Markov models for manpower planning, some remarks and an application on a problem of decreasing growth

EINDHOVEN UNIVERSITY OF TECHNOLOGY

Department of Industrial Engineering

Department of Mathematics

Markov models for manpower planning, some remarks and an application on a problem of decreasing growth

by

J. Wijngaard

Manpower Planning Reports no. 11

Eindhoven, May 1978

Markov models for manpower planning, some remarks and an application on a problem of decreasing growth.

J. Wijngaard

1. Introduction

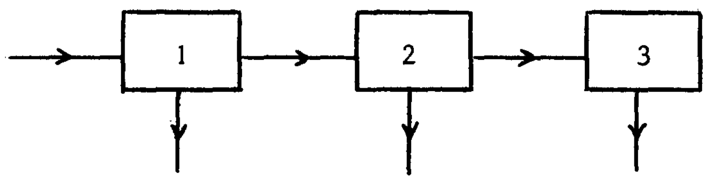
Last years there is a growing interest in manpower planning. This has to do with the decreasing growth of most organizations. Each organization has medium and long term plans for future activities. These plans determine the requirements for manpower in the various categories, categories defined by characteristics like age, education, experience, etc. On the other hand the organization tries to maintain rather constant promotion possibilities. If the promotion possibilities are decreasing too fast there is, for instance, a risk that the best people will leave. But maintaining promotion possibilities gives constraints on the availability of personnel in the various categories. As long as the organization is growing it is rather easy to fill the gap between requirements and availability by recruitment or by enlarging the promotion possibilities (improving the usage of the personnel potential). But if the growth decreases the higher grades will become overoccupied even without recruitment.

Management is interested in questions as, which promotion possibilities can be offered under these circumstances, what is the future age distribution, how one has to distribute the recruits over grade, age and educational level to keep the necessary changes in the promotion possibilities as small as possible.

A class of models suitable to answer these questions are the Markov models. In these models the total personnel is subdivided in a number of categories, categories defined by characteristics like grade, age, grade-age (time spend in a certain grade), education, etc. In this paper the contribution of Markov models to the just mentioned problems will be discussed. In section 2 some general remarks will be made on the different types of manpower planning models. In section 3 some methodological points concerning Markov models are considered. The computer system FORMASY developed at the Eindhoven University of Technology is discussed in section 4. In section 5 some (personnel) problems of decreasing growth are illustrated and in section 6 the use of FORMASY in the analysis of a real problem of this type is shown.

2. Different types of manpower planning and forecasting models

In all manpower planning models there are two types of variables. In the first place the variables $x_i(t)$ representing the number of people in category i at time t and in the second place the variables $y_{ij}(t)$ representing the number of people going from category i to category j in period t (between time t-1 and time t). The turnover from category i can be denoted by $y_{io}(t)$ and the number of recruits in category i by $y_{oi}(t)$. The flows $y_{ij}(t)$ can be determined by a push mechanism or by a pull mechanism. A flow from category i to category j is called a push flow if $y_{ij}(t)$ is a fixed fraction of $x_i(t-1)$, $y_{ij}(t) = p_{ij}x_i(t-1)$. The fractions p_{ij} have to be given (parameters of the model). To show the idea of a pull flow we consider a strictly hierarchical system with three grades.

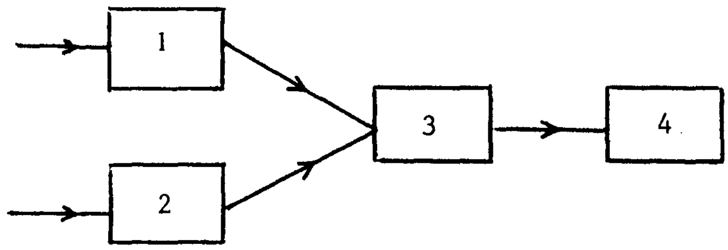


Let p_{10}, p_{20}, p_{30} be the fractions of turnover from each of the grades 1, 2, 3, so $y_{io}(t) = p_{io}x_i(t-1)$.
If one has to maintain the occupation in each of the grades at a certain, eventually time dependent,level $n_i(t)$ then the flows from grade 2 to grade 3 and from grade 1 to grade 2 are determined by the following relations

$$y_{23}(t) = n_3(t) - n_3(t-1) + y_{30}(t)$$
$$y_{12}(t) = n_2(t) - n_2(t-1) + y_{20}(t) + y_{23}(t)$$
$$y_{01}(t) = n_1(t) + n_1(t-1) + y_{10}(t) + y_{12}(t)$$

The flows y_{23}, y_{12} and y_{01} are called pull flows, that means that these flows are determined by the necessaty to maintain a certain given occupational level in each of the categories.

If there are more inflows in one category the flows are not determined uniquely by the levels $n_i(t)$. Consider for instance the following system



Then the necessaty to maintain grade 3 at a certain level only determines the total inflow in grade 3, $y_{13}(t) + y_{23}(t)$. In this case one has to indicate how the total inflow has to be distributed over $y_{13}(t)$ and $y_{23}(t)$. This can be done by maintaining a fixed ratio between y_{13} and y_{23} or by pulling first from category 1 and then if necessary from category 2.

A model with only push flows is called a Markov model, a model where all internal flows are pull flows is called a renewal model. The parameters of a Markov model are the fractions p_{ij}, the number of recruits at each time in each of the categories and the actual number of people in each of the categories. The parameters in a renewal model are the fractions p_{io} which determine the turnover, the levels $n_i(t)$ and (if there are categories with more inflows) the parameters which determine the distribution of the total inflow in a category over the different inflows.

The output of a Markov model are the functions $x_i(t)$ and $y_{ij}(t)$, the output of a renewal model are the flows $y_{ij}(t)$.

Markov models as well as renewal models are simulation models. These simulation models can be extended to optimisation models, for instance linear programming models. Then $x_i(t)$ and $y_{ij}(t)$ are the decision variables. Instead of push and pull flows one has push and pull restrictions. A push restriction is a restriction of the following type

$$\bar{p}_{ij}x_i(t) \le y_{ij}(t+1) \le \bar{\bar{p}}_{ij}x_i(t)$$

and a pull restriction is a restriction of the following type

$$\bar{n}_i(t) \leq x_i(t) \leq \bar{\bar{n}}_i(t)$$

As objective function one can use something like

$$\sum_{t=0}^{T} \{\sum_i c_i x_i(t) + \sum_i \sum_j d_{ij} y_{ij}(t+1) + \sum_i r_i |x_i(t) - n_i(t)|\}$$

The parameters c_i can be interpreted as the salary costs in category i, the parameters d_{ij} as the costs of one transition from category i to category j and the parameters r_i as the penalty for a difference between x_i and its norm n_i.

It is very difficult in general to translate the preferences of management with respect to manpower flows in costs and restrictions of a linear programming model. The decision makers will not be completely satiesfied by the "optimal" solution. They want to see other possibilities. That is possible of course but only in a very indirect way by changing costs and restrictions. A second difficulty is to make the "optimal" decisions operational. The real decision variables are in fact for each category the time until a transition and the fraction of people which will make that transition. That means that one has to distinguish between people with different grade-age (grade-age has to be a category defining characteristic). But the transition percentages per grade-age may not be chosen independently of each other and the output of a linear programming model is difficult to operationalize therefore. It is possible to circumvent this difficulty by sharpening the restrictions but this causes very often infeasabilities. For all these reasons linear programming is not very suitable for most personnel planning problems. The difference between Markov models and renewal models is in fact not so large. If the norms $n_i(t)$ are very sharp a renewal model is the most natural choice, if the restrictions on the transition fractions are important a Markov model is the most natural choice. But one can use Markov models also for systems with very hard occupational norms. By varying the transition fractions one can realize the required occupations. In the same way one can use renewal models if the restrictions on the transition fractions are sharp. By varying the norms $n_i(t)$ and the dis-

tribution of the total inflow in a certain category over all possible inflows the required transition fractions can be realized. The advantage of Markov models is that the fractions p_{ij} are of somewhat more significance to the personnel manager than the parameters in a renewal model which regulate the distribution of the total inflow in some category over all possible inflows. Therefore in the rest of this paper we restrict our attention to Markov models.
Another comparison of the three different types of models is given by Draper and Merchant [4]. For an extensive treatment of quantitative models for manpower planning see Batholomew [1].

3. Methodological points

The most important methodological questions in using Markov models for manpower planning are:

a. Which part of the personnel should be considered (all engineers, all people in a certain plant, ..)?
b. By which characteristics the categories have to be defined (grade, age, grade-age, educations, ...)?

Both questions have in the first place to do with

- the structure of the personnel system, the factors which determine transitions and turnover.
- the problems which one will investigate with aid of the model.

With respect to the last point, there are two types of problems, which can be analyzed with aid of Markov models:

I. The career possibilities of a certain group of personnel (characterized for instance by age or educational degree).
II. The future development of the distribution of the people occupying a given set of functions over characteristics like grade, age, education, experience.

If one wants to analyze the promotion possibilities of a certain group the Markov model has to contain at least that group. The promotion possibilities are expressed in general in the probability to make a promotion and the time spend in a grade before promotion. This implies that at least grade and grade-age have to be included as category-defining characteristics. So a minimal model to analyze a problem of type I is a model which only describes the group of personnel in which one is interested and in which grade and grade-age are the only category-defining characteristics. It can be usefull of course to include a larger population and more characteristics. Suppose one wants to analyze the promotion possibilities of people with educational degree A. If these people occupy functions which can also be occupied by people with another type of education then it might be usefull to include these people in the model and to add education as category-defining characteristic. One can use the model then also to get some insight in the future distribution over educational degree of the people occupying a certain set of functions (a problem of type II).

An important factor in the promotion possibilities is of course the turnover. If grade and grade-age are the only category-defining characteristics then the percentage of turnover may only depend on these characteristics. Although a part of the turnover is more directly determined by age it is not always necassary to add age as characteristic, especially not if the conditional age distribution, given grade and grade-age, is constant over time. But if the age distribution is unstable it is better to add age as category-defining characteristic. One can distinguish then two types of turnover, people who leave because of retirement and people who leave for other reasons. The first type of turnover is only determined by age, the second may depend on grade, age and grade-age but not necessarily on all three characteristics. If one is interested in age distributions (problems of type II) then it is necessary of course to include age as one of the category-defining characteristics.

4. Formasy

If one wants to use Markov models in manpower planning it is very important that these models can be used by the personnel managers themselves. That means that the model has to be translated into a conversational computer system and that input and output have some significance for personnel managers. We have tried to realize this in FORMASY, a conversational computer system for manpower planning and forecasting, developed at the Eindhoven University of Technology (see [6] for a detailed description).
The category-defining characteristics in this system are grade, grade-age, age and a free characteristic which can be used for instance for educational level or for time spend in the organization. The first step in the application of this system is the input of the structure of the model, the categories and possible transitions. The second step is the input of transition percentages, numbers of recruits in the various categories and the actual distribution of the people over the categories. This last type of information can be changed in a conversational way. Transition percentages can be changed in the following three ways.

1. Changing single percentages.
2. Lenghtening or shortening the time to spend in a certain grade.
3. Increasing or decreasing the fraction of people in a certain grade which will reach a next grade or leave the system.

Let i, j, k, ℓ represent the category-defining characteristics, with i the grade and ℓ the grade-age. Let $p_{ijk\ell}$ be the transition percentages to the next grade and $q_{ijk\ell}$ the percentage of turnover. Then in the first way one changes only one percentage $p_{ijk\ell}$ or $q_{ijk\ell}$ at a time. In the second and third way one changes a whole set of percentages. Lengthening the time to spend in grade i with one year means that the rows of percentages

$$p_{ijk1},\ p_{ijk2},\ p_{ijk3},\ \cdots$$

for all j, k are changed into the rows

$o, p_{ijk1}, p_{ijk2}, p_{ijk3}, \dots$

By using the third way of changing transition percentages all transition percentages $p_{ijk\ell}$ or $q_{ijk\ell}$ for fixed i are multiplied with a constant α.

The number of recruits in the future can be given per category, but it is also possible to use a planning procedure which determines the number of recruits necessary to satisfy a given future need for personnel in the various grades.

5. Problems of decreasing growth

To illustrate some problems of decreasing growth we consider here the simple situation of a set of functions A which have to be occupied by people with educational level *A*. The set of functions is divided into the subsets $A_1, A_2, \dots, A_n$ in increasing order of required experience and/or capabilities. To each subset of funactions A_i is adjoint a grade i. Grade is assumed to be the only relevant characteristic. The only possible transitions are the transitions from grade i to grade i+1 and the turnover from each of the grades. The promotion fractions are denoted by $p_{i\ i+1}$, $i = 1, \dots n-1$, $p_{i\ i+1}$ is the fraction of people in grade i which will be in grade i+1 next year. The fraction of turnover from grade i is given by α_i. Then $p_{ii} := 1 - \alpha_i - p_{i\ i+1}$ is the fraction of people who stay in grade i.

Let $n_i(t)$ be the number of people in grade i at time t and N(t) the total number of people. If N(t) has to stay constant (N(t) = N) each leaver has to be replaced by a recruit. Let $r_i(t)$ be the fraction of new people which is recruited in grade i. Then we have the following relations

(1) $$n_i(t+1) = n_i(t)(1-\alpha_i-p_{i\ i+1}) + n_{i-1}(t)p_{i-1\,i} + \left(\sum_{i=1}^{n} \alpha_i n_i(t)\right)r_i(t)$$

The fractions $p_{i\ i+1}$ and $r_i(t)$ have to be chosen such that the $n_i(t)$ are equal or at least almost equal to the number of (available) functions in the subsets $A_1, A_2, \dots, A_n$.

If the organization has to grow at rate β then $N(t+1) = (1+\beta)N(t)$ and the total number of recruits has to be $\sum_{i=1}^{n} (\alpha_i+\beta)n_i(t)$. For simplicity we assume that $\alpha_i = \alpha$ for all i (the turnover does not depend on the grade) and that the number of functions available in each of the subsets $A_1, \ldots, A_n$ increases proportional, that means that the fraction $n_i(t)/\sum_{i=1}^{n} n_i(t)$ has to stay (almost) constant.
Let $\pi_i := n_i/\sum_{i=1}^{n} n_i$, then

(2) $$\pi_i = \pi_i(1-\alpha-\beta-p_{ii+1}) + \pi_{i-1}p_{i-1i} + \alpha r_i, \quad i = 1, \ldots n$$

, or

(3) $$r_i = \pi_i + \frac{\pi_i p_{ii+1} - \pi_{i-1}p_{i-1i}}{\alpha+\beta}, \quad i = 1, \ldots n$$

which is equivalent to

(4) $$\sum_{i=1}^{j} r_i = \sum_{i=1}^{j} \pi_i + \frac{\pi_j p_{jj+1}}{\alpha+\beta}, \quad j = 1, \ldots n$$

In these expressions the parameters p_{01} and p_{nn+1} have to be taken equal to 0 of course.
Notice that α and β play the same role in these expressions, an increase in α (increased turnover) has the same effect as an increase in β (increased growth). The same sort of expressions can be found in Forbes [5].
From (4) we see that if $\alpha + \beta$ increases one can keep the fractions π_i constant by increasing the p_{jj+1} and/or by a shift of the r_i to be the higher functions. Increased promotion rates are possible as long as there is a surplus of experience and capabilities in the organization. Recruitment in higher grades (changing the r_i) can be realized in two ways. In the first place one can try to recruit people with the same educational level (*A*) but with experience in other organizations. In the second place one can recruit people with no or not much experience but with a higher educational level (say level *B*). This presupposes of course a certain substitutability of experience and educational level.

In formalized organizations one has in general a specific group of grades and functions for each educational level. That means that in this case to apply the second possiblity (recruitment of people with higher educational level) it is necessary to shift some of the higher functions in A to the group of functions B which have to be occupied by people with educational level *B*. If this second possiblity is used the increase in demand for people with a higher education is even larger than the growth of the organization.

In case of a decreasing growth rate, a more common phenomenon last years, one has the reverse problems.

A reduction in $\alpha + \beta$ can be compensated by a reduction in the promotion possibilities and/or by a shift of the recruitment to the lower grades. In most organizations a reduction of promotion possiblities will give a lot of problems.

It causes a surplus of experience and capabilities in the organization and if this is not used too long there is a risk that the best people will leave. A shift in the recruitments is a much better solution in general. But if most people are already recruited in the lower grades this does not help much. In case of an increased growth rate a way to solve the problem is by recruiting people with educational level *B* instead of *A* and shifting functions from A to B. In case of a decreased growth rate the reverse treatment can help. One can try to shift some of the lower functions in B to A. This implies that the decrease in growth rate in the total number of available functions in A is weakened and furthermore that there are relatively more higher functions in A (a shift in the π_i to the higher grades). Both effects improve the promotion possibilities.

The highest group of grades is the most difficult one. In case of decreasing growth rate the number of functions available in this group is reduced even extra since some of the functions are shifted to the lower group.

The aim of this section is to illustrate some problems of decreased growth and some possibilities to solve these problems. The model which is used here is too poor for real applications. In the next section some of the results are shown of an application of FORMASY on a problem of decreased growth at the Dutch Ministry of Public Works. The most important aspect of that application is the shift of functions from one group to another. See [2] for a more detailed description, see also [3].

6. A case study

In the Ministry of Public Works the engineers of three different educational levels have their own grade system. For all three groups there are five grades (after lumping some of the less used topgrades). The two higher level groups of engineers are homogeneous with respect to education. For each of these groups we used a model with grade and grade-age as category-defining characteristics. The lower level group of engineers (called surveyors) consists of a less homogeneously educated set of people. For this group we used a model with grade, grade-age and educational level as category-defining characteristics. In figure 1 the transition possibilities between the different combinations of grade and educational level are shown.

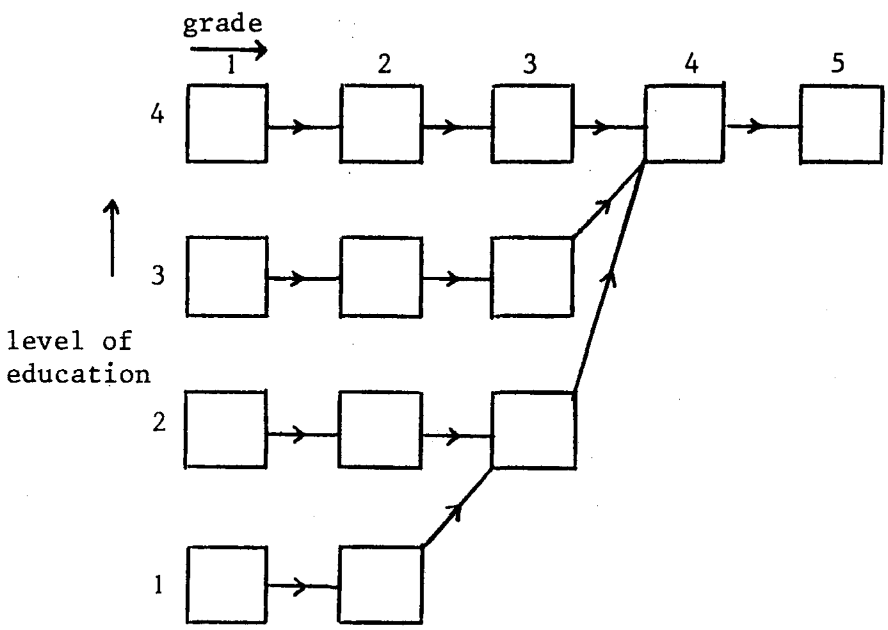


Fig. 1.

The number of functions available at each of the three levels has been increasing in the past but has to stay about constant from now on. In tables 1, 2 and 3 the current and forecasted grade occupations for the three groups of engineers are given based on historical promotion and turnover percentages.
For the surveyors recruitment is also based on historical data but is adapted to keep the total number of surveyors constant. For the middle level engineers there is recruitment in the two lowest grades, 48 each year in grade 1 and 20 each year in grade 2. For the top level engineers there is only recruitment in the lowest grade, 17 each year.

	grade 1	grade 2	grade 3	grade 4	grade 5	total
1977	276	651	563	218	118	1826
1984	147	570	627	303	181	1827
1990	135	503	620	339	229	1826

Table 1. Current and forecasted grade occupations for surveyor grades, historical promotion possibilities.

	grade 1	grade 2	grade 3	grade 4	grade 5	total
1977	238	548	380	183	73	1422
1984	134	427	490	267	120	1438
1990	133	375	434	323	163	1428

Tabel 2. Current and forecasted occupations for middle level engineers, historical promotion posssibilities.

	grade 1	grade 2	grade 3	grade 4	grade 5	total
1977	96	163	97	70	56	482
1984	92	54	159	107	74	486
1990	92	61	97	132	103	485

Tabel 3. Current and forecasted grade occupations for top level engineers, historical promotion possibilities.

In all three groups the future occupation of the highest grades exceeds the current occupation. That means that the past promotion possibilities can not be continued. Table 4 shows how rigorously the promotion possibilities have to be changed to keep the grade occupations about constant. In this table the forecasted grade occupations are given for the middle level engineers based on a promotion scheme which implies, compared to the historical situation, a delay of 6 years for promotion from grade 2 to grade 3 and a delay of 5 years for promotion from grade 3 to grade 4 and for promotion from grade 4 to grade 5.

	grade 1	grade 2	grade 3	grade 4	grade 5	total
1977	238	548	380	183	73	1422
1984	140	713	332	174	64	1423
1990	138	613	428	168	81	1428

Table 4. Current and forecasted grade occupations for middle level engineers, modified promotion possibilities.

We see that as long as the three groups have to be considered isolated it will be very difficult to keep the grade occupations constant. However, with regard to tasks as well as salaries there is a considerable overlap between the three groups. The top grades of the surveyors have the same salary level as the lower grades of the middle level engineers and similarly for the top grades of the middle level engineers and the lower grades of the top level engineers (see fig. 2).

salary level	surveyors	middle level engineers	top level engineers
11			grade 5
10			grade 4
9			grade 3
8		grade 5	grade 1/2
7		grade 4	
6		grade 3	
5	grade 5	grade 1/2	
4	grade 4		
3	grade 3		
2	grade 2		
1	grade 1		

Fig. 2. Relation between group, grade and salary level.

The overlap in functions between the groups makes it possible to let surveyors occupy middle level functions and to let middle level engineers occupy top level functions.
This implies that the organisational restriction are not on the numbers per grade per group but on the numbers per salary level.
The forecasts for the three different groups have to be integrated.
The number of grades in the integrated system is equal to the number of salary levels, the number of educational levels is six (top level, middle level and four surveyor levels).
In table 5 the tables 1, 2 and 3 are integrated.

	salary level											
	1	2	3	4	5	6	7	8	9	10	11	total
1977	276	651	563	218	904	380	183	332	97	70	56	3730
1984	147	570	627	302	742	490	267	266	159	107	74	3751
1990	135	503	620	339	737	434	323	316	97	132	103	3739

Table 5. Integration of tables 1, 2, 3, salary levels instead of grades.

These results can be improved by the recruitment of more surveyors and less top level engineers, by the delay of the promotion scheme in the lower grades and by fastening the promotion in the higher surveyor and middle level grades. Some results are given in table 6.
The promotion scheme used here is based on a modified historical promotion scheme. Some of the promotions are delayed and some others fastened. (See table 7). The recruitment used is given in table 8.

	salary level											
	1	2	3	4	5	6	7	8	9	10	11	total
1977	276	651	563	218	904	380	183	332	97	70	56	3730
1984	274	626	559	233	932	321	210	393	47	85	59	3739
1990	272	647	548	232	873	391	211	316	113	68	64	3735

Table 6. Results for the integrated model with modified promotion scheme.

surveyor grades	middle level grades	top level grades
1 → 2 → 3 → 4 → 5	1 → 2 → 3 → 4 → 5	1 → 2 → 3 → 4 → 5
+1 +1 -2	+4 -2	+4 +3

Table 7. Modifications in the promotion scheme, (+ is delay, - is fastening), used for the results in table 6.

surveyor grades					middle level grades					top level grades				
1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
45	62	-	-	-	45	20	-	-	-	12	-	-	-	-

Table 8. Recruitment used for the results in table 6.

From table 6 it is clear that especially in the two lowest groups (surveyors, middle level engineers) the integration of the system can have a very positive effect on the promotion possibilities. For the surveyors the new promotion scheme is as fast as it was, for the middle level engineers the new promotion scheme is delayed in total only two years.

The number of surveyors with a middle level function and the number of middle level engineers with a top level function is given in table 9.

	surveyors with salary level 5	middle level eng. with salary level 9
1977	118	73
1984	201	140
1990	241	152

Table 9.

That means that a shift of about 120 middle level functions to the surveyor grades is necessary and a shift of about 80 top level functions to the middle level grades. The question if that is possible can only be answered with aid of a detailed function analysis. In the Ministry of Public Works one has started such an analysis.

References

[1] D.J. Bartholomew, Stochastic models for social processes (2nd ed), Wiley, New York 1973.

[2] E. van der Beek, Markov modellen in de personeelsplanning: theorie en praktijk (in Dutch), Master's thesis, Eindhoven University of Technology, 1977.

[3] E. van der Beek, C.J. Verhoeven, J. Wessels, Some applications of the manpower planning system FORMASY, Memorandum COSOR 77-21, Eindhoven University of Tecynology, 1977.

[4] J. Draper, J.R. Merchant, Selecting the most appropriate manpower model, NATO conference on manpower planning and organisation design, Stresa, 1977.

[5] A.F. Forbes, Promotion and recruitment policies for the control of quasi-stationary hierarchical systems, in Smtih (ed.) Models of manpower systems, English University Press, London, 1970.

[6] C.J. Verhoeven, FORMASY 2, een conversationeel personeelsplannings-systeem (in Dutch), Memorandum COSOR 77-19, Eindhoven University of Technology, 1977.