally, "tf" indicates the user-set end of the simulation. On the right of figure 8, the number of buckles versus the steered in-plane displacement $u_{z=a/2}$ is shown. The number of buckles have been determined by hand, studying deformation contour plots and scaled plots of the deformed plate.

Figure 8: Compressed flange model with BC1 for experiment 42. On the left load vs. deformation (i.e. representative load), on the right number of buckles vs. deformation.

Figure 9 on the left shows—for "ty", "tu", and "tf" in figure 8—, the maximum value of the Von Mises stresses at the bottom and top surfaces. The contour colours have been set such that any colour indicates yielding. On the right, out-of-plane deformations are shown. The figure shows clearly that yielding starts not along the longitudinal edge but at the top or bottom of a buckle. At the ultimate load and beyond, the longitudinal edges also yield, together with yield zones (in the figure being vertical) at buckle tops or bottoms.

Figure 9: On the left yielding starts in the middle, different from the effective-width approach. On the right the number of buckles change as buckles move inwards to the middle from both sides.
To obtain more insight in the yielding pattern, figure 10 plots the Von Mises stress for the bottom, middle, and top layers of the compressed flange, both at first yield (ty) and at failure (tu). It can be seen that first yield is due to local bending (as only the outer fibres yield) and at ultimate load the longitudinal edges are yielding due to compression (as the middle surface yields) and local bending, in combination with (in the figure vertical) yield lines at the top and bottom of the buckles. The behaviour of the compressed plate can be compared with the compressed flange from the sheet-section finite element model, figure 11. In the sheet-section, first yield is shown to occur due to the indentation of the plate by the load bearing plate, whereas failure occurs due to yield lines (due to local bending) in a circular shape adjacent to the load bearing plate. This is clearly different from the behaviour of the compressed flange only. This needs not to be surprising, but on the other hand, the theoretical model (see section 1) uses only a square part of the compressed flange to predict first yield of a sheet-section very well.

![Figure 10: Compressed flange model: First yield is due to local bending of the compressed flange. Failure is due to compression along the longitudinal edges and local bending at these edges and in the figure vertical yield lines at the top and bottom of the buckles.](image)

![Figure 11: Sheet-section model: First yield is due to indentation of load bearing plate. Failure is related to small zones of yielding due to compression (middle layer) and two circular yield line zones due to bending, both to the left and right of the load bearing plate.](image)

Figure 12 presents the out-of-plane displacements (so in y-direction) at the longitudinal symmetry line, for the positions along the length equal to -a/2, 0, and a/2. These out-of-plane displacements are measured relatively to the other longitudinal edge and are thus not influenced by e.g. beam deflection in the sheet-section finite element model. The displacements are plotted against the representative load. The compressed flange finite element simulation shows clearly out-of-plane buckling deformations after the buckling load (dotted horizontal line). The change of sign of the out-of-plane deformations quickly after onset are due to the shift of buckle positions (see figure 9). Completely different, the sheet-
section finite element model shows strong downwards deflections due to the action of the load bearing plate.

This compressed flange simulation with BC1 was also carried out with a static solution procedure for verification. Boundary conditions BC2 and BC3 were also tested, the latter with using several imperfection sizes and different end times (i.e. different end speeds) for the explicit procedure. Finally six simulations were carried out for the yield-eye and the transition mode, for all three boundary conditions. The sheet-section models and the bottom flange models were compared for the Eigen mode (symmetric or asymmetric), the buckling load, the yield pattern development, the ultimate load, and the out-of-plane deformations, and did not relate clearly for any of these investigated characteristics [39].

The buckling and ultimate loads are predicted too high by the compressed flange only, possibly due to the absence of stresses caused by the load bearing plate. As the load bearing plate may also correct the different out-of-plane displacement behaviour, as typically shown in figure 12, the next subsection presents a possible improvement of the compressed flange model by incorporating a load bearing plate.

![Figure 12: Sheet-section out-of-plane displacement can only occur in positive y-direction, as load-bearing plate restricts the other direction, snap-back occurs due to the forming of yield-lines. For the plate, displacements start around buckling and change sign due to the shifting of the buckles, see figure 9.](image)

**4.2 Long plate without rounded bottom corner radius and with fixed load bearing plate**

This bottom flange finite element model is equal to the model in section 4.1 except that it includes a fixed load bearing plate with contact modelled between the load bearing plate and the bottom flange. The bearing plate and the contact are modelled completely following the techniques as used for the sheet-section finite element models.

The models with load bearing plate, for all failure modes and for all boundary conditions, did not improve compared to the models without load bearing plate [39]. The difference in failure load between the sheet-section model and bottom flange model including load bearing plate was even larger than for the model excluding the load bearing plate. This is probably due to the fact that the compressed flange is more restricted in its displacements: out-of-plane deformation graphs and contour plots show that the load bearing plate effectively restricts out-of-plane flange movements away from the plate. This possibly also led also to the fact that load vs. deformation plots show more ductile behaviour after ultimate load, often with a rigid-plastic like path. This is very different from the strong decrease in strength for the flanges without load bearing plate. Further data and graphs, similar to the simulation presented in the previous subsection, are available in [39].
4.3 Long plate with rounded bottom corner

To investigate whether modelling the rounded bottom corner could improve the basic model of subsection 4.1, this model was extended with the rounded bottom corner, as shown in figure 13 on the left. In the model the prescribed displacements according to the displacement function is applied on the corner-flange junction and the boundary conditions on the outer corner longitudinal edge. Different from the previous models, buckled shapes as part of the imperfection cannot be seen before actual buckling, figure 13 on the right. This can be explained as follows. In the model of section 4.2, without a rounded corner even light compression amplified the imperfection shape strongly, which was visible in the contour plots and the (amplified) deformed flange (not shown here). However, for the model in this section the corner will carry a large part of the compression force, the flange and corner being much stiffer, and consequently an amplification of the imperfection buckles such that they become visible only takes place after buckling.

![Diagram](image)

Figure 13: On the left: displacements are prescribed at the corner-flange junction and boundary conditions along the rounded corner edge. A single point is fixed in z-direction to avoid rigid body movements for all flange models. On the right: buckles are only visible after buckling.

This model predicts the load at first yield and the ultimate load reasonably well if BC3 is used. Unfortunately, corresponding yield line and out-of-plane displacement patterns do not match the sheet-section models [39]. Therefore in the next section this model is also extended with a fixed load bearing plate.

4.4 Long plate with rounded bottom corner and fixed load bearing plate

This model combines the models of subsection 4.3 with the fixed load bearing plate as used in subsection 4.2. As for all models, three boundary conditions for three different failure modes were tried. Except that the location and moment of first yield were predicted reasonably well, at least for BC3, all other comparisons failed in achieving a resemblance between the behaviour of the compressed flange model and the flange in the sheet-section models. In a logical attempt to further increase the performance of the compressed flange models, a model was developed with rounded bottom corner and a moving (by means of a prescribed displacement) load bearing plate, to be presented in the next section.

4.5 Long plate with rounded bottom corner and moving load bearing plate

In this model, the load bearing plate starts with its loading surface coincident with the bottom flange surface, and is then given a linearly increasing speed, from 0 to -6.67 mm/s from the beginning at 0 s to the end time of 0.15 s. This is equivalent to a constant
acceleration equal to 44.44 m/s\(^2\). This results in a displacement at the end time equal to 0.5 mm, and this is equal to the amount for which the rounded corners will be indented.

For all simulations (3 modes \(\times\) 3 BC's), out-of-plane deformations are similar for the flange and sheet-section model up to the failure load of the sheet-section model, as shown in figure 14 on the left for the yield eye mode, BC2. To investigate the ratio between the in-plane compression of the bottom flange and the out-of-plane indentation of the load bearing plate, the constant acceleration was modified to 100 mm/s\(^2\) and 200 mm/s\(^2\). A larger acceleration resulted in larger out-of-plane deformations compared to the unmodified in-plane load, and as can be seen in figure 14 on the right. This also resulted in larger relative out-of-plane deformations. This could be helpful for further calibration of this specific model, were it not that for first yield and ultimate loads this model gave as inconsistent results as the other models. Also the model did not indicate symmetric or asymmetric behaviour corresponding to the sheet-section models. However, naturally the final deformed shape of the bottom flange resembled the sheet-sections as both are dominated by the indentation by the load bearing plate.

![Graphs](image)

Figure 14: On the left: for the first part of loading, the bottom flange model now follows the behaviour of the flange in the sheet-section mode (compared with figure 12). On the right: higher acceleration means larger load bearing plate displacements, which in turn increases relative out-of-plane deformations.

### 4.6 Long plate with rounded bottom corner and elastic foundation

Despite the increasingly advanced attempts to model the compressed flange such that it can represent the behaviour of the flange in a sheet-section, the models developed so far cannot predict the first yield and failure loads correctly. Yield line patterns and their symmetric or asymmetric character also do not match for flange and sheet-section models. A subsequent attempt was made by realising that in reality the compressed bottom flange is significantly curved during loading. To take this effect into account in the bottom flange model, a strategy is followed as shown in figure 15. Along the longitudinal edges to each node a spring is fixed, effectively creating a beam on elastic foundation. Hereafter, prescribed displacements at midspan result in a curvature of the bottom flange. Finally, the boundary conditions as used before are applied (either BC1, BC2, or BC3), after which the springs and the prescribed displacements at midspan can be removed. As such the compressed flange has a curved geometry, as is the case for a loaded sheet-section, and thus the geometry of the compressed flange is described more precisely. Comparisons between the deformed shapes of the bottom flange models and the bottom flange of the sheet-section models lead to a spring stiffness of
the elastic foundation equal to 0.0001 N/mm and a prescribed displacement equal to 6.31, 10.63, and 8.30 mm for the yield-arc, yield-eye, and transition modes respectively.

Figure 15: The bottom flange is fixed to an elastic foundation, curved by prescribed displacements, and then boundary conditions are applied.

By this model using BC3, the first yield load is predicted reasonably well. However, the location of first yield may be different from the location as found in the sheet-section model. The ultimate load is predicted as larger than that of the sheet-section model, and this overprediction varies for the boundary condition type and failure mode inconsistently. Out-of-plane deformations closely follow the Eigen mode and are not influenced by the initial curvature. Yield line patterns are similar to the uncurved version of this model (subsection 4.3).

4.7 Long plate with rounded bottom corner, elastic foundation, and load-bearing plate

The most extended model investigated is presented in figure 16. Again an elastic foundation was provided to the bottom flange with bottom corner ("Step 1" in the figure). For displacement control, a load bearing plate is used on which a prescribed displacement is set equal to the displacement mentioned in the previous section. In step 3, the bottom flange is loaded in longitudinal direction as before, and the load bearing plate is displaced further such that the corner indentation (the difference in height between the rounded corner top and bottom) equals this indentation in the sheet-section model if both models have the same in-plane displacements. Equally for load control, the load bearing plate is loaded load-controlled such that in step 2 the same displacement is found as for the displacement control option. In step 3 the load at the load bearing plate is controlled such—along the prescribed displacements following the displacement function—that the indentation of the rounded corners are the same for bottom flange model and the sheet-section model for equal in-plane displacements (of the displacement function).

Figure 16: The bottom flange is fixed to an elastic foundation, curved by a load bearing plate, and then boundary conditions are applied with further load bearing plate action.

For the yield-arc experiment (nr. 42) all three types of boundary conditions (BC1, BC2, and BC3) were used in combination with both displacement and load control. BC1 with displacement control obtained a good prediction of the ultimate load. Therefore also the
transition mode (nr. 56) was simulated with BC1 and displacement control. This resulted, however, in a prediction of the ultimate load 2.4 times too large, which led to the decision to stop further simulations of this type. For the simulations carried out, out-of-plane deformation resembled the bottom flange behaviour in the sheet-section reasonably up to the failure load. Due to the lack of a complete set of simulations, no conclusions can be made about the relation of symmetric or asymmetric behaviour of the compressed flange model and the sheet-section models. However, all previous models (section 4.1 to 4.6), together including all the features of this model of section 4.7, could not established this relation. Thus it is not likely that the model in this subsection will perform differently.

5 FLANGE BEHAVIOUR FOR SECOND-GENERATION SHEET-SECTIONS

As mentioned earlier in the introduction, one of the reasons to investigate whether compressed flange behaviour is representative for sheet-section behaviour is that the complex behaviour of second-generation sheet-sections, with stiffeners, could be studied more conveniently, namely for the bottom flange in isolation. For this a start was made by developing sheet-section finite element models for stiffened sections as researched in the past [8] as shown in figure 17. Although finite element simulations were also carried out in the past research, the simulations here provide new information as they simulate both the stiffened sections and the same sections without stiffeners.

![Figure 17: Cross-sections of second-generation sheet-sections investigated in [8].](image)

As this paper is focussed on flange behaviour, here only the simulations for W0-F1w and W0-F1s, see figure 17, are discussed. The stiffened sheet-section analyses were performed with an implicit dynamic and an explicit solver, all very similar to the simulations presented in section 3. Results for W0-F1s are shown in figure 18, which shows that reasonable agreement exists between the two solvers and the experiments. On the right of figure 18, the implicit dynamic simulation is shown, but now together with the same simulation without stiffeners, as also shown in figure 17. The introduction of a stiffener increases the ultimate load significantly for a short period along the deformation path, but the load quickly falls back to levels very comparable to the unstiffened section, which may limit the redistribution of bending moments. Graphs for W0-F1w are similar and yield the same conclusions [39]. The "envelope" Von Mises stresses are shown in figure 19 for "t1" and "t2" as marked in the plots in figure 18. In these plots it can be seen that both the top and bottom flange yield due to bending. In the stiffened case the shortening of the bottom flange leads to a folding yield line pattern in the flat parts and yielding due to compression in the stiffener. As soon as the yield line pattern and yielding due to compression enable shortening across the complete width, strength is very similar to the unstiffened case. It is interesting to note that the unstiffened
case shows web-crippling, evidenced by the folding yield line pattern both at t1 and t2, but the stiffened case not. As the sections are similar in overall geometry and loading, a reasonable explanation seems that the web-crippling yield lines in the web are a compatible mechanism to the bottom flange yield line pattern—which is indeed different from the stiffened case—rather than an indication of significant web-crippling itself.

Figure 18: On the left: New simulations agree with experiment W0-F1s. On the right: a stiffener increases the ultimate load, but the load quickly degrades to nearly unstiffened values.

Figure 19: At t1 (see figure 18 on the right) the stiffener starts to yield, the section starting to lose the extra strength compared to the unstiffened section. At t2 shortening of the flange is completely possible, thanks to yield line patterns in the unstiffened parts and/or yielding of the stiffener.

6 CONCLUSIONS

A finite element model has been presented for a hat-section with boundary conditions such that it models sheeting, and it has been tailored for the yield-arc, transition, and yield-eye failure modes.
Finite element models have been developed that model only the compressed flange of the sheet-section, applying a prescribed displacement field along the flange longitudinal edges that simulate compression due to sheet-section bending.

Results of the two model types have been compared for (a) onset of yielding; (b) ultimate load; (c) yield line patterns; and (d) out-of-plane deformation patterns. The first type compressed flange finite element model, a flat plate only, predicts (a) and (b) not very well, and it is shown that adding a fixed load bearing plate—to suppress deformations beyond the bearing plate surface—nor modelling the rounded corner improves the results.

If the fixed load bearing plate is allowed to displace so as to result in web-crippling deformations comparable to those of the sheet-section models, aspect (d) improves, but (a) and (b) may still be inaccurate. Additionally modelling the curvature of the compressed flange at failure does not improve the situation. However, then incorporating a load-bearing plate results in a more accurate prediction of the location of first yield.

Interestingly, a theoretical model that only models a square part of the compressed flange predicts the ultimate load of first-generation sheet-sections very well [36]. This should be due to the main difference between the theoretical model and the finite element models presented here: In the theoretical model, very large imperfections are used, in fact representing elastic full web-crippling deformations. Differently, in the finite element simulations very small imperfections are used, representing realistic imperfections in practice. Thus further research should investigate the introduction of web-crippling deformation like imperfections in the bottom flange models.

Simulations of second-generation stiffened sheet-sections revealed information about their bottom flange behaviour compared to that for first-generation unstiffened sheeting. For two investigated sections the ultimate load was related to yielding of the bottom flange edges. This indicates that it could be worthwhile to further develop a future successful bottom flange model for first-generation sheeting into one for second generation sheeting.

REFERENCES


