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Positive Stabilizability of a Linear Continuous-Time System

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Abstract

Positive stabilizability of a continuous-time linear system $\dot{x} = Ax + Bu$ means that for all initial conditions there exists a positive L_2 -input function resulting in an L_2 -state trajectory. In this paper, easily verifiable necessary and sufficient conditions for positive stabilizability are presented. Moreover, attention will be paid to a feedback variant of this stabilization problem.

1 Introduction

Control problems involving a positivity constraint on the control variables are studied extensively in the literature. One encounters studies on controllability and reachability issues in both continuous time [4, 11, 22, 23] and discrete time [7, 8], design of positive stabilizing controllers [26, 28], time-optimal positive control [17, 30], analysis [10, 18] and numerics [25] of linear-quadratic optimal control problems with positive controls. Also in the setting of positive and compartmental systems [21, 29], in which additionally the state and output of the system are restricted to positive values, many control problems have been considered.

The continuing interest in these control problems is well explained and motivated by many applications in which the attainable values of the control function are inherently constrained in the sense that the direction of its influence cannot be changed. One might think of electrical networks with diode elements, mechanical systems with one-way valves (e.g. compressors [27]), economics, (investments and taxes are positive controls), population control [15], ecological (soil fertilization) and medical systems (drug infusion), etcetera.

In this paper, a stabilization problem of a continuous-time linear system is considered, where the control restraint set is a closed and convex polyhedral cone such as the positive orthant in a Euclidean space. Necessary and sufficient conditions are presented guaranteeing for each initial state the existence of an open-loop positive control function that steers the state asymptotically to zero. A system having this property is called positively stabilizable.

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Of course, many feedback variants of the above stabilization problem are of interest. Zaslavsky [28] shows the existence of a piecewise continuous state feedback that renders the origin locally stable (under the condition of ‘positive controllability’). A more general result has been proven by Smirnov [26]. Smirnov shows that the conditions for ‘open-loop positive stabilizability’ are necessary and sufficient for the existence of a stabilizing Lipschitz continuous state feedback. In contrast with [26, 28] we will a priori impose a simple and easily implementable structure of the feedback. To be specific, the feedback consist of the componentwise maximum of a linear state feedback and zero. Under certain requirements we can actually show that the conditions for ‘open-loop positive stabilizability’ are necessary and sufficient to guarantee the existence of a stabilizing feedback of such a simple form. For solving the general problem, stability techniques might be needed from hybrid systems theory [1, 20], since the closed-loop system displays switching between several dynamical regimes. The general problem is still open and we will show that many common stabilization techniques will even fail in the simplest case of interest.

The following notational conventions will be in force. \mathbb{R}_+ denotes the positive real numbers (including zero) and \mathbb{C}_+ denotes the set of complex numbers with real parts in \mathbb{R}_+ . The spectrum of a matrix $A \in \mathbb{R}^{n \times n}$ is denoted by $\sigma(A)$. A square matrix A is called stable, if $\sigma(A) \cap \mathbb{C}_+ = \emptyset$ and called antistable if $\sigma(A) \subset \mathbb{C}_+$. The projection operator Π from the vector space \mathbb{R}^m to the positive orthant \mathbb{R}_+^m is defined for $v \in \mathbb{R}^m$ by

$$(\Pi v)_i = \max(v_i, 0) = \begin{cases} v_i, & \text{if } v_i \geq 0 \\ 0, & \text{if } v_i \leq 0 \end{cases} \quad (1)$$

2 Problem formulation

We consider a linear system (A, B) given by

$$\dot{x}(t) = Ax(t) + Bu(t),$$

where $u(t) \in \mathbb{R}^m$ is the input function and $x(t) \in \mathbb{R}^n$ the state at time t . $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$ are constant matrices. The input functions are assumed to belong to the Lebesgue space of square integrable, measurable functions on \mathbb{R}_+ taking values in \mathbb{R}^m , denoted by L_2^m . For every input function $u \in L_2^m$ and initial state $x_0 \in \mathbb{R}^n$, the solution of (2) with $x(0) = x_0$ is an absolutely continuous state trajectory, denoted by $x_{x_0, u}$.

The closed convex cone of positive functions in L_2^m is defined by

$$P^m := \{u \in L_2^m \mid u(t) \in \mathbb{R}_+^m \text{ for almost all } t \in \mathbb{R}_+\},$$

Considering a control restraint set equal to the positive orthant \mathbb{R}_+^m may seem quite restrictive. However, the obtained results apply to arbitrary convex polyhedral cones Ω containing 0. Parametrizing Ω as $\Omega = F\mathbb{R}_+^m$, where F is an a matrix of appropriate dimensions and taking $(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}) = (A, BF, C, DF)$ transforms the problem to one with control restraint set \mathbb{R}_+^m (also used in [18]).

A control $u \in L_2^m$ is said to be stabilizing for initial state x_0 , if $x_{x_0,u} \in L_2^n$. Note that $u \in L_2^m$ and $x_{x_0,u} \in L_2^n$ imply $\dot{x}_{x_0,u} \in L_2^n$, which guarantees that $x_{x_0,u}(t) \rightarrow 0$ ($t \rightarrow \infty$).

Definition 2.1 (Stabilizability) (A, B) is said to be stabilizable, if for every $x_0 \in \mathbb{R}^n$ there exists a stabilizing control $u \in L_2^m$.

Necessary and sufficient conditions for stabilizability of (A, B) are specified by Hautus [9]. (A, B) is stabilizable if and only if

$$\text{rank}(A - \lambda I; B) = n \text{ for all } \lambda \in \mathbb{C}_+,$$

where I denotes the identity matrix. In this note, the objective is to give explicit conditions for positive stabilizability.

Definition 2.2 (Positive Stabilizability) (A, B) is said to be positive stabilizable, if for every $x_0 \in \mathbb{R}^n$ there exists a stabilizing control $u \in P^m$.

It is evident, that positive stabilizability implies stabilizability. The converse statement is not true as is demonstrated by the following example.

Example 2.3 Consider the system $\dot{x} = x + u$ with initial condition $x_0 = 1$. It is obvious that no $u \in P^m$ exists such that $x_{x_0,u} \in L_2^n$. However, the system is stabilizable.

A stabilizable system admits a linear state feedback $u(t) = Fx(t)$ with $F \in \mathbb{R}^{m \times n}$ stabilizing the system in the sense that all solution trajectories of $\dot{x}(t) = (A + BF)x(t)$ are contained in L_2^n [9]. In case the system is subject to a positivity constraint, one may raise the question whether a simple stabilizing feedback structure can be obtained as well. One of the first forms one might think of is given in the following definition.

Definition 2.4 (Positive Feedback Stabilizability) (A, B) is said to be positive feedback stabilizable, if there exists an $F \in \mathbb{R}^{m \times n}$ such that all solution trajectories of

$$\dot{x}(t) = Ax(t) + B\Pi(Fx(t))$$

are contained in L_2^n .

It is obvious, that positive stabilizability is a necessary condition for positive feedback stabilizability, but it is not clear if this condition is also sufficient. Note that in the unconstrained case (i.e. u may take values in \mathbb{R}^m) feedback stabilizability (by a linear state feedback) is equivalent to stabilizability.

3 Positive stabilizability

Theorem 3.1 *The system (A, B) is positive stabilizable if and only if (A, B) is stabilizable and all real eigenvectors v of A^\top corresponding to a positive eigenvalue of A^\top have the property that $B^\top v$ has at least one strictly positive component.*

Remark 3.2 Note that if v is a real eigenvector of A^\top , then $-v$ is also an eigenvector corresponding to the same eigenvalue. Hence, in the theorem above $B^\top v$ must also have at least one strictly negative component.

Corollary 3.3 Consider a system (A, B) with scalar input ($m = 1$). Then (A, B) is positive stabilizable if and only if (A, B) is stabilizable and $\sigma(A) \cap \mathbb{R}_+ = \emptyset$.

Proof. If (A, B) is stabilizable and A (and hence A^\top) has no positive real eigenvalues, then the conditions in Thm. 3.1 for positive stabilizability are satisfied. Conversely, if (A, B) is positive stabilizable, then (A, B) is stabilizable. Suppose that $\lambda \in \sigma(A^\top) \cap \mathbb{R}_+$. According to Thm. 3.1 and Remark 3.2, a corresponding real eigenvector v must satisfy that $B^\top v$ has both negative and positive components. However, this is impossible, because $B^\top v$ is a scalar. \square

Proof of the necessity part of Theorem 3.1 Obviously, positive stabilizability implies stabilizability of (A, B) . Suppose that there exists an eigenvector v of A^\top corresponding to an eigenvalue $\lambda \geq 0$ of A^\top with the property that all the components of $B^\top v$ are smaller than or equal to zero. Take x_0 such that $v^\top x_0 < 0$. We claim that for all $u \in P^m$, $x_{x_0, u} \notin L_2^n$. Let $u \in P^m$ and consider $s(t) := v^\top x_{x_0, u}$. Since $B^\top v \leq 0$ (inequality holding for all components) and $u(t) \geq 0$ almost everywhere,

$$\dot{s}(t) = v^\top A x_{x_0, u}(t) + v^\top B u(t) = \lambda v^\top x_{x_0, u}(t) + v^\top B u(t) \leq \lambda s(t)$$

Combining this expression for $\dot{s}(t)$ with $\lambda \geq 0$ and $s(0) = v^\top x_0 < 0$, yields $s(t) \leq s(0) < 0$ contradicting that $x_{x_0, u} \in L_2^n$. Hence, (A, B) is not positive stabilizable. \square

To prove the sufficiency of Theorem 3.1, some preliminary results are needed. For the moment, let Ω be an arbitrary subset of \mathbb{R}^m and let L_∞ be the space of essentially bounded Lebesgue measurable functions with norm denoted by $\|\cdot\|_\infty$. Define

$$U := \{u \in L_\infty^m \mid u(t) \in \Omega \text{ for almost all } t \in \mathbb{R}_+\}.$$

The system (A, B) with control restraint set Ω is called *0-controllable*, if for every initial state x_0 there exists a control input $u \in U$ such that the corresponding state trajectory x satisfies $x_{x_0, u}(T) = 0$ for some $T \geq 0$. If $\Omega = \mathbb{R}^m$, we call (A, B) controllable and if $\Omega = \mathbb{R}_+^m$, we call (A, B) positive controllable.

Theorem 3.4 (A, B) is positive controllable if and only if

1. (A, B) is controllable;
2. All real eigenvectors v of A^\top have the property that $B^\top v$ has at least one strictly positive component.

Proof. The theorem is a simple translation of the results obtained in [4, 11]. \square

The following result is well-known, see e.g. [13].

Lemma 3.5 *Let (A, B) be stabilizable. There exists a nonsingular transformation S and a decomposition of the new state variable $\bar{x} = Sx$ in $(x_1^\top, x_2^\top)^\top$ such that $\bar{A} := SAS^{-1}$ and $\bar{B} = SB$ can be decomposed accordingly as:*

$$\bar{A} = \begin{pmatrix} A_{11} & 0 \\ 0 & A_{22} \end{pmatrix} ; \quad \bar{B} = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}$$

with A_{11} antistable, A_{22} stable and (A_{11}, B_1) controllable.

Proof of the sufficiency part of Theorem 3.1. According to Lemma 3.5, there exist a nonsingular transformation S and a decomposition of the new state space variable, such that the new system can be written as

$$\dot{x}_1(t) = A_{11}x_1(t) + B_1u(t) \quad (2a)$$

$$\dot{x}_2(t) = A_{22}x_2(t) + B_2u(t) \quad (2b)$$

with A_{11} antistable, A_{22} stable and (A_{11}, B_1) controllable.

We claim that the system

$$\dot{x}_1(t) = A_{11}x_1(t) + B_1u(t) \quad (3)$$

is positive controllable. According to Thm. 3.4 and the controllability of (A_{11}, B_1) , it suffices to verify condition 2 of Thm. 3.4. Suppose there exists a real eigenvector v of A_{11}^\top with (real) eigenvalue λ satisfying $B_1^\top v \leq 0$. We see that $z := S^\top \begin{pmatrix} v \\ 0 \end{pmatrix}$ is a real eigenvector of A^\top with eigenvalue λ . From

$$0 \geq B_1^\top v = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}^\top \begin{pmatrix} v \\ 0 \end{pmatrix} = B^\top S^\top \begin{pmatrix} v \\ 0 \end{pmatrix} = B^\top z$$

we conclude that z is a real eigenvector of A^\top with $B^\top z \leq 0$. Since $\lambda \in \mathbb{R}_+$ (because $\lambda \in \sigma(A_{11})$ and λ is real), the hypotheses of the theorem are contradicted. Hence, condition 2 of Thm. 3.4 is satisfied and system (3) is positive controllable.

Take an arbitrary initial state $x_0 = (x_{01}^\top, x_{02}^\top)$ of the transformed system (\bar{A}, \bar{B}) . Positive controllability of (A_{11}, B_1) implies the existence of a L_∞ -function u in the cone of positive functions such that the corresponding state trajectory $x = x_{x_0, u}$ satisfies $x_1(T) = 0$ for certain $T > 0$. Taking the function $\bar{u} \in L_2$ as $\bar{u}(t) := u(t)$, $t \in [0, T]$ and $\bar{u}(t) = 0$, $t \in (T, \infty)$ results in the state trajectory $x_1(t) = 0$, $t \in [T, \infty)$ and $x_2 \in L_2$, because A_{22} is stable. \square

4 Positive Feedback Stabilizability

We recall the definition of positive feedback stabilizability (PFS) (Def. 2.4). (A, B) is PFS, if there exists a matrix $F \in \mathbb{R}^{m \times n}$ such that

$$\dot{x}(t) = Ax(t) + B\Pi(Fx(t)) \quad (4)$$

only admits L_2 -solution trajectories. Stability of a system as in (4) (given F) is most likely studied in the context of ‘switching’ or ‘hybrid systems’ [1, 20]. In case of a scalar input ($m = 1$) the system description becomes

$$\dot{x}(t) = Ax(t) + B \max(0, Fx(t)). \quad (5)$$

It is clear that this system switches between two dynamics (sometimes called ‘modes’), to wit $\dot{x}(t) = Ax(t)$ and $\dot{x}(t) = (A + BF)x(t)$, depending on the sign of $Fx(t)$.

The stability of similar switching systems, where two linear dynamics $\dot{x}(t) = A_1x(t)$ and $\dot{x} = A_2x(t)$, are separated by a hyperplane $c^\top x = 0$, i.e.

$$\dot{x}(t) = \begin{cases} A_1x(t) & \text{if } c^\top x(t) \geq 0 \\ A_2x(t) & \text{if } c^\top x(t) \leq 0 \end{cases} \quad (6)$$

are studied extensively [3, 6, 12, 16]. However, the proposed techniques do not apply here.

The determination of stability for the discrete time equivalent of (6) is even computationally intractable (NP-hard, for a definition of NP-hardness see [19]) as has been shown in [2]. Moreover, [2] mentions that the question of decidability is a major open problem.

The ‘state of the art’ in stability theory [5, 6] does not lead to necessary and sufficient conditions in a constructive or easy way, since it is based on the existence of suitable Lyapunov functions. Even in the simplest case of interest (A being unstable and single input ($m = 1$)) many of the common techniques fail. Trying to find a common (sometimes called “simultaneous”) quadratic Lyapunov function (see e.g. [3, 16, 24]) will not work due to instability of A . This implies that the well-known circle criterion [14] (note that the nonlinearity in (5) is sector-bounded) will fail, because its proof is based on a common quadratic Lyapunov function. The Popov criterion [14] does also not lead to an answer of the stabilization problem, because the sector-bounded nonlinearity will always allow a system that is governed by the unstable dynamics $\dot{x} = Ax$ (even after a transformation). An approach related to the Popov criterion (at least in simple cases) is the use of piecewise quadratic functions [12]. This method tries to reduce conservatism by exploiting the S-procedure and take Lyapunov functions that are piecewise quadratic. The technique in [12] needs the existence of two positive definite matrix P_- and P_+ such that

$$x^\top (A + BF)^\top P_+ x + x^\top P_+ (A + BF)x < 0 \quad \text{if } Fx \geq 0 \quad (7)$$

$$x^\top A^\top P_- x + x^\top P_- Ax < 0 \quad \text{if } Fx \leq 0 \quad (8)$$

(and $x^\top P_- x = x^\top P_+ x$ when $Fx = 0$). Observe that the inequalities must in fact hold in the whole state space. Indeed, if the first inequality in (8) holds for x with $Fx \geq 0$, then it holds also for $-x$. Hence, the instability of A prevents that the condition (8) can be satisfied by any $P_- > 0$. The multiple Lyapunov method proposed in [6] requires the existence of two Lyapunov functions for each discrete state (i.e. on each side of the switching surface). However, it is well-known that the construction of Lyapunov functions can be very awkward. Especially regarding the fact that [6] requires that the value of the Lyapunov function at the entry time of one mode must be lower than the previous entry time of this mode. Such a result seems hard to establish, because one has to determine the effect of a complete cycle of Fx being successively negative and positive. The net result depends heavily on the dwelling time in each mode and

the rate of decrease/increase (see also the proof of the theorem below, where we indeed use this explicitly). For the problem at hand this means that one almost needs explicit expressions for its trajectories. Consequently, the results could then be established without using Lyapunov functions.

However, there is an approach which seems very promising. Under the conditions of (open loop) positive stabilizability Smirnov [26] shows the existence of a feedback that is Lipschitz continuous and positive. Furthermore, the existence of a corresponding Lyapunov function (convex and positively homogeneous) is also guaranteed. His approach does not reveal whether or not a stabilizing feedback of the structure as given in Def. 2.4 is possible. However, future research will focus on verifying if Smirnov's approach allows such a specific feedback structure.

For simple cases the conditions for positive stabilizability (PS) turn out to be necessary and sufficient for PFS.

Theorem 4.1 *Suppose that (A, B) has scalar input ($m = 1$) and A has at most one pair of unstable, complex conjugate eigenvalues. The problem of positive feedback stabilizability is solvable if and only if (A, B) stabilizable and $\sigma(A) \cap \mathbb{R}_+ = \emptyset$.*

Proof. Note that positive stabilizability is a necessary condition for positive feedback stabilizability. In combination with Corollary 3.3 this proves the necessity part.

In case A has no unstable complex eigenvalues, the result is trivial. Since $\sigma(A) \cap \mathbb{R}_+ = \emptyset$, A is necessarily stable and $F = 0$ results in a stable closed loop system (4). Hence, consider the case where A has one pair of complex conjugate eigenvectors $\sigma_0 \pm j\omega_0$ with nonzero imaginary parts. Apply a similarity transformation as specified in Lemma 3.5 yielding a system description as in (2). The stability of A_{22} implies that for all $u \in L_2$ the corresponding state trajectory $x_2 \in L_2$ (for arbitrary initial state). Hence, if we can construct a feedback of the form $u = \max(0, Fx_1)$ (only depending on x_1) that positively stabilizes (2a), the proof is complete.

We can concentrate on (2a) which is controllable. Note that the vector x_1 has dimension 2. Since (A_{11}, B_1) is controllable, the eigenvalues of $A_{11} + B_1F$ can be placed arbitrarily by suitable choice of F . We claim that if F is designed such that the eigenvalues of $A_{11} + B_1F$ are contained in $\{\lambda = \sigma + j\omega \in \mathbb{C} \mid \sigma < 0 \text{ and } |\frac{\omega}{\sigma}| < |\frac{\omega_0}{\sigma_0}|\}^1$, then the resulting closed-loop system (2a) is stable. A solution to (2a) corresponding to initial state x_0 will be denoted by x_{x_0} . Note that we omitted the subscript 1 to indicate that we only consider (2a). We distinguish two cases.

1. The eigenvalues of $A_{11} + B_1F$ are real. There are now three possibilities.

- (a) $x_0 \in K_+ := \{z \in \mathbb{R}^2 \mid Fz \geq 0\}$ and $x_{x_0}(t) \notin K_+^c := \{z \in \mathbb{R}^2 \mid Fz < 0\}$ for all $t > 0$. Hence, x_{x_0} satisfies $\dot{x}_{x_0}(t) = (A_{11} + B_1F)x_{x_0}(t)$ for all $t > 0$. Stability of $A_{11} + B_1F$ gives $x_{x_0} \in L_2$.
- (b) $x_0 \in K_+^c$. A sign switch in Fx_{x_0} will occur. Indeed, as long as $Fx_{x_0}(t) \leq 0$,

$$Fx_{x_0}(t) = Fe^{A_{11}t}x_0 = Ce^{\sigma_0 t} \cos(\omega_0 t + \phi) \quad (9)$$

¹In case $\sigma_0 = 0$ it is sufficient to place the eigenvalues such that $\sigma(A_{11} + B_1F) \subset \mathbb{C}_-$.

for certain real constants $C \neq 0$ and ϕ . This implies that a sign switch must occur. Say the time of and state at the first sign switch are equal to t_0 and \tilde{x}_0 , respectively. For a positive time interval the system (2a) evolves according to the dynamics of the ‘stable mode’ $\dot{x}(t) = (A_{11} + B_1F)x(t)$. Observe that $F e^{(A_{11} + B_1F)(t-t_0)} \tilde{x}_0$ can have at most one zero, because the eigenvalues of $A_{11} + B_1F$ are real and the dimension of x_1 is 2. Since there is a zero for $t = t_0$, there will be no switch of dynamics beyond t_0 and the system stays in the stable mode.

(c) $x_0 \in K_+$ and $x_{x_0}(t) \in K_+^c$ for some $t > 0$ is similar to the previous possibility.

2. The eigenvalues of $A_{11} + B_1F$ are complex, say $\sigma \pm j\omega$. Eventually, the system will switch between the two dynamics as long as the state $x_{x_0}(t)$ does not become equal to zero. This is most easily seen from (9). For the stable mode similar arguments can be used. From this it can also be seen that the time spent in the stable mode equals $\frac{\pi}{|\omega|}$ and similarly, in the unstable mode $\frac{\pi}{|\omega|}$. The norm of the state decays in one complete cycle of stable and unstable mode by $e^{\frac{\pi\sigma}{|\omega|}} \cdot e^{\frac{\pi\sigma_0}{|\omega_0|}}$, which is strictly less than 1 due to the choice of the eigenvalues of $A_{11} + B_1F$.

□

The proof above relies on the fact that either no switching occurs eventually or that the duration in the separate modes is known exactly. This approach becomes quite complicated for higher dimensional state spaces and different design techniques have to be exploited.

5 Conclusions

In this note, necessary and sufficient conditions for positive stabilizability of continuous-time linear systems have been presented. The results have been based on well-known conditions for positive controllability. Besides the open-loop version of stabilizability, a feedback variant has been stated. For a relatively simple case, conditions could be given guaranteeing the solvability of the problem. However, the general problem is still open and will be the object of further research. Especially the work of Smirnov [26] could play a major role in the solution of the problem.

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