Aggregation in manpower planning

EINDHOVEN UNIVERSITY OF TECHNOLOGY

Department of Industrial Engineering

Department of Mathematics and Computing Science

no 22

AGGREGATION IN MANPOWER PLANNING

by

J. Wijngaard

September 1981

BDK/ORS/80/10 (revised version)

1. Introduction

In manpower planning, as in all other kinds of planning, important parameters are the planning horizon and the level of aggregation. These two are related to each other. Generally there are more planning levels. The lower level planning is detailed and has a short planning horizon. In the higher level planning the variables are more aggregated and the planning horizon is longer. The higher level planning determines restrictions (budgets) for the lower level planning. The structure of the complete planning process necessary to control an organization depends on one hand on the flexibility of the organization and on the other hand on the instability of the environment. With respect to manpower planning, flexibility is mainly mobility, the capability of personnel to execute different types of jobs, and instability of the environment is mainly instability of the manpower requirement.

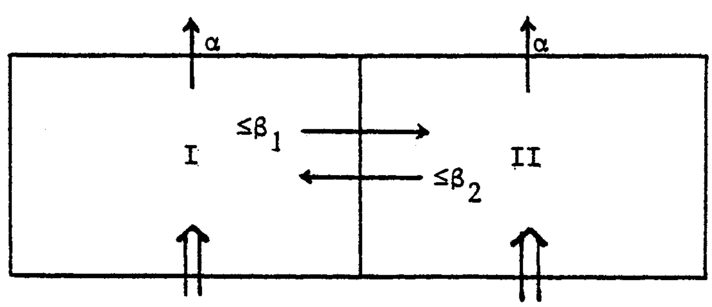
An extreme case is the case of an organization where each employee can do all kinds of jobs, where vacancies can be filled directly by recruitment and where firing is easily possible. In such an organization only short--term planning is necessary. One only has to make an assignment plan. Another extreme case is the case of an organization with very specific functions and employees (which causes a low mobility and makes it difficult to fill vacancies directly by recruitment), no possibilities to fire people and a very unstable manpower requirement. In this case the appropriate planning process is much more complex. Since most decisions have a longlasting impact one also needs medium- and long-term manpower planning. And by the lack of mobility it may be necessary to make these medium- and long-term plans rather detailed. Of course planning is only useful if there is at least some flexibility.
Another relevant characteristic with respect to the structure of the planning process is the predictability of the environment.
Predictable instability requires a complete different planning process than unpredictable instability. If in the second extreme case the future manpower requirement is very unpredictable then long-term manpower planning may be of no use. In cases with a high flexibility it is <u>not necessary</u> to have medium- and long-term plans, in cases with high uncertainty it is of <u>no use</u> to have medium- and long-term plans because the information on which these plans are based is too unreliable. See also Verhoeven [7], sections 1.6, 5.4 and 5.5.

This paper is concentrated on flexibility. The effect of unpredictability is considered in Smits, Wijngaard [6]. We consider a goal programming approach of manpower planning and investigate the relationship of the right structure of the planning process to the mobility of the personnel and the instability of the goals on the contents of the categories. Goal programming is widely used in manpower planning, especially in defense organizations (see for instance Charnes et al. [2], [3]), but there has never been paid much attention to the problem of aggregation and disaggregation. The related problem of the right planning horizon has been considered in Nuttle, Wijngaard [5]. Similar problems in the field of production planning have been considered in Wijngaard [9]. The whole approach is also related to the work of Hax and Meal on hierarchical planning systems [4].

In the manpower systems considered in this paper the categories of personnel are characterized by two dimensions, level and function group. If there is sufficient mobility over the function groups one may aggregate over the different function groups. Instead of a long term detailed model one may use a long term aggregated model and a detailed model with planning horizon one. In this detailed model the results of the long term aggregated model serve as restrictions (budgets). The advantages of such a stratification of the planning process are not only numerical. The numerical advantages are only secondary. The main advantage is that it is not necessary to forecast the requirement in each of the categories but that it is sufficient to forecast only aggregated requirements. This not only reduces the forecasting load of the organization, but also makes the forecasts more reliable.
In section 2 the one-level case is treated, in section 3 the multi-level case. In section 4 a numerical example is worked out and some remarks are made on the applicability of the results.

2. One level system

The system we will consider first is depicted in the following figure



There are two function groups (I and II). The forecasted future requirements for these categories in period t are $n_I(t)$ and $n_{II}(t)$. The "cost" of deviations from these "goals" is expressed by a penalty function which is linear in the deviations, so

$$|x_I(t) - n_I(t)| \text{ and } |x_{II}(t) - n_{II}(t)|$$

where $x_I(t)$, (respectively $x_{II}(t)$) is the planned number of people in category I (resp. II) in period t. The turnover fraction from each of these categories is α. The population can be controlled by recruitment and by transferring people from one function group to another. Recruitment is nonnegative but further unrestricted and gives no cost. Mobility is restricted; each year one may move not more than a fraction β_1 of the people in function group I to function group II and a fraction β_2 of the people in function group II to function group I. Usage of this mobility gives no cost as long as it remains within these restrictions. We assume that the actual content of the categories is equal to $n_I(0)$, $n_{II}(0)$. The planning problem for the system is represented by the following model (model A):

$$\text{Minimize} \quad \sum_{t=1}^{T} \{|x_I(t)-n_I(t)| + |x_{II}(t)-n_{II}(t)|\}$$

such that

$$x_I(t+1) \geq (1-\alpha-\beta_1)x_I(t), \qquad t=0,1,\ldots,T-1 \qquad (\text{with } x_I(0) = n_I(0))$$

$$x_{II}(t+1) \geq (1-\alpha-\beta_2)x_{II}(t), \qquad t=0,1,\ldots,T-1 \qquad (\text{with } x_{II}(0) = n_{II}(0))$$

$$x_I(t+1) + x_{II}(t+1) \geq (1-\alpha)(x_I(t)+x_{II}(t)), \qquad t=0,1,\ldots,T-1$$

The variables $x_I(t), x_{II}(t)$, $t=1,\ldots,T$ are the decision variables, while $x_I(0)$, $x_{II}(0)$ represent the actual number of employees in I, II. T is the planning horizon.

The obvious aggregated version of model A is the following model (model B):

$$\text{Minimize:} \quad \sum_{t=1}^{T} |s(t) - n_I(t) - n_{II}(t)|$$

such that

$$s(t+1) \geq (1-\alpha)s(t), \quad t=0,1,\ldots,T-1 \quad (\text{with } s(0) = n_I(0)+n_{II}(0))$$

Each feasable solution $x_I(t)$, $x_{II}(t)$ of model A generates a feasable solution $s(t)$ of model B by $s(t) := x_I(t) + x_{II}(t)$. Of course

$$|s(t)-n_I(t)-n_{II}(t)| \leq |x_I(t)-n_I(t)|+|x_{II}(t)-n_{II}(t)|$$

and there is equality if and only if $x_I(t)-n_I(t)$ and $x_{II}(t)-n_{II}(t)$ do not have opposite signs.
Each feasable solution $s(t)$ of model B generates a feasable solution $x_I(t),x_{II}(t)$ of model A by executing subsequently, for $t=1,2,\ldots,T$, the following minimizations:

$$\text{Minimize } |x_I(t) - n_I(t)| + |x_{II}(t) - n_{II}(t)|$$

such that

$$x_I(t) + x_{II}(t) = s(t)$$

$$x_I(t) \geq (1-\alpha-\beta_1)x_I(t-1)$$

$$x_{II}(t) \geq (1-\alpha-\beta_2)x_{II}(t-1)$$

This is called disaggregation. Let $s^*(t)$ be an optimal solution of model B and let $x_I^*(t)$, $x_{II}^*(t)$ be the solution of model A following from disaggregation of $s_I^*(t)$. If (for all $t=1,\ldots,T$) $x_I^*(t) - n_I(t)$ and $x_{II}^*(t) - n_{II}(t)$ do not have opposite signs then the cost of $x_I^*(t),x_{II}^*(t)$ in model A is equal to the cost of $s^*(t)$ in model B and $x_I^*(t),x_{II}^*(t)$ is optimal therefore. The following proposition gives conditions on the mobility of the personnel (β_1,β_2) and the instability of the goals $(n_I(t),n_{II}(t))$ under which disaggregation of the optimal solution $s^*(t)$ of model B gives and optimal solution of model A,

<u>Proposition 2.1.</u> Let the following conditions be satisfied

(1) $n_I(t+1) \geq (1-\alpha-\beta_1)n_I(t), \quad t=0,1,\ldots,T-1$
(2) $n_{II}(t+1) \geq (1-\alpha-\beta_2)n_{II}(t), \quad t=0,1,\ldots,T-1$

(3) $s^*(t+1)-(1-\alpha)s^*(t) + \beta_1 n_I(t)+(1-\alpha)n_{II}(t) \geq n_{II}(t+1)$, $t=0,\dots,T-1$

(4) $s^*(t+1)-(1-\alpha)s^*(t) + \beta_2 n_{II}(t)+(1-\alpha)n_I(t) \geq n_I(t+1)$, $t=0,\dots,T-1$

where $s^*(t)$ is an optimal solution of model B. Then there is a disaggregation $x_I^*(t)$, $x_{II}^*(t)$ of $s^*(t)$ which is optimal in model A.

Proof Let $x(t),y(t)$, $t=1,\dots,S(<T)$, be such that for $t=1,\dots,S$

$$x(t)+y(t) = s^*(t)$$
$$x(t) \geq (1-\alpha-\beta_1)x(t-1)$$
$$y(t) \geq (1-\alpha-\beta_2)y(t-1)$$

$x(t) - n_I(t)$ and $y(t)-n_{II}(t)$ do not have opposite signs.

The conditions (1), (2), (3), (4) imply the existence of numbers v, w such that

$v \geq (1-\alpha-\beta_1)x(S)$, $w \geq (1-\alpha-\beta_2)y(S)$, $v+w = s^*(S+1)$ and

$v-n_I(S+1)$ and $w-n_{II}(S+1)$ do not have opposite signs.

Applying this for $S=0,1,\dots,T-1$ shows that it is possible to construct a disaggregation $x_I^*(t)$, $x_{II}^*(t)$ of $s^*(t)$ sich that (for all $t=1,\dots,T$) $x_I^*(t)-n_I(t)$ and $x_{II}^*(t)-n_{II}(t)$ do not have opposite signs. The solution $x_I^*(t)$ $x_{II}^*(t)$ is optimal therefore. □

It is important to note that the conditions (1), (2), (3), (4) have indeed to do with the mobility of the personnel (β_1,β_2) and the instability of the goals ($n_I(t),n_{II}(t)$). The turnover α works as a kind of indirect mobility.
The conditions (1) and (2) imply that if function group I (II) has a shortage in period t then it is possible to prevent that this function group has a surplus in period t+1. The conditions (3) and (4) imply that if both function groups have a surplus in period t then it is possible to prevent that functiongroup I or II has a shortage in period t+1.
It is easy to generalize the result of proposition 2.1 to the case with more functiongroups. Consider a case with m function groups. Let $n_i(t)$ be the goal for function group i. The mobility in function group is β_i, that means that one may remove each period at most a fraction β_i of the people in function group

i and distribute these people freely over all function groups. The turnover in each function group is α. Let $x_i(t)$ be the planned content of function group i and let $n_i(0) := x_i(0)$ be the actual content of function group i. The T-period planning problem may be represented by the following model (model C):

$$\text{Minimize} \sum_{t=1}^{T} \sum_{i=1}^{m} |x_i(t)-n_i(t)|$$

such that

$$x_i(t+1) \geq (1-\alpha-\beta_i)x_i(t), \quad t=0,\ldots,T-1, \quad i=1,\ldots,m$$

$$\sum_{i=1}^{m} x_i(t+1) \geq (1-\alpha) \sum_{i=1}^{m} x_i(t), \quad t=0,\ldots,T-1$$

The obvious aggregated version of the model is again model B. Disaggregation of a solution s(t) of model B can be executed by subsequently, for t=1,...,T, solving the following minimization problems:

$$\text{Minimize} \sum_{i=1}^{m} |x_i(t)-n_i(t)|$$

such that

$$\sum_{i=1}^{m} x_i(t) = s(t)$$

$$x_i(t) \geq (1-\alpha-\beta_i)x_i(t-1), \quad i=1,\ldots,m$$

The next proposition is a straightforward generalization of proposition 2.1 to the case with more function groups.

Proposition 2.2. Let the following conditions be satisfied for all t=0,...,T-1

(5) $n_i(t+1) \geq (1-\alpha-\beta_i)n_i(t)$ for all i=1,...,m

(6) $s^*(t+1) - (1-\alpha)s^*(t) + \sum_{i\in I'} \beta_i n_i(t) + (1-\alpha) \sum_{i\in I} n_i(t) \geq \sum_{i\in I} n_i(t+1)$

for all nonempty sets I of {1,2,...,m} and I' the complement of I, where $s^*(t)$ is an optimal solution of model B.

Then there is a disaggregation $\{x_i^*(t)\}$ of $s^*(t)$ which is optimal in model C.

Proof The proof is analogous to the proof of proposition 2.1. □

Essential in this stratification of the planning problem is that it never occurs that one function group has a surplus while another function group has a shortage. The conditions (1), (2), (3), (4) or (5) and (6) are sufficient to guarantee that but certainly not necessary.
Another set of conditions is worked out in the next proposition. This set of conditions is sufficient to keep the differences between function group content and function group goal equal over all function groups. This implies also that the differences will never have opposite signs. The proposition is formulated for the case with 2 function groups, but can be generalized to the case with more function groups.

Proposition 2.3. Let $\beta_{max} := \max(\beta_1,\beta_2)$ and $\beta_{min} := \min(\beta_1,\beta_2)$
Suppose the following condition is satisfied for $t=0,1,\dots,T-1$

$$(7)\quad |n_I(t+1)-n_{II}(t+1)-(1-\alpha-\beta_{min})(n_I(t)-n_{II}(t))| \; +$$

$$+\;(\beta_{max}-\beta_{min})s^*(t) \le s^*(t+1)-(1-\alpha-\beta_{min})s^*(t)$$

where $s^*(t)$ is the optimal solution of model B.
Then there is a disaggregation $x_I^*(t),x_{II}^*(t)$ of $s^*(t)$ such that $x_I^*(t)-n_I(t) = x_{II}^*(t)-n_{II}(t)$ for all $t=1,\dots,T$ and $x_I^*(t),x_{II}^*(t)$ is an optimal solution of model A therefore.

Proof Let $x(t),y(t)$, $t=1,\dots,S\;(<T)$, be such that for $t=1,\dots,S$

$$x(t) + y(t) = s^*(t)$$
$$x(t) \ge (1-\alpha-\beta_1)x(t-1)$$
$$y(t) \ge (1-\alpha-\beta_2)y(t-1)$$
$$x(t)-n_I(t) = y(t)-n_{II}(t)$$

Then $(1-\alpha-\beta_1)x(S)-n_I(S+1) = x(S)-n_I(S)-(\alpha+\beta_1)x(S)-(n_I(S+1)-n_I(S))$

and $(1-\alpha-\beta_2)y(S)-n_{II}(S+1) = y(S)-n_{II}(S)-(\alpha+\beta_2)y(S)-(n_{II}(S+1)-n_{II}(S))$

It is possible then to find v,w such that $v \geq (1-\alpha-\beta_1)x(S)$, $w \geq (1-\alpha-\beta_2)y(S)$, $v+w = s^*(S+1)$ and $v-n_I(S+1) = w-n_{II}(S+1)$ if

$$|(\alpha+\beta_1)x(S)+n_I(S+1)-n_I(S)-(\alpha+\beta_2)y(S)-n_{II}(S+1)+n_{II}(S)| \leq$$

$$\leq \beta_1 x(S)+\beta_2 y(S)+s^*(S+1)-(1-\alpha)s^*(S)$$

Using the definition of β_{min} and β_{max} and that $x(S)-n_I(S) = y(S)-n_{II}(S)$ yields that the lefthand side of this inequality is less than or equal to

$$(\beta_{max}-\beta_{min})s^*(S)+|n_I(S+1)-n_{II}(S+1)-(1-\alpha-\beta_{min})(n_I(S)-n_{II}(S))|$$

while the righthand side of the inequality is greater than or equal to

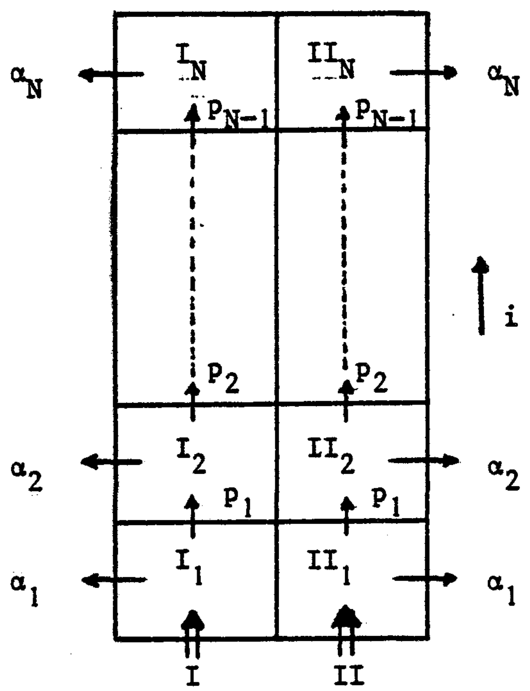
$$s^*(S+1)-(1-\alpha-\beta_{min})s^*(S)$$

That means (see condition (7)) that there is indeed such a pair (v,w). Applying this for S=0,1,...,T-1 shows that it is possible to construct a disaggregation $x_I^*(t), x_{II}^*(t)$ of $s^*(t)$ such that $x_I^*(t)-n_I(t) = x_{II}^*(t)-n_{II}(t)$ □

In proposition 2.3. the linearity of the penalties is not used. Suppose that, instead of $|x_I-n_I|$ and $|x_{II}-n_{II}|$, we have penalties $p(x_I-n_I)$ and $p(x_{II}-n_{II})$ where p is some convex function with its minimum in 0. Under condition (7) it is possible to keep the differences x_I-n_I and $x_{II}-n_{II}$ equal. That means that the total penalty is equal to $2p\,\frac{x_I+x_{II}-n_I-n_{II}}{2}$ It is possible therefore to stratify the planning problem in the same way as in case of linear penalties. In the aggregate model (model B) $|s(t)-n_I(t)-n_{II}(t)|$ has to be replaced by
$2p((s(t)-n_I(t)-n_{II}(t))/2)$ and in the disaggregation step the penalties $|x_I(t)-n_I(t)|$ and $|x_{II}(t)-n_{II}(t)|$ have to be replaced by $p(x_I(t)-n_I(t))$ and $p(x_{II}(t)-n_{II}(t))$.

3. More function levels

In this section we will extend some of the results of the previous section to a system with more function levels. We will consider the case with two function groups (see fig.) but, as in the previous section, the results are generalizable to the case with more function groups.



The category I_i (II_i) indicates function level i and function group I (II). The forecasted future requirement for category I_i (II_i) in period t is equal to $n_{Ii}(t)$ ($n_{IIi}(t)$). The costs of deviations from these (forecasted) requirements are assumed to be linear in the size of the deviations. The penalties for the categories I_i and II_i are equal. The turnover fraction from level i is α_i. The promotion policy is very strict; each year a fraction p_i of all people at level i has to be promoted to level i+1. But one is free to promote people to either function group I or II, independent of where they come from. That means that mobility is coupled with promotion. More general cases are considered at the end of this section. To make notation easier we define $p_N := 0$. Recruitment is only at the lowest level. We assume that the actual personnel population matches the requirement.
The planning problem for this system is represented by the following model (model D):

$$\text{Minimize} \sum_{t=1}^{T} \sum_{i=1}^{N} c_i\{|x_{Ii}(t)-n_{Ii}(t)|+|x_{IIi}(t)-n_{IIi}(t)|\}$$

such that

$$x_{Ii}(t+1) \geq (1-\alpha_i-p_i)x_{Ii}(t), \quad t=0,1,\dots,T-1; \quad i=1,2,\dots,N$$

$$x_{IIi}(t+1) \geq (1-\alpha_i-p_i)x_{IIi}(t), \quad t=0,1,\dots,T-1; \quad i=1,2,\dots,N$$

$$x_{Ii}(t+1) + x_{IIi}(t+1) = (1-\alpha_i-p_i)(x_{Ii}(t)+x_{IIi}(t)) + p_{i-1}(x_{Ii-1}(t) +$$

$$+ x_{IIi-1}(t)), \quad t=0,1,\dots,T-1; \quad i=2,\dots,N$$

$$x_{Ii}(0) = n_{Ii}(0),\ x_{IIi}(0) = n_{IIi}(0), \quad i=1,\dots,N$$

Aggregation of this model gives the following model (model E):

$$\text{Minimize} \sum_{t=1}^{T} \sum_{i=1}^{N} c_i|s_i(t)-n_{Ii}(t)-n_{IIi}(t)|$$

such that

$$s_1(t+1) \geq (1-\alpha_1-p_1)s_1(t), \quad t=0,1,\dots,T-1$$

$$s_i(t+1) = (1-\alpha_i-p_i)s_i(t) + p_{i-1}s_{i-1}(t), \quad t=0,1,\dots,T-1; \quad i=2,\dots,N$$

$$s_i(0) = n_{Ii}(0) + n_{IIi}(0), \quad i=1,\dots,N$$

Each feasable solution $x_{Ii}(t)$, $x_{IIi}(t)$ of model D generates a feasable solution of model E by $s_i(t) := x_{Ii}(t) + x_{IIi}(t)$. The cost of this solution in model E is less than or equal to the cost of $x_{Ii}(t)$, $x_{IIi}(t)$ in model D, and there is equality only if $x_{Ii}(t) - n_{Ii}(t)$ and $x_{IIi}(t) - n_{IIi}(t)$ do not have opposite signs.

Each feasable solution $s(t)$ of model generates a feasable solution $x_{Ii}(t)$, $x_{IIi}(t)$ of model D by executing subsequently, for $t=1,2,\dots,T$, the following minimization (disaggregation):

$$\text{Minimize} \sum_{i=1}^{N} c_i\{|x_{Ii}(t)-n_{Ii}(t)|+|x_{IIi}(t)-n_{IIi}(t)|\}$$

such that

$$x_{Ii}(t) + x_{IIi}(t) = s_i(t), \quad i=1,\ldots,N$$

$$x_{Ii}(t+1) \geq (1-\alpha_i-p_i)x_{Ii}(t), \quad i=1,\ldots,N$$

$$x_{IIi}(t+1) \geq (1-\alpha_i-p_i)x_{IIi}(t), \quad i=1,\ldots,N$$

Let $s_i^*(t)$ be an optimal solution of model E and let $x_{Ii}^*(t)$, $x_{IIi}^*(t)$ be a disaggregation of $s_i^*(t)$ (a solution of model D following from disaggregation of $s_i^*(t)$). If (for all $t=1,\ldots,T$ and for all $i=1,\ldots,N$) $x_{Ii}^*(t) - n_{Ii}(t)$ and $x_{IIi}^*(t) - n_{IIi}(t)$ do not have opposite signs then x_{Ii}^*, x_{IIi}^* is an optimal solution of model D. In the following two propositions we consider sufficient conditions for the optimality of $x_{Ii}^*(t)$, $x_{IIi}^*(t)$. The propositions are analogous to the propositions 2.1 and 2.3 and will be given without proof.

<u>Proposition 3.1.</u> Let $s_i^*(t)$ be an optimal solution of model E. Suppose the following conditions are satisfied for $t=0,\ldots,T-1$ and $i=1,\ldots,N$.

(8) $n_{Ii}(t+1) \geq (1-\alpha_i-p_i)n_{Ii}(t)$

(9) $n_{IIi}(t+1) \geq (1-\alpha_i-p_i)n_{IIi}(t)$

(10) $s_i^*(t+1) - (1-\alpha_i-p_i)s_i^*(t) + (1-\alpha_i-p_i)n_{IIi}(t) \geq n_{IIi}(t+1)$

(11) $s_i^*(t+1) - (1-\alpha_i-p_i)s_i^*(t) + (1-\alpha_i-p_i)n_{Ii}(t) \geq n_{Ii}(t+1)$

Then there is a disaggregation $x_{Ii}^*(t)$, $x_{IIi}^*(t)$ of $s_i^*(t)$ which is optimal in model D.

<u>Proposition 3.2.</u> Let $s_i^*(t)$ be an optimal solution of model E. Suppose the following condition is satisfied for $t=0,\ldots,T-1$ and $i=1,\ldots,N$

(12) $|n_{Ii}(t+1) - n_{IIi}(t+1) - (1-\alpha_i-p_i)(n_{Ii}(t)-n_{IIi}(t))| \leq s_i^*(t+1) - (1-\alpha_i-p_i)s_i^*(t)$

Then there is a disaggregation $x_{Ii}^*(t)$, $x_{IIi}^*(t)$ of $s_i^*(t)$ such that for all

$i=1,\ldots,N$ and all $t=1,\ldots,T$ the differences $x^*_{Ii}(t) - n_{Ii}(t)$ and $x^*_{IIi}(t) - n_{IIi}(t)$ are equal. The solution $x^*_{Ii}(t)$, $x^*_{IIi}(t)$ is optimal (in model D) therefore.

In the system considered here the mobility of the personnel is coupled with a promotion. It is easily possible to generalize the results to systems where there is also horizontal mobility (as in the one-level case). In case of a maximum horizontal mobility at level i of β_i the conditions (8) - (11) have to be modified in the following way:

(8') $n_{Ii}(t+1) \geq (1-\alpha_i-p_i-\beta_i)n_{Ii}(t)$

(9') $n_{IIi}(t+1) \geq (1-\alpha_i-p_i-\beta_i)n_{IIi}(t)$

(10') $s^*_i(t+1)-(1-\alpha_i-p_i)s^*_i(t) + (1-\alpha_i-p_i)n_{IIi}(t) + \beta_i n_{Ii}(t) \geq n_{IIi}(t+1)$

(11') $s^*_i(t+1)-(1-\alpha_i-p_i)s^*_i(t) + (1-\alpha_i-p_i)n_{Ii}(t) + \beta_i n_{IIi}(t) \geq n_{Ii}(t+1)$

The condition (12) has to be modified as follows:

(12') $|n_{Ii}(t+1)-n_{IIi}(t+1)-(1-\alpha_i-p_i-\beta_i)(n_{Ii}(t)-n_{IIi}(t))| \leq s^*_i(t+1)-(1-\alpha_i-p_i-\beta_i)s^*_i(t)$

Condition (12) can also be generalized to the case where only part of the promoted people can be transferred to another function group. Let f_i be the fraction of the people promoted from level i which can be transferred and let β_i again be the maximal horizontal mobility at level i. Then condition (12) has to be modified as follows:

(12'') $|n_{Ii}(t+1)-n_{IIi}(t+1)-(1-\alpha_i-p_i-\beta_i)(n_{Ii}(t)-n_{IIi}(t)) +$

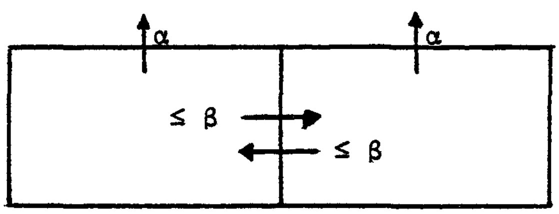
$+ (1-f_{i-1})p_{i-1}(n_{Ii-1}(t)-n_{IIi-1}(t))| \leq f_{i-1}p_{i-1}s^*_{i-1}(t)+\beta_i s^*_i(t)$

For i=1 the component $f_{i-1}p_{i-1}s^*_{i-1}(t)$ in the right hand side of (12'') has to be replaced by the total recruitment.
In case it is possible to recruit at more levels the conditions for aggregation

/disaggregation remain the same; only in condition (12'') the total recruitment at the levels 2,3,...,N has to be added to the right hand side. Restricted recruitment does not change the conditions either as long as the restrictions are on the total recruitment per level.
Also in cases where the promotion fractions are not fixed (p_i) but only bounded (by p_i^{min} and p_i^{max}) it is possible to derive the same kind of conditions.

4. Numerical example. Application.

In the introduction we stated that the main advantage of aggregation is that it reduces the forecasting load of the organization. Instead of working with detailed goals one may work with aggregated goals. From the results in the previous section we see that to check the conditions sufficient for optimal aggregation/disaggregation we need information about the detailed goals. However, it is not necessary to forecast these detailed goals precisely but only to check whether the goals will stay within the range where optimal aggregation/disaggregation is possible.
The optimal solution of the aggregated model is also necessary to check the conditions. That means that one has to solve the aggregated model before one knows whether the first period disaggregation of this solution is optimal.
Consider the following example:



We have to solve first the aggregated model (model B). Let $\alpha = 0.1$, T (planning horizon) = 3 and let the aggregated goals be 220, 180 and 150. The total actual content is 200. The optimal solution for the aggregated model is $s^*(1) = 200$, $s^*(2) = 180$, $s^*(3) = 162$. To disaggregate $s^*(1)$ we have to know

the maximal mobility β, the actual content in each of the function groups ($x_I(0)$, $x_{II}(0)$) and the goals for each of the function groups in the first period.
Suppose $\beta = 0.1$, $x_I(0) = x_{II}(0) = 100$, $n_I(1) = 100$, $n_{II}(1) = 120$. Then $s^*(1)$ has to be disaggregated by the following minimization:

$$\text{Minimize } \{|100-x_I(1)| + |120-x_{II}(1)|\}$$

such that

$$x_I(1) + x_{II}(1) = 200$$

$$x_I(1) \geq 80$$

$$x_{II}(1) \geq 80$$

There are many optimal solutions for this minimization problem, namely, all feasable solutions with the property that $x_I(1) \leq 100$ and $x_{II}(1) \leq 120$. A solution where the total shortage is distributed evenly over both function groups is $x_I^0(1) = 90$, $x_{II}^0(1) = 110$. To know whether this solution is optimal we have to check the conditions for optimal aggregation/disaggregation. Consider for instance condition (7):

$$|100-120-0,8\cdot(100-100)| \leq 200-0,8(200) = 40$$

$$|n_I(2)-n_{II}(2)-0,8(100-120)| \leq 180-0,8(200) = 20$$

$$|n_I(3)-n_{II}(3)-0,8(n_I(2)-n_{II}(2))| \leq 162-0,8(180) = 18$$

The condition for the first period is satisfied indeed. The condition for the second period is satisfied if $n_{II}(2)-36 \leq n_I(2) \leq n_{II}(2)+4$. The condition for the third period is satisfied if $n_I(3)-n_{II}(3)$ lies between $0,8(n_I(2)-n_{II}(2))-18$ and $0,8(n_I(2)-n_{II}(2))+18$.
Based on these conditions one may conclude whether or not aggregation/disaggregation is optimal.

The conclusion that aggregation/disaggregation is allowed implies that it is not necessary to distinguish the two function groups in the medium and long term manpower planning. An important point in medium and long term manpower planning is always the way to define the categories. Which characteristics have to be included in the definition of the categories. The first criterion for inclusion of certain characteristics is the contribution of these characteristics to the predictabilty of certain transitions, turnover for instance. That is why age is important in general. The second criterion is organizational. It must be possible to decide whether or not a projected personnel population, categorized in a certain way, meets the organizational requirements. To make this evaluation possible the categorization needs to be sufficiently detailed. The results of this paper show that what is sufficiently detailed depends as well on the mobility over the categories as on the instability of the goals. If for all reasonable patterns of the future requirement the optimal solution of the aggregated model is such that the conditions for aggregation/disaggregation are satisfied then one may consider the aggregated function groups as the basic function groups in medium and long term manpower planning.

It is important that condition (7) is also sufficient for aggregation in case of non linear penalty functions. Linear penalty functions are typical for goal programming, but goal programming is not the only approach in manpower planning. Simulation models (Markov or renewal models) are also frequently used (see Bartholomew [1] and Verhoeven et al. [8]). In case of a simulation approach one does not work with explicit penalty functions. The projected personnel populations under various strategies are evaluated by management. This evaluation is in general mainly based on the differences between projected function groups contents and future requirements per function group; the evaluation is based on implicit penalty functions for differences between future availability and future requirement. That means that also in this approach one may use conditions like condition (7) to check whether it is allowed to aggregate over certain function groups.

The main points of criticism on the use of quantitative methods in manpower planning are that

1. The future requirement is so unpredictable that it does not make any sense to use forecasts for this requirement in quantitative planning models, and that

2. functions and people are so unique that it is not possible to smooth out this unpredictability by working with large categories.

It is only possible to meet this last point of criticism by analyzing the mobility required to aggregate. That analysis was the subject of this paper.

References

[1] Bartholomew, D.J. (1973) "Stochastic Models for Social Processes" (2nd ed.) John Wiley, New York.

[2] Charnes, A., Cooper, W.W. and Niehaus, R.J. (1972) "Studies in Manpower Planning" U.S. Navy, Office of Civilian Manpower Management, Washington D.C.

[3] Charnes, A., Cooper, W.W., Lewis, K.A. and Niehaus R.J. (1978) "A Multi-level Coherence Model for EEO Planning" in TIMS-Studies in the Management Sciences (North Holland-American Elsevier), vol. 8.

[4] Hax, A.C. and Meal H.C. (1975) "Hierarchical Integration of Production Planning and Scheduling" in TIMS-Studies in the Management Sciences (North Holland-American Elsevier), vol. 1.

[5] Nuttle, H.L.W and Wijngaard, J. (1979) "Planning Horizons for Manpower Planning, a theoretical analysis" Manpower Planning Reports 19, Eindhoven University of Technology.

[6] Smits, A.J.M. and Wijngaard J. (1981) "Rolling Plans and Aggregation in Manpower Planning", to appear.

[7] Verhoeven, C.J. (1980) "Instruments for Corporate Manpower Planning, Applicability and Applications" Ph.D.-thesis, Eindhoven University of Technology.

[8] Verhoeven, C.J., Wessels, J. and Wijngaard, J. (1979) "Computer-aided Design of Manpower Policies" Manpower Planning Reports 16, Eindhoven University of Technology.

[9] Wijngaard, J. (1981) "On Aggregation in Production Planning" Report BDK/ORS/81/02, Eindhoven University of Technology.