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Effect of flow and fluid properties on the mobility of multiphase flows through porous media

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HIGHLIGHTS

- 3D fully resolved pore-scale simulations of multiphase flow using a VOF-IBM method.
- Validation test cases for multiphase porous media flows with wettability effects.
- Water flooding simulations through regular single and random multi-pore structures.
- Effect of capillary number, contact angle and viscosity ratio on the mobility of flow.

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ABSTRACT

In this paper we quantify the effect of capillary number \( \text{Ca} \), contact angle \( \theta \) and viscosity ratio \( M \) on the mobility of multiphase flow through porous media. The focus is mainly on oil-water flows through porous rocks observed during the water flooding process. Simulations are performed using a finite volume method employing a staggered grid formulation. Interactions between fluids and complex solid boundaries are resolved by a direct forcing, implicit and sharp interface immersed boundary method (IBM). The fluid-fluid interface is tracked by a mass conservative sharp interface volume of fluid (VOF) method. IBM and VOF are coupled by imposing the contact angle as a boundary condition at the three phase contact line. Our methodology has been verified/validated for several test cases including multi-phase Poiseuille flow in a channel, a viscous finger in a channel and mesh convergence of the contact force. Two types of porous structures are considered: (i) a repeated single pore and (ii) a random multi-pore arrangement. Temporal evolution of phase pressure difference and oil saturation have been studied as viscous fingers penetrate the pores. We observed that the residual oil saturation for different capillary numbers shows exactly the opposite trend for the single and multi-pore arrangement. The residual oil saturation for multi-pore shows a well defined linear trend with \( \log \text{Ca} \), \( \theta \) and \( \log M \).

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1. Introduction

Multiphase flow through porous media is a subject of significant interest among several scientific communities and industries. Examples include underground water flows (Høst-Madsen and Jensen, 1992), enhanced oil recovery (EOR) (Alvarado and Manrique, 2010), fluidized bed reactors (Pyle, 1972), microfluidics devices (Stone et al., 2004), soil remediation (Mulligan et al., 2001; Kumpiene et al., 2008) and many more.

The oil recovery process is generally carried out in multiple stages. In the primary stage, oil flows out of the porous rocks automatically due to its own natural pressure. Gradually, this natural flow decreases/stops due to rock pressure normalization. A lot of oil still remains trapped inside the porous rocks after the primary recovery due to capillary effects. To recover this residual oil variety of secondary and ternary (EOR) processes are used e.g. water flooding (Sheng, 2014; Jerald et al., 2006), surfactant or polymer flooding (Shah, 2012; Keshkar et al., 2016; De et al., 2017), thermal recovery (Prats, 1982; Zhu, 2011), gas injection, etc. Water flooding is one of the most common secondary oil recovery processes in which a high pressure water is used to displace the oil out of the porous rocks. This paper focuses on direct numerical simulations (DNS) of oil-water flow through porous media. To perform these simulations the following challenges need to be resolved efficiently: (i) oil-water interface tracking, (ii) modeling of the interactions between fluid (oil or water) and porous media and (iii) wettability effects at three phase contact lines.

Numerical simulations of multi-fluid interfaces are especially challenging because of the requirements of mass conservative
interface advection and accurate computation of interfacial tension forces. A wide range of numerical methods has been developed and successfully tested for this purpose and an overview is presented by Scardovelli and Zaleski (1999). Frequently applied methods are front-tracking (Tryggvason et al., 2001), level-set (Sussman et al., 1994), volume of fluid (Hirt and Nichols, 1981; Youngs, 1982), Lattice-Boltzmann (Chen and Doolen, 1998) and phase field (Singer-Loginova and Singer, 2008). The Volume of fluid (VOF) method is used for the present simulations as it has a sharp interface representation and is the most mass conservative method among all multi-fluid interface tracking/capturing methods. Immersed boundary methods (IBM) (Mittal and Iaccarino, 2005) use non-body conforming structured (mostly Cartesian) computational grids to resolve fluid-solid interactions. Main advantage of IBM is the ease of grid generation, discretization of the Navier-Stokes equations and computer code implementation. IBM uses simple data structures due to structured grids which increases computational efficiency and decreases computational time. IBM can be categorized into two types: (i) continuous forcing approach where an explicit forcing function is applied to the no-slip boundary condition at solid boundaries and (ii) direct forcing approach where the no-slip boundary condition is applied at the level of the discretized momentum equation. Present IBM is direct forcing, 2nd order accurate and implicit which sharply resolves fluid-solid interactions. There are other methods available to model fluids-solid interactions but they either require calibration of the geometry (Lattice-Boltzmann method) or produce a diffuse interface (level-set method). Three phase contact line dynamics plays a major role in wetting-dewetting phenomena when a multi-fluid interface comes in contact with a solid boundary. The contact line motion is determined by the microscopic physico-chemical interactions of the interface with substrate and it can drastically alter the bulk flow (Snoeijer and Andreotti, 2013). The effect of contact line dynamics at the macroscopic length scale can be represented by an apparent contact angle which may also depend on surface topology, surface roughness or contact line hysteresis. For fully-resolved numerical simulations, this effect can be incorporated by imposing a single/static (Renardy et al., 2001) or a dynamic (Saha and Mitra, 2009) value for the apparent contact angle as a boundary condition at three-phase contact line. A review on multiphase flow simulations with moving contact line is presented by (Sui et al., 2014).

Accurate and efficient simulations of heterogeneous oil reservoir flow poses many challenges (Gerritsen and Durlufsky, 2005): (i) length scale of the reservoir (kilometers) compared to that of pores (micrometers) (ii) presence of many phases in flow e.g. gas pockets, water, soil particles, oil (iii) uncertain rock properties and (iv) inter-solubility of fluid phases. Usage of coarse-scale models offers one option to model multiphase flow through oil reservoirs where a grid cell is (many fold) larger than the finest scale of the porous media. Effects of fine-scale interactions e.g. wettability, pore size distribution, pore arrangement are imposed in a volume averaged manner as a source term for coarse-scale simulations. Although coarse-scale models can simulate flow through heterogeneous reservoirs, a lot of work is still required in coarse-graining of multiphase flows with wettability effects and material transport. Recently, fully resolved fine-scale simulations of (a small part of) oil reservoirs have attracted many technologists and researchers. These simulations can accurately model wettability and material transport which may further help in developing upscaling models. The Lattice-Boltzmann method (Yiotis et al., 2007; Huang and Lu, 2009; Huang et al., 2009) is the most common technique used for fully resolved simulations. Pores are typically modeled by randomly arranging multiple solid unit squares/cubes in 2D/3D and the oil-water flows through the voids between them. Even though the Lattice-Boltzmann method is computationally fast because of being fully explicit, it is unable to strictly enforce continuity of velocity and shear-stress at the fluid-fluid interface. Very few attempts have been made to simulate pore-scale multiphase flow with wettability effects using VOF. Lv and Wang (2015) used the VOF method to simulate hot water flooding process through a single 2D pore and quantified the residual oil for the flooding at different temperatures. Ferrari and Lunati (2013) simulated multiphase flow through a 2D porous bed consisting of multiple cylindrical particles to link capillary pressure and total surface energy. Both of these papers use body-fitted unstructured grid for simulations in CFD software packages e.g. Fluent or OpenFOAM.

In this paper, we present simulations of multiphase flow through 3D porous media with wettability effects on a non-body fitted Cartesian computational grid using a coupled IBM-VOF method developed in-house. First, we will introduce the governing equations and the implementation details for the individual methods (IBM or VOF) and then for the IBM-VOF coupling in brief. Further, we will present some verification/validation test cases specifically pertaining to the oil-water flows through the porous media. Finally, we will simulate multiphase flows with wettability effects through a single and multi pore structures to quantify its mobility.

2. Numerical methodology

The mass and momentum conservation equations for unsteady, incompressible, Newtonian, multiphase flow can be expressed in a single field formulation as follows,
\[ \nabla \cdot \mathbf{u} = 0 \]

\[ \rho \frac{D \mathbf{u}}{Dt} + \rho \nabla \cdot (\mathbf{u} \mathbf{u}) = -\nabla p + \nabla \cdot \mathbf{\tau} + \mathbf{F}_s \]

where \( \mathbf{\tau} = \mu \left[ \nabla \mathbf{u} + (\nabla \mathbf{u})^T \right] \) is the fluid stress tensor. \( \mathbf{F}_s \) is the volumetric interfacial tension force which acts in the vicinity of the fluid-fluid interface. \( \rho \) and \( \mu \) are local phase averaged density and dynamic viscosity, respectively computed from the following equations:
\[ \rho = \rho_1 + (1 - F) \rho_2 \]
\[ \mu = \frac{\rho_1}{\mu_1} + (1 - F) \frac{\rho_2}{\mu_2} \]

where the two different immiscible fluids are denoted by subscripts 1 and 2. \( F \) is the local phase fraction which shows the fractional amount of a particular fluid present in a certain computational cell. In our study, \( F = 1 \) for a computational cell means that it is fully occupied by fluid 1. Advection of \( F \) is governed by the following equation:
\[ \frac{DF}{Dt} = \frac{\partial F}{\partial t} + \mathbf{u} \cdot \nabla F = 0 \]

The volume of fluid (VOF) method is used for the numerical solution of Eq. (5) using geometrical advection. A finite volume methodology with pressure-velocity coupling on a staggered Cartesian computational grid is used to discretize of Eqs. (1) and (2). No-slip boundary conditions at non-body fitting solid boundaries are applied by a sharp interface immersed boundary method (IBM). IBM and VOF are coupled by imposing the contact angle as a boundary condition at the three phase contact line. Our IBM, VOF and IBM-VOF coupling are described with all necessary implementation details and verification/validation test cases in Das et al. (2016), Van Sint Annaland et al. (2005) and Patel et al. (2017), respectively. Here we will only outline them for completeness.
A 2nd order, sharp interface, implicit and direct forcing IBM imposes the no-slip boundary condition for the fluids-solid interface at the level of the discretized momentum Eq. (2). Initially, all computational cells inside the solid bodies are marked as solid-cells and the remaining ones as fluid-cells. An example is shown in Fig. 1(a) where s is a solid-cell and f1,f2 are fluid-cells. While discretizing velocity (in any arbitrary $\xi$ direction) at f1, the velocity at s is expressed as a linear combination of the velocities at f1 and f2 in such a way that the no-slip boundary condition is satisfied at a solid surface ($\xi = \xi_s$). The same procedure is carried out for all the fluid-cells neighboring the immersed boundary.

The VOF method uses a piecewise linear interface construction (PLIC) (Youngs, 1982) to represent a fluid-fluid interface by a plane in a computational cell. Location and orientation of this plane can be uniquely defined by the combination of phase fraction in a computational cell. Location and orientation of this plane can be defined at three phase contact line.

The VOF method uses a piecewise linear interface construction (PLIC) (Youngs, 1982) to represent a fluid-fluid interface by a plane in a computational cell. Location and orientation of this plane can be uniquely defined by the combination of phase fraction ($F$) and a unit normal vector ($n = \nabla F / |\nabla F|$). Note that a smoothed phase fraction ($\tilde{F}$) is used (instead of $F$) for computing $n$ to eliminate the difficulties arising in the numerical differentiation of a discontinuous $F$ field. The interface is advected under the influence of local velocity by solving the hyperbolic $F$-advection Eq. (5) numerically; using the pseudo-Lagrangian geometrical advection schemes to minimize numerical diffusion. Computational cells in the vicinity of the interface ($q$ in Fig. 1(a)) will experience interfacial tension force. The density-scaled continuum surface tension ($C_1$) model (Brackbill et al., 1992) computes volumetric interfacial tension force ($F_{\alpha} = \frac{1}{2} \sigma n \cdot \nabla F$) which is added as an explicit source term in the momentum Eq. (2). $\sigma$ is an interfacial tension coefficient and $\rho = 0.5(\rho_1 + \rho_2)$ is an average density. $\kappa$ is the local interface curvature computed from the divergence of the unit normal vector ($\kappa = -\nabla \cdot n$) to the interface.

Contact line dynamics becomes important during wetting/dewetting of the solid surface. This effect can be accommodated in the CFS model by imposing the contact angle ($\theta$) as a boundary condition at the contact line. This is achieved by modifying the interface normal at the solid boundaries as follows:

$$n = n_i \cos \theta + t_i \sin \theta$$

where $n_i$ is the unit solid normal pointing towards the solid region as seen from the fluid region, $t_i$ is the unit tangent which is normal to both $n_i$ and the contact line. This contact angle boundary condition is applied to all the solid-cells which possess at least one neighboring fluid-cell containing a fluid-fluid interface. Solid-cell $p$ in Fig. 1(a) has at least one neighboring fluid cell, $q$, that contains the fluid-fluid interface. So, the contact angle boundary condition will be applied to solid-cell $p$.

In the present implementation, we extrapolate the phase fraction ($F$) field inside the solid region along the tangent to the fluid-fluid interface at the contact line. This way the extended phase fraction ($F_{\alpha}$) field also inscribes the angle $\theta$ with the solid surface. For all simulations reported in this paper, extrapolation is performed up to 4 grid cells normal to the solid boundary in the solid region. Note that $F_{\alpha}$ has no physical meaning inside the solid region and is used only to compute the smooth phase fraction ($\tilde{F}$) field. The smoothing process is carried out using a smoothing kernel (Patel et al., 2017) and produces $\tilde{F}$ with a typical interface thickness of 2–3 grid cells. $\tilde{F}$ is continuous across the fluid-solid interface which improves the contact angle boundary condition for a ‘diffused’ fluid-fluid interface on ‘sharp’ fluids-solid interface. Fig. 1(b) shows the $F$ and $\tilde{F}$ field for the typical problem setup where a fluid-fluid interface makes a contact angle ($\theta$) with the solid surface.

3. Verification and validation

The methodology described in the previous section has been extensively verified/validated using a wide range of test cases. The IBM and VOF methods have been tested individually by Das et al. (2016) and Van Sint Annaland et al. (2005), respectively. The coupling between IBM and VOF with contact line dynamics has been tested by Patel et al. (2017) with static and/or dynamic contact angles for (i) the equilibrium shape of a droplet on a flat surface, (ii) the equilibrium shape of a droplet on a spherical surface and (iii) temporal evolution of droplet spreading on a flat surface. Their results revealed an excellent agreement with analytical, numerical and/or experimental results published in literature.

In this paper, we investigate additional verification/validation test cases pertaining to the multiphase flows in porous media. First, a test case of multiphase Poiseuille flow is considered to compare the velocity profiles and relative permeabilities with analytical expressions. Further, a viscous fingering phenomenon is simulated in a channel and finger characteristic parameters, i.e. dimensionless finger width and tip radius have been compared with results published in literature. Finally, a grid independence study for the contact force is presented for 3D coupled IBM-VOF implementation.

3.1. Multiphase Poiseuille flow

In this section, we investigate the co-current multiphase Poiseuille flow of two immiscible fluids in a 2D channel of width $L$. Fig. 1. Numerical implementation of the present IBM-VOF coupling: (a) 2D schematic representation of the fluid-fluid interface in contact with a solid surface on a staggered Cartesian computational grid. • = scalar (pressure/phase fraction) cell-center and ■ = x-velocity cell-center. Interface between two fluids is piecewise linear according to the PLIC algorithm (VOF). (b) Contours of phase fraction field ($F$) and extended smooth phase fraction field ($\tilde{F}$) for a typical case. Contact angle ($\theta$) is applied as a boundary condition at three phase contact line.
The wetting fluid (w) is in contact with the channel walls (a < |y|< L) whereas the non-wetting fluid (nw) resides between the layers of wetting fluid (0 < |y|< a) as shown in Fig. 2. The no-slip boundary condition is applied at the channel walls (|y| = L) and the flow in x-direction is periodic. Both fluids have the same density (ρw = ρnw) and different dynamic viscosities. The viscosity ratio (M) is defined as M = μnw/μw. A constant and equal body force G is applied to both fluids in the x-direction such as the flow remains in the Stokes regime (Re ≪ 1). Here, Re = ρnwUnwL/μw with Unw being the average interstitial velocity of non-wetting fluid through the channel. Due to the existence of a flat fluid-fluid interface, the interfacial tension force is essentially zero in this problem.

For the given wetting and non-wetting saturations (Sw = (L - a)/L and Snw = a/L), the analytical expressions for the velocity of the wetting and non-wetting fluids (uw and unw) are given by:

\[
\begin{align*}
uw(y) & = \frac{C_0}{L} \left( L^2 - y^2 \right) \\
unw(y) & = \frac{C_0}{L} \left( L^2 - a^2 \right) + \frac{C_0}{L} \left( a^2 - y^2 \right)
\end{align*}
\]  

(7)

Permeability (k) is a proportionality constant in Darcy’s law \((Q = -k\nabla p/\mu)\) which relates volumetric flow rate \((Q)\) through porous media with fluid viscosity \((\mu)\) and pressure gradient \((\nabla p)\). In multiphase porous media flows, the relative permeability is defined as a ratio of effective permeability of a particular fluid at a particular saturation to the absolute permeability of that fluid at total saturation. For the present case, analytical expressions for the relative permeability of each fluid (k_w and k_nw) can be given as,

\[
k_w = \frac{1}{2} S_w (3 - S_w)
\]

\[
k_nw = S_nw \left[ \frac{3}{2} M + S_nw \left( 1 - \frac{1}{2} M \right) \right]
\]  

(8)

Readers are referred to Yiotis et al. (2007) for the detailed derivation of Eqs. (7) and (8). Eq. (8) suggests that k_w is always bounded between 0 and 1. However, k_nw may become greater than 1 in case M > 1 due to the ‘lubricating’ effect of the wetting fluid.

Simulations have been performed for M = 0.01, 1 and 100 with Snw ranging from 0 to 1 with increment of 0.1. Total 100 grid cells have been taken across L. Fig. 3 shows analytical and numerical velocity profiles for the case of M = 100 and Snw = 0.5. Fig. 4 compares analytical and numerical relative permeabilities for wetting and non-wetting fluids for M = 100. Our simulations show an excellent agreement with analytical results with a maximum error in relative permeability to be less than 0.2%. Huang and Lu (2009) reports this error to be nearly 7% using the multiphase Lattice-Boltzmann method. Also, in their results, continuity of the shear stress is not maintained at the interface and hence a velocity jump is observed. In the present simulations, continuity of the shear stress at the interface can be attributed to the harmonic averaging of the kinematic viscosities (Eq. (4)) as suggested by Prosperetti (2002).

### 3.2. Viscous finger in a channel

A viscous finger is an instability that may occur when a low viscosity fluid displaces a high viscosity fluid. In this section, we simulate the single viscous finger formation phenomenon in a 2D channel. Initially, a channel with finite width H is fully saturated with high viscosity fluid 2. Fluid 1 with low viscosity is introduced at the channel inlet with a fully developed velocity profile and displaces Fluid 2 from channel. During this displacement process the viscous finger becomes fully developed and produces a steady state shape (constant finger tip velocity) as shown in Fig. 5.

Average velocity of the inlet fluid 1 is V1. Viscosity ratio \(M = \mu_2/\mu_1\) is 20 which is generally experienced during the water
flooding process where water displaces oil. No-slip boundary conditions are applied at the channel walls in y-direction whereas velocity inlet and pressure outlet boundary conditions are applied in x-direction. Densities of fluid 1 and 2 have been chosen equal \( \rho_1 = \rho_2 = \rho \) for simplicity as the density ratio does not affect the finger formation and its properties. Capillary and Reynolds numbers defined using the steady state finger tip velocity \( V_t \) are

\[
Ca = \frac{l_2 V_t}{\sigma} \quad \text{and} \quad Re = \frac{\rho V_t H}{\mu_c}.
\]

From the mass balance, one can find \( V_t = \frac{V_1 H}{W} \). To remain in the Stokes regime, the values of \( Ca \) and \( Re \) have been chosen such that \( ReCa/C_2 < 10^3 \).

Simulations have been performed with different \( Ca \) ranging from 0.025 to 3 with 64 grid cells across the height \( H \). The length of the channel \((=8H)\) is sufficient for the finger to attain steady shape. The steady state finger width \( W \) and finger tip radius \( R \) have been obtained from the simulations. Fig. 6 compares the same finger characteristic parameters in dimensionless form with the results from literature obtained using different numerical methods. Note that Halpern and Gaver (1994) uses a boundary element method; Yang and Boek (2013) uses a color gradient and free energy model of Lattice-Boltzmann method and Kang et al. (2004) uses the Shan-Chen model within Lattice-Boltzmann framework for the same simulations. Our results of \( W/H \) show an excellent match with maximum deviation to be less than 2%. Also, \( R/H \) shows an excellent match at low \( Ca \). However at higher \( Ca \), the observed deviation is higher due to higher tip curvatures.

### 3.3. Contact force calculation

In numerical simulations of multiphase flows involving solids and multiple fluids, it is essential to compute the contact force at the three phase contact line accurately. Hence, we present a grid independence study of the contact force in 3D for our IBM-VOF coupling. Initially, an oil droplet of equivalent radius \( R_{eq} = 1 \) mm \((\text{Volume } V = \frac{4}{3} \pi R_{eq}^3)\) is placed on the solid sphere of radius \( R_s = 1 \) mm such that it inscribes \( \theta = 60^\circ \) as shown in Fig. 7. In this position, droplet radius \( R_d = 1.1082R_{eq} \), contact radius \( R_c = 0.9069R_{eq} \) and \( \varphi = 35.08^\circ \). Readers are referred to Patel et al. (2017) for the detailed calculation. The oil droplet is surrounded by water with \( \sigma = 0.03 \) N/m. Here, physical properties of the fluids as well as droplet location are chosen such that the computations proceed efficiently. Changing these parameters doesn’t affect the final results. The analytical expressions for the contact force derived by a simple force balance are as follows,

\[
F_y;\text{analytical} = 2\pi \sigma R_c \sin \varphi
\]

\[
F_x;\text{analytical} = F_z;\text{analytical} = 0
\]

Simulations are performed to compute the numerical value of contact force in all three directions using the method proposed by Washino et al. (2013). The relative error in the contact force in the y-direction is given by following expression,

\[
\text{Error (\%)} = \left| \frac{F_y;\text{analytical} - F_y;\text{numerical}}{F_y;\text{analytical}} \right| \times 100\%
\]
4. Results

In this section, we present results on the mobility of multiphase flow through porous media. Our focus will be specifically on the oil-water flow through porous media as experienced in water flooding process. Depending upon the capillary number of the flow and viscosity ratio of oil and water, four different types of flow regimes are observed (Zhang et al., 2011; Lenormand et al., 1988): (i) stable displacement, (ii) unstable displacement, (iii) viscous fingering and (iv) capillary fingering. In the stable displacement regime almost all oil is removed from a porous media, whereas in the viscous and capillary fingering regimes a (nearly) constant amount is displaced. Present simulations are performed in the unstable displacement regime as the mobility of fluids has a strong correlation with its physical properties in this regime. Two types of pore structures have been selected for this purpose: (i) a repeated single pore and (ii) a random multi-pore arrangement. The reason for this selection is to understand the difference how viscous fingers penetrate through an individual pore in contrast to a pore network. For the present results, readers are advised to consider ‘oil’ and ‘water’ as two general immiscible fluids with their own physical properties rather than any specific fluids.

4.1. Repeated single pore

In this section, we simulate a viscous finger penetrating an array of 3D periodically repeated single pores as shown in Fig. 9. An individual pore is the void in the center of the cube with size $H$ that has spheres of diameter $D$ at its eight vertices. Here the ratio $H/D$ is chosen such that the porosity ($\phi$) is 0.6. A channel like computational domain comprises of two regions: (i) the entry region and (ii) the pore region. Initially, the whole computation domain is considered to be fully saturated with oil (a high viscosity fluid). Water (a low viscosity fluid) is introduced into the domain from the entry region with a constant velocity $V_m$. A no-slip boundary condition is applied at the channel walls in $y$- and $z$-directions for the entry region. This will result into the generation of a viscous finger in the entry region as water displaces oil. Sufficient length ($5H$) is provided in the entry region for the viscous finger to achieve steady state before it penetrates the pore region. Periodic boundary conditions are applied in $y$- and $z$-directions for the pore region and a constant pressure boundary condition is applied at the outlet.

Simulations are performed with different capillary numbers ($Ca = \mu_{oil}V_m/\sigma$) ranging from 0.1 to 2.5. The viscosity ratio of oil and water ($M = \mu_{oil}/\mu_{water}$) is 20, densities are equal ($\rho_{oil} = \rho_{water} = \rho$) and the Reynolds number ($Re = \rho V_m H/\mu_{oil}$) is 1. The spheres’ surfaces and no-slip channel wall (in entry region) are oil-wet with a contact angle of 30°. However, Kang et al. (2004) shows that the wetting properties of the no-slip wall doesn’t affect the finger properties. It is important to note that there is no contact between the viscous finger and pore structure during penetration for the given set of simulation parameters. We look at mainly two criteria to quantify the mobility of oil-water flows through porous media: (i) oil saturation in the pore ($S_{oil}$) and (ii) dimensionless phase pressure difference ($\Delta P_{phase}$). $S_{oil}$ is a fraction of oil present among the total fluid (oil + water) in a pore. From Eq. 10, $\Delta P_{phase}$ is defined as below:

$$\Delta P_{phase} = \frac{(P_{water}) - (P_{oil})}{(P_{oil}) - (P_{out})}$$

where $(P_{water})$ and $(P_{oil})$ are volume averaged water and oil phase pressures respectively for multiphase flow. $(P_{in})$ and $(P_{out})$ are area averaged pore inlet and outlet pressures respectively for single phase flow of oil in the same operating conditions. Note that the phase pressure difference ($\Delta P_{water} - \Delta P_{oil}$) for multiphase flow is non-dimensionalized by the pore pressure difference ($P_{oil} - P_{out}$) of single phase flow to obtain $\Delta P_{phase}$.

Fig. 9 shows that the pore region of the computational domain has total 7 pores arranged in series. Our first target is to verify weather this many pores is enough to mimic the periodic behavior of the viscous finger in a single pore. For this purpose, a simulation is performed at $Ca = 0.5$ with 50 grid cells across $H$ to compute $S_{oil}$ and $\Delta P_{phase}$ for each individual pore. Fig. 11 shows a plot of $\Delta P_{phase}$ with $S_{oil}$ for the individual pores in the pore region. Note that the numbers 1–7 indicate the pores in ascending order along the flow direction. That means viscous finger will first penetrate pore 1 and then travel further downstream towards pore 7. During the initial penetration, plots for pore 1 and 2 shows a large deviation. However, as the finger goes further downstream the deviation between

![Fig. 8. Grid independence study for contact force in y-direction: present (●) and 1st order line (–) for reference.](image-url)
subsequent pores (4–7) decreases. The finger attains periodic behavior in the pore region from pore 4 and hence all the further results will be presented only for the representative pore 4. Note that the simulation is performed until the viscous finger reaches the outlet to eliminate the outlet effects on final results.

Simulations are also performed to check for sufficient grid independence of the present results. Total three grid resolutions ($H/\Delta = 32, 50$ and $75$) with $Ca = 0.1$ and $0.5$ are chosen for this purpose. Obtained results are presented in Fig. 12 as plots of (a) $\Delta P_{\text{phase}}$ with $S_{\text{oil}}$ and (b) $S_{\text{out}}$ with $\tau$. Note that, $\tau = t/T_{\text{res}}$ is a dimensionless time with $T_{\text{res}} = \phi H/V_{\text{in}}$ being the pore residence time. Characteristics of these plots will be explained in the subsequent text. The $L^2$ relative error norm (defined with respect to the finest grid) is used to quantify the difference as follows:

$$L^2 = \sqrt{\sum_{i=1}^{N} \left( q_{\text{grid}} - q_{i} \right)^2 \sum_{i=1}^{N} q_{\text{grid}}^2}$$

where $G75$ is the finest grid corresponding to $H/\Delta = 75$, $G$ is any grid corresponding to $H/\Delta = 32$ or $50$, $q$ is the quantity on which the norm is defined, $i$ is the index for sampling points and $N$ is total number of sampling points. Table 1 lists the $L^2$ relative error norm for the data presented in Fig. 12(a) and (b). It is clear that $H/\Delta = 50$ can produce sufficiently grid independent results considering both accuracy and computational time. Hence, all the subsequent simulations are performed with $H/\Delta = 50$ which is equivalent to $D/\Delta = 45.7078$.

Further simulations are performed to quantify the effect of $Ca$ on the single pore water flooding process. A wide range of $Ca$ (from 0.1 to 2.5) is chosen for this purpose and relevant quantities ($\Delta P_{\text{phase}}$ and $S_{\text{out}}$) are measured up to $\tau = 1$. Fig. 13(a) shows a plot of $\Delta P_{\text{phase}}$ with $S_{\text{out}}$ and (b) a plot of $\delta_{\text{out}}$ with $\tau$ for all $Ca$. Initially, the pore is fully saturated with oil and hence the results starts with $S_{\text{out}} = 1$. As the viscous finger tries to enter the pore, it has to squeeze through the pore and overcome the pore resistance. While doing so, the shape of the finger changes and becomes more concave (increased curvature). Due to this shape change the capillary pressure of the water increases which in-turn increases $\Delta P_{\text{phase}}$ during the initial penetration. We term this initial resistance as an ‘entry barrier’ ($\Delta P_{\text{phase max}}$) which a finger needs to overcome to penetrate the pore. With decrease in $Ca$, this ‘entry barrier’ keeps on increasing due to the increase in capillary pressure. As the finger passes through the pore at fixed $Ca$, $\Delta P_{\text{phase}}$ keeps on decreasing mainly due to two reasons: (i) oil (with higher viscosity) is being replaced by water (with lower viscosity) which has a lower hydrodynamic pressure gradient and (ii) the finger shape relaxes and its curvature reduces which decreases the capillary pressure of the water phase. We consider the pore drainage process to have ended at $\tau = 1$ as the oil displacement process mostly occur up to this time. At the end of the drainage process, $\Delta P_{\text{phase}}$ for higher $Ca$ shows some oscillations. Capillary forces are lower compared to hydrodynamic forces at higher $Ca$. Hence the finger can’t hold its shape steady and keeps on oscillating around its mean position. At lower $Ca$, finger retains its shape and hence these oscillations are minimal or non-existent. One interesting thing to note is that at the end of drainage process, $S_{\text{out}}$ is higher for higher $Ca$. In other words,
more oil is remaining in the pore after drainage at higher \( \text{Ca} \). This is contrary to what is observed in the multi-pore setup, details of which will be discussed in Section 4.2.1.

### 4.2. Random multi-pore

In this section, we present the drainage of oil from a model porous media (multi-pore structure) as shown in Fig. 14. The pore structure of porosity 0.6 is made of multiple spherical particles (total 100) with diameter \( D \) arranged in random fashion. The random arrangement has been obtained from the standard hard sphere Monte-Carlo method (Frenkel and Smit, 2001) and is periodic in \( y \) - and \( z \) -directions. The computational domain (lateral dimension \( H \)) is comprised of total 3 regions: (i) entry region \( H \), (ii) pore region \( 3H \) and (iii) exit region \( 3H \). The values between brackets indicate the length of a particular region in the flow direction. Similar to the single pore simulations, the whole computational domain is fully saturated with oil initially. Water is introduced into the domain from the entry region with a constant velocity \( \left( V_{\text{in}} \right) \). Flow is periodic in \( y \) - and \( z \) -directions. Water from the entry region penetrates the pore region with multiple fingers and displaces oil into the exit region. Sufficient length is provided in the exit region to avoid the direct contact of displaced oil (coming out of the pore region) with the flow outlet. Constant pressure boundary condition is applied at the outlet.

Simulations are performed to quantify the effect of capillary number \( \text{Ca} = \frac{\mu_{\text{oil}} V_{\text{in}}}{\sigma} \), contact angle \( \theta \) and viscosity ratio \( \left( M = \frac{\mu_{\text{oil}}}{\mu_{\text{water}}} \right) \) on the mobility of oil. Densities of both fluids are set equal \( \left( \rho_{\text{oil}} = \rho_{\text{water}} = \rho \right) \) for the sake of simplicity and the

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Fig. 12. Grid independence study for single pore simulations with grid resolutions \( (H/\Delta) \) of 32, 50 and 75: (a) \( \Delta P_{\text{phase}} \) with \( S_{\text{oil}} \) plot and (b) \( S_{\text{oil}} \) with \( \tau \) plot for \( \text{Ca} = 0.1 \) and 0.5.

Fig. 13. Effect of \( \text{Ca} \) on single pore water flooding process: (a) \( \Delta P_{\text{phase}} \) with \( S_{\text{oil}} \) plot and (b) \( S_{\text{oil}} \) with \( \tau \) plot for \( \text{Ca} = 0.1 \) to 2.5.

Fig. 14. Schematic diagram of water flooding process through multi-pore structure.
Reynolds number \((Re = \rho V_a D / \mu_a)\) is 1. 80 grid cells are placed across the lateral height of the computational domain \((H/\Delta = 80)\) which results into \(D/\Delta = 22.72\). Each simulation is performed on multi-pore structures of three different random arrangements to obtain better averaged results. Standard deviation of the obtained results with respect to their individual mean value are presented in Table A1 of the appendix A. \(\Delta P_{\text{phase}}, S_{\text{sat}}\) and \(\tau\) are measured during the simulations to quantify the mobility of oil. These quantities are defined similar to Section 4.1 but calculated over the full multi-pore structure. Obtained results are discussed in the subsequent sections.

4.2.1. Effect of capillary number

To quantify the effect of \(Ca\) on the mobility of oil, water-flooding simulations are performed over a range of \(Ca\) from 0.01 to 1 with a fixed \(M = 20\) and \(\theta = 30^\circ\) (oil-wet structure). The results are presented in Fig. 15 as plots of (a) \(\Delta P_{\text{phase}}\) with \(S_{\text{sat}}\) and (b) \(S_{\text{sat}}\) with \(\tau\). Both plots start with \(S_{\text{sat}} = 1\) as the multi-pore structure is fully saturated with oil initially. Fig. 15(a) shows that the ‘entry barrier’ \((\Delta P_{\text{phase, max}})\) keeps on increasing as \(Ca\) is decreased. It becomes progressively more difficult for the viscous fingers to penetrate the multi-pore structure as \(Ca\) is decreased. After the initial penetration, \(\Delta P_{\text{phase}}\) continuously reduces with \(S_{\text{sat}}\). The reason for this behavior in multi-pore structure is similar to what is observed for the single-pore case (Section 4.1). Note that simulations are performed until \(S_{\text{sat}}\) in the multi-pore structure attains a (near) constant value with time. This is referred as residual oil saturation \((S_{\text{sat}})\) subsequently. We consider the flow to be at steady state when the difference of average \(S_{\text{sat}}\) between two consecutive 5000 time-steps windows is less than 1%. This criterion is used to define steady state for the simulations presented in Sections 4.2.2 and 4.2.3 as well.

Fig. 15 shows that the \(S_{\text{sat}}\) keeps on increasing as \(Ca\) is decreased. This means that water flooding at higher \(Ca\) yields more oil from the multi-pore structure. Li et al. (2005) and Lenormand et al. (1988) reported a similar trend in their simulations and experiments. However, Fig. 13 shows the exactly opposite trend for the single pore simulations. The reason for this behavior is as follows. In the single pore water flooding simulations a viscous finger can only travel one path i.e. subsequent pores arranged in series. This arrangement is similar to the test case presented in Section 3.2 where a viscous finger travels into a 2D channel. Hence, the results obtained for a viscous finger penetrating an aligned pore arrangement (Fig. 13) and a 2D channel (Fig. 6) show a similar trend i.e. higher residual oil at higher \(Ca\). However, in the present random multi-pore arrangement viscous fingers have choice to select their own path depending on local flow conditions and pore structure. Naturally, these viscous fingers choose to penetrate the pores with lower ‘entry barriers’ and desaturate them. In this process, the pores with higher ‘entry barrier’ don’t get penetrated and the oil in them remains as it is. Due to the random nature of the pore geometry, the ‘entry barrier’ for each individual pore of the multi-pore structure will be different. These differences will become even more pronounced at lower \(Ca\) due to the higher absolute value of the ‘entry barrier’ (Fig. 13(a)). Hence, viscous fingers in the multi-pore structure at lower \(Ca\) will be thin and elongated. They will only penetrate the series of (random) pores with the least ‘entry barriers’ and keep the rest of the pores untouched which will result into higher \(S_{\text{sat}}\). At higher \(Ca\), differences of ‘entry barriers’ between pores are small. So the viscous fingers will penetrate almost all pores of the multi-pore structure and desaturate them resulting into lower \(S_{\text{sat}}\). Fig. 15(b) shows that the plot of \(S_{\text{sat}}\) for different \(Ca\) follows the same path up to a fixed \(\tau\) (< 1) and after this time the difference becomes apparent. The oil recovery process at higher \(Ca\) is quite slow and takes a lot of time to reach steady state. Note that \(\tau\) is plotted on a logarithmic scale in Fig. 15(b).

Finally, Fig. 16 concludes the investigation into the effect of \(Ca\) on the water flooding process through multi-pore structure. It shows the \(S_{\text{sat}}\) with \(Ca\) along with the best fit line. Note that \(Ca\) is plotted on a logarithmic scale. The coefficient of fitting \((R^2 = 0.9934)\) is very close to 1 which shows the linear trend of \(S_{\text{sat}}\) with \(\log Ca\) is intact for the selected range of parameters. A similar trend has been observed by Lenormand et al. (1988).

4.2.2. Effect of contact angle

In this section, we will discuss the effect of the contact angle \((\theta)\) on the water flooding process through a multi-pore structure. Simulations are performed with different \(\theta\) varying from 15° (oil-wet structure) 90° (neutrally-wet structure) with a fixed \(Ca = 0.1\) and \(M = 20\). The contact angle boundary condition is applied at the three-phase (oil-water-solid structure) contact line as discussed in Section 2. For the parametric range selected in this paper, the water-wet structure shows (close to) full oil recovery hence all simulation are performed for either oil-wet or neutrally-wet structures.

Fig. 17(a) shows a plot of \(\Delta P_{\text{phase}}\) with \(S_{\text{sat}}\) for different \(\theta\). Similar to the previous simulations, this plot also starts with \(S_{\text{sat}} = 1\) as the multi-pore structure is fully saturated with oil initially. Examination of the plot reveals that the ‘entry barrier’ or \(\Delta P_{\text{phase, max}}\) is the same for all \(\theta\). In other words, the ‘entry barrier’ is independent of \(\theta\). Plots for different \(\theta\) follows almost the same path with different end points and hence different \(S_{\text{sat}}\). Fig. 17(b) shows a plot of \(S_{\text{sat}}\) with \(\tau\). Similar to the results of Section 4.2.1, plots for different \(\theta\) follows a same path up to a certain \(\tau\) (< 1) and after this time they deviate from each other. As \(\theta\) is decreased, the porous

![Fig. 15](image-url) Effect of capillary number \((Ca)\) on multi pore water flooding process: (a) \(\Delta P_{\text{phase}}\) with \(S_{\text{sat}}\) plot and (b) \(S_{\text{sat}}\) with \(\tau\) plot for \(Ca = 0.01\) to 1 with \(\theta = 30^\circ\) and \(M = 20\).
structure becomes more oil-wet and oil tends to stick to the porous structure due to higher contact forces. This will lead to lower oil recovery and higher soil. For the neutrally-wet structure ($\theta = 90^\circ$), contact forces between oil and solid structure are not as strong and hence almost all oil gets displaced from the structure. The oil recovery process at higher $\theta$ lasts for a longer time whereas it completes faster for smaller $\theta$. However, the difference of drainage time with $\theta$ is not as sharp as observed in Section 4.2.1 with Ca. Note that $\theta$ is plotted on a linear scale in Fig. 17(b). Fig. 18 presents the soil with $\theta$ along with best fit line. The coefficient of fitting $R^2 = 0.9928$ is very close to 1 which shows that soil follows a well defined linear trend with $\theta$ for the selected parametric range.

4.2.3. Effect of viscosity ratio

In this section, we will quantify the effect of viscosity ratio ($M = \mu_{oil}/\mu_{water}$) on the mobility of oil through a multi-pore structure. Simulations have been performed for $M$ ranging from 0.3125 to 80 with a fixed Ca = 0.025 and $\theta = 45^\circ$ (oil-wet structure). Note that $M < 1$ would mean that the oil is less viscous than water.

Fig. 19(a) shows a plot of $P_{phase}$ with $S_{sat}$ for different $M$. First thing to note is that $\Delta P_{phase}$ is strongly influenced by $M$. As $M$ decreases, $\Delta P_{phase,max}$ rapidly increases and attained at progressively lower $S_{sat}$. Results presented in Fig. 19(a) match quite well with those obtained by Ferrari and Lunati (2013) using 2D multiphase simulations through porous media. Fig. 19(b) shows a plot of $S_{sat}$ with $\tau$ for different $M$. As $M$ decreases, $S_{sat}$ in the multi-pore structure decreases. In other words, lower $M$ would result in better oil recovery. Note that no specific trend is found in the time to reach the steady state. As $M$ decreases, oil becomes less viscous compared to water. Specifically for the case of $M < 1$, oil with lower viscosity is displaced out of the pore by water with higher viscosity. Due to the higher viscosity of the pore fluid i.e. water, $\Delta P_{phase}$ increases. Even though a higher amount of oil is recovered at lower $M$, it is achieved at the cost of higher $\Delta P_{phase}$. Obtained results for the steady state are concluded in Fig. 20 with a plot of $S_{sat}$ with $M$. Note that $M$ is plotted on a logarithmic scale. Best fit line ($R^2 = 0.9921$) shows that a linear trend of $S_{sat}$ with log $M$ is well maintained for the present parametric range.

**Fig. 16.** Residual oil saturation ($S_{sat}$) with Ca at $\theta = 30^\circ$ and $M = 20$ for multi pore water flooding process: (i) Present ($\circ$) and (ii) best fit line ($\cdots$): $S_{sat} = -0.2454\log_{10}(Ca) + 0.06890$ with $R^2 = 0.9934$.

**Fig. 17.** Effect of contact angle ($\theta$) on multi pore water flooding process: (a) $\Delta P_{phase}$ with $S_{sat}$ plot and (b) $S_{sat}$ with $\tau$ plot for $\theta = 15^\circ$ (oil-wet) to $90^\circ$ (neutrally-wet) with $Ca = 0.1$ and $M = 20$.

**Fig. 18.** Residual oil saturation ($S_{sat}$) with Ca at $\theta = 45^\circ$ and $M = 20$ for multi pore water flooding process: (i) Present ($\circ$) and (ii) best fit line ($\cdots$): $S_{sat} = -0.2454\log_{10}(Ca) + 0.06890$ with $R^2 = 0.9934$. Results presented in Fig. 18 match quite well with those obtained by Ferrari and Lunati (2013) using 2D multiphase simulations through porous media. Fig. 18(b) shows a plot of $S_{sat}$ with $\tau$ for different $M$. As $M$ decreases, $S_{sat}$ in the multi-pore structure decreases. In other words, lower $M$ would result in better oil recovery. Note that no specific trend is found in the time to reach the steady state. As $M$ decreases, oil becomes less viscous compared to water. Specifically for the case of $M < 1$, oil with lower viscosity is displaced out of the pore by water with higher viscosity. Due to the higher viscosity of the pore fluid i.e. water, $\Delta P_{phase}$ increases. Even though a higher amount of oil is recovered at lower $M$, it is achieved at the cost of higher $\Delta P_{phase}$. Obtained results for the steady state are concluded in Fig. 20 with a plot of $S_{sat}$ with $M$. Note that $M$ is plotted on a logarithmic scale. Best fit line ($R^2 = 0.9928$) shows that a linear trend of $S_{sat}$ with log $M$ is well maintained for the present parametric range.
5. Conclusions

In this paper, we have simulated multiphase flow through porous media to quantify the effect of capillary number ($Ca$), contact angle ($\theta$) and viscosity ratio ($M$) on its mobility. Fully resolved simulations are performed using a finite volume method with pressure-velocity coupling on a staggered Cartesian computational grid. This grid structure is non-body fitting to the complex solid boundaries hence an immersed boundary method (IBM) is used to resolve the interactions between (multiple) fluids and solid. The fluid-fluid interface is tracked by a mass conservative and sharp interface volume of fluid (VOF) method. The fluid-fluid interface gives rise to three-phase contact lines at the solid boundaries and experiences contact forces due to wettability effects. IBM and VOF are coupled by imposing the contact angle as a boundary condition at the contact line to include wettability effects on the macroscopic length scale (Patel et al., 2017). The present IBM-VOF coupling is verified/validated in this paper for the following test cases specifically related to the multiphase flow through porous media: (i) multiphase Poiseuille flow in a channel to validate the continuity of velocity and shear stress at the fluid-fluid interface, (ii) viscous finger formation in a channel to verify the accuracy of the interfacial tension model and (iii) contact force calculation to check its grid convergence.

Water flooding simulations are performed on two type of periodic porous structures: (i) a repeated single pore and (ii) a random multi-pore arrangement. We first presented the pore selection (to mimic periodic behavior) and grid convergence study for the single pore. Temporal evolution of dimensionless phase pressure difference ($\Delta P_{\text{phase}}$) and oil saturation ($S_{\text{oil}}$) have been measured for different $Ca$ as a viscous finger penetrates the single pore. With decrease in $Ca$, the ‘entry barrier’ ($\Delta P_{\text{phase,max}}$) for the viscous finger to penetrate the single pore increases and residual oil saturation ($S_{\text{oil}}$) decreases.

For the random multi-pore arrangement we observed a similar trend for the ‘entry barrier’ with $Ca$ as for the single pore but opposite trend for $S_{\text{oil}}$. The reason for this behavior was discussed in detail as well. Even though a higher amount of oil is removed at higher $Ca$, the drainage time also increases with $Ca$. As the contact angle ($\theta$) varies from $15^\circ$ (oil-wet structure) to $90^\circ$ (neutrally-wet structure), the contact force between the oil phase and solid structure decreases and hence higher oil recovery resulting into lower $S_{\text{oil}}$. Drainage time increases with $\theta$ but the increase is not as sharp as observed with $Ca$. The ‘entry barrier’ remains unaffected with $\theta$. With decrease in viscosity ratio ($M = \mu_{\text{oil}}/\mu_{\text{water}}$), water viscosity increases compared to oil and hence it becomes easier to displace more oil out of the multi-pore structure. $\Delta P_{\text{phase}}$ is greatly influenced by $M$; the ‘entry barrier’ increases rapidly and attained at lower $S_{\text{oil}}$ as $M$ decreases. For the present parametric range $S_{\text{oil}}$ shows a well defined linear trend with $\log Ca \cdot \theta$ and $\log M$ which matches well with the numerical and experimental results presented in Lenormand et al. (1988) and Ferrari and Lunati (2013).

Acknowledgments

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This paper is an extension of a conference paper that was presented at the 12th International Conference on CFD in Oil & Gas, Metallurgical and Process Industries (CFD2017), Trondheim, Norway.
Appendix A. Standard deviation of the reported data

Table A1

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References