Towards optimal aerodynamic design of vertical axis wind turbines: Impact of solidity and number of blades

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A B S T R A C T
The current study systematically analyzes the impact of solidity (s) and number of blades (n) on the aerodynamic performance of 2-, 3- and 4-bladed Darrieus H-type vertical axis wind turbines (VAWTs). Solidity varies within the wide range of 0.09–0.36. A large number of operational parameters, i.e., tip speed ratio (λ), Reynolds number (Re), turbulence intensity and reduced frequency (K) are investigated to provide a deeper insight into the impact of s and n on the dynamic loads on blades, the turbine performance and the wake. High-fidelity unsteady Reynolds-averaged Navier-Stokes (URANS) simulations, extensively validated with experiments, are employed. The results show that the turbine optimal tip speed ratio (λopt) is invariant to a newly-introduced parameter ‘α2’, regardless of the turbine geometrical and operational characteristics. In addition, a new correlation is derived to estimate λopt as a function of s, which can also be employed to predict the optimal s for a turbine with a given λ. It is also found that: (i) for constant-speed urban VAWTs, which due to the low mean wind speed in the urban environment, frequently operate at moderate to high λ, a relatively-low s is optimal; (ii) an optimal VAWT is a moderately-high-solidity variable-speed rotor maintaining a relatively-low λ, where due to the large blade chord length the resulting Re and K are favorably high; (iii) within the turbine optimal operational range, turbine power coefficient (Cp) is almost independent of n. The present findings support the optimal aerodynamic design of small-to large-scale VAWTs.

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1. Introduction

Vertical axis wind turbines (VAWTs) have recently received renewed interest for wind energy harvesting in two new potential locations, i.e., far offshore and in urban and rural environments [1–6]. In far offshore, VAWTs are promising for deep-water floating offshore windfarms [7–10]. In the urban and rural environments, VAWTs can be mounted on the building roof, between the buildings, integrated into the urban and rural settings, inside the wind catchers and ventilation ducts [11–18]. VAWTs have several advantages compared to horizontal axis wind turbines (HAWTs): omni-directionality, low noise, simple design, low manufacturing, installation and maintenance costs, scalability, compactness, small shadow flickering, less visual aesthetic disturbance and higher bird and bat safety [19–24]. However, their aerodynamic performance is currently lower than HAWTs due to the comparatively small amount of research that they have received and due to their more complex unsteady aerodynamics [25–31]. Therefore, to benefit from their many advantages, their aerodynamic performance needs to be further improved.

The complex aerodynamic performance of VAWTs is influenced by flow phenomena such as unsteady separation and dynamic stall [32–40], blade-wake interactions [41], flow curvature effects [42,43] and rotational and Coriolis effects on boundary layer and shed vortices [44]. Such complex flow phenomena are driven by the geometrical and operational characteristics of the turbine, which therefore necessitate a sound understanding of the individual and combined impacts of these characteristics on the aerodynamic performance of the turbine. The impact of the operational characteristics, namely tip speed ratio λ, Reynolds number Re, and turbulence intensity Tt for VAWTs have been extensively studied by Bachant and Wisnik [45], Millet et al. [46] and Rezaeiha et al. [20]. The geometrical characteristics include solidity [47,48], number of blades, airfoil shape [49,50], blade surface roughness, location of blade-spoke connection, pitch angle [51], blade aspect ratio [7], shaft diameter and surface roughness [52].
Solubility and number of blades are two important geometrical parameters, which significantly affect the aerodynamic performance of VAWTs. Solidity shows the ratio of the overall area of the blades over the swept area of the turbine and is defined as \( \sigma = n c / d \), where \( n \), \( c \), and \( d \) are number of blades, blade chord length, and turbine diameter, respectively. An optimal turbine solidity would maximize the output power where the optimum value could be dependent on the operational conditions. The impact of solidity and number of blades has been studied numerically and experimentally \cite{47, 48, 53–61}. However, to the best of our knowledge, the existing literature has the following limitations:

(i) The conclusions have been derived using a limited number of test cases and are dependent on the operational conditions;
(ii) The impact of solidity and number of blades at fixed \( Re_c \) has not yet been investigated. It should be noted that as changing solidity naturally modifies \( Re_c \), therefore, the findings in previous studies could have been influenced by the Reynolds number effects;
(iii) The dependency of the optimal tip speed ratio on the solidity has not been yet elucidated;
(iv) The focus has been mainly on the average turbine performance, i.e., \( C_p \). Detailed analyses of the blade dynamic loads and the turbine wake, however, has been investigated either incompletely or not at all.

Therefore, in this paper:

(i) The impact of solidity is studied within a wide range from a low value of 0.09 to a relatively high value of 0.36 for three turbines with different number of blades, i.e., 2-, 3- and 4-bladed turbines;
(ii) A wide range of operational conditions is considered. Tip speed ratio varies from a low value of 1.5, where the flow is dominated by the dynamic stall and the blade-wake interactions, to a high value of 5.5, where although the flow is mostly attached, the rotational effects are significant. Turbulence intensities vary from 0%, representative of high-quality low-turbulence wind tunnels, to 25%, representative of relatively high turbulence level environments. The chord Reynolds number varies from 0.51 \( \times 10^5 \) to 6.41 \( \times 10^5 \), which covers the relevant range for small-to-medium-scale VAWTs. Reduced frequency \( (K) \) varies from 0.02 to 0.18, which covers weak-to-highly-unsteady flows;
(iii) The dependency of the optimal solidity on tip speed ratio is comprehensively investigated;
(iv) A new parameter is introduced to which the turbine optimal tip speed ratio \( \lambda_{opt} \) is invariant, regardless of the turbine geometrical and operational characteristics;
(v) A new correlation is derived to estimate \( \lambda_{opt} \) only as a function of \( \sigma \), which can be helpful during the initial design phase of VAWTs;
(vi) Detailed analyses are performed at constant \( Re_c \) to better clarify the impact of solidity and number of blades, unaffected by any potential Reynolds number effects;
(vii) In addition to the turbine averaged performance, the turbine wake and dynamic loads on the blades are also investigated for various solidities and number of blades;
(viii) Reduced frequency is shown to be an influential parameter on the aerodynamic performance of VAWTs.

The evaluation is based on high-fidelity CFD simulations, extensively validated with experiments. In total 296 simulations are performed. The findings of this study are intended to provide an in-depth understanding of the impact of solidity and number of
blades on the aerodynamic performance of VAWTs and to support the optimal design of VAWTs by elucidating the influence of various operational parameters.

The outline of the paper is as follows: the computational settings and parameters are described in Section 2. Section 3 briefly reviews the validation studies. The test cases are described in Section 4. The results and discussion of the impact of solidity and number of blades are presented in Sections 5 and 6, respectively. Conclusions are provided in Section 7.

2. Computational settings and parameters

The reference turbine is a Darrieus H-type VAWT with the same geometrical and operational characteristics as in the wind tunnel experiment by Tesione et al. [62], (Table 1 and Fig. 1a). The blade cross-section is the symmetric NACA0018 airfoil. The location of the blade-spoke connection is half-chord, where a single connection is used. The shaft rotates at the same rotational velocity and direction as the turbine, i.e., counter-clockwise.

As the focus of the study is on the midplane of a turbine with high blade aspect ratio, h/c > 10 [7,62], where the 3D blade tip effects are avoided, 3D simulations are not considered. An extensive comparison of 2D and 2.5D results revealed that the 3D boundary layer effects result in a systematic difference of less than 6% and 2% in the calculated turbine CP and CT, respectively, where the difference is weakly sensitive to the operational conditions [20]. Therefore, with respect to the large number of simulations performed in the present work for the extensive parametric study, i.e., 296 URANS simulations, 2D simulations are selected in view of limiting the computational costs.

The 2D computational domain is 35d × 20d and consists of a rotating core and a fixed domain surrounding the core as shown in Fig. 1b. The 2D blockage ratio (d/W) is 5%. The size of the computational domain is based on the guidelines for CFD simulations of VAWTs [63,64].

The computational grid, shown in Fig. 1c–e, consists of approximately 400,000 quadrilateral cells with a maximum and an average y+ value of 3.8 and 1.4 on the blade surfaces, respectively. The grid is based on a grid sensitivity analysis using three uniformly refined grids with a linear refinement factor of γ/2. The Grid Convergence Index (GCI) [65], calculated using CP values with a safety factor (Fg) of 1.25, is 3.5 × 10^-2, which is 0.85% of the respective CP value. Further detailed information about the grid-sensitivity analysis is presented in Ref. [64].

The boundary conditions at the inlet and outlet of the domain are a uniform velocity inlet and zero gauge static pressure outlet. Symmetry condition is used for the side faces. No-slip condition holds on the airfoil and shaft walls. Sliding grid interface is employed for the interface between the rotating and fixed grids.

In the CFD simulations, the approach-flow (i.e., inlet) total turbulence intensity is 5% with a turbulence length scale equal to the turbine diameter, i.e., 1 m. The incident-flow total turbulence intensity is 4.42% representing the real value experienced by the turbine. The incident-flow value corresponds to the total turbulence intensity at the turbine incidence calculated using an empty domain and is lower than the approach-flow value due to the turbulence decay in the computational domain [64,66,67].

The four-equation transition SST model [68], also known as Y-Reθ model, is employed to model the turbulence. This model is developed based on the two-equation SST k-ω turbulence model [69,70] which has been further improved to predict the laminar-to-turbulent transition for wall-bounded flows. The improvement is obtained by solving two other transport equations, one for the intermittency γ and one for the momentum-thickness Reynolds number Reθ. The model includes empirical correlations developed by Langtry and Menter [68] to trigger the transition model so that the model can recognize the flows where the laminar-to-turbulent transition might not be present in the flow, e.g. low freestream turbulence flows or high turbulence standard bypass transition flows. Further details of the model can be found in Refs. [68,71].

The intermittency-based transition models have been shown to improve the flow prediction in transitional regimes of wall-bounded flows, such as flow on airfoils, cylinders, etc., [71–76]. Previous studies have shown the good performance of such transition models for different applications, e.g. Suzen et al. [73,74], Walters and Leylek [77], Cutrone et al. [78].

Incompressible unsteady Reynolds-Averaged Navier-Stokes (URANS) simulations are performed using the commercial CFD software ANSYS Fluent 16.1 [79] with the SIMPLE scheme [80] for pressure-velocity coupling and the second-order temporal and spatial discretization. The azimuthal increment, dθ, is 0.1°, which is based on CFD guidelines for VAWTs [63,64]. The number of iterations per time step is 20. A number of 20 turbine revolutions are performed, and the results are sampled at the 21st turbine revolution. This value is based on a comprehensive convergence analysis and allows the results to reach a statistically steady-state condition [63,64].

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**Table 1**

Geometrical and operational characteristics of the reference turbine.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of blades, n</td>
<td>2</td>
<td>Airfoil chord, c [m]</td>
<td>0.06</td>
</tr>
<tr>
<td>Diameter, d [m]</td>
<td>1</td>
<td>Shaft diameter, d1 [m]</td>
<td>0.04</td>
</tr>
<tr>
<td>Height, h [m]</td>
<td>1</td>
<td>Freestream velocity, U∞ [m/s]</td>
<td>9.3</td>
</tr>
<tr>
<td>Swept area, A [m²]</td>
<td>1</td>
<td>Rotational velocity, Ω [rad/s]</td>
<td>27.9–102.3</td>
</tr>
<tr>
<td>Solidity, σ</td>
<td>0.12</td>
<td>Tip speed ratio, λ</td>
<td>1.5–5.5</td>
</tr>
<tr>
<td>Blade aspect ratio, h/c</td>
<td>16.67</td>
<td>Chord Reynolds number, Reₜ</td>
<td>0.69 × 10³–2.14 × 10⁵</td>
</tr>
</tbody>
</table>
3. Validation study

Two sets of validations have been performed to ensure the accuracy of the CFD simulations, where the CFD results are compared with the experimental data by Tescione et al. [62] and Castelli et al. [81]. The two turbines employed in the two validations have different number of blades, solidity, airfoil shape, tip speed ratio, and Reynolds number. A detailed comparison of the CFD results with the experimental data and a comprehensive explanation of the possible reasons for the deviations between the numerical and experimental data are discussed in Refs. [20,51,52,64], which for brevity are only briefly repeated here:

1) Wake velocity of the 2-bladed reference turbine: the time-averaged (over one turbine revolution) streamwise and lateral normalized velocities at different downstream positions, x/R = 2.0–3.0, along the lateral direction, −0.75 ≤ y/R ≤ 0.75, in the wake of a 2-bladed turbine (reference turbine) were compared with the experimental data by Tescione et al. [62]. The deviations are 8.6%–11.8% for the streamwise velocity component and 2.3%–2.5% for the lateral velocity component corresponding to the least and the most downstream positions.

2) Power coefficient of a 3-bladed turbine: the power coefficient of a 3-bladed turbine with a diameter of 1.03 m, solidity of 0.25 and blade cross-section of NACA0021 airfoil was compared with the experimental data by Castelli et al. [81] at different tip speed ratios from 2.04 to 3.08 (see Table 2). A good agreement between the numerical results and the experimental data was achieved where the minimum deviation of 3.42% is present at λ = 2.04. The maximum deviation of 23.2% is observed at the highest λ of 3.08.

4. Test cases

The present work is a characterization study aimed at revealing the previously-unknown aspects of the impact of solidity and number of blades on the aerodynamic performance of VAWTs. Therefore, the two parameters are systematically studied within a wide and meaningful range. Such a systematic analysis is needed to provide a comprehensive picture of the influence of the two aforementioned parameters under various operational conditions.

The impact of the turbine solidity on the aerodynamic performance of VAWTs is studied for a wide range of solidities from a low value of 0.09 to a relatively-high value of 0.36 (see Table 3). Note that, in the present study, the solidity is changed via the blade chord length, which at a given turbine operational condition, inevitably modifies Re. Note that in the present study for turbines with different solidities but identical number of blades and tip speed ratio, Re is kept constant by scaling the blade chord length, freestream and rotational velocities, see Section 5.3.

The study is performed for turbines with different number of blades, i.e. 2, 3 and 4, and at different tip speed ratios ranging from a low value of 1.5, where dynamic stall occurs, to a high value of 5.5, where the flow is mostly attached but rotational effects are considerable, in order to analyze the corresponding impact for each regime. The Re varies from 0.51 × 10⁵ to 6.41 × 10⁷, which covers the range for the small-to medium-scale VAWTs. Different values of approach-flow (i.e. inlet) turbulence intensities from 0%, representative of low-turbulence wind tunnel tests, to 25%, representative of relatively-high turbulence operating environments, are investigated. The incident-flow total turbulence intensities are presented in Table 4. The K varies from 0.02 to 0.18 covering a wide range of unsteadiness in the flow.

A detailed analysis of the effect of solidity on the turbine wake and the dynamic loads on the blades at constant Re is performed for a 2-bladed turbine, see Section 5.3.

<table>
<thead>
<tr>
<th>n</th>
<th>σ [cm]</th>
<th>c [cm]</th>
<th>K</th>
<th>λ = 1.5–5.5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Reᵢ × 10⁵</td>
</tr>
<tr>
<td>2-bladed</td>
<td>0.09</td>
<td>4.5</td>
<td>0.045</td>
<td>0.51–1.60</td>
</tr>
<tr>
<td></td>
<td>0.12</td>
<td>6</td>
<td>0.06</td>
<td>0.69–2.14</td>
</tr>
<tr>
<td></td>
<td>0.18</td>
<td>9</td>
<td>0.09</td>
<td>1.03–3.20</td>
</tr>
<tr>
<td></td>
<td>0.24</td>
<td>12</td>
<td>0.12</td>
<td>1.38–4.27</td>
</tr>
<tr>
<td></td>
<td>0.30</td>
<td>15</td>
<td>0.15</td>
<td>1.72–5.34</td>
</tr>
<tr>
<td></td>
<td>0.36</td>
<td>18</td>
<td>0.18</td>
<td>2.07–6.41</td>
</tr>
<tr>
<td>3-bladed</td>
<td>0.09</td>
<td>3</td>
<td>0.03</td>
<td>0.51–1.60</td>
</tr>
<tr>
<td></td>
<td>0.12</td>
<td>4</td>
<td>0.04</td>
<td>0.69–2.14</td>
</tr>
<tr>
<td></td>
<td>0.18</td>
<td>6</td>
<td>0.06</td>
<td>1.03–3.20</td>
</tr>
<tr>
<td></td>
<td>0.24</td>
<td>8</td>
<td>0.08</td>
<td>1.38–4.27</td>
</tr>
<tr>
<td></td>
<td>0.30</td>
<td>10</td>
<td>0.10</td>
<td>1.72–5.34</td>
</tr>
<tr>
<td></td>
<td>0.36</td>
<td>12</td>
<td>0.12</td>
<td>2.07–6.41</td>
</tr>
<tr>
<td>4-bladed</td>
<td>0.09</td>
<td>2.25</td>
<td>0.0225</td>
<td>0.51–1.60</td>
</tr>
<tr>
<td></td>
<td>0.12</td>
<td>3</td>
<td>0.03</td>
<td>0.69–2.14</td>
</tr>
<tr>
<td></td>
<td>0.18</td>
<td>4.5</td>
<td>0.045</td>
<td>1.03–3.20</td>
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<tr>
<td></td>
<td>0.24</td>
<td>6</td>
<td>0.06</td>
<td>1.38–4.27</td>
</tr>
<tr>
<td></td>
<td>0.30</td>
<td>7.5</td>
<td>0.075</td>
<td>1.72–5.34</td>
</tr>
<tr>
<td></td>
<td>0.36</td>
<td>9</td>
<td>0.09</td>
<td>2.07–6.41</td>
</tr>
</tbody>
</table>
The impact of the number of blades on the aerodynamic performance of VAWTs is studied by comparing 2-, 3-, and 4-bladed turbines at different tip speed ratios and solidities (see Table 3). Note that for turbines with different number of blades but identical solidity and tip speed ratio, Re is kept constant, by scaling the freestream and rotational velocities, to avoid the Reynolds number effects [20].

5. Impact of solidity

5.1. Turbine performance

Fig. 2a shows the power coefficient versus tip speed ratio for 2-bladed VAWTs with different solidities from 0.09 to 0.36. It can be seen that a low-solidity turbine obtains its highest $C_p$ ($C_{p_{\text{max}}}$) at moderately-high $\lambda$ while a high-solidity turbine works optimally at relatively-low $\lambda$. For instance, a turbine with $\sigma = 0.09$ works optimally at $\lambda = 4.5$, while the optimal $\lambda$ ($\lambda_{\text{opt}}$) of a turbine with $\sigma = 0.36$ shifts to $\lambda = 2.0$. This implies that the geometrical design of a constant-speed VAWTs, which inevitably has variable $\lambda$ due to the changes of the incoming flow, is only optimal for one particular $\lambda$ and the turbine will be working sub-optimally in the majority of the operating conditions. Fig. 2a also indicates that with the increase of $\sigma$ from 0.09 to 0.36, $C_{p_{\text{max}}}$ increases by 22.5%.

For constant-speed urban VAWTs, which during their design condition most frequently operate at moderately-high $\lambda$ regime due to the low mean wind speed in the urban environment, a low solidity is shown to be optimal, e.g. for a turbine dominantly operating at $4.0 \leq \lambda \leq 5.5$, a solidity range of $0.1 \leq \sigma \leq 0.15$ is preferred. This is while a higher solidity will result in a dramatically unfavorable $C_p$. Note that the design condition, which is the focus of this study, refers to when the wind speed has already exceeded the

![Fig. 2. Power coefficient versus (a) tip speed ratio and (b) solidity for 2-bladed VAWTs (TI = 5%).](image)

![Fig. 3. Contours of power and thrust coefficients in $\lambda - \sigma$ space for 2-, 3- and 4-bladed VAWTs with the same Re, at identical $\lambda - \sigma$ positions. Each contour plot is based on 48 simulations.](image)

![Fig. 4. Contours of power coefficient in $\lambda - \sigma$ space for 2-bladed VAWTs with different approach-flow turbulence intensities. Each contour plot is based on 48 simulations.](image)
turbine cut-in velocity and the start-up phase, during which the turbine starts to rotate and reaches the designed rotational speed, has passed. Also note that for constant-speed urban VAWTs, moderate to high λ is unavoidable because, given their small size and blade chord length, in order to avoid the undesirable low Reynolds number effects, they need to rotate relatively fast to keep a sufficiently high Re (≥10^5).

Fig. 2b shows the power coefficient versus solidity for 2-bladed VAWTs at constant λ, which is the case for variable-speed VAWTs. It can be seen that for variable-speed (constant λ) VAWTs, the optimal σ, where C_p,max is delivered, decreases with the increase of λ. For instance, the optimal σ for a turbine constantly operating at λ = 3.0 is σ = 0.24. The optimal σ reduces to 0.18 when the turbine is operated at a higher λ of 3.5. Moreover, the value of C_p,max slightly increases for turbines with higher optimal σ (operating at lower λ). The increment in C_p,max for higher σ is due to the Reynolds number effects, i.e., higher Re of the blades [20]. Detailed analysis provided in Section 5.3 further confirms this. The finding implies that a turbine with a higher optimal σ slightly benefits from the Reynolds number effects due to the larger blade chord length. Therefore, from an aerodynamic point of view, an optimal variable-speed VAWT is a relatively-high-solidity turbine operating at a moderately-low λ. In the present study, the optimum occurs for a turbine with σ = 0.3 operating at λ = 2.5. However, a turbine with a higher optimal σ = 0.36 operating at λ = 2.0 has a comparatively lower Re, and C_p,max.

The optimal design of a VAWT can be more clearly inferred from the contours of C_p illustrated in λ – σ space shown in Fig. 3a. Fig. 3b and c show similar C_p contours for 3- and 4-bladed turbines where a similar trend is observed. The contours explicitly highlight the optimal operation of the turbine with respect to the solidity and tip speed ratio while also elucidating the regions of the poor turbine performance which need to be avoided, i.e., a high-solidity turbine at moderate to high λ or a low-solidity turbine at low λ. These figures can serve as a design guide for designers and manufacturers of VAWTs for different operating conditions.

The turbine thrust coefficient C_T in λ – σ space for 2-, 3- and 4-bladed VAWTs are exhibited in Fig. 3d–f. It can be seen that the C_T values grow by increasing both σ and λ, where the minimum and the maximum C_T values correspond to the lowest and the highest σ and λ combinations.

The impact of solidity on the turbine performance as a function of TI is further investigated at TI = 0%, 15%, and 25%. Fig. 4 shows the contour of power coefficient in λ – σ space for 2-bladed VAWTs with turbulence intensity from 0% to 25%. Fig. 4 reveals that the trend of C_p in λ – σ space is independent of TI. This is of significant importance for the design of variable-speed (constant λ) VAWTs as the optimal aerodynamic design, irrespective of the turbulence level, will remain a relatively high-solidity turbine operating at a moderately-low tip speed ratio regime.

As presented in Figs. 3 and 4, the optimal performance of the turbine is obtained either at a combination of low solidity and high tip speed ratio or vice versa. This means that, at a given wind speed, when the turbine is rotating faster (higher λ), the optimal performance is achieved with a smaller blade area (lower σ) while if the turbine is rotating slower (lower λ) the optimal performance is obtained for a larger blade area (higher σ). The shift could be correlated with a balance in the output power of the turbine as follows. One can consider λ and σ as dimensionless measures of blade velocity and surface area, respectively. As the power is proportional to area times the cubic velocity, i.e., C_p can be considered as C_p = f(σλ^3). Therefore, the newly-defined parameter, σλ^3, can be employed to better investigate the invariance of the turbine C_p values. Fig. 5 illustrates C_p versus σλ^3 for 2-, 3- and 4-bladed turbines. The figure shows that the σλ^3 value corresponding to optimal performance, i.e. maximum C_p is almost independent of the number of blades and solidity. This is an important finding, which highlights the invariance of the optimal operation to the newly-defined parameter σλ^3.
Fig. 6 presents contours of $C_P$ in $s/C_0$-$s_l$ space illustrating the $s_l$ (approximate) invariance of the $C_{P,max}$ for turbines with different geometrical and operational characteristics.

### 5.2. Optimal tip speed ratio

Optimal tip speed ratio $\lambda_{opt}$ is an important design parameter for both constant-speed (variable $\lambda$) and variable-speed (constant $\lambda$) VAWTs. For constant-speed VAWTs, the turbine can be designed to have the $\lambda_{opt}$ in the vicinity of the dominant $\lambda$ to maximize the power output. For variable-speed VAWTs, the turbine can be designed to maintain the $\lambda_{opt}$ during the majority of possible operating conditions. Previous experimental and numerical results have shown the Re-independency of $\lambda_{opt}$ [20,45,46]. However, $\lambda_{opt}$ could be dependent on the other parameters, such as solidity. Therefore, it is of prime importance to analyze such dependencies.

Table 5 presents the optimal performance versus solidity for 2-, 3- and 4-bladed VAWTs (TI – 5%).

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>0.09</th>
<th>0.12</th>
<th>0.18</th>
<th>0.24</th>
<th>0.30</th>
<th>0.36</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Re_0$ ($\times 10^5$)</td>
<td>1.32</td>
<td>1.57</td>
<td>2.09</td>
<td>2.42</td>
<td>2.57</td>
<td>2.56</td>
</tr>
</tbody>
</table>

**Table 6**
The optimal tip speed ratio versus solidity for 2-bladed VAWTs with different approach-flow turbulence intensities.

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>0.09</th>
<th>0.12</th>
<th>0.18</th>
<th>0.24</th>
<th>0.30</th>
<th>0.36</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Re_0$ ($\times 10^5$)</td>
<td>1.32</td>
<td>1.57</td>
<td>2.09</td>
<td>2.42</td>
<td>2.57</td>
<td>2.56</td>
</tr>
</tbody>
</table>

**Table 7**
Details of the data sets employed for the curve fitting.

<table>
<thead>
<tr>
<th>Method</th>
<th>Numerical</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>High-fidelity URANS</td>
<td>Wind tunnel</td>
<td>Water channel</td>
</tr>
<tr>
<td>Turbine type</td>
<td>Darrieus H-type</td>
<td>3</td>
</tr>
<tr>
<td>$n$</td>
<td>3</td>
<td>0.09–0.36</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.51–0.64</td>
<td>1.5–5.5</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.5, 15, 25</td>
<td>0.5, 15, 25</td>
</tr>
<tr>
<td>TI [%]</td>
<td>0, 5, 15, 25</td>
<td>0, 5, 15, 25</td>
</tr>
<tr>
<td>$Re_0$ ($\times 10^5$)</td>
<td>0.51–0.64</td>
<td>1.5–5.5</td>
</tr>
<tr>
<td>$# \lambda_{opt}$ points for fit</td>
<td>36</td>
<td>1</td>
</tr>
<tr>
<td>$#$ test cases to identify $\lambda_{opt}$</td>
<td>288</td>
<td>= 100</td>
</tr>
</tbody>
</table>

Fig. 7. Optimal tip speed ratio vs. solidity with a curve fit using Eq. (1) based on data sets in Table 7.
turbulence. At a given solidity, $\lambda_{\text{opt}}$ is only weakly dependent on $\text{TI}$.

Given the paramount importance of $\lambda_{\text{opt}}$ for design purposes, a quick first approximation of $\lambda_{\text{opt}}$ based on the turbine geometrical parameters is very much desired by the designers and manufacturers of wind turbines. Therefore, in this study, a new correlation between $\lambda_{\text{opt}}$ and $\sigma$ is derived, see Eq. (1). A two-term power function (with 95% confidence levels) is used to fit the CFD results (presented in Tables 5 and 6) and four other experimental data sets. The data sets and the fitted curve are shown in Table 7 and Fig. 7.

$$\lambda_{\text{opt}} = 2.693\sigma^{-0.329} - 1.605$$  \hspace{1cm} (1)

The correlation can be employed in the following two ways:

1) To estimate the $\lambda_{\text{opt}}$ for a turbine with a known geometrical design: for instance when the geometrical design of the turbine, i.e., $\sigma$, is already constrained by the other design drivers, such as the structural requirements. In that case, the turbine optimal rotational velocity can be estimated using the provided correlation for a given wind condition of any potential installation location.

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$c$ [cm]</th>
<th>$K$</th>
<th>$\lambda$</th>
<th>$\text{Re}_t \times 10^5$</th>
<th>$U_\infty$ [m/s]</th>
<th>$\Omega$ [rad/s]</th>
<th>TI [%]</th>
<th>TI [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.12</td>
<td>6</td>
<td>0.06</td>
<td>0.69–2.14</td>
<td>9.3</td>
<td>27.90–102.30</td>
<td>5</td>
<td>4.42</td>
<td></td>
</tr>
<tr>
<td>0.24</td>
<td>12</td>
<td>0.12</td>
<td>4.65</td>
<td>13.95–51.15</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.30</td>
<td>15</td>
<td>0.15</td>
<td>3.72</td>
<td>11.16–40.92</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 8
Description of cases for detailed analysis of the impact of solidity at constant $\text{Re}_t$.

Fig. 8. (a–c) Contours of instantaneous moment coefficient versus $\lambda$ (note the different range of colormaps); (d–f) instantaneous moment coefficient line plot for different $\lambda$; (g) instantaneous power coefficient ($\lambda_{\text{Cm}}$) during the last turbine revolution for different $\sigma$ at their $\lambda_{\text{opt}}$; and (h–i) resulting power and thrust coefficients. Note that $\text{Re}_t$ is constant at identical $\lambda$. 

---

2) To identify the optimal \( \sigma \) for a turbine with a desired \( \lambda_{\text{opt}} \): for instance when the turbine rotational velocity is already constrained by the other design drivers, e.g., the drivetrain rotational speed, and thus the dominant \( \lambda \) is known for a given wind condition. In that case, the optimal turbine \( \sigma \) can be estimated using the provided correlation by solving the equation for \( \sigma \).

Given the Re-independency of \( \lambda_{\text{opt}} \) (see Refs. [20,45,46]), its weak dependency to Ti and the large diversity of the geometrical and operational characteristics of the data sets employed for the curve fitting (Table 7), the proposed correlation can be used for the Darrieus H-type VAWTs with different geometrical characteristics, namely number of blades, airfoil shapes, solidities, and operational conditions, namely \( \lambda \), Ti, K and Re<sub>c</sub>.

5.3. Analysis at constant Re<sub>c</sub>

In order to further elucidate how the solidity influences the turbine performance and wake, a more detailed analysis is performed for 2-bladed turbines with different solidities of 0.12, 0.24 and 0.30 at constant Re<sub>c</sub> for identical \( \lambda \). Note that, for a given \( \lambda \), Re<sub>c</sub> is kept constant by adjusting both U<sub>∞</sub> and \( \Omega \) in the same manner. For instance at \( \lambda = 1.5 \), for a turbine with \( \sigma = 0.12 \) and chord length of 0.06 m, U<sub>∞</sub> and \( \Omega \) are 9.3 m/s and 27.9 rad/s, respectively. This yields Re<sub>c</sub> = 0.69 \times 10<sup>5</sup>. For a turbine with \( \sigma = 0.24 \) and chord length of 0.12 m, the modified U<sub>∞</sub> and \( \Omega \) are 4.65 m/s and 13.95 rad/s, which similarly yields Re<sub>c</sub> = 0.69 \times 10<sup>5</sup>. Table 8 presents the description of the cases.

Fig. 8a–c show contours of the instantaneous moment coefficient during the last turbine revolution versus \( \lambda \) for the different solidities. To further clarify the contours, three additional typical C<sub>m</sub> vs \( \theta \) plots are also presented in Fig. 8d–e, corresponding to the \( \lambda_{\text{opt}} \) of the different solidities. The following observations are made:

- Increasing the solidity is found to substantially delay the early drop in C<sub>m</sub> and avoid the subsequent fluctuations at low \( \lambda \), which occur due to the dynamic stall on the blades (see Fig. 8a and d). For \( \sigma = 0.12 \), the load fluctuations due to the dynamic stall are observable for 1.5 \( \leq \lambda \leq 2.5 \) while they are already absent at \( \lambda = 2.0 \) for \( \sigma = 0.30 \). This is thought to be due to the higher reduced frequency, K, for the higher solidity turbine. For instance, increasing \( \sigma \) from 0.12 to 0.30 scales up K from 0.06 to 0.15, resulting in much higher unsteady aerodynamics over the blade. This consequently delays the flow separation and dynamic stall on blades. The observed trend for the reduced frequency is also in line with the studies on pitching airfoils [84,85]. Further analysis to highlight such impact follows in this section where the boundary layer events are discussed to confirm the reduced frequency effect.

- Increasing the solidity substantially reduces the contribution of the turbine aft half 180° < \( \theta < 360° \) in the total power production. This is explained as a higher solidity turbine extracts more energy from the flow in the turbine fore half 0° < \( \theta < 180° \), which therefore leaves less energy in the flow to be exploited downstream. In addition, higher solidity results in higher blockage to the incoming flow, which eventually increases the induction by the turbine and further slows down the incoming flow. The lesser contribution by the turbine aft half for \( \sigma = 0.30 \) is more pronounced for \( \lambda \geq 3.0 \) where the turbine aft half yields zero to negative C<sub>m</sub> values (see Fig. 8f and f).

Fig. 8g and i show power and thrust coefficients versus \( \lambda \) for the different solidities. Note that Re<sub>c</sub> is constant at identical \( \lambda \). As already discussed in Section 5.2, increasing the solidity shifts \( \lambda_{\text{opt}} \) to lower values while interestingly, at constant Re<sub>c</sub>, C<sub>P,\text{max} \) remains almost independent of \( \sigma \). On the other hand, increasing solidity is found to substantially increase the C<sub>T</sub> due to the higher total blade surface area.

Fig. 8g shows the instantaneous power coefficient (\( \lambda\text{C}_{\text{p}} \)) for the respective solidities at their optimal operating condition, i.e., \( \lambda_{\text{opt}} \). As already shown, the turbines have almost the same C<sub>P,\text{max} \) at \( \lambda_{\text{opt}} \), implying that the integrals of the three curves shown in Fig. 8g are approximately the same.

Fig. 9 shows the normalized cumulative sum of the instantaneous moment coefficient and the contribution of each turbine quartile for the different solidities at their \( \lambda_{\text{opt}} \). It can be seen that a high solidity turbine at the optimal condition produces more power in the upwind quartile, benefitting from the larger blade area, while leaving less energy to be extracted in the downwind quartile. On the other hand, a low solidity turbine can yield less power in the upwind quartile allowing more energy extraction further downstream in the downwind quartile. This somehow shows that due to the inherent unsteady nature of VAWTs, the maximum achievable efficiency using a static design (blades with no active mechanism to modify the local flow over the blade and to modify the angle of attack during the revolution) is basically limited, and regardless of the turbine geometrical design, the optimal efficiency will stay
Fig. 10. Variations of experienced angle of attack, normalized relative velocity, lift and drag coefficients during the last turbine revolution for different σ at constant Re for identical λ.

Fig. 11. Spatiotemporal contours of friction coefficient along the blade suction side (denoted with '-' sign) over the half-revolution for different σ with constant Re, at identical λ.
almost constant. This urges the need for active mechanisms for further improvement of the aerodynamic performance of VAWTs.

Fig. 10 shows the variations of the experienced angle of attack ($\alpha$), normalized relative velocity ($V_{rel}$), lift and drag coefficients ($C_l$ and $C_d$) during the last turbine revolution for different solidities at various $\lambda$. The comparison shows that:

- For different $\lambda$, increasing $\sigma$ decreases the variations of $\alpha$. This is due to the comparatively high blockage of a turbine with higher $\sigma$, which further decelerates the incoming flow. The comparatively high induced velocity by the turbine is also apparent from the lower (higher) values of relative velocity $V_{rel}$ at $0^\circ \leq \theta \leq 180^\circ$ and $270^\circ < \theta \leq 360^\circ$ ($180^\circ < \theta < 270^\circ$). Note that for $180^\circ < \theta < 270^\circ$, as the incoming flow is opposing the blade rotational speed, an incoming flow with lower velocity (due to the higher induced velocity) will lead to a comparatively high $V_{rel}$.
- The reduction in the absolute value of $\alpha$ by increasing $\sigma$ is observed at both turbine fore and aft halves, where the reduction for the latter is more pronounced. Higher (absolute) differences in $V_{rel}$ for different $\lambda$ are also apparent in the turbine aft half.
- Increasing $\alpha$, at a given $Re$, is found to influence the $C_l$ and $C_d$ in two ways: first due to a reduction in the absolute value of $\alpha$ and second due to the higher $K$. The former is more pronounced in the turbine aft half, where $C_l$ is significantly lower for higher $\sigma$. The latter is found to decrease the slope of the $C_l - \alpha$ curve during the upstroke, delay the flow separation and stall and diminish the load fluctuations. The load fluctuations occur due to the blade stall and the blade-wake interactions (BWI). Such fluctuations are observed for $\lambda = 2.5$ and $3.0$, where separation is noticeable. In addition, the higher $K$ substantially increases the $C_{l,max}$ and significantly reduces the $C_d$ and the $C_{d,max}$, which occurs due to stall and BWI, for all cases except $\lambda = 4.0$ where the flow is mostly attached, and stall is avoided.

In order to further elucidate how changing the solidity, at a given $Re$, affects the dynamic loads on the blades, i.e. $C_l$ and $C_d$, it is essential to analyze the boundary layer events. Such analysis can also confirm the role of the reduced frequency in comparison to the studies on pitching airfoils. Fig. 11 shows the spatiotemporal contours of friction coefficient $C_f$ along the blade over the last turbine revolution for different solidities at various $\lambda$. Figs. 12 and 13 show the contours of dimensionless $z$-vorticity with superimposed streamlines during the last turbine revolution at various azimuthal positions. The figures explicitly show that increasing $\sigma$: (i) significantly delays the formation of the laminar separation bubble (LSB) to later $\theta$ as well as further towards the trailing edge along the blade; (ii) substantially postpones the bursting of the LSB and the...
follow-up deep stall, which occurs at $\lambda = 2.5$; (iii) diminishes the trailing-edge roll-up vortices; and (iv) delays the trailing-edge separation. These observations highlight the mechanisms in the boundary layer through which changing the solidity influences the turbine aerodynamic performance. Such mechanisms are thought to occur due to the reduced frequency effect and are in line with the experimental studies on pitching airfoils [84,85]. The clarification of the impact of the $K$ due to changing the solidity is an important aspect of the solidity effect on the turbine aerodynamic performance, which to the best of the authors' knowledge, has not yet been identified in the literature and needs to be considered as a favorable characteristics of the higher solidity turbines.

Fig. 14 illustrates the time-averaged (over the last turbine revolution) normalized streamwise velocity at different downstream locations ($x/d = 1.0, 1.5, 2.0, 2.5, 3.0$ and $4.0$) in the turbine wake along several lateral lines ($-0.75 \leq y/d \leq 0.75$) for different solidities at various $\lambda$. Fig. 15 illustrates the contours of the instantaneous normalized velocity magnitude. The comparison exhibits that increasing $\sigma$:

- noticeably increases the velocity deficit in the wake due to much higher turbine $C_T$ (see Fig. 8j) and higher turbine induction (see Fig. 10b) which tends to much further decelerate the incoming flow, irrespective of the power extracted from the flow.
- significantly expands the wake, which is thought to be due to larger blade chord length and, therefore, larger influenced region in the most windward and leeward sides.
- diminishes the oscillations observed in the near wake for $x/d = 1.5$ at $\lambda = 2.5$, which appear due to the presence of the vortices shed from the stalled blades.
- reduces the impact of the shaft on the mean $x$-velocity profile in the wake, which is thought to be due to the comparatively high relative ratio of the blade chord length to the shaft diameter for higher $\sigma$.
- reduces the turbine wake length (see Fig. 15). It should be noted that in this study the turbine wake length is defined as the position in which $V/U_\infty = 0.95$. The reduction could be due to the lower freestream velocity, and consequently lower $Re_D$, employed for the turbines with higher solidities to keep $Re_c$ constant, and probably not a direct impact of the turbine solidity. It is found that the turbines with different $\sigma$ have almost the same wake length at their $\lambda_{opt}$. $L_w/d$ at $\lambda_{opt}$ remains approximately $8-12$ for various $\sigma$.
- enlarges the turbine upstream induction field.
6. Impact of number of blades

6.1. Turbine performance

Fig. 16 shows the power coefficient versus tip speed ratio and solidity for 2-, 3-, and 4-bladed VAWTs with constant Re, at identical solidities and tip speed ratios. It can be seen that except for \( \lambda < 2.5 \), the turbine CP is weakly sensitive to the number of blades. In the optimal operating range of turbines with different solidities, i.e., in the vicinity of \( \lambda_{opt} \), CP is almost \( n \)-independent. This is an important finding which implies that for variable-speed VAWTs maintaining their \( \lambda_{opt} \) at a given constant Re, the number of blades could be selected based on the other design parameters such as uniformity of the output power, structural loads and vibrations, and the cost. The same applies to constant-speed low-solidity urban VAWTs, at a given constant Re, frequently operating at moderate to high \( \lambda \) where the CP is almost insensitive to the number of blades.

In addition, at a given solidity, the turbine with less number of blades will operate at a higher \( K \) due to the larger blade chord length. As discussed in Section 5.3, the higher \( K \) will improve the turbine aerodynamic performance at low \( \lambda \) by delaying/avoiding the flow separation and dynamic stall and the consequent load fluctuations. The higher CP for the turbine with less number of blades at \( \lambda < 2.5 \), therefore, could be a result of the higher \( K \). For instance, this is more pronounced for the 2-bladed turbine compared to the 3- and 4-bladed turbines at \( \lambda = 1.5 \) and 2.0, see Fig. 16b. Further discussions are provided in Section 6.2.

6.2. Analysis at constant Re

In order to further elucidate the impact of the number of blades, a more detailed analysis is performed for a set of cases described in

However, in practice for small- to medium-scale turbines due to the low mean wind speed, e.g., in the urban environment, Re is typically in the low to moderate regime, \( \lesssim 2 \times 10^5 \), where Re number effects are significantly influencing the turbine CP [20]. Therefore, to avoid the low Reynolds number effects, the blade chord length needs to be sufficiently large. Hence, from an aerodynamic point of view and for a given solidity, the smaller number of blades will yield a higher CP due to the Reynolds number effect. Therefore, the choice of the optimal number of blades with respect to the turbine CP needs to consider the Re. Note that, as already mentioned, to select the number of blades there exists other important considerations, i.e., uniformity of structural loads, turbine vibrations, power uniformity, and the cost.
Table 9. The analysis is performed at two different solidities: $\sigma = 0.12$ and $\sigma = 0.24$. In addition, two $\lambda$ values of 2.5 and 4.0 are considered, where the former represents the operation in dynamic stall, and the latter denotes the optimal operational range. Note that for turbines with a different number of blades, $Re_z$ is kept constant at identical $\sigma$ and $\lambda$.

Fig. 17 shows the instantaneous moment coefficient for a single blade and its normalized cumulative sum during the last turbine revolution and also the instantaneous moment coefficient for the sum of all blades. It can be seen that, at a given $\sigma$ and $\lambda$, the length scale of the load fluctuations, which occurs due to the dynamic stall on blade and the blade-wake interactions, is larger for the turbine with smaller number of blades, i.e., $n = 2$, due to the larger blade chord length, this is while the frequency of the fluctuations, consequently, is comparatively lower (Fig. 17a–b).

The $C_m$ plots for all the blades (Fig. 16b and d) shows that a higher number of blades results in higher uniformity in the instantaneous output power.

The normalized cumulative sum of $C_m$ values of a single blade over the last turbine revolution reveals that increasing the number of blades reduces the contribution of the upwind quartile in the total power production, which instead grows the contribution of the downwind quartile. This shift, which is found to occur for different $\sigma$ and $\lambda$, implies the more gradual energy extraction from the flow using more blades, which consequently results in more uniform output power.

Fig. 18 shows the variations of the experienced angle of attack, normalized relative velocity, lift and drag coefficients during the last turbine revolution for turbines with different number of blades.

The comparison shows that:

- The experienced angle of attack $\alpha$ and relative velocity $V_{rel}$ experienced by the blade are almost not influenced by the number of blades. Small differences are observed in $90^\circ < \theta < 260^\circ$, which are due to the differences in the length scale and frequency of the shed vortices further upstream from the blades for turbines with different number of blades. This results in differences in the blade-wake interactions which are more pronounced for $\lambda = 2.5$ due to the larger variations of $\alpha$ and the consequent dynamic stall.

- In the turbine fore half, $0^\circ < \theta < 180^\circ$, higher $C_l$ values and delayed drop in $C_l$ is observed for the turbine with less number of blades at both $\sigma = 0.12$ and 0.24. This is due to the higher $K$ of the turbine with less number of blades. Studies on pitching airfoils and flat plates have revealed that higher $K$ improves the $C_{l,max}$ and delays the stall angle [84,85].

- For $\lambda = 2.5$ in dynamic stall, due to the smaller blade chord length for the turbine with a higher number of blades, the post-stall load fluctuations are of smaller length scale and higher frequency. This is more pronounced for $\sigma = 0.24$ at $\theta > 105^\circ$.

In order to further elucidate how changing the number of blades, at a given $Re_z$, affects the dynamic loads on the blades, i.e. $C_l$ and $C_d$, the boundary layer events need to be analyzed. Fig. 19 shows the spatiotemporal contours of the friction coefficient $C_f$ along the blade over the last turbine revolution for turbines with different number of blades at $\lambda = 2.5$. It can be seen that by reducing the number of blades: (i) the laminar separation bubble (LSB) forms at a later azimuthal position; (ii) for $\sigma = 0.12$, the LSB bursting is also postponed (for example, the LSB bursting for $n = 3$ in Fig. 19b occurs at $\theta = 74^\circ$ while it delays to $\theta = 84^\circ$ for $n = 2$ in Fig. 19a); (iii) for $\sigma = 0.24$, the extent of the trailing-edge separation is diminished; and (iv) for $\sigma = 0.24$, the dynamic stall vortex (DSV) is shed later and travels slower towards the trailing edge. These
observations reveal that changing the number of blades, even at a given Re, substantially influences the separation onset and the chordwise extent of the separated region on the blades. This is an important finding as it highlights that the chordwise position of any active flow control device aimed to control the flow separation on the VAWT blades is dependent on the number of blades even if $\sigma$ and $Re$ are kept constant. The delay in separation for the turbine with less number of blades occurs because of the higher $K$ (see Table 9). Higher $K$ is known to result in higher resistance to separation in the boundary layer [84,85].

In addition, increasing the number of blades is found to increase the number of the trailing-edge roll-up vortices and their shedding frequency while reducing their size. This is more pronounced at $\sigma = 0.12$.

Table 9
Description of the cases for further analysis of the impact of number of blades.

<table>
<thead>
<tr>
<th>n</th>
<th>$\sigma$</th>
<th>c [cm]</th>
<th>$\lambda = 2.5, 4.0$</th>
<th>$Re_s (\times 10^5)$</th>
<th>$U_{\infty}$ [m/s]</th>
<th>$\Omega$ [rad/s]</th>
<th>$TI%$</th>
<th>$TI_i%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.12</td>
<td>6</td>
<td>0.06</td>
<td>1.03, 1.58</td>
<td>9.3</td>
<td>46.5, 74.4</td>
<td>5</td>
<td>4.42</td>
</tr>
<tr>
<td></td>
<td>0.24</td>
<td>12</td>
<td>0.12</td>
<td>2.06, 3.15</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>0.04</td>
<td>1.03, 1.58</td>
<td>13.95</td>
<td>69.8, 111.6</td>
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<td>4.42</td>
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<td></td>
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</tr>
<tr>
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<td>3</td>
<td>0.03</td>
<td>1.03, 1.58</td>
<td>18.6</td>
<td>93.0, 148.8</td>
<td>5</td>
<td>4.42</td>
</tr>
<tr>
<td></td>
<td>0.24</td>
<td>6</td>
<td>0.06</td>
<td>2.06, 3.15</td>
<td></td>
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</tbody>
</table>

Fig. 17. Instantaneous moment coefficient and its normalized cumulative sum during the last turbine revolution for turbines with a different number of blades at constant $Re_s$ for identical $\sigma$ and $\lambda$.

Fig. 20 illustrates the time-averaged (over the last turbine revolution) normalized streamwise velocity at different downstream locations ($x/d = 1.0, 1.5, 2.0, 2.5, 3.0$ and $4.0$) in the turbine wake along several lateral lines ($-0.75 \leq y/d \leq 0.75$) for turbines with a different number of blades. The comparison reveals that:

- At the windward side ($0 > y$), increasing the number of blades also increases the velocity deficit in the wake, while in the leeward side ($0 < y$) the reverse applies and a larger velocity deficit is observed for the less number of blades.
- The wake profile is more sensitive to the number of blades in the leeward side ($0 < y$), compared to the windward.
- The sensitivity of the wake profile to the number of blades is higher for lower $\lambda$ and higher $\sigma$. The impact of $\lambda$ is more pronounced.
- The impact of the number of blades on the wake profile persists as the wake travels downstream to $x/d = 4.0$.
- The turbine wake length is found to be almost insensitive to the number of blades (not shown in this paper).

7. Summary and conclusions

Solidity ($\sigma$) and number of blades ($n$) are the two important geometrical parameters, which significantly drive the turbine performance. While an optimal solidity can be identified based on an aerodynamic analysis, the selection of the number of blades for a turbine is a function of several important parameters, namely uniformity of output power, turbine loads and vibrations, cost as well as the turbine aerodynamic performance. The focus of the present study is, therefore, confined to investigating the impact of $\sigma$ and $n$ on the aerodynamic performance of the turbine to provide a profound understanding and to help the designers and the manufacturers with this aspect of the design.

In the present study, high-fidelity CFD simulations, extensively validated with experimental data, are employed to study the impact of the aforementioned two parameters. $\sigma$ varies by changing the blade chord length within $0.09 \leq \sigma \leq 0.36$ for turbines with $2, 3$ and $4$ blades. The study covers a wide range of tip speed ratios, $1.5 \leq \lambda \leq 5.5$, freestream chord-based Reynolds numbers, $0.51 \times 10^5 \leq Re_s \leq 6.41 \times 10^5$, approach-flow (i.e. inlet) total turbulence intensities, $0\% \leq TI \leq 25\%$, and reduced frequencies, $0.02 \leq K \leq 0.18$.

The main findings of the study can be summarized as follows:

1. Solidity:
   - Increasing $\sigma$ from $0.09$ to $0.36$, (i) shifts the optimal tip speed ratio $\lambda_{opt}$ from $4.5$ to $2.0$; and (ii) increases the $C_{p_{max}}$ by 22.5%.
   - The increment in $C_{p_{max}}$ is due to the Reynolds number effects. This is confirmed as the results show that such an increment is not present when $\sigma$ increases at constant $Re_s$. 

- At the windward side ($0 > y$), increasing the number of blades also increases the velocity deficit in the wake, while in the leeward side ($0 < y$) the reverse applies and a larger velocity deficit is observed for the less number of blades.
- The wake profile is more sensitive to the number of blades in the leeward side ($0 < y$), compared to the windward.
- The sensitivity of the wake profile to the number of blades is higher for lower $\lambda$ and higher $\sigma$. The impact of $\lambda$ is more pronounced.
- The impact of the number of blades on the wake profile persists as the wake travels downstream to $x/d = 4.0$.
- The turbine wake length is found to be almost insensitive to the number of blades (not shown in this paper).
The trend of reduction in lift with the increase of solidity is similarly observed for turbines with 2, 3 and 4 number of blades. A similar trend is also observed for different turbulence intensities. 

$CT$ grows asymptotically with the increase of solidity.

At a given $Re_c$, increasing $\sigma$ is found to: (i) noticeably reduce the absolute $\alpha$ and increase the induced velocity by the blade. For instance, when $\sigma$ increases from 0.12 to 0.24, $\alpha$ reduces up to 2.5°, and 6° in the turbine fore and aft halves, respectively; (ii) increase the $C_{l,max}$ while reducing the $C_{d,max}$, especially for the lower $\lambda$ values due to the higher $K$; (iii) increase the turbine upstream induction field, the velocity deficit in the wake, the wake expansion and diminish the oscillations in the wake velocity profile, which occur due to shed vortices from the blades and the shaft wake, and decrease the turbine wake length.

Fig. 18. Variations of experienced angle of attack, normalized relative velocity, lift and drag coefficients during the last turbine revolution for turbines with a different number of blades at constant $Re_c$ for identical solidities and tip speed ratios.
(2) Number of blades:
- At a given $\Re_t$, the turbine $C_p$ is independent of $n$ within the optimal operational range, i.e., in the vicinity of $\lambda_{opt}$.
- For a given $\sigma$ and at low $\lambda$ where dynamic stall is present, $C_p$ values are dependent on the number of blades due to the impact of $K$. This means that at a given solidity, the smaller number of blades (higher chord length) delivers a higher $K$ and thus a higher $C_p$.
- At a given $\Re_t$, $a$ and $V_{rel}$ are almost independent of number of blades.
- Decreasing $n$ is found to increase the $C_{l,max}$ and reduce the $C_d$ due to the higher $K$.

It is shown that the $\lambda_{opt}$ is invariant to the newly-defined parameter $'a\lambda^3'$, regardless of the turbine geometrical and operational conditions.

A new correlation is derived based on a large set of numerical and experimental data sets to estimate the $\lambda_{opt}$ solely based on the turbine solidity: $\lambda_{opt} = 2.693\sigma^{-0.329} - 1.605$. The correlation can be very helpful for the turbine designers and manufacturers either to estimate the $\lambda_{opt}$ for a given $\sigma$ or vice versa to choose the optimal $\sigma$ for a desired $\lambda_{opt}$.

The optimal geometrical design, namely the optimal solidity, is highly dependent on the target operating conditions of the turbine to be designed. For constant-speed small-to medium-scale (urban) VAWTs, which frequently operate at moderate to high $\lambda$ due to the low mean wind speeds at the potential sites, a
low solidity is preferred. On the other hand, variable-speed VAWTs are optimal with relatively-high solidity maintaining a moderately-low $\lambda$.

- For a given turbine diameter and $n$ and under a given operating condition, changing the $\sigma$ inevitably modifies the $Re_n$ and $K$ where the two parameters have significant influences on the flow topology and the boundary layer events on the blades, namely unsteady laminar leading-edge and turbulent trailing-edge separations, reattachment and laminar-to-turbulent transition. Such impacts are more pronounced in the low $\lambda$ where due to the higher variations of $\alpha$, dynamic stall might occur.

- Increasing $\sigma$ also increases the contribution of the upwind quartile to the total power production while reducing the contribution of the downwind quartile. At constant $Re_n$, turbines with different $\sigma$ yield approximately the same $C_{P,max}$, where the instantaneous moment coefficient is only shifted between the upwind and downwind quartiles. This implies that due to the unsteady nature of the VAWTs, a non-dynamic geometrical design for VAWTs can barely further enhance the turbine efficiency and unsteady active mechanisms have to be devised.

- From an aerodynamic point of view, at a given $\sigma$, the less number of blades is favorable due to the higher $Re_n$ and $K$. However, as discussed above, the choice of the number of blades is also driven by several other important design parameters, such as uniformity of the output power and structural loads and the cost. A larger number of blades yields more uniform instantaneous loads and power during the revolution while the length scale of the load fluctuations is also comparatively smaller due to the smaller blade chord length at the given $\sigma$.

In the present study, the turbine design condition is considered, which refers to the condition where the wind speed has already exceeded the turbine cut-in velocity and the start-up phase, during which the turbine starts to rotate and reaches the designed rotational speed, has passed. A dedicated analysis of the impact of solidity and number of blades at off-design conditions, e.g., start-up, is recommended.

In addition, a fixed airfoil shape has been considered in the present analysis. Further investigation should be performed to study the dependency of the derived conclusions for different airfoil shapes.

The findings of this study help to better understand the impact of the geometrical characteristics on the aerodynamic performance of VAWTs and support to design optimum VAWTs and more widespread wind energy harvesting.

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