Model and experimental validation of a unidirectional phase modulator

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Abstract: A unidirectional phase modulator consisting of tandem phase modulators is studied in detail for use as an integral part of an integrated optical isolator. The effects of non-linearity and residual amplitude modulation in the modulators, as well as the effect of the RF driving signals are captured in a phenomenological model for the first time. The model has been verified experimentally using a device realized in a generic InP based photonic integration platform and is used to study the operating range of the device. Design parameters of the modulator are derived such that modulation side bands in the forward propagating light are less than 40 dB, while isolation is maximized.

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1. Introduction

Semiconductor lasers are very sensitive to external optical feedback (EOF) [1–5]. Effects include broadening of the laser linewidth, increased relative intensity noise and even pulsations of the laser output power on time scales of tens of MHz under special conditions [1, 6]. In many applications, EOF is considered undesirable and is suppressed by placing a strong optical isolator between the laser and the optical system it is connected to. In this way the laser light can propagate into the system but any reflections back into the laser are suppressed. The maximum amount of EOF that can be tolerated by a laser is dependent on its cavity design. In general, 60 dB isolation is considered sufficient as was experimentally found for DFB lasers in [1].

In photonic integrated circuits (PICs) it is possible and desirable to integrate one or more lasers onto a chip with additional components such as modulators, splitters and filters. This extent of integration allows for a complex circuit with only electrical inputs and a single optical output that can be directly coupled to a fiber, such as a high speed transmitter for telecom applications [7]. The complexity of assembling the full system can thereby be greatly reduced and circuit functionality can be increased. Other advantages of PICs include reduced size, weight and power consumption of systems as well as increased yield and reliability [8, 9]. The stability of many systems is however critically dependent on the stability of the light source and thus on the presence of EOF.

To make the integrated laser immune to EOF it is currently not possible to place an optical isolator in series with the laser as is commonly done in fiber- and free space optics since a suitable integrated optical isolator is not available [10]. Isolation has been demonstrated, but it requires either significant extra steps in the processing of the wafer [11, 12], provides limited isolation [13, 14], or presents a high insertion loss [15]. Many of these isolators are narrow-band [13–16] requiring some tuning mechanism to align them to the lasing wavelength.

Recently, we predicted that 10 dB optical isolation is sufficient to stabilize the relative intensity noise of an integrated laser to within 3 dB and the linewidth and optical output power to 0.1 % for EOF up to −0.1 dB [17]. This finding indicates that it is possible to obtain a laser that is insensitive to EOF using e.g. the isolator presented in [13]. We proposed to achieve this insensitivity by placing an optical isolator in the cavity of a ring laser. The isolator forces the ring...
laser into unidirectional operation, allowing only one propagating direction to reach threshold. Since EOF propagates in the backward direction it will return to the counter-propagating mode in the laser. This mode is below threshold and EOF therefore has a greatly reduced impact on the characteristics of the lasing mode. Additionally the narrow-band isolator is inherently aligned to the lasing wavelength, facilitating the tuning of the operating wavelength.

In this paper we study the properties of the unidirectional phase modulator (UPM), that is an essential part of the isolator presented in [13]. The impact of non-linearity, residual amplitude modulation (RAM) and imperfect driving signals on the quality of the complete optical isolator are obtained. The UPM and its ideal parameters are described in section 2. In section 3 a detailed, phenomenological model is presented that includes non-linearity and RAM in the electro-refractive phase modulators (ERMs). The experimental verification of the model is reported in section 4 for ERMs manufactured in a generic integration platform [18]. Section 5 presents a detailed, theoretical analysis of the impact of the operating conditions on the performance of the UPM on the basis of the validated model. This section also highlights a number of trade-offs that need to be made in the design of such a device. The paper is concluded by a set of requirements on the UPM that yield a side mode suppression ratio of 40 dB when the component is placed in the laser cavity.

2. UPM concept

The isolator presented in [13] consists of two tandem ERMs and a spectral filter and is schematically shown in Fig. 1. Ideally the tandem ERMs act as a single phase modulator in the backward propagating direction, while not affecting the light in the forward direction, hence the name unidirectional phase modulator. The phase modulated, backward propagating light is passed through the filter and the generated side bands are attenuated, reducing the total power in this propagation direction. Because light in the other direction is not modulated, this light passes the filter without attenuation. Combined, the UPM and the spectral filter therefore provide optical isolation.

This simplified model is only valid when the voltage response of the ERMs is strictly linear, the ERMs do not exhibit RAM and when the RF modulation of the ERMs is perfectly sinusoidal with a very specific amplitude and phase. When this is not the case, the performance of the UPM is suboptimal. In this case the forward propagating light will be modulated, resulting in the generation of side bands. This effect is especially important when the UPM is integrated into the cavity of a laser as these side bands can build up over several round trips of the light. The side bands can also interact with the charge carriers in the amplifier of the laser reducing the spectral quality of the laser output. In the reverse direction, suboptimal operating conditions result in a reduced suppression of light the central frequency, limiting the amount of isolation.
that can be achieved. A laser containing such a UPM will therefore show increased sensitivity to EOF. Because of their impact on laser performance, this paper focuses on ways to reduce the modulation of the forward propagating light and on ways to increase the suppression of the backward propagating light.

3. Model description

The construction of the phenomenological model of the UPM starts from a single ERM. This component is commonly used in InP based PICs as a voltage-driven phase modulator [7, 19]. Each of the two ERMs is modeled as a lumped element, which is justified because their lengths are shorter than one tenth of the wavelength of the RF driving signals. A change in refractive index inevitably changes the absorption of the material. Therefore, it can be expected that the modulators introduce unwanted amplitude modulation as was experimentally found in [20, 21]. For that reason, RAM is included in the model. The ERMs have a non-linear phase-voltage relationship, which we approximate by a second order polynomial. Throughout the model we assume that the non-linearity is instantaneous.

The transmission of each single ERM is then modeled as

\[ T_{-}^{\pm}(t) = A(V) \exp\left( i \Phi(V) \right) \]

\[ = \left( A_{p0} + A_{p1} \sin(\omega t + \psi_{p}^{\pm}) + A_{p2} \sin^2(\omega t + \psi_{p}^{\pm}) \right) \exp \left( i (\Phi_{p0} + \Phi_{p1} \sin(\omega t + \psi_{p}^{\pm}) + \Phi_{p2} \sin^2(\omega t + \psi_{p}^{\pm})) \right), \tag{1} \]

where the superscripts + and − refer to propagation in the forward and backward directions, respectively, Φ denotes the phase modulation amplitude, A denotes the modulation amplitude of the optical field amplitude as caused by RAM, the index p = L, R denotes the left or right ERM respectively, the subscripts 0, 1 and 2 indicate the order of the modulation, ω is the angular frequency of the electrical driving signal and t is time.

Both modulators are operated at a different effective phase as indicated by $\psi_{p}^{\pm}$. The difference between the effective phases of the two ERMs determines the modulation amplitude of the ensemble. For forward propagating light, the light first passes through ERM$_{L}$ and only later through ERM$_{R}$, resulting in an effective phase difference $\psi_{+}^{\pm} = \phi + \omega D/v_{g}$, where the superscript denotes the propagation direction, D is the path length difference between the centers of the two ERMs as indicated in Fig. 1 and $v_{g}$ is the group velocity. For backward propagating light, the light goes through the ERMs in the opposite order, resulting in an effective phase difference $\psi_{-}^{\pm} = \phi - \omega D/v_{g}$. The direction dependence of the effective phase difference $\psi^{\pm}$ ultimately provides the basis for the optical isolator.

The model for the complete UPM is obtained by multiplying the transmission of the two individual ERMs. This model can be simplified by grouping terms of similar modulation order. In this way an effective ERM is obtained that shows the same behavior as the UPM. Because of the direction dependence of the effective phase difference, this effective ERM is different for both directions. The respective transmissions in forward (+) and backward (−) direction can be expressed as

\[ T_{-}^{\pm}(t) = T_{L}^{\pm}(t)T_{R}^{\pm}(t) \]

\[ = \left( A_{0} + A_{1}^{\pm} \sin(\omega t + \Psi_{1}^{\pm}) + A_{2}^{\pm} \sin(2\omega t + \Psi_{2}^{\pm}) \right) \exp \left( i (\Phi_{0} + \Phi_{1}^{\pm} \sin(\omega t + \Psi_{1}^{\pm}) + \Phi_{2}^{\pm} \sin(2\omega t + \Psi_{2}^{\pm})) \right) \] \tag{2}
where

$$\Phi_0 \equiv \Phi_{L0} + \Phi_{R0} + \frac{1}{2} \Phi_{L2} + \frac{1}{2} \Phi_{R2}$$  \hspace{1cm} (3)$$

$$\Phi^+_1 \equiv \sqrt{\Phi_{L1}^2 + \Phi_{R1}^2 + 2\Phi_{L1} \Phi_{R1} \cos \psi^\pm}$$  \hspace{1cm} (4)$$

$$\Phi^+_2 \equiv \frac{1}{2} \sqrt{\Phi_{L2}^2 + \Phi_{R2}^2 + 2\Phi_{L2} \Phi_{R2} \cos 2\psi^\pm}$$  \hspace{1cm} (5)$$

$$A_0 \equiv (A_{L0} + \frac{1}{2} A_{L2})(A_{R0} + \frac{1}{2} A_{R2})$$  \hspace{1cm} (6)$$

$$A^+_1 \equiv \sqrt{A_{L1}^2 + A_{R1}^2 + 2A_{L1} A_{R1} \cos \psi^\pm}$$  \hspace{1cm} (7)$$

$$A^+_2 \equiv \frac{1}{2} \sqrt{A_{L2}^2 + A_{R2}^2 + 2A_{L2} A_{R2} \cos 2\psi^\pm}$$  \hspace{1cm} (8)$$

$$\tan \Psi^\pm_m = \frac{(1 - \Phi_m) \sin m\psi^\pm}{(1 + \Phi_m) \cos m\psi^\pm}$$  \hspace{1cm} (9)$$

and \( \Phi_m \equiv (\Phi_{Lm} - \Phi_{Rm})/(\Phi_{Lm} + \Phi_{Rm}) \) is a measure for the imbalance between the modulation amplitudes imposed on the light by both modulators.

To gain more insight into the effect of the UPM, the spectral profile of Eq. (2) can be calculated. To this end, the exponent in Eq. (2) is expanded using \( \exp(i x \sin \phi) = \sum_{n=-\infty}^{\infty} J_n(x) \exp(in\phi) \) [22, page 22], where \( J_n \) is the Bessel function of the first kind \( n \)th order. The sines are expanded using \( \sin(x) = (i/2) \exp(-i x) - (i/2) \exp(i x) \) and the resulting expressions are grouped by their frequency components, yielding

$$T^\pm(t) = \sum_{n=-\infty}^{\infty} \exp(in\omega t) \left( \sum_{m=-\infty}^{\infty} \sum_{k=-2}^{2} J_{n-2m-k}(\Phi^+_1) J_m(\Phi^+_2) A_k \times \exp \left(i((n-2m-k)\Psi^+_1 + m\Psi^+_2)\right) \right).$$  \hspace{1cm} (10)$$

From this Eq. it follows that single-frequency input light becomes a spectral comb that is centered at the optical frequency of the input light and with lines spaced by the electrical modulation frequency \( \omega \) after it has propagated through the UPM once. If the comb lines are numbered from the center (index 0), the relative power in each of the comb lines is found as

$$P^\pm_n = \left| \sum_{m=-\infty}^{\infty} \sum_{k=-2}^{2} J_{n-2m-k}(\Phi^+_1) J_m(\Phi^+_2) A_k \times \exp \left(i((n-2m-k)\Psi^+_1 + m\Psi^+_2)\right) \right|^2.$$  \hspace{1cm} (11)$$

This solution will be referred to as the full solution in the remainder of this paper.

When the modulators are linear (\( \Phi^+_2 = 0 \)) and do not introduce RAM (\( A_{k1} = A_{k2} = 0 \)) Eqn. (11) can be studied analytically as only the term with \( m = k = 0 \) contributes to the sum. In that case it follows that the transmission of the UPM is identical to that of a single ERM that is modulated with amplitude \( \Phi^+_1 \), and reads

$$P^\pm_{n,\text{simple}} = \left| J_n(\Phi^+_1) \right|^2.$$  \hspace{1cm} (12)$$

Note that the effective phase difference \( \psi^\pm \) is dependent on the propagation direction of the light. Equation (12) is referred to as the simple model in the remainder of this paper. When the modulators are balanced and have equal modulation amplitude, this equation simplifies further to

$$P^\pm_n = \left| J_n \left( \frac{\Phi^2}{2} \sqrt{2} \cos(\psi^\pm) \right) \right|^2.$$  \hspace{1cm} (13)
Table 1. Overview of the model parameters.

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total phase modulation amplitude</td>
<td>$\Phi$</td>
</tr>
<tr>
<td>Modulation imbalance</td>
<td>$\tilde{\Phi}$</td>
</tr>
<tr>
<td>Non-linearity</td>
<td>$\tilde{\Phi}$</td>
</tr>
<tr>
<td>RAM</td>
<td>$\tilde{\Phi}$</td>
</tr>
<tr>
<td>Non-linear RAM</td>
<td>$\tilde{\Phi}$</td>
</tr>
<tr>
<td>Forward effective phase difference</td>
<td>$\psi^+$</td>
</tr>
<tr>
<td>Backward effective phase difference</td>
<td>$\psi^-$</td>
</tr>
<tr>
<td>RF phase difference</td>
<td>$\phi$</td>
</tr>
<tr>
<td>Additional phase difference</td>
<td>$\omega D/v_g$</td>
</tr>
</tbody>
</table>

Finally, it will prove convenient later to define the total phase modulation amplitude, phase non-linearity, RAM and non-linear RAM for the case of identical ERMs as

$$\Phi \equiv \Phi_{L1} + \Phi_{R1} \quad (14)$$
$$\tilde{\Phi} \equiv \Phi_{L2}/\Phi_{L1} = \Phi_{R2}/\Phi_{R1} \quad (15)$$
$$\tilde{\Phi} \equiv A_{L1}/\Phi_{L1} = A_{R1}/\Phi_{R1} \quad (16)$$
$$\tilde{A} \equiv A_{L2}/\Phi_{L2} = A_{R2}/\Phi_{R2}. \quad (17)$$

Table 1 provides an overview of the names and symbols of the most important parameters in the model.

4. Experimental characterization and validation of the model

In this section, the characterization results of a fabricated UPM are used to verify the phenomenological model and to derive relevant parameter values. The PIC containing the UPM was fabricated in the SMART photonic integration platform that is based on InP technology [18]. The layout of the components on the PIC is presented in Fig. 2. The total on chip loss is estimated to be 8 dB. Each ERM in the UPM is implemented in three sections. In the figure this can be recognized as the two groups of three ERMs. The two center sections are 1 mm long, while the four outer sections are 1.08 mm long. Due to electrical limitations, only the two center ERMs were contacted in this work and the other four are left unbiased. The only difference between

![Fig. 2. Layout of the UPM that was fabricated by SMART Photonics [18]. ERMs are indicated in red, contact pads in yellow, electrical isolation sections in black and deeply etched waveguides in blue.](image)
these unbiased ERMs and regular waveguides is a metal contact pad that is located far from the optical mode. It is therefore assumed that the unconnected ERMs behave the same as passive waveguides.

The ERMs are connected using a bond wire, printed circuit board (PCB) and coaxial cable. Figure 3 shows a photograph of these electrical connections as well as the aligned lensed fibers that are used for the optical in- and outputs. Both ERMs are DC-biased at $-6 \text{ V}$ and each modulator is individually connected to an RF signal generator (Anritsu MG3691B and Rohde & Schwarz SGS100A). The maximum modulation amplitude of the RF sources is $3.2 \text{ Vpp}$. Together with the modulation efficiency of the ERMs, $V_L = 9 \text{ Vmm}$, the maximum suppression of the central line that can be expected is $99\%$. Electrical losses, mainly those in the PCB, were too high to obtain meaningful results, limiting us to operation at $3.5 \text{ GHz}$. This frequency was used for the experiments, as opposed to $5 \text{ GHz}$ for which the UPM was designed. As will be shown later, this limits the maximum isolation that is obtained experimentally, but does not impact the validity of the conclusions. The signal generators are both phase locked to the $10 \text{ MHz}$ clock of the Anritsu signal generator such that they operate at the same frequency. The modulation amplitude and phase difference between the two signal generators is varied between measurements.

For characterization, lensed fibers are aligned to the angled and anti-reflection coated input and output facets of the PIC using nanoblocks (Thorlabs MAX300). This results in an estimated coupling loss of $5 \text{ dB}$ and facet reflections of less than $-40 \text{ dB}$. The input fiber is connected to a laser that is lasing at a wavelength of $1550 \text{ nm}$ with a power of $10 \text{ mW}$ (HP 81940A). The output spectrum is determined using an optical spectrum analyser (OSA) with a resolution of $20 \text{ MHz}$ (APEX 2641-B), yielding spectra such as the one presented in Fig. 4(a). The total optical power in the five central peaks in this spectrum is then extracted, essentially using the OSA as five parallel, high resolution, spectral filters.

First, the signal generator connected to ERM$_L$ is set to its maximum signal strength of $20 \text{ dBm}$ or $3.2 \text{ Vp}$ while the power supplied by the second signal generator is increased from $65\%$ ($2.1 \text{ Vp}$) until $100\%$ ($3.2 \text{ Vp}$) of this value in steps of $5\%$ ($0.16 \text{ Vp}$). Then a similar experiment is done using a fixed RF power of $20 \text{ dBm}$ for ERM$_R$ while sweeping the signal on ERM$_L$. When both modulators and RF signal paths are identical, which is not the case in the experiment, this corresponds to sweeping the imbalance in modulation depth $\Phi_1$ from $-0.21$ to $0.21$. For each amplitude setting the RF phase difference $\phi$ is swept from $0$ to $2\pi$ rad in steps of $\pi/18$ rad or $10^\circ$ resulting in a sweep of the effective phase difference $\psi^\pm$. This whole sequence is then repeated four times to obtain an estimate for the measurement error. Finally, the model is fitted to all the data simultaneously. This yields the parameters shown in Table 2.

Figures 4(b)–4(d) present the power in each of the five colored peaks of Fig. 4(a) as a function of $\psi^\pm$ and for three distinct modulation imbalances: stronger modulation in the second modulator ($\Phi_2 = -0.093$), approximately equal modulation amplitude ($\Phi_1 = 0.005$) and stronger modulation in the first modulator ($\Phi_1 = 0.034$) respectively. Each color in the subplots corresponds to the spectral line of the same color in Fig. 4(a). It should be noted that phase modulation depth ratio $\Phi_1^+/\Phi_2^+$ was not equal to $V_L/V_R$ in these measurements. This is attributed to a difference in
Fig. 4. Measured spectra for various RF imbalances and RF phase differences. Subfigure (a) presents a single measured spectrum, where the relative frequency, \( F \), is the optical frequency relative to the frequency of the incoming light wave. Subfigures (b)-(d) show the power in the central five peaks in these spectra as a function of effective phase difference \( \psi_{\pm} \) for three values of modulation imbalance \( \hat{\Phi}_1 \). The colors used for the data in subfigures (b)-(d) correspond to colors of the peaks in subfigure (a). The solid lines represent the result of the model fitted to all data simultaneously (parameters as in Table 2) for various modulation imbalances (\( \Phi_{L1}/\Phi_{R1} \)) as specified in the captions. Note that the fitted curve falls inside most of the indicated error intervals, indicating that our model includes all relevant parameters.
Table 2. Parameter values resulting from fitting our model to the measured data.

<table>
<thead>
<tr>
<th>Description</th>
<th>Fit value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total phase modulation amplitude</td>
<td>0.96</td>
<td>rad</td>
</tr>
<tr>
<td>Modulation imbalance</td>
<td>0.005</td>
<td>1</td>
</tr>
<tr>
<td>Non-linearity</td>
<td>5.5</td>
<td>%</td>
</tr>
<tr>
<td>RAM</td>
<td>8.4</td>
<td>%</td>
</tr>
<tr>
<td>Non-linear RAM</td>
<td>0.15</td>
<td>%</td>
</tr>
</tbody>
</table>

electrical losses in the paths between the signal generators and ERMs and to different phase modulation efficiencies of the individual ERMs. In the fitting procedure this is taken into account by scaling the modulation imbalances by a single fitting parameter.

In Figs. 4(b)–4(d) the experimental data are presented as error bars that span two standard deviations of the measurement error. In these figures the fit to the full model is presented as a solid line of the same color. Very good agreement is obtained indicating the model captures all relevant effects.

A number of features of these figures can be understood from Eq. (13). First of all, the effective modulation amplitude is greatest for an effective phase difference of \( \psi^+ = 0 \) and smallest for \( \psi^+ = \pi \). This translates to strongest and weakest phase modulation, respectively. In Figs. 4(b)–4(d) is represented by the reduction of the central peak (blue line) and the increase of the other peaks for \( \psi^+ = 0 \). From the blue line it can also be seen that the central peak was suppressed by maximally 2.2 dB, corresponding to the maximum isolation that can be achieved using the available operating conditions. This is caused by the limited phase modulation amplitude mentioned before.

The figures show only a small impact of RAM due to the limited modulation amplitude. The experiment did not provide us with an accurate measure for the RAM parameters \( \tilde{A} \). Our theory does however predict that RAM becomes much more visible when the ideal modulation amplitudes are applied. For this reason we do not neglect it. In order to obtain quantitative results, the value found in [21] for an ERM fabricated in the same platform, but obtained using DC signals is used. It is equal to \( \tilde{A} \equiv \tilde{A}_{L1}/\Phi_{L1} = \tilde{A}_{R1}/\Phi_{R1} = 0.05 \).

Finally, Fig. 4 shows features of residual phase modulation that are caused by imbalanced modulation and residual modulation due to non-linearity. To understand the impact of these effects, a more detailed understanding of the model is required however. They will be explained in subsections 5.2 and 5.3 of the next section.

5. Analysis of the performance of a UPM

The verified model will now be used to study the effect of the non-linearity and the RAM of the ERMs and the effect of the RF signals on the maximum isolation that can be achieved using the UPM, as well as the intensity of the side bands generated on the forward propagating light. As indicated before, the modulation amplitude that was achieved during the experiment was insufficiently to fully deplete the central peak, and is was therefore not possible to obtain complete isolation. During this analysis we use our model to extrapolate the experimental data to higher phase modulation amplitudes. In practice this could be achieved by elongating the ERMs or by increasing the amplitude of the RF voltage supplied to the ERMs. Also, the efficiency of the modulators could be increased, e.g. by using quantum well ERMs instead of the bulk ERMs used in this work. However, this last option would however have an impact on the linearity and RAM of the modulators as well, invalidating the parameters obtained in the previous section.
Even for this case, the general model would still apply.

Various interesting properties of the model will be studied in the following subsections. For each property it is assumed that the UPM is otherwise ideal to allow for more clear results. Since the imperfections in the modulator parameters are small, the separate study of the properties is justified.

The remainder of this section first derives the set of operating parameters that allow for complete suppression of the backward propagating light. These values agree with the values presented in [13], indicating that our model matches the model presented in [13] for the cases studied in that paper. Then the effect of any deviation from the operating points is studied. From the results of this study an understanding of the maximum allowed deviation from the optimal operating parameters of the UPM is developed. Values of this maximum allowed deviation are derived for a specific requirement on the spectral quality of the forward transmitted light: the intensity of all side bands is at least 40 dB below that of the central wavelength.

5.1. Trade-off: amplitude, phase, frequency, length

Ideally an isolator does not affect the forward propagating light. This is only true when the transmission spectrum of the UPM is a single peak centered at 0. In other words, \( P_0^n = 1 \) for \( n = 0 \) and 0 otherwise. The contrast, this peak is ideally completely suppressed for backward propagating light, ensuring that all power is converted to side bands which can subsequently be filtered to achieve isolation. This requires \( P_{0}^{-} = 0 \). As was already presented in [13] this is achieved when there is no non-linearity in the phase-response \( (\Phi_2^+ = 0) \), when there is no RAM \( (A_{k1} = A_{k2} = 0) \), when the modulators are perfectly balanced \( (\Phi_{L1} = \Phi_{L2}) \) and when the phase relations obey \( \phi = \omega D/v_g = \pi/2 \). As will be shown from Eq. (11), these are however not the only solutions that allow for perfect isolation.

From this equation it is found that a trade-off can be made between the RF modulation amplitude, phase difference and the modulation frequency, and the spacing of the ERMs. It is most insightful to study this trade-off assuming the modulators do not show a non-linear phase response or RAM and the modulation amplitudes are balanced as this yields the most easily interpreted Eqs., without introducing significant errors in the trade-offs. In this special case Eq. (4) can be simplified to

\[
\Phi_{\text{ideal}}^{\pm} = \Phi \sqrt{\frac{1}{2} + \frac{1}{2} \cos \psi^\pm}
\]  

(18)

where \( \Phi \equiv \Phi_{L1} + \Phi_{R1} \) is the summed phase modulation amplitude. To prevent modulation of the forward propagating light it follows that the effective modulation amplitude for the forward propagating light equals 0, or \( \sqrt{\frac{1}{2} + \frac{1}{2} \cos \psi^+} = 0 \). This implies that effective phase difference for the forward propagating light equals \( \pi \). In section 3, it was shown that this phase difference equals \( \psi^+ = \phi \pm D\omega/v_g \). Minimal influence on the forward propagating light can therefore be obtained by tuning one of the three parameters: the phase difference between the RF signal generators \( (\phi) \); the modulation frequency \( (\omega) \); or the optical path length between the centers of the two ERMs \( (cD/v_g) \). To further illustrate the influence of the effective phase difference for the forward propagating light as well as the validity of the simple model, Fig. 5 shows the calculated power in the side bands in the forward propagating light generated by the UPM when the effective phase difference is varied. The results are presented for both calculations using the simplified model and calculations using the full model which uses the non-linearity parameters obtained from a fit to our experimental data and the RAM parameters obtained from [21]. Both effects have a negligible effect on the first order side band of the modulation of the forward wave as can be observed from the overlap between the two curves in the figure. As can be expected, the power levels in the second order side bands are however determined by the non-linear phase response and the RAM. Taking 40 dB as the maximum intensity of the side bands, the forward
Effective phase difference $\psi^{+}$ needs to be accurate up to 0.017 rad.

From Eq. (18) it also follows that maximum isolation requires an effective modulation amplitude of $\bar{J}$, where $J_0(\bar{J}) \equiv 0$ is the first zero of the Bessel function. This requirement can be expressed as $\Phi \sqrt{1/2 + 1/2 \cos \psi^{-}} = \bar{J}$, which shows that there is a trade-off between the RF modulation amplitude $\Phi$ and the effective phase difference $\psi^{-}$. Usually it is desirable to reduce the modulation amplitude as much as possible to lower power consumption for the RF sources and to reduce non-linearity in the modulators. Figure 6 shows the modulation amplitude $\Phi$ that yield maximum isolation as a function of the effective phase difference in the backward direction $\psi^{-}$. The figure shows a minimum required modulation amplitude at $\psi^{-} = 0$. The UPM will commonly be designed to operate at this point as power consumption of the RF sources and linearity of the modulators usually deteriorates with increased modulation amplitude. The required modulation amplitude for maximum isolation only slightly increases for small deviations from the optimum effective phase difference. Small error in effective phase difference can therefore be compensated by driving the ERM's harder without incurring a big penalty. In practice this could mean that the effective phase difference is optimized for optimum performance in the forward direction by tuning the phase difference between the RF sources, while simultaneously optimizing performance in the backward direction by tuning the modulation amplitude. Finally the figure shows an asymptote at the least optimal effective phase difference, $\psi^{-} = \pm \pi$. This is expected as the two ERM's completely cancel each other’s effects in this case. No matter how hard they are driven, no phase modulation will occur for this setting and no isolation will be possible.

Again allowing side bands of maximum 40 dB as an example, this yields an upper limit on the modulation of the forward propagating light of $|J_0(\Phi^{-})|^2 < 1 \times 10^{-4}$, or $\Phi^{-} < 0.02$. Assuming the minimum value for the RF modulation amplitude $\Phi = \bar{J}$ this yields a maximum allowable error of 0.017 rad in the effective phase difference in the reverse direction, $\psi^{-}$.
Effective phase difference [rad]

Total required modulation amplitude [rad]

Fig. 6. The total phase modulation amplitude, \( \Phi \) that is required to obtain maximum suppression of the central wavelength (optimum isolation) in the backward direction as a function of effective RF phase difference in the backward direction, \( \psi^- \). The ideal value is reached for \( \psi^- = 0 \). It can be seen that deviations as large as \( \pi/2 \) from this ideal value do not impact the modulation depth requirements greatly, but larger deviations from the optimum value should be avoided.

5.2. Amplitude imbalance

Both modulators in the UPM do not necessarily impose the same modulation amplitude on the light. In practice this can result from a different electrical path for both RF signals, or from a different response of both modulators. Any imbalance results in an increase of the side bands imposed on the forward propagating wave and in a reduction of the suppression of the central peak in the backward propagating wave.

To study this effect Eq. (4) is rewritten as

\[
\Phi_1^\pm = \Phi \sqrt{\frac{1 + \Phi_1^2}{2} + \frac{1 - \Phi_1^2}{2} \cos \psi^\pm}
\]

where \( \Phi_1 = (\Phi_{L1} - \Phi_{R1})/(\Phi_{L1} + \Phi_{R1}) \) is the modulation imbalance as defined before. The relative effective modulation amplitude \( \Phi_1^\pm/\Phi \) is plotted in Fig. 7 as a function of the modulation imbalance \( \Phi_1 \) for various effective phase differences \( \psi^\pm \).

The suppression of any residual modulation of the forward propagating light can only be achieved when the modulation amplitudes are balanced and the effective phase difference for the forward propagating light is ideal. For 40 dB side mode suppression ratio (SMSR), a maximum imbalance of \( \Phi_1 = 0.02/\Phi \) is found. For the minimum modulation amplitude \( \Phi = J \) this yields \( \Phi_1 < 0.008 \). It should be noted that tolerances on the modulation imbalance become more tight if the modulation amplitude is increased to compensate for sub-optimal phase settings as discussed in the previous subsection.

In Figs. 4(b)–4(d) the effect of imbalance is most apparent in the first order side bands. For well balanced modulation (Fig. 4(c)) it can be seen that the power in these bands tends to 0 for an effective phase difference of \( \psi^e = \pi \). For unbalanced modulation (Figs. 4(b) and 4(d)), the residual modulation results in an increase in the side bands at \( \psi^e = \pi \).
Fig. 7. The effective phase modulation amplitude of the UPM relative to the sum of the modulation amplitudes of the two ERMs, $\Phi_{\pm}^1/\Phi$, for a number of values of effective phase differences $\psi_{\pm}$ and as a function of the imbalance in the effective modulation depth of the refractive index, $\Phi_1$. As shown, no modulation on the forward propagating wave requires equal modulation amplitude and an effective phase difference of $\pi/2$ ($\Phi_{\pm}^1 = 0$ and $\psi_{\pm}^+ = \pi$ respectively). In the backward direction, ideally $\psi_{\mp} = 0$. Any deviations from this value can be compensated by increasing the modulation amplitude of the two ERMs, $\Phi$.

For the backward propagating light the requirement is $\Phi_{\mp}^1 = \bar{J}$ such that the central spectral peak is fully suppressed. It is possible to achieve this condition for any value of modulation imbalance $\Phi_1$ by increasing the total modulation amplitude. For effective phase difference values $\psi_{\mp}$ that are relatively close to the ideal 0 and reasonably balanced modulation, the effect of the imbalance in the modulation can readily be compensated by a minor increase in the total modulation amplitude. Therefore, the effect of the UPM on the backward propagating light is not affected by a modulation imbalance and requirements on the modulation imbalance are derived fully from its effects on forward propagating light.

5.3. Non-linearity and residual amplitude modulation

Due to phase non-linearity in the ERMs, the light effectively experiences a modulation at $2\omega$. It is not possible to suppress both the linear and the quadratic modulation at the same time, as this would require $\phi + \omega D/v_g = \pi$ and $\phi + 2\omega D/v_g = \pi$ simultaneously. As a result, side bands will be generated at $2\omega$. Figure 4(c) shows this most clearly in our experimental data. The first order side bands are strongly suppressed in this case, while the second order side bands remain clearly visible. Furthermore, non-linearity combined with unbalanced modulation results in an asymmetric transmission as show in Figs. 4(b) and 4(d) and is most clear for an effective phase difference slightly above and below $\psi_{\pm}^+ = \pi$.

For further analysis of the impact of non-linearity and RAM, it is assumed that both modulators show identical non-linearity in the phase and amplitude response such that $\Phi \equiv \Phi_{L2}/\Phi_{L1} = \Phi_{R2}/\Phi_{R1}$. The intensity of each of the peaks at $2\omega$ is found as $|J_1(\Phi_{\pm}^1)|^2$, which is graphically shown for otherwise ideal parameters in Fig. 8. It is shown that RAM only has a significant effect for a phase non-linearity $\Phi < 0.01$.

Tolerable values are again obtained using $\text{SMSR} > 40 \text{ dB}$ as a guideline for the required laser light quality, and it is found that the requirement on phase non-linearity equals $\Phi_{\pm}^1 < 0.02$. When the minimal modulation amplitude $\Phi_{\pm}^1 = \bar{J}$ is assumed, this implies non-linearity should be less
than $\Phi^+_2/\Phi^+_1 = 0.8 \%$, which in turn is satisfied if $\Phi^+_{L2}/\Phi^+_{L1}$ and $\Phi^+_{R2}/\Phi^+_{R1}$ are both smaller than 1.7\%.

In the backward direction non-linearity in the phase and amplitude modulation has a more complicated impact and influences the maximum achievable isolation which can be calculated using Eq. (11). This expression does not lend itself to simplification. Figure 9 shows the maximum isolation that can be obtained using the UPM as a function of RAM (Å) for optimized modulation amplitude $\tilde{\Phi}$, effective phase difference $\psi$ and modulation imbalance $\hat{\Phi}_1$. A maximum isolation of approximately $-22$ dB is found for the parameters obtained in the experiment. This demonstrates that RAM and non-linearity limit the performance of the UPM and the importance of optimizing the design to minimize these effects. Both the phase non-linearity and RAM depend on the bias voltage and RF amplitude. Optimum performance can therefore be achieved by tuning these parameters. It should be noted that a reduction in RF amplitude will result in increased length for the modulators to ensure that the phase modulation amplitude remains unchanged.

5.4. Quality of the RF source

Finally, we analyze the impact of the spurious signals and harmonics in the RF signals supplied to the ERMs. The performance of the UPM is simulated with modulators that have a perfectly linear response which does not change over time (linear and time-invariant). This allows the driving signals to be treated as a superposition of their frequency components. As long as the power in the spurious signals is much smaller than the signal power, this approximation is accurate.

In section 5.1 it was explained that a driving frequency $\omega$ that differs from $\omega_D \equiv \pi \nu_g/2D$ results in residual phase modulation on the forward propagating light. The amplitude of this modulation is dependent on the frequency. Therefore the tolerable amount of power in harmonics and spurious signals from the RF signal generator is dependent on the frequency of those signals. Again taking a maximum allowable side band power ratio of $-40$ dB, Fig. 10 shows the maximum allowed level of harmonics and spurious signals as a function of the frequency in units of the
Phase non-linearity

RAM

(a) Without second order RAM ($\hat{A} = 0$).

(b) With second order RAM ($\hat{A} = 2.5\%$).

Fig. 9. Maximum achievable isolation as function of phase non-linearity $\Phi$ and RAM $\hat{A}$ without and with second order RAM. The cross indicates the operating point for the ERM used in the experiment. Both the phase non-linearity and RAM reduce the maximum achievable attenuation that can be achieved for the backward propagating light. As such, both limit the maximum isolation that can be achieved. For isolation above 37 dB the non-linear RAM limits performance.

electrical frequency $\omega_D$ for which the UPM was designed.

At $\omega = (1 + 4q)\omega_D$, with $q$ an integer, modulation has no effect on the forward propagating light as both modulators will cancel each other’s effects at these frequencies. This explains the unlimited maximum permissible modulation strength at these frequencies. In these cases, the linear approximation of the ERM does not hold anymore and in practice non-linearities an RAM determine the limit for spurious signals at these frequencies. For $\omega = (3 + 4q)\omega_D$ the modulators amplify each other’s effects and the amplitude requirement on the spurious signals is most strict, $\sim 20.8$ dBc. The backward propagating wave is also affected by any impurities in the RF driving signals, resulting in reduced suppression of the central peak and thus reduced isolation.

6. Conclusion

In conclusion, a UPM was characterized for use as part of an integrated optical isolator. A phenomenological model was used to describe the performance of the UPM. The model includes the non-linear response of the ERM and RAM and was able to accurately describe our experimental data. It was found that RAM had little effect at the modulation amplitude used, but it is predicted to play an important role for the modulation amplitude that is required to achieve maximum isolation. Subsequently the maximum achievable isolation and the residual phase modulation on forward propagating light were derived as a function of the characteristics of the RF source and the ERM. For the RF source, the effect of the RF modulation amplitude, imbalance, frequency and phase difference as well as harmonics and spurious signals was studied. The effect of non-linearity and RAM in the transfer function of the ERM was also studied in detail.

Taking $-40$ dB as the maximum permissible power in the side bands of the laser, a maximum allowable phase error of 0.017 rad in the control of the RF signals, a maximum imbalance in the RF signal amplitudes of 1.66 % and a maximum non-linear term of 0.017 was found. The amount of harmonics and spurious signals that can be tolerated in the RF driving signals was found to be frequency dependent. When these signals are below $-20.8$ dBc the generated side bands are
Maximum allowed amplitude $\Phi_{\text{max}}$ of an additional electrical signal at frequency $\omega$ in units of the design frequency $\omega_D$. All frequency components in the RF driving signals modulate the light in both the ERMs. Because of the nature of the UPM, certain frequencies completely cancel out, while others do not. Therefore, the tolerable amount of harmonics and spurious signals is frequency dependent and is shown here for a desired SMSR of 40 dB.

below $-40$ dB, but this requirement is relaxed for most frequencies. These requirements on the RF source can be met using current electronics. The effect on light propagating through the UPM in the forward direction can therefore be limited to acceptable levels. Based on these results it is predicted that the performance of a UPM-based isolator is sufficient for the inclusion in an EOF-insensitive laser.

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**References**


