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2D Semi-Analytical Modeling of Eddy Currents in Multiple Non-Connected Conducting Elements

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The paper concerns the semi-analytical modeling of parasitic eddy current distributions and forces in electromagnetic devices. The harmonic, Fourier based, model is extended to incorporate a position dependent conductivity, and restrictions on the induced current density are added to model eddy currents in non-connected conducting elements. To model time dependent behavior of eddy currents, multiple time harmonics can be incorporated in the solutions. The developed method is verified using the finite element method (FEM) for a wide range of frequencies. The model is applied to a coreless linear motor with electrically conducting cooling plates and the parasitic force is analyzed.

Index Terms—Eddy current, Fourier analysis, Permanent Magnet Machines, Analytical Modeling.

I. INTRODUCTION

Accurate modeling of parasitic forces in synchronous permanent-magnet machines is necessary to optimize their performance. Parasitic forces are often ignored during design procedures because they are relatively small compared to the main force. However, high precision (nano-meter accurate) positioning devices have ever higher precision and acceleration demands. To meet these demands, parasitic forces can not be ignored because they are impairing the performance of the device. One of the parasitic forces, the eddy current force, is caused by induced currents in conductive parts of the motor. These eddy currents also cause power losses. Examples of conductive parts in a motor are magnets, cooling plates and back iron plates. To decrease the power losses due to eddy currents, the conductive parts are often segmented [1], [2].

To model parasitic forces and power losses due to eddy currents in electromagnetic devices, several techniques can be used. An often used method is the finite element (FE) method. Because of the relatively high computation time of the finite elements method, alternative analytical methods, such as the harmonic (Fourier) model [3], [4], can be used. However, to include the reaction field of eddy currents in an element which is not spanning the entire periodic width, the classical Fourier model is not suited. In [5], eddy currents in a segmented structure are approximated for low frequencies by applying the method of images to the results of the Fourier analysis technique applied to the non-segmented structure. In [6] a technique has been developed to include the spatially dependent conductivity of a region in the solutions of the Fourier model, thereby modeling the eddy currents and their reaction field in segmented structures. However, when applied to a geometry with multiple conducting elements, the model calculates the currents as if the elements are electrically connected.

In this paper, the 2D Fourier analysis technique from [6] is extended to analytically model eddy currents in multiple non-connected conductive elements for a wide range of frequencies. The developed method is applied to the geometrical model shown in Fig. 1 for verification. The forces on conducting parts due to induced eddy currents and the power losses are calculated and compared to FE results.

II. MODEL FORMULATION

The solution to the magnetic field and current density in the conducting region is the main focus of this paper. In the other regions, the classical solutions are used as presented in [6] and [7].

The geometric model of Fig. 1 consists of a three phase coil set with above it two conducting elements, surrounded by air. The time varying field, originating from the coils induces eddy currents in the conducting parts. The model is assumed periodic in the x-direction. The model is divided into horizontal regions and the permeability is assumed constant.
as a function of position for each region. Because of the periodicity, all quantities dependent on \( x \) are written as a Fourier series

\[
g(x) = \sum_{n=-N/2}^{N/2} g(n) e^{jknx},
\]

where \( g \) is a vector containing the Fourier coefficients of the series and \( N \) is the number of harmonics considered in the truncated Fourier series. The spatial frequency \( k_n \) is given by

\[
k_n = \frac{n\pi}{\tau_x},
\]

where \( \tau_x \) is half of the periodic width. It is assumed that the problem is quasi-static, and the time derivative is equal to \( j\omega \).

III. SEMI-ANALYTICAL SOLUTION

A. Maxwell equations

To obtain the solutions of magnetic field quantities, the vector potential is introduced. The magnetic vector potential \( \vec{A} \) is defined by

\[
\vec{B} = \nabla \times \vec{A},
\]

where \( \vec{B} \) is the magnetic flux density. Because of the 2D Cartesian problem, the vector potential has only a \( z \)-component, and currents, either induced or source currents, flow in the \( z \)-direction. The induced (eddy) current density is now defined for each conducting element separately

\[
J_{\text{ind}}^{z_i} = -\sigma_i f_i(x) \frac{\partial A_z}{\partial t} - f_i(x)c_i,
\]

where \( i \) denotes the number of the element, \( \sigma_i \) is the value of the electrical conductivity and \( f_i \) describes the spatial dependency of the conductivity of element \( i \). The total number of conducting elements is denoted by \( I \). The functions for elements 1 and 2 of the geometric model are shown in Fig. 1. An unknown, denoted by \( c_i \), is added for each element. The introduction of the extra unknown per conducting element, makes it possible to put a restriction on the induced current density (Section III-D). With this novel approach it is possible to model the eddy currents as if there is no electrical connection between the separate elements, which was not achieved in [6].

Based on Maxwell’s equations and the constitutive relation the diffusion equation for the conducting region is found

\[
\frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} = -\mu_0 \frac{\partial M_y}{\partial x} + \mu \sum_{i=1}^{I} \sigma_i f_i(x) \frac{\partial A_z}{\partial t} + \mu \sum_{i=1}^{I} f_i(x)c_i,
\]

where \( M_y \) is the magnetization in the \( y \)-direction (which is not present in the model of Fig. 1), \( \mu \) is the permeability and \( \mu_0 \) the permeability of vacuum. It is assumed that no imposed currents are present in a conducting region.

B. Vector potential solution

The solution to the vector potential is found by applying the method of separation of variables. The vector potential solution can be divided into a homogeneous, and a particular solution which depends on the source terms. The coefficients of the homogeneous solution are denoted by \( h \) and are obtained through

\[
h = Q \left( E^+(\lambda, y) a + E^- (\lambda, y) b \right),
\]

where

\[
E^+(\lambda, y) = \text{diag} \left( e^{\lambda(y-yT)} \right),
\]

\[
E^- (\lambda, y) = \text{diag} \left( e^{-\lambda(y-yB)} \right),
\]

where \( a \) and \( b \) are vectors containing unknowns per harmonic which have yet to be determined. The variables \( yT \) and \( yB \) represent the \( y \)-values of the top and bottom boundary of a region respectively. By subtracting \( yT \) or \( yB \) in the exponent, the result of this exponent is always scaled between 0 and 1 which enhances the stability of the model. The mathematical function \( \text{diag}(\cdot) \) in (7) and (8) forms a diagonal matrix using the entries of the input vector on the diagonal. The functions \( f_i \) that represent the spatial dependence of the conductivity are written as a Fourier series

\[
f_i = \sum_{n=-N}^{N} \Psi_i(n) e^{jknx},
\]

where \( \Psi_i \) are the Fourier coefficients of element \( i \). To obtain the propagation variables \( \lambda \) and \( Q \), \( h \) and (9) are substituted into (5). Because of the Fourier series description, the double derivative to \( x \) is equal to \( -k_n^2 \). Then, (5) can be written as

\[
\sum_{n=-\infty}^{\infty} \frac{\partial^2 h(n)}{\partial y^2} e^{jknx} = \sum_{n=-\infty}^{\infty} k_n^2 h(n) e^{jknx} + j\omega \mu \left( \sum_{n=-\infty}^{\infty} \sum_{i=1}^{I} \sigma_i \Psi_i(n) e^{jknx} \right) \sum_{n=-\infty}^{\infty} h(n) e^{jknx}.
\]

The multiplication of two series in (10) can be handled using Laurent’s multiplication rule [8], [9], as explained in [6]. The multiplication of the two time series is in fact a convolution of the Fourier coefficients, and therefore (10) is given in the spectral domain

\[
\frac{\partial^2 h}{\partial y^2} = \left( K_n^2 + j\omega \mu \sum_{i=1}^{I} \sigma_i \Psi_i \right) h,
\]

where \( K_n \) is a diagonal matrix with the spatial frequencies \( k_n \) on the diagonal and \( \Psi \) is a Toeplitz matrix which represents the convolution of coefficients, [8], [9]. An eigenvalue decomposition is applied to the right hand side of (11) to obtain the vector with the propagation variable \( \lambda \)

\[
Q \Lambda^2 Q^{-1} = K_n^2 + j\omega \mu \sum_{i=1}^{I} \sigma_i \Psi_i,
\]

where \( \Lambda \) is a diagonal matrix containing the vector \( \lambda \). The matrix \( \Lambda \) is squared because it represents the double derivative.
to \( y \) in (10). The matrix \( \mathbf{Q} \) contains the eigenvectors belonging to each eigenvalue.

In the particular solution the magnetization terms and terms containing \( c_i \) have to be taken into account. Substituting \( \mathbf{p} \), the vector containing the Fourier coefficients of the particular solution, which only depends on \( x \) and \( t \), into (5) the following is obtained

\[
\frac{\partial^2}{\partial x^2} \mathbf{p} + \frac{\partial^2}{\partial y^2} \mathbf{p} = -\mu_0 \frac{\partial \mathbf{M}_y}{\partial x} + \mu \sum_{i=1}^{l} \sigma_i f_i(x) \frac{\partial}{\partial t} \mathbf{p} + \mu \sum_{i=1}^{l} f_i(x)c_i,
\]

which, in the spatial domain, leads to the matrix equation

\[
\left( \mathbf{K}_n^2 + j\omega \mu \sum_{i=1}^{l} \sigma_i \bar{\Psi}_i \right) \mathbf{p} = j\mu_0 \mathbf{K}_n \mathbf{m}_y - \mu \sum_{i=1}^{l} \mathbf{p}_i c_i,
\]

where \( \mathbf{m}_y \) is a vector containing the coefficients of the magnetization in the \( y \)-direction. From (14), the coefficients of the particular solution are equal to

\[
\mathbf{p} = \left( \mathbf{K}_n^2 + j\omega \mu \sum_{i=1}^{l} \sigma_i \bar{\Psi}_i \right)^{-1} \left( j\mu_0 \mathbf{K}_n \mathbf{m}_y - \mu \sum_{i=1}^{l} \mathbf{p}_i c_i \right).
\]

\[ \text{C. Magnetic flux density and induced currents solution} \]

From the expressions obtained for the vector potential in the previous section, the Fourier coefficients of the magnetic flux density components, \( \mathbf{b}_x \) and \( \mathbf{b}_y \), and induced current density, \( \mathbf{j}_x \) and \( \mathbf{j}_y \), in each element can be derived using respectively (3) and (4)

\[
\mathbf{b}_x = \mathbf{Q} \mathbf{A} \left( \mathbf{E}^+(\lambda, y) \mathbf{a} - \mathbf{E}^-(\lambda, y) \mathbf{b} \right),
\]

\[
\mathbf{b}_y = -j \mathbf{K}_n \left( \mathbf{Q} \left( \mathbf{E}^+(\lambda, y) \mathbf{a} + \mathbf{E}^-(\lambda, y) \mathbf{b} \right) + \left( \mathbf{K}_n^2 + j\omega \mu \sum_{i=1}^{l} \sigma_i \bar{\Psi}_i \right)^{-1} \left( j\mu_0 \mathbf{K}_n \mathbf{m}_y - \mu \sum_{i=1}^{l} \mathbf{p}_i c_i \right) \right),
\]

\[
\mathbf{j}_x = -j\omega \sigma_i \bar{\Psi}_i \left( \mathbf{Q} \left( \mathbf{E}^+(\lambda, y) \mathbf{a} + \mathbf{E}^-(\lambda, y) \mathbf{b} \right) - \left( \mathbf{K}_n^2 + j\omega \mu \sum_{i=1}^{l} \sigma_i \bar{\Psi}_i \right)^{-1} \mu \sum_{i=1}^{l} \mathbf{p}_i c_i \right) - \mathbf{p}_i c_i,
\]

\[ \text{D. Boundary Conditions and Current Density Restrictions} \]

In non-conducting regions the expressions for magnetic fields are obtained using the vector potential formulation as explained in [6] and [7]. To calculate the final solution to the vector potential and thus all other magnetic field quantities, the unknowns (\( a, b \) and \( c_i \)) per region have to be determined by solving a set of equations. This set of equations is formed by applying boundary equations between the regions in the model.

For regions adjacent to a infinitely permeable material (\( \mu = \infty \)) at a certain height \( y_{\mu=\infty} \), the tangential component of \( H \) is forced to zero on the boundary

\[
H_x = 0 \quad |y = y_{\mu=\infty}.
\]

A continuous boundary condition is applied to adjacent regions at a certain boundary height \( y_{\text{cont}} \). It ensures continuation of the normal \( B \) field and tangential \( H \) field

\[
H_x^1 = H_x^1 \quad |y = y_{\text{cont}},
\]

\[
B_y^1 = B_y^1 \quad |y = y_{\text{cont}}.
\]

The third boundary condition is the Dirichlet boundary condition which specifies the solution to a certain value at a boundary. Here it is used to force all field components of a certain region to zero if the region extends to infinity

\[
\bar{H} = \bar{B} = 0 \quad |y = \pm \infty.
\]

This boundary condition implies that one of the two unknowns (\( a \) or \( b \)) in the solution is canceled.

In Section III-A an extra unknown \( c_i \) is introduced for each element in the conducting region. To construct a system of equations that is solvable, the number of equations has to be equal to the number of unknowns. As a consequence, for each element an additional condition has to be constructed.

The induced current in one conducting element can only flow through the element itself and not into another element. As a result, the total induced current in an element should be zero when integrated over the surface of the element. Since the induced current is described by a Fourier series in the \( x \)-direction, the only component that can give a non-zero result after integration over \( x \) is the dc component (the coefficient of the 0th spatial harmonic). Therefore, the expression of the 0th coefficient of the induced current of a conducting element is integrated over \( y \) and forced to zero as presented in (23). In (23), \( \bar{\Psi}_i \{0\} \) is the center row of matrix \( \bar{\Psi}_i \). After substituting the obtained expressions for \( \bar{B} \) and \( \bar{H} \) into the boundary conditions and adding the conditions described in this section, a system of equations can be constructed. By solving this system of equations all unknown coefficients can be obtained.

\[ \text{E. Periodical time dependent behavior} \]

To model movement of one of the motor parts (e.g. the coil array) and current excitation profiles the model is extended using a time harmonic Fourier series. This means that any quantity dependent on \( x \) and time is now written as

\[
g(x, t) = \sum_{m=-M/2}^{M/2} \sum_{n=-N/2}^{N/2} g \{n, m\} e^{i(k_n x + \omega_m t)},
\]

where

\[
\omega_m = \frac{m \pi}{T},
\]

where \( T \) is the time period and \( M \) is the number of time harmonics considered. The time derivative changes now from \( j\omega \) to \( j\omega_m \) per time harmonic \( m \). The source currents are calculated per time harmonic as presented in [6]. In the source
\[-j\omega\sigma_i\bar{\Psi}_i\{0\}\left(Q\Lambda^{-1}\left(\left(E^+(\lambda, yT) - E^+(\lambda, yB)\right)a - \left(E^-\left(\lambda, yT\right) - E^-\left(\lambda, yB\right)\right)b\right) - \int (yT - yB)\left(K_n^2 + j\omega\mu\sum_{i=1}^{l}\sigma_i\bar{\Psi}_i\right)s^{-1}\mu\sum_{i=1}^{l}\Psi_i c_i\right) - \left((yT - yB)\Psi_i\{0\} c_i = 0\right)\]

IV. FORCE AND POWER CALCULATION

The forces are calculated using the Maxwell Stress tensor for complex fields which are, in the 2D problem, defined by

\[F_x = \frac{L}{\mu_0 \int C B_x B_y dx dy}, \quad (26)\]

\[F_y = \frac{L}{2\mu_0 \int C B_x B_y^* - B_y B_x^* dx dy}, \quad (27)\]

where \(L\) is the depth of the domain and * denotes the complex conjugate. For the 2D problem under consideration, the contour integral simplifies to an integral over a line in the \(x\)-direction in the airgap.

The power losses due to eddy currents is calculated by

\[P = \frac{L}{\sigma} \int S J_z^{ind}(J_z^{ind})^* dS. \quad (28)\]

The surface integration is performed on the surfaces of the conducting elements where the eddy currents are flowing.

V. MODEL VERIFICATION

In this section the results of the developed method are compared to results obtained with finite element (FE) analysis. The developed method is applied to the geometric model, depicted in Fig. 1. The surface below the coils is assumed to be infinitely permeable and non-conductive. The dimensions of the model are given in Table I, and the conductivity of the two conducting elements is equal to \(20 \text{ A/mm}^2\) and the frequency is varied.

To verify the results obtained with the developed semi-analytical (ANA) model, the geometric model of Fig. 1 is implemented in the finite element method (FEM) software package Flux 2D [10]. In total 47408 second-order surface mesh elements are created and the mesh size inside the conducting elements is equal to 0.5 mm.

In Fig. 2 the magnetic flux density in the center of the airgap calculated by the two different methods is shown. The frequency of the currents and magnetic fields for this result is equal to 100 Hz. The total number of harmonics used for this calculation is 81 \((N = 80)\), and it has been verified that the solution is converged with this number of harmonics. The semi-analytical solution is obtained in around 0.35 s. The results are in good agreement, with a rms error less than 1%. The top figure of Fig. 3 shows the current density in the two conducting elements of the model. In the bottom figure, the absolute error between the semi-analytical model and the FEM is shown. Also these results are in good agreement and also here the rms error is less than 1%. The calculated power loss due to the eddy currents in the two elements is 92 W, with an error compared to FEM of 1.2%.

To verify the developed method over multiple frequencies, the force between the conducting elements and the coils, due to the induced eddy currents, is calculated for a range of frequencies. The frequency is varied from 10 Hz to 10 kHz, and the result is depicted in Fig. 4. It can been seen that the force calculation is accurate over the whole range of frequencies with a rms error less than 2%.

VI. TRANSIENT CORELESS MOTOR SIMULATION

To simulate the forces created by eddy currents induced in cooling plates of a coreless linear motor, the geometric model shown in Fig. 5 has been implemented. Only the top-half of the problem is shown, because the problem is symmetric around the \(x\)-axis. The coils of the linear motor are cooled by five cooling plates (numbered from left to right), separated from each other by a 1 mm distance, made of titanium with a conductivity of 1.8 MS/m. Dimensions of the geometric model are presented in Table II. The magnets are assumed non-conducting in this simulation and are accelerated in the \(x\)-direction. On the top of the magnets, non-magnetic materials are assumed (modeled as an air region), and the weight of the moving part is equal to 2 kg. The currents are commutated based on the position of the moving magnet array in such a way that it is accelerated with 50 m/s² in the \(x\)-direction. Half of the motion profile is depicted in Fig. 6. For this simulation, the number of spatial harmonics is set to 101 \((N = 100)\)
Fig. 2. Magnetic flux density components calculated in the center of the airgap ($f=100$ Hz).

Fig. 3. Current density in both conducting elements calculated by the semi-analytical model and the absolute error compared to FEM ($f=100$ Hz).

Fig. 4. Force in two directions calculated for a range of frequencies by different methods.

Fig. 5. Geometric model of a half coreless motor. The model is symmetric around the $x$-axis.

As can be seen the results are, also for a simulation with multiple time harmonics in the semi-analytical model, in good agreement. The rms error compared with FEM is around 6.5%. This error is larger than the errors for the single frequency simulations of the previous section. This is due to the fact that the errors per time harmonic are now summed. The peak force due to eddy currents in the $x$-direction is around 1.2 N. With an acceleration of 50 m/s$^2$ and a mover of 2 kg, the motor produces 100 N main force. The parasitic force due to the eddy currents in the cooling plates is, hence, around 1% of the main force and could be impairing accuracy for nano-meter positioning applications.

<table>
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<th>Dimension</th>
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<td>mm</td>
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<td>Height of cooling plates</td>
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non-connected conducting elements. The spatial dependence of the conductivity is included in the magnetic field solution and an extra restriction on the induced current density ensures that the current flows in each element separately. Due to the inclusion of time harmonics it is possible to simulate periodical time dependent behavior of electromagnetic devices. With the developed method, parasitic eddy current forces are accurately modeled and power losses due to eddy currents can be calculated.

The developed method has been verified using FEM up to a frequency of 10 kHz. For the whole range of frequencies, the semi-analytical model is in good agreement with FEM with an error less than 2%. Secondly, the model has been applied to a coreless linear motor and compared to FEM. The transient results show an error around 6.5% compared to FEM. The computation time of the developed method is smaller than the computation time of FEM. For the periodical model of a coreless linear motor under consideration, in transient simulations, dependent on the number of spatial and transient harmonics, accurate results can be obtained in less than 60 s with the developed method.

VII. CONCLUSION

In this paper a 2D magnetic field modeling technique has been presented to calculate induced current distributions in

REFERENCES