Towards acoustic metafoams: The enhanced performance of a poroelastic material with local resonators

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ARTICLE INFO

Article history:
Received 2 May 2018
Revised 7 September 2018
Accepted 3 October 2018
Available online 12 October 2018

Keywords:
Acoustic metamaterials
Poroelastic materials
Local resonance
Viscothermal dissipation
Acoustic foams
Metafoams

ABSTRACT

Acoustic foams are commonly used for sound attenuation purposes. Due to their porous microstructure, they efficiently dissipate energy through the air flowing in and out of the pores at high frequencies. However, the low frequency performance is still challenging for foams, even after optimisation of their microstructural design. A new, innovative, approach is therefore needed to further improve the acoustic behaviour of poroelastic materials. The expanding field of locally resonant acoustic metamaterials shows some promising examples where resonating masses incorporated within the microstructure lead to a significant enhancement of low frequency wave attenuation. In this paper, a combination of traditional poroelastic materials with locally resonant units embedded inside the pores is proposed, showing the pathway towards designing acoustic metafoams: poroelastic materials with properties beyond standard foams. The conceptual microstructural design of an idealised unit cell presented in this work consists of a cubic pore representing a foam unit cell with an embedded micro-resonator and filled with a viscothermal fluid (air). Analysis of complex dispersion diagrams and numerical transmission simulations demonstrate a clear improvement in wave attenuation achieved by such a microstructure. It is believed that this demonstrates the concept, which serves the future development of novel poroelastic materials.

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1. Introduction

Protection against noise pollution is nowadays a compelling problem due to progressing industrialisation and omnipresent sources of disturbing and harming sounds. In particular, low frequency noise attenuation still awaits for an efficient solution, preferably based on a material suitable for mass production.

In terms of sound absorption, porous materials like acoustic foams, fiber glass or mineral wool are usually quite efficient due to their large microstructural air-solid interface area, which results in high viscothermal dissipation. However, the efficiency of these materials for low frequencies is significantly lower than for mid and high frequency sounds (Yang and Sheng, 2017). Moreover, studies investigating the relationship between the microstructure of conventional acoustic foams and their performance (Chevillotte and Perrot, 2017; Doutres et al., 2011; 2014; Gao et al., 2017) show that the shift of the

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https://doi.org/10.1016/j.jmps.2018.10.006
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absorption peak towards lower frequencies by means of a detailed pore design or by increasing the reticulation rate is rather limited (if it can be achieved, it comes at a price of lower efficiency).

An alternative to improvements at the microstructural level is to modify the materials at a scale larger than the pore size (i.e. meso scale). It has been observed that meso-perforations (air inclusions) introduced to a porous material may be beneficial for its acoustic performance at low frequencies (see the review paper Sgard et al. (2005) on double porosity media and the references therein). Embedding other types of meso-scale inclusions in foam-like materials has been explored for the same purpose. A distribution of Helmholtz resonators in the bulk of a poroelastic material has been introduced in Lagarrigue et al. (2013) and Doutrès et al. (2015), and was recently optimised in Park et al. (2017). Note that, in order to achieve low frequency performance of such panels, the sizes of resonating cavities need to be rather large (order of centimetres). Moreover, inspired by the concept of phononic crystals (Deymier, 2013) (i.e. a periodic arrangement of scatterers showing sound attenuation via destructive wave interference), a periodic arrangement of metal rods has been proposed for example in Weisser et al. (2016) and a double porosity material with an array of mass inclusions has been reported in Cui and Harne (2017). Although such solutions, based on meso-scale modifications of porous materials, can offer an improvement of the attenuation level at low frequencies, they also require significantly different designing and manufacturing steps.

Acoustic metamaterials are another class of materials that are promising in the context of low frequency sound attenuation. The dedicated microstructural design of these materials results in higher attenuation level relative to what is achieved with traditional materials. A number of metamaterials achieve their extraordinary properties through a local resonance phenomenon. The prime manufactured example of a solid material generating so called band gaps and acting as a total wave reflector due to resonating inclusions, which effectively prohibits the propagation of waves at certain frequencies, has been reported rather recently in Liu et al. (2000). Boutin and Becot (2015) have proposed a material entirely composed of Helmholtz resonators (of significant size in order to attain low frequency performance), also achieving an improvement in sound attenuation. This concept was further developed in Griffiths et al. (2017), proposing a poro-granular material design with resonators made of soft elastomer shells. Recently, the presence of shear wave band gaps and left-handed behaviour of a cellular material entrained with water have been demonstrated experimentally in Dorodnitsyn and Van Damme (2016), where the dispersive properties of the material are achieved through resonance of the lattice walls.

A special type of acoustic metamaterials are the structures incorporating membrane resonance. Such materials are typically designed to improve absorption performance as is the case for a “dark” acoustic metamaterial (Mei et al., 2012) or in the work of Ma et al. (2014) where hybrid resonance of a decorated membrane is exploited. Yang et al. (2015) have proposed a combination of a membrane-type and a Helmholtz resonator, showing that such degenerate resonators, when properly coupled, can also serve as an effective absorber. Recently, atypical acoustic behaviour of rigid permeable materials with thin elastic membranes embedded in a rigid microstructure has been demonstrated numerically and experimentally in Venegas and Boutin (2017).

In many studies concerning metamaterials, especially those involving fluid structure interaction, material (fluid) losses are either not included in the model or incorporated in a simplified way through a phenomenological parameter. For some materials, incorporating the influence of realistic damping occurring through thermal and viscous dissipation is not just a refinement of the model but may be crucial for the correct predictions of their response. For instance, in the study by Molerón et al. (2016), focusing on a metamaterial consisting of rigid slabs embedded in air, the inclusion of fluid losses changed the predicted acoustic behaviour from perfect transmission to perfect reflection. Furthermore, Henríquez et al. (2017) have reported that viscothermal dissipation in the air, even for geometries much larger than the boundary layer thicknesses, may completely suppress the double negative behaviour of the analysed rigid periodic structure.

In this paper, the acoustic performance of a poroelastic material enriched with resonators embedded in the pores is investigated. Unlike previous studies (Chevillotte and Perrot, 2017; Doutrès et al., 2011; 2014; Gao et al., 2017) aiming at optimising the pore geometry and morphology for low frequency performance, here, a major change in terms of micro-dynamical phenomena is introduced. The micro-resonator is represented by a cantilever beam with a heavy mass attached at its tip, which represents a particle embedded in the pore during the manufacturing process. Bloch analysis performed for the proposed unit cell design predicts a significant attenuation in the low frequency range for both fast and slow compressional waves propagating through the system. This behaviour is also confirmed by a transmission analysis of a finite size set-up by direct simulations. Moreover, it is shown that the shear viscosity of the fluid is crucial for revealing the resonance-related attenuation mechanisms. In order to numerically demonstrate the concept of embedding local resonators within the pores of poroelastic material, a simple cubic unit cell is used. Supporting the recent progress in innovative approaches to manufacturing poroelastic/cellular materials (Kaur et al., 2017), this work contributes towards the development of a new type of foams: acoustic metafoams.

The paper is organised as follows. In Section 2, the unit cell and the modelling approach are described. In Section 3, the simulation results are shown, including the analysis of the dispersion diagrams and transmission studies of the response of a single unit cell as well as a finite size configuration. In Section 4 the main results are further discussed, after which conclusions are presented.
Fig. 1. (a) Unit cell without a resonator, (b) unit cell with a light resonator, (c) unit cell with a resonator with a heavy mass. Light grey represents polyurethane (PU), dark grey is the heavy mass at the tip of the resonator, light blue is the air domain. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

2. Modelling approach

2.1. Geometry of the unit cell

The geometry and dimensions of the unit cell have been chosen on the basis of polymer foams (Gao et al., 2017), aiming at high absorption for low frequencies. A cubic idealised representation of the pore has struts with a square cross-section along all edges and membranes in the faces (Fig. 1). A small cell size of \( a = 100 \mu m \) is adopted, with the thicknesses of the struts \( t_s = 25 \mu m \) and of the membranes \( t_m = 1 \mu m \). These values are close to the parameters measured experimentally in Gao et al. (2016). The membranes of four out of six faces are partially open, with a small opening ratio of 1% (where the opening ratio represents the area fraction of the hole in the face).

A local resonator is introduced within the microstructure of the idealised cubic cell in the form of a cantilever decorated at its tip with a heavy (relatively to the unit cell weight) mass of \( m_{\text{heavy}} = 1 \times 10^{-10} \text{kg} \). In order to reduce the role of the geometry in this study and to focus on the contribution of the local resonance effect to the material performance, the dimensions of the resonating mass are minimised to 0.01 mm \( \times 0.01 \text{mm} \times 0.01 \text{mm} \). In this way, the dissipation is not being enhanced due to an increase of the solid-fluid interface. Moreover, two reference cases are considered, the case of a unit cell without a resonator and a unit cell with a light resonator (i.e. the cantilever only, omitting any external mass at its tip). The three geometries are depicted in Fig. 1.

2.2. Material description

The microstructure of the unit cell (Fig. 1) consists of fluid and solid phases which are described in the frequency domain. Conventional descriptions for each component are adopted from Gao et al. (2015) and Yamamoto et al. (2011).

The solid part is modelled as in general an isotropic elastic material. Within the solid domain, the equation of motion holds:

\[
-\rho^s \omega^2 \mathbf{u} = \nabla \cdot \mathbf{\sigma},
\]

where \( \mathbf{u} \) is the displacement vector, \( \omega \) the angular frequency, \( \rho^s \) the solid density and \( \mathbf{\sigma} \) is the stress tensor given by \( \mathbf{\sigma} = 4 \mathbf{C} : \nabla \mathbf{u} \), with \( 4 \mathbf{C} \) being the total symmetric fourth order elasticity tensor, symbol \( \nabla \) denotes the gradient operator.

At the microstructural level, dissipative effects result from the viscothermal behaviour of the fluid. Therefore a complex description of the fluid domain is adopted. The governing equations for a lossy compressible fluid can be written as:

\[
\begin{align*}
\rho_0^f \omega \mathbf{v} &= -\nabla p + \nabla \cdot \left( \mu^f (\nabla \mathbf{v} + (\nabla \mathbf{v})^T) - \frac{\kappa^f}{\omega} (\nabla \cdot \mathbf{v}) \mathbf{I} \right) \quad \text{(2)} \\
\rho_0^f \mu^f \theta &= \mu_0^f \mathbf{p} = -\nabla \cdot (-\kappa^f \nabla \theta) \quad \text{(3)} \\
\rho_0^f \omega \frac{\mathbf{p}}{p_0} &= \omega \frac{\theta}{\theta_0} - \nabla \cdot \mathbf{v} \quad \text{(4)}
\end{align*}
\]

which represent the balance of momentum, the energy balance and mass conservation (considering the ideal gas law), respectively. In these equations, \( \mathbf{v} \) denotes the velocity vector, \( p \) the pressure, \( \theta \) the temperature, \( \rho_0^f \) the equilibrium density, \( \mu^f \) the viscosity, \( c_p^f \) the heat capacity at constant pressure, \( \kappa^f \) the thermal conductivity, \( i \) the imaginary unit, \( p_0 \) and \( \theta_0 \) the ambient pressure and temperature, respectively.

In order to reduce the computational costs, external fluid domains (outside the unit cells), used in transmission simulations (see Section 2.4), are modelled in a simplified way, neglecting viscothermal damping. Then, the local momentum
balance and continuity equations are:
\[ i \omega \rho_0^f \mathbf{v} + \nabla p = \mathbf{0}, \]  
\[ i \omega p + c_0^2 \rho_0^f \nabla \cdot \mathbf{v} = \mathbf{0}, \]  
where \( c_0 \) denotes the wave speed in the fluid. By combining Eqs. (5) and (6), the acoustic Helmholtz equation is obtained:
\[ \nabla \cdot (\nabla p) + k_0^2 p = 0, \quad \text{with the wavenumber} \quad k_0 = \frac{\omega}{c_0}. \]  
This domain will be referred to as inviscid isothermal.

The coupling between the domains is prescribed in the following way. At the interface between the viscothermal fluid and the solid, continuity of velocities and tractions is assumed, along with an isothermal condition (justified by the large difference between thermal conductivity of the solid and fluid phases). Continuity of tractions and normal acceleration is adopted at the interfaces between the solid and the inviscid isothermal fluid as well as between the viscothermal and inviscid isothermal fluid. At the latter interface, also adiabatic conditions for the temperature are assumed.

The material parameters adopted for the fluid (air) and solid (polyurethane, PU) domains are presented in Tables 1 and 2, respectively.

### 2.3. Bloch analysis

Free wave propagation in an infinite periodic medium is typically analysed based on dispersion diagrams. By considering the complex band structures, the assessment of the wave dispersion in the presence of local resonators is possible, as well as the description of the wave attenuation due to (thermo) viscous dissipation inside the unit cell. The wave attenuation is characterised by the imaginary part of the wavenumber (attenuation factor), where higher values indicate stronger attenuation in space. In this study, the complex dispersion diagrams are obtained with the standard \( k(\omega) \) approach (Hussein et al., 2014; Krushynska et al., 2016; Wang et al., 2015) based on the Bloch-Floquet theorem, which states that the wave solution inside the Brillouin Zone (Kittel, 1971) is a superposition of a periodic function and a plane wave (Deymier, 2013). The Bloch-Floquet theorem is applied to all considered field variables and substituted in the governing Eqs. (1), (2), (4) and (7) of the domains. Although thermal effects are included in the transmission analysis, due to their marginal contribution demonstrated in Section 3.3, they are neglected in this part of the study. COMSOL Multiphysics is used to solve the coupled eigenvalue problem, exploiting a user defined weak form for a unit cell with periodic boundary conditions. The details of the strong and weak formulations used for the Bloch analysis are given in the Appendix. The study is restricted to the \( \Gamma X \) direction of the wave propagation (Deymier, 2013) since it coincides with the conditions realised in the corresponding transmission analysis.

### 2.4. Transmission analysis

A numerical three-dimensional transmission set-up (Cui and Harne, 2017) is used in this paper to assess the performance of the material. The finite element method simulations have also been conducted with COMSOL Multiphysics. The set-up is inspired by an impedance tube test, typically utilised for measuring the acoustic properties of poroelastic materials. A three point method (Henriques et al., 2017; Ho et al., 2005) is used to measure transmission, absorption and reflection. The tested sample is placed in between two air domains, with a pressure plane-wave excitation assigned to the left boundary.
and a perfectly matched layer (PML) used at the right boundary to reduce spurious reflections. Lateral boundaries (in y and z directions) are periodic in this analysis. The mid-plane cross-section of the numerical model with the position of the measurement points (mic) is shown in Fig. 2. Following Henriquez et al. (2017), reflection $r(\omega)$ and transmission $t(\omega)$ coefficients are obtained by:

$$r(\omega) = \frac{p_2 \exp(-ikx_3) - p_1 \exp(-ikx_2)}{p_1 \exp(i k x_2) - p_2 \exp(i k x_1)},$$

$$t(\omega) = \frac{p_3 (\exp(-ikx_2) + r(\omega) \exp(i k x_2))}{\frac{p_2 \exp(-i k x_3)}{\exp(-i k L)},}$$

where $p_1$, $p_2$ and $p_3$ are complex pressure values calculated at three measurement points $x_i$ (at the positions of the microphones 1, 2 and 3, respectively), $L$ is the length of the sample, $k$ denotes the wavenumber.

Power transmittance and reflectance are then expressed as: $R(\omega) = |r(\omega)|^2$ and $T(\omega) = |t(\omega)|^2$, respectively, and sum up to 1 in the absence of losses, due to the conservation of energy. In the presence of losses, the dissipated energy is given by an absorption term: $A(\omega) = 1 - R(\omega) - T(\omega)$. In order to distinguish the mechanisms underlying the absorption, the fractions of viscous and thermal dissipated powers are respectively obtained by (Cambonie et al., 2018; Pierce, 1981):

$$P_v = \int_V \sigma : \nabla \mathbf{v} \, dV,$$

$$P_t = \int_V \frac{k_0}{k} \Delta T \, dV,$$

and will be normalised by incident power. In the above expressions, $V$ denotes the volume of the viscothermal fluid and the symbol $\Delta$ is the Laplace operator.

3. Results

In this section, the numerical results are presented. First, the behaviour of an infinite periodic arrangement of unit cells is assessed based on the Bloch analysis. Next, the acoustic performance of a finite material sample (considering single and multiple cells) is studied using the transmission set-up.

3.1. Dispersion diagrams

Fig. 3 shows the dispersion diagrams obtained for the three considered 3D unit cell geometries of Fig. 1, using an inviscid isothermal fluid. The wave polarisations are identified by the ratio between the amplitudes of the displacement in the solid along the x axis and the total displacement in the solid, both integrated over the solid domain. The compressional and shear wave polarisations are then distinguished by colours varying from red (1) to blue (0), respectively. As can be seen in Fig. 3, two compressional waves propagate in the material consisting of 3D unit cells with the solid and fluid phases, where two shear waves $(S_1, S_2)$ relate to the two perpendicular displacement directions, and two compressional (longitudinal) waves $(L_1, L_2)$ correspond to the in-phase (fast, fluid-born) and out-of-phase (slow, solid-born) motion of the fluid and solid, respectively (Allard and Atalla, 2009; Bruneau and Potel, 2013).

Dispersion curves for the inviscid cases without the resonator and with the light resonator presented in Fig. 3a overlap in the considered frequency range and do not exhibit any dispersive or dissipative effects (imaginary parts of the wavenumbers all equal zero). Yet, significant dispersion can be observed for the case with the heavy resonator in Fig. 3b. A band gap is formed in the frequency range 445–560 Hz for two wave polarisation: the slow (solid-born) compressional wave $(L_2)$ and one of the shear waves $(S_2)$, as evident from the absence of the dispersion curves in the real plane and high imaginary values of the wavenumber. It should be emphasised that due to the presence of the two other wave types $(L_1, S_1)$ inside the
band gap region, a complete band gap is not formed. Note, that due to the small dimensions of the unit cell, the dispersive effects related to the cavity resonance for the wave type L1 occur at much higher frequencies, exceeding the considered range (not shown here).

Fig. 4 presents the 3D band structure and its 2D projections obtained for the geometry with the heavy resonator when a viscous fluid is considered (with the viscosity of air). As a reference, the lossless case discussed previously is also shown in the graph (in black). The viscosity of the fluid has a significant influence on the dispersion curves. In particular, both compressional waves (L1, L2) are attenuated in the considered frequency range, with attenuation peaks located around 440 Hz. The 3D view of the band structure (Fig. 4a) reveals how both dispersion curves associated with these waves bend towards the complex wavenumber domain, exhibiting a spin in the band gap region, characteristic for dissipative systems (Krushynska et al., 2016; Lewinska et al., 2017). The shear wave S2 (forming a band gap in the lossless case) now shows a slight broadening of the attenuation regime in the viscous case. No influence of the air viscosity can be observed for the other shear wave polarisation (S1).

In order to assess the effect of the resonance on the attenuation performance of the unit cell with the heavy resonator entrained with viscous fluid, the corresponding dispersion curves obtained are compared with those calculated for the unit cell with a light resonator (also accounting for the fluid viscosity). Fig. 5 shows that for low frequencies (below 450 Hz),
The fast (fluid-born) compressional wave (L1) reveals a higher level of attenuation in the case with the heavy resonator. On the other hand, for frequencies above 450 Hz the attenuation factor is higher for the light resonator. An attenuation peak located around frequency 440 Hz, which is observed for the slow (solid-born) wave (L2) with the heavy resonator, is not present in the case of the light resonator, for which the attenuation is slightly lower at the higher frequencies as well.

The attenuation of the compressional waves shown in the dispersion diagrams can be associated with the dynamic behaviour of the unit cell. Therefore, in Fig. 6 the velocity fields in the longitudinal direction at the mid-plane cross section for different points (marked in Fig. 5b by A, AL, B, BL), are presented for both unit cells. The mode shape for the fast wave, for which the fluid motion is in-phase with the solid, shows high velocity gradients occurring around the membrane opening, if the light resonator is considered (Fig. 6a). For the heavy resonator unit cell, the resonating cantilever significantly enhances the velocity gradient field, in particular, through its out-of-phase oscillations (Fig. 6b). Analogous effects can be observed for the slow wave (Fig. 6c,d). The presence of the heavy resonator contributes to the higher attenuation level by, enhancing both dissipative effects and increasing reflection, as will be demonstrated in the following sections.

In Figs. 7 and 8, the band structures obtained for the unit cell with heavy resonators and viscous fluid with different viscosities are presented. For clarity, only the compressional waves are shown and the distinction between fast (fluid-born) and slow (solid-born) waves is introduced using colours, where red and green colours denote fluid-born and solid-born waves, respectively. Intermediate colours reflect the ratio between the fluid and solid velocities in the longitudinal direction integrated over the front face of the unit cell and averaged over the solid and fluid parts of this surface, allowing to identify the in-phase and nearly out-of-phase motion between fluid and solid.

The 3D dispersion diagram (Fig. 7a) also shows dispersion curves obtained for a fluid viscosity lower than that of air i.e. \( \mu = 0.1 \mu_{\text{air}} \). In this case, both fluid-born and solid-born waves exhibit less attenuation preserving the general shape of the dispersion curves (with attenuation peaks around 440 Hz for both wave polarisations). The velocity fields for the points
Fig. 7. Complex dispersion diagrams for the heavy resonator unit cell and different fluid viscosities: $\mu = \mu_{\text{air}}$ marked with filled circles, $\mu = 0.1\mu_{\text{air}}$ marked with open circles: (a) 3D structure; (b) 2D projections. Colours represent compressional wave types from solid-born (green) to fluid-born (red). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Fig. 8. Complex dispersion diagrams for the heavy resonator unit cell and different fluid viscosities: $\mu = \mu_{\text{air}}$ marked with filled circles, $\mu = 10\mu_{\text{air}}$ marked with open circles: (a) 3D structure; (b) 2D projections. Colours represent compressional wave types from solid-born (green) to fluid-born (red). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

marked in Fig. 7b as A01 (Fig. 9a) and C01 (Fig. 10a) reveal a reduced coupling between the solid and fluid domains, with a lower viscosity: higher velocity gradients are present in the unit cell in comparison with the reference air viscosity case (cf. Figs. 6b and 10b).

Fig. 8 shows that the influence of an increased fluid viscosity ($\mu = 10\mu_{\text{air}}$) on the dispersion diagrams depends on the wave type. For the fast wave smoothing of the attenuation factor is observed, with a decrease of the attenuation peak at 440 Hz and a minor increase of the imaginary parts of the wavenumbers in the frequency range 450–650 Hz. For the slow wave, the attenuation increases in almost the entire frequency range. Velocity fields for the points marked in Fig. 8 are shown in Fig. 9 and 10. At the point A10 significant motion of the entire solid skeleton can be observed together with reduced velocity gradients, in comparison with the point A01 (Fig. 9a) and A (Fig. 6b). On the other hand, the velocity field for the point C10 (Fig. 10c) is characterised by significant velocity concentrations around the membrane opening region. Note, that the attenuation of the slow wave is stronger than the attenuation of the fast wave for all considered viscosities due to the relative viscous flow of the pore-fluid relative to the solid frame (see Fig. 10) (Allard and Atalla, 2009; Borocin, 2003).
3.2. Single cell performance

The dispersion analysis allows to assess the wave propagation and attenuation in the infinite material domain. In order to analyse the behaviour of finite structures and distinguish between the mechanisms underlying the wave attenuation, a transmission calculation is conducted using the numerical set-up detailed in Section 2.4. In this section, the performance of a single unit cell as a material sample is investigated.

Fig. 11 shows the acoustic properties of a single unit cell with viscothermal losses for the unit cells with and without resonators. The heavy resonator cell reveals, a transmission dip at 440 Hz, which is not visible for the case with a light or no resonator (Fig. 11a). Based on the reflection (Fig. 11b) and absorption (Fig. 11c) curves, it can be stated that at this frequency part of the energy is dissipated within the fluid and a similar part is reflected.

In Fig. 12, the velocity fields at the mid-plane cross-section of the three unit cells are depicted. The unit cells without the resonator (Fig. 12a) and with the light resonator (Fig. 12b) behave quite similar. A minor increase of the velocities is present around the membrane opening, and additionally around the resonator tip for the corresponding unit cell. The presence of the heavy resonator significantly changes the response of the unit cell (Fig. 12c). At a frequency of 440 Hz, the elastic cantilever resonates and high velocity gradients can be observed within the fluid. Note, that this velocity field qualitatively resembles the one obtained through the dispersion analysis (Fig. 6b), suggesting that the dominant role for the transmission analysis in the considered set-up is played by the fast (fluid-born) compressional wave. As a consequence, a high level of viscous dissipation is obtained, as well as an increase in the acoustic impedance, resulting in a reflection peak, see Fig. 11b.

In Fig. 13, the acoustic properties obtained for the unit cell with a heavy resonator and viscothermal fluid are compared with those obtained for the inviscid isothermal fluid (described by Eq. (21)). A clear difference between both unit cells emerges. The transmission dip observed with the viscothermal fluid is not present in the analysis without viscothermal losses. Only an isolated reflection peak is found at 550 Hz (Fig. 13b), which can be associated with the global eigenfrequency of the finite system. Note, that the reflection dips are present in both cases at the frequency 560 Hz (see logarithmic scale insert graph in Fig. 13c), which is the frequency closing the band gap in Fig. 3c. Naturally, no absorption is observed if the losses are not considered (Fig. 13c).

Based on the above results it can be therefore concluded that viscothermal losses play a key role for the performance of the unit cell. To further scrutinise this effect, in Fig. 14, the results of a variation of the fluid viscosity around the regular
Fig. 11. Transmission, reflection and absorption of a single unit cell with a viscothermal fluid: without resonator (dashed line); with the light resonator (red line); with heavy resonator (green line). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Fig. 12. Velocity field in the longitudinal x direction at the mid-plane cross section of the unit cells in the transmission simulation at 440 Hz for (a) the unit cell without a resonator, (b) the unit cell with the light resonator and (c) the unit cell with the heavy resonator.

Fig. 13. Transmission, reflection and absorption of a single unit cell with a heavy resonator and a viscothermal fluid (black line) and an inviscid fluid (grey line).
air viscosity are presented for the unit cell with a heavy resonator. Both an increase or a decrease of the viscosity results in smoothening of the transmission spectra and a reduction of the transmission dip. The change of viscosity initially influences the reflection spectrum, where for both $\mu = 0.1 \mu_{\text{air}}$ and $\mu = 10 \mu_{\text{air}}$ the peak values are already significantly reduced compared to the value with air viscosity (Fig. 14b). The absorption level is only affected strongly if the viscosity is modified considerably ($\mu = 0.01 \mu_{\text{air}}$ and $\mu = 100 \mu_{\text{air}}$). If the viscosity reduction is relatively small (e.g. $\mu = 0.1 \mu_{\text{air}}$) absorption is not influenced much. This amount of viscous dissipation is still achieved due to the local resonance effect resulting in high local velocity gradients.

In Fig. 15, the amplitudes of the longitudinal velocity fields are depicted for three different fluid viscosities, showing that the highest resonator amplitudes can be observed if the viscosity of air is adopted, which corresponds with the largest transmission dip. This suggests that for this particular system, with the unit cell dimensions and material parameters based on the optimisation study of Gao et al. (2017), the viscosity of air is the best among the values considered. This is also consistent with the predictions obtained in the dispersion analysis where the highest attenuation peak for the fast (fluid-born) compressional wave has been achieved with the viscosity of air. Both reducing and enhancing the viscosity leads to the mitigation of the resonator dynamics, with a stronger effect for higher losses $\mu = 10 \mu_{\text{air}}$. The interplay between the viscosity level and the resonance magnitude also explains the lower absorption level for the higher fluid viscosity, see Fig. 14c.

The small opening ratio of the membrane in this study has been used in order to induce the viscothermal dissipation at low frequencies. In Fig. 16, transmission, reflection and absorption for different opening ratios are shown, from fully open to closed unit cells. It is clear that for the high attenuation to occur, a sufficiently small membrane opening is required, since the increase of the membrane opening size reduces both reflection and absorption levels. On the other hand, the presence of small membrane openings results in broadening of the absorption zone also beyond the regime of local resonance.
3.3. Multiple cell performance

In this section, results obtained for the transmission set-up with multiple unit cells as a material sample are presented. In Fig. 17, transmission, reflection and absorption are shown for a row of ten unit cells with heavy resonators and a viscothermal fluid with the viscosity of air. The performance of the unit cells with light resonators is also depicted. In analogy to the single unit cell study, a transmission dip around 440 Hz can be observed, which is not present in the case without added mass. Moreover, in this case, the attenuation range spans a broader frequency range. Note, that the behaviour of the transmission spectra for both light and heavy resonators can be directly related to the attenuation factor for the fast compressional wave (L1) shown in Fig. 5. According to Fig. 17b, the main mechanism underlying the reduction in transmission is reflection, since a high reflection peak exceeding 0.8 emerges around 440 Hz. The reflection peak is followed by a reflection dip, which explains the increase of transmission observed around 560 Hz. Such a behaviour is typical for locally resonant materials, where at the end of the band gap region there is again a high transmissibility (e.g. Lewińska et al. (2017); Liu et al. (2000)). The absorption performance of the multiple cell set-up is rather moderate (Fig. 17c). Moreover, the absorption peak is shifted to higher frequencies in comparison with the single cell behaviour, which is a result of the strong reflection at the resonance frequency.

Among the different dissipative mechanisms occurring in the unit cells, the major role is played by viscous losses, as can be seen in Fig. 18, where the lines denoting total absorption and viscous dissipation practically overlap. Indeed, as stated in Gao et al. (2017), for small unit cell sizes, thermal dissipation is highly reduced. This justifies why this contribution was neglected in the Bloch analysis.
In Fig. 19, the acoustic performance of set-ups of different sizes is shown. The increase in the number of unit cells results in an increase of the wave attenuation in the frequency range around 440 Hz. An increase of the reflection can be observed which is accompanied by a decrease and a shift of a broadened absorption peak.

4. Discussion

The results of this analysis show that the proposed microstructural configuration, in spite of its simplicity clearly indicates a potential way for improving the acoustic attenuation of foams by combining the mechanisms of viscothermal dissipation with local resonance. This combination constitutes a pathway towards the design of acoustic metafoams.

In this study, based on transmission, reflection and absorption spectra, as well as complex dispersion diagrams, the behaviour of foam-like unit cells with and without resonators has been assessed, showing that the presence of resonating masses locally improves the noise isolation properties of the material.

The enhancement of the sound attenuation is based on the increase of both reflection and absorption properties due to the specific design of this coupled solid-fluid system. Moreover, by increasing the number of cells in the material, the reflection mechanism becomes the dominating one.

To obtain the desired transmission dip, it is necessary to properly incorporate the losses in the fluid, i.e. a realistic viscothermal description of air needs to be adopted. A complex description of the fluid domain is fully justified since the characteristic lengths of the pores are comparable with the thicknesses of the viscous and thermal boundary layers. Coupling between the fluid and solid domains plays an important role in inducing the resonance and amplifying viscothermal dissipation. Although the idea of designing a dynamic absorber using tuned resonators (also called Lanchester damper) introduced...
by Den Hartog (Den Hartog, 1985), is well known in the literature (and could be explored further in the metamaterial community), the concept proposed here goes clearly beyond that and relies on the more complex interaction between solid and fluid including viscous effects resulting from the non-slip interface condition. Particularly, due to the presence of the viscous stresses, the solid and fluid domains are strongly coupled and the local resonance has a visible effect on the performance of the material. As demonstrated through the dispersion analysis, the combination of the local resonance of the solid and fluid viscosity allows for the attenuation of the fast (fluid-born) compressional wave, which is key for the development of the next generation acoustic porous materials.

This study has also identified the main parameters governing the attenuation performance of acoustic metafoams, namely the characteristic size of the pores, the membrane opening ratio and the viscosity of the fluid. The characteristic pore size determines the frequencies at which absorption can be expected (Gao et al., 2017). The opening ratio influences the attenuation performance (see Fig. 16). In particular, small membrane openings broaden the absorption peak, but they should be sufficiently small for obtaining high attenuation levels. Finally, the fluid viscosity is essential for enhancing attenuation by ensuring a strong coupling between solid and fluid phases. Depending on other material and geometrical parameters, an optimal level of viscosity exists for which dissipation in the fluid is enhanced by sufficiently high amplitudes of the local resonator (in analogy to viscoelastic solid metamaterials, as shown in Lewińska et al. (2017)). In other words, a viscosity value exists for which the attenuation factor for the fast compressional wave is (locally) the highest. Therefore, an optimal design of the geometry could be pursued, by controlling the characteristic dimensions of the unit cell and the viscosity of the fluid such that the effect of the attenuation is maximised. Intrinsic parameters of the resonators (mass, stiffness) can also be varied in order to tune the operating frequency range, as done for purely solid metamaterials e.g. in Krushynska et al. (2014). Moreover, material damping (viscosity) of the solid should also be taken into account, since it directly influences the efficiency of the local resonator as demonstrated e.g. in Krushynska et al. (2016).

In terms of applications, the microstructural design proposed in this paper can be used to improve the performance of porous materials by the distribution of resonating particles inside their pores. This work might open new paths for designing porous materials, especially considering recent advancements in the control of foam manufacturing processes (Lesov et al., 2014) and the developments in 3D printing techniques for cellular materials (Kaur et al., 2017).

Limited by the computational resources for direct simulations of multiphase lossy material with fine geometrical features, the performance of up to fifteen identical unit cells in a row has been analysed, whereas the typical thickness of foam panels is of the order of several centimetres using high-performance computing tools e.g. van Tuijl et al. (2018), these larger thickness could be reached. On the other hand, based on the studies of band structures, the behaviour of an infinite periodic material has been assessed, leading to consistent conclusions. For the proposed unit cell geometry, the observed transmission dip is followed by a transmission peak, which is typical for locally resonant acoustic metamaterials (Lewińska et al., 2017; Liu et al., 2000) and coherent with the attenuation spectra obtained based on Bloch analysis. This fact implies that the proposed idealised structure is specifically effective for wave attenuation at a selected low frequency range. A random microstructure with non-uniform resonators, however, may have the potential of providing even broader frequency attenuation ranges. This would require further analyses which will be the subject of future investigations.

5. Conclusion

This paper presented an acoustic metafoam concept, based on a poroelastic unit cell with an embedded resonating mass. The acoustic attenuation at low frequencies has been improved by combining the effects of viscothermal dissipation in the fluid (air) with local resonance of the solid. In contrast to many previous studies, the resonators are introduced within the pore and in the form of a resonating particle at the tip of an elastic micro-cantilever.

Analysis of complex dispersion diagrams and numerical transmission spectra showed that the proposed unit cell enriched with a resonator performs significantly better at low frequencies (below 450 Hz) than its light or non-resonating equivalent. The resulting transmission dip is profound even for a single unit cell. However, due to the resonant origin of the attenuation the affected frequency range remains limited.

It has been shown that the enhanced attenuation only emerges if viscous losses in the fluid are included. This underlines the role of the fluid-solid coupling due to which not only the local resonance is induced but also the viscous dissipation is increased. The complex fluid description is another feature distinguishing this work among other metamaterials involving acoustic-structure design (Kook and Jensen, 2017), where a phenomenological description of losses is usually sufficient.

This work contributes to a novel design towards acoustic metafoams and to the development of new porous materials, with improved performance at low frequencies.

Acknowledgements

The research leading to these results has received funding from the European Research Council under the European Union’s Seventh Framework Programme (FP7/2007-2013) / ERC grant agreement no 339392.
Appendix

In the appendix the details concerning the complex band structure calculation for a two phase unit cell consisting of a solid frame entrained with either a complex viscous or an inviscid fluid are presented. For this purpose, the weak forms of governing equations with the application of Bloch theorem are presented.

Acoustic field with viscous effects

The governing equations (see Eqs. (2), (3), and (4) assuming that thermal dissipation is negligible allowing to discard the temperature field) are:

\[ i \omega \rho_0^f v = -\nabla p + \nabla \cdot \left( \mu^f (\nabla \mathbf{v} + (\nabla \mathbf{v})^T) - \frac{2}{3} \mu^f (\nabla \cdot \mathbf{v}) \mathbf{l} \right), \tag{12} \]

\[ \rho_0^f \nabla \cdot \mathbf{v} = -i\omega \frac{p}{c_0^2}. \tag{13} \]

Next, Bloch wave solutions are assumed in the form:

\[ p = \tilde{p} \exp(i\mathbf{k} \cdot \mathbf{x}), \tag{14} \]

\[ \mathbf{v} = \tilde{\mathbf{v}} \exp(i\mathbf{k} \cdot \mathbf{x}). \tag{15} \]

\[ \nabla p = \left( \nabla \tilde{p} + i\tilde{p} \mathbf{k} \right) \exp(i\mathbf{k} \cdot \mathbf{x}), \tag{16} \]

\[ \nabla \mathbf{v} = \left( \nabla \tilde{\mathbf{v}} + i\mathbf{k} \tilde{\mathbf{v}} \right) \exp(i\mathbf{k} \cdot \mathbf{x}). \tag{17} \]

\[ \nabla \cdot \mathbf{v} = \left( \nabla \cdot \tilde{\mathbf{v}} + i\mathbf{k} \cdot \tilde{\mathbf{v}} \right) \exp(i\mathbf{k} \cdot \mathbf{x}), \tag{18} \]

where \( \tilde{p} \) and \( \tilde{\mathbf{v}} \) are the Bloch pressure and velocity functions with spacial periodicity (identical to the periodicity of the structure), \( \mathbf{x} \) is the position vector and \( \mathbf{k} \) is the wavevector. In these equations, the dyadic product is denoted as: \( \mathbf{ab} = a_b b_a \mathbf{e}_a \mathbf{e}_m \).

The final weak form describing the acoustic field with viscous effects is obtained after the substitution of the Bloch wave solutions in Eqs. (12) and (13), followed by the multiplication with their respective test functions \( \phi^f, \phi^f \) and integration over the fluid domain \( \mathcal{V}^f \):

\[ -i \int_{\mathcal{V}^f} \omega \rho_0^f \tilde{\mathbf{v}} \cdot \phi^f d\mathcal{V} - i \int_{\mathcal{V}^f} \tilde{p} \mathbf{k} \cdot \phi^f d\mathcal{V} + \int_{\mathcal{V}^f} \tilde{p} \nabla \cdot \phi^f d\mathcal{V} \]
\[ - \int_{\partial \mathcal{V}^f} (\nabla \phi^f)^T : \left( \frac{\mu^f}{2} \left( (\nabla \tilde{\mathbf{v}} + i\mathbf{k} \tilde{\mathbf{v}}) + (\nabla \tilde{\mathbf{v}} + i\mathbf{k} \tilde{\mathbf{v}})^T \right) \right) \cdot \phi^f d\Gamma \]
\[ + i \int_{\partial \mathcal{V}^f} \mathbf{k} \cdot \left( \frac{\mu^f}{2} \left( (\nabla \cdot \tilde{\mathbf{v}} + i\mathbf{k} \cdot \tilde{\mathbf{v}}) \mathbf{l} \right) \right) \cdot \phi^f d\Gamma \]
\[ - i \int_{\partial \mathcal{V}^f} \mathbf{n}^f \cdot \left( \tilde{\mathbf{p}} \phi^f \right) d\Gamma - i \int_{\partial \mathcal{V}^f} \mathbf{n}^f \cdot \left( \frac{\mu^f}{3} \left( (\nabla \cdot \tilde{\mathbf{v}} + i\mathbf{k} \cdot \tilde{\mathbf{v}}) \mathbf{l} \right) \cdot \phi^f \right) d\Gamma \]
\[ + \int_{\partial \mathcal{V}^f} \mathbf{n}^f \cdot \left( \mu^f \left( (\nabla \tilde{\mathbf{v}} + i\mathbf{k} \tilde{\mathbf{v}}) + (\nabla \tilde{\mathbf{v}} + i\mathbf{k} \tilde{\mathbf{v}})^T \right) \cdot \phi^f \right) d\Gamma = 0, \tag{19} \]

\[ \rho_0^f \int_{\mathcal{V}^f} \nabla \cdot (\tilde{\mathbf{v}} \phi^f) d\mathcal{V} - \rho_0^f \int_{\mathcal{V}^f} (\nabla \phi^f) \cdot \tilde{\mathbf{v}} d\mathcal{V} + i \rho_0^f \int_{\mathcal{V}^f} \tilde{\mathbf{v}} \cdot \mathbf{k} \phi^f d\mathcal{V} + i\omega \frac{c_0^2}{\rho_0^f} \int_{\mathcal{V}^f} \tilde{p} \phi^f d\mathcal{V} = 0, \tag{20} \]

where \( \mathbf{n}^f \) is the unit outward normal to the boundary \( \partial \mathcal{V}^f \) of the domain \( \mathcal{V}^f \).
Acoustic inviscid isothermal fluid

The acoustic Helmholtz equation is given (see Eq. (7)) by:
\[ \nabla \cdot (\nabla p) + k_0^2 p = 0, \quad \text{with} \quad k_0 = \frac{\omega}{c_0}. \tag{21} \]

The weak form obtained for Eq. (21) after the substitution of Bloch wave solution is written as:
\[
- \int_{V^o} \nabla \phi^a \cdot (\nabla \tilde{p} + i \kappa \kappa) dV + i \int_{\partial V^o} \kappa \phi^a d\Gamma = 0
\]
\[
+ \int_{V^o} k_0^2 \tilde{p} \phi^a dV + \int_{\partial V^o} \mathbf{n} \cdot (\nabla \tilde{p} + i \kappa \kappa) \phi^a d\Gamma = 0
\]
where \( \phi^a \) is a test function, \( V^o \) denotes the fluid domain, \( \mathbf{n} \) is the unit outward normal to the boundary \( \partial V^o \) of the domain \( V^o \).

Elastic solid

The equation of motion for elastic solid is given (see Eq. (1)) by:
\[ -\rho \omega^2 \mathbf{u} = \nabla \cdot (\mathbf{C} : \nabla \mathbf{u}). \tag{23} \]

The Bloch wave solutions assumed in this case are:
\[ \mathbf{u} = \tilde{\mathbf{u}} \exp (i \kappa \cdot \mathbf{x}), \tag{24} \]
\[ \nabla \mathbf{u} = (\nabla \tilde{\mathbf{u}} + i \kappa \tilde{\mathbf{u}}) \exp (i \kappa \cdot \mathbf{x}), \tag{25} \]
where \( \mathbf{u} \) is the Bloch displacement function with spatial periodicity (identical to the periodicity of the structure).

The weak form obtained for Eq. (23) after the substitution of the Bloch wave solution is written as:
\[
- \int_{V^s} \rho \omega^2 \tilde{\mathbf{u}} \cdot \mathbf{\Phi} dV - i \int_{\partial V^s} \mathbf{\Phi} \cdot \mathbf{C} : (\nabla \tilde{\mathbf{u}} + i \kappa \tilde{\mathbf{u}}) d\Gamma
\]
\[
+ \int_{V^s} (\nabla \tilde{\mathbf{u}}) \cdot (\nabla \tilde{\mathbf{u}} + i \kappa \tilde{\mathbf{u}}) dV - \int_{\partial V^s} \mathbf{n} \cdot (\mathbf{C} : (\nabla \tilde{\mathbf{u}} + i \kappa \tilde{\mathbf{u}}) \cdot \mathbf{\Phi}) d\Gamma = 0, \tag{26} \]
where \( \mathbf{\Phi} \) is a test function, \( V^s \) denotes the solid domain, \( \mathbf{n} \) is the unit outward normal to the boundary \( \partial V^s \) of the domain \( V^s \).

Coupling conditions at the interface between fluid and solid are adopted as described in Section 2.2 through the boundary terms in the weak forms of the governing equations (using weak contributions assigned to surfaces). A set of fully coupled equations is solved similarly as done for instance in Matuszyk et al. (2012).

In order to obtain an eigenvalue problem, the \( \kappa \)-vector is written as \( \kappa = k \alpha \). where \( k \) is the amplitude of the wave vector along the unit propagation direction \( \kappa \). Since only the \( \Gamma \) direction of wave propagation is considered here, wavenumbers \( k = k_i \alpha \) are calculated for given frequencies.

References


