Factors influencing students’ proficiency development in the fraction domain: the role of teacher cognitions and behaviour.

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Factors influencing students’ proficiency development in the fraction domain: the role of teacher cognitions and behaviour

Maaike Koopman, Marieke Thurlings and Perry den Brok

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ABSTRACT

In this study, we investigated which teacher characteristics influence student proficiency development regarding fractions in Grade 5 of Dutch primary education. At least three domains of research (i.e. perspectives) on effective teaching can be distinguished: studies focusing on teachers' background characteristics, on their knowledge and conceptions regarding the subject they are teaching and on the domain-specific and general pedagogical characteristics of their teaching. In this study, effects of the three perspectives on student fraction proficiency were examined simultaneously using multilevel analyses. Findings revealed that teachers' age and experience in the upper grades, their pedagogical content knowledge and the degree of student participation in their lessons had positive effects. Their subject matter knowledge, quality of their concept maps and the richness of the mathematics in their lessons had negative effects. Thus, effects were found pertaining to all three perspectives on teachers and their teaching we included in our study.

Introduction

In this study, we investigated which teacher factors influence student proficiency regarding fractions in Grade 5 of primary education. Fractions are amongst the most difficult topics that students encounter in primary mathematics education. This is caused, amongst others, by the fact that prior knowledge about natural numbers has to be expanded and reconciled with knowledge about rational numbers (Stafylidou and Vosniadou 2004). Student proficiency regarding fractions is often problematic. In Grade 5 in the Netherlands, for example, many students find it difficult to solve problems related to improper fractions and mixed numbers or addition and subtraction including fractions, even though instruction has been provided to them about these topics (Bruin-Muurling 2010).

As teachers are assumed to have substantial impact on student learning, in order to be able to improve students’ fraction proficiency it is necessary to examine the role of their teachers. There seems to be consensus that teachers make a difference regarding student mathematics learning, but it is less clear which aspects matter most (Palardy and Rumberger 2008). Effective
mathematics teaching ‘is likely to be a conglomerate of behaviours’ (Muijs and Reynolds 2000, 278; see e.g. Brophy 1986). At least three domains of research (i.e. perspectives) on effective teaching can be found: studies focusing on teachers’ background characteristics, on their knowledge and conceptions regarding the subject they are teaching and on the characteristics of their teaching (Ernest 1988; Palardy and Rumberger 2008; Boonen, Van Damme, and Onghena 2014). Many previous studies focus on effects of variables originating from only one or two of these domains. As all three of the perspectives have their merits, in this study, they are investigated in combination (cf. Palardy and Rumberger 2008; Boonen, Van Damme, and Onghena 2014).

From a policy viewpoint and regarding the first perspective, some studies have focused on the role of teacher background characteristics such as the amount of training teachers received, teacher certification, experience (Kyriakides, Christoforou, and Charalambous 2013; Boonen, Van Damme, and Onghena 2014). There is relatively little evidence about the actual effects of such variables (Palardy and Rumberger 2008), though some studies showed that experience does matter (Hanushek and Rivkin 2006). Regarding the second perspective, studies paid attention to the role of teacher knowledge and conceptions in relation to student achievement, such as teacher attitudes towards their ability to teach or beliefs about what mathematics teaching and learning should look like (Askew et al. 1997; Boonen, Van Damme, and Onghena 2014). Results of effectiveness studies including knowledge and conceptions are not univocal, but sometimes positive effects of teacher knowledge are found (see e.g. Ball, Hill, and Bass 2005).

Regarding the third perspective, research has frequently shown that a large proportion of variance in student achievement can be attributed to teaching behaviour or students’ perceptions thereof (see e.g. Bosker and Scheerens 1997; Kyriakides, Christoforou, and Charalambous 2013). Several studies have addressed the effectiveness of different aspects of mathematics teaching with respect to student achievement (see e.g. Reynolds and Muijs 2000; Hattie 2009; Kyriakides, Christoforou, and Charalambous 2013). Some of these characteristics may be considered more general pedagogical strategies suitable almost irrespective of the subject matter that is taught, such as optimising time on task and communicating high expectations (Reynolds and Muijs 2000). Other characteristics are more domain-specific and relate particularly to effective mathematics education, such as certain teaching strategies that are often used in mathematics lessons (for example, teaching multiple solution strategies; see e.g. Hill et al. 2012). To our knowledge, however, no studies have focused on the effectiveness of mathematics instruction dealing particularly with student fractions achievement. Moreover, in the effectiveness studies that dealt with mathematics instruction, usually general pedagogical and domain-specific aspects of teaching were not investigated simultaneously.

This study intends to investigate the effects of the three perspectives on teachers’ effects on student fraction proficiency simultaneously. The context of the study is primary education – Grade 5 (age 10–11) – in the Netherlands, where there is a need to improve student proficiency in mathematics (Royal Netherlands Academy of Arts and Sciences 2009). Fraction lessons, as a mathematical topic that students often perceive as difficult, are the focus of this study. The main research question was: which characteristics of Grade 5 primary school teachers and their teaching regarding fractions have an influence on their students’ proficiency development in the fraction domain? Multilevel analyses were conducted to answer this question. The results of this study should contribute to which elements of the three perspectives matter most for student achievement, particularly related to the topic of fractions, integrating both general pedagogical and domain-specific aspects of teaching.
The results of the study can be used to further improve education regarding fractions, but may also very well be relevant for mathematics education dealing with other topics. Also, the results may be used as input for teacher education.

**Teacher factors influencing student achievement in mathematics**

In the following sections the three perspectives that are studied are defined and described, that is: teacher background characteristics, teacher knowledge and conceptions and the instructional characteristics of their lessons. The perspectives and their hypothesised inter-relations are displayed in Figure 1. As can be derived from the model, we expect all perspectives to have a direct effect on students’ fraction proficiency. The model also shows that we assume there are inter-relations between the perspectives, that might result in interaction effects on students’ fraction proficiency. Regarding inter-relations between teacher background characteristics and their knowledge and beliefs for example, sometimes relations between teaching experience and their level of expert knowledge have been described (see e.g. Leinhardt and Smith 1985), but these relations were not always supported by empirical data. Assumptions about inter-relations between teachers’ knowledge and beliefs and their instructional behaviour have frequently been described (see e.g. Pajares 1992). However, large-scale studies on teacher effectiveness generally focus on teacher behaviour and mostly do not include possible effects of teacher knowledge and beliefs on their actual behaviour, though the importance is sometimes acknowledged (see e.g. Kyriakides, Creemers, and Antoniou 2009). In some small-scale studies on effects of domain-specific strategies for teaching mathematics on students’ mathematics achievement, correlations between teacher

![Figure 1. Conceptual model.](image-url)
knowledge and behaviour were found (see e.g. Hill, Kapitula, and Umland 2011). In the present research, we expect that teacher behaviour has the largest effect on students’ fraction proficiency, as the relation between teacher cognition and background characteristics and student performance is expected to run largely via teacher characteristics.

The perspectives that are part of this model were investigated in combination before within primary mathematics education (see Palardy and Rumberger 2008; Boonen, Van Damme, and Onghena 2014), but – to our knowledge – never specifically in the domain of fractions. Investigating the effects of the perspectives for fraction proficiency is particularly relevant as within the curriculum of primary mathematics education fractions are, both for teachers (regarding the knowledge they need to adequately teach fractions) and students, a complicated topic to teach and learn.

**Teacher background characteristics**

Some studies have shown effects of teacher background characteristics on student achievement. Background variables that have often been included in such studies are, for example, teacher gender, degrees and certification and years of experience in teaching in general or in a specific classroom (Wayne and Youngs 2003). The results of these studies are inconclusive: sometimes direct or indirect effects are indeed found, but often not.

Teacher gender, for example, is sometimes controlled for in effectiveness studies (see e.g. Ehrenberg, Goldhaber, and Brewer 1995). It is not exactly clear if teacher gender matters for student outcomes. Studies of Ehrenberg, Goldhaber, and Brewer (1995) and Lamb and Fullarton (2002), for example, show no effect of gender on student mathematics achievement.

Regarding teaching degrees, amount of coursework teachers did and teacher certification differential results are found, except for high-school mathematics education where students generally benefit from teachers’ own education in mathematics (Wayne and Youngs 2003). In primary mathematics education though, Boonen, Van Damme, and Onghena (2014), for example, reported a (small) negative effect of the amount of in-service training teachers had regarding their students’ learning. Conversely, Kyriakides et al. (2006) found a positive effect of teachers that graduated in education on primary student mathematics achievement. In a study of Hill, Rowan, and Ball (2005) effects of certification were also found, but these were either positive or negative depending on the grade level of students (positive effects for first grade and negative for third grade). Finally, Lamb and Fullarton (2002) and Palardy and Rumberger (2008) did investigate the role of credentials, educational qualifications and training but found no effects.

The literature suggests that the amount of teaching experience a teacher has, matters for student achievement (Darling-Hammond 2000). Inexperienced teachers typically seem to be less effective than senior teachers. Sometimes curvilinear relations are found, possibly caused by very experienced teachers growing tired of their jobs or stopping to learn (Darling-Hammond 2000). Regarding primary school teachers’ experience and student mathematics achievement, again, inconclusive results are reported. Boonen, Van Damme, and Onghena (2014) and Hill, Rowan, and Ball (2005) found positive effects of teaching experience on student achievement in primary education. In contrast, Kyriakides et al. (2006), Lamb and Fullarton (2002), and Palardy and Rumberger (2008) also investigated possible effects of years of experience in teaching, but found no effects.
Teacher knowledge and conceptions

The perspective of teacher knowledge and conceptions and their effects on student achievement has been investigated with different foci, ranging for example, from teachers' general efficacy beliefs about their teaching to the domain-specific knowledge they have (Boonen, Van Damme, and Onghena 2014). As our study focuses on primary mathematics education, we decided to focus on two features of teacher knowledge and conceptions, that is teachers' mathematical knowledge for teaching (MKT) and their conceptions about mathematics education.

Mathematical knowledge for teaching

Wayne and Youngs (2003) showed that teacher knowledge, as reflected in test scores, has a positive effect on student outcomes. Regarding knowledge that is relevant for mathematics education, often a distinction is made between subject matter knowledge (SMK) and pedagogical content knowledge (PCK; Hill, Ball, and Schilling 2008; Loewenberg Ball, Thames, and Phelps 2008). These concepts are combined into the MKT framework, doing justice to the specific knowledge that is needed for teaching mathematics. The SMK that is part of MKT consists, for example, of common content knowledge that everyone needs, for instance, for shopping and the more specific content knowledge that is necessary for teachers. The PCK pertains to knowledge about what students find difficult, knowledge about (dis)advantages of certain teaching methods and knowledge about how mathematical concepts in the curriculum relate (Hill, Ball, and Schilling 2008).

Teachers differ in the amount and structure of knowledge they possess. This may be related to the amount of course work related to mathematics they did (Hill 2007). Izsak et al. (2012) showed that, regarding primary education teachers' knowledge about fractions, there were large differences between teachers in the degree to which they were able to solve problems regarding division and multiplication of fractions (SMK). Additionally, there were even larger differences in the degree to which they were able to reason about fraction problems and explain these problems properly (PCK). Hill and Charalambous (2012a) showed that in more complex domains of primary education mathematics (such as ratios and fractions) it is all the more important that teachers have sufficient MKT, in order to for example, help the student build connections between learning content. Some studies showed that the amount of MKT a teacher possesses, has an influence on student achievement (Hill, Rowan, and Ball 2005; Baumert et al. 2009; Shechtman et al. 2010; Hill, Kapitula, and Umland 2011; Campbell et al. 2014).

Conceptions about mathematics education

Conceptions of teachers about mathematics and how it should be taught and learned are believed to have an influence on their teaching (Thompson 1984; Ernest 1988) and on student learning (Askew et al. 1997). Conceptions may be considered a general mental structure – conscious or subconscious – encompassing beliefs, meanings and concepts about mathematics and mathematics teaching and learning that teachers hold (cf. Pepin 1999). Teachers' conceptions of mathematics concern what mathematics encompasses. Teachers' conceptions about mathematics education concern how mathematics should be taught and how mathematics is learned (Thompson 1984; Andrews and Hatch 2000). Three orientations may be distinguished (Askew et al. 1997). First, a connectionist orientation encompassing
conceptions valuing both pupils’ methods and teaching strategies, with an emphasis on connections within learning content. This orientation may be related to viewing mathematics as a continually expanding field, in which patterns are distilled into knowledge (Ernest 1988; see also Lerman 1983). Second, a transmission orientation focusing primarily on teaching and a view of mathematics as a collection of unrelated, yet utilitarian routines and procedures. Third, a discovery orientation is distinguished focusing primarily on students’ (problem-driven) learning; in this view mathematics is being discovered by students. Askew et al. (1997) found that, compared to the other orientations, teachers holding connectionist conceptions were most effective regarding student achievement.

**Instructional characteristics**

As stated in the Introduction, regarding instructional characteristics in this study both general pedagogical aspects and domain-specific aspects of teaching are combined.

**General pedagogical aspects of teaching**

Much research has been conducted dealing with which aspects of instruction are effective regarding student achievement in primary mathematics education (see e.g. Reynolds and Muijs 2000; Kyriakides, Christoforou, and Charalambous 2013). Many of these studies focused on the effectiveness of a set of instructional characteristics, consisting of (a) time on task, (b) certain teaching strategies, (c) the use of the direct instruction model, (d) communicating high expectations and (e) the use of textbooks.

First, creating as much time for learning as possible seems to be one of the most influential factors when student learning is concerned. In many studies, this factor is operationalised as time on task or opportunity to learn (see e.g. de Jong, Westerhof, and Kruiter 2004; Reynolds and Muijs 2010; Kyriakides, Christoforou, and Charalambous 2013; Boonen, Van Damme, and Onghena 2014). Time on task is generally related to the amount of classroom work that is actually related to the subject being taught, and, as such, to classroom management (Reynolds and Muijs 2000). In case of mathematics education, this factor may be defined as the degree to which the content of a lesson is connected to mathematics (Learning Mathematics for Teaching Project 2010). Second, Sammons, Hillman, and Mortimore (1995) showed that specific teaching strategies such as orienting, guided instruction, checking for understanding, summarising were highly effective for learning (see also Ellis and Worthington 1994). Third and related to the phasing of a lesson, the direct instruction model is often found to be an effective teaching approach in primary mathematics education (see e.g. Houtveen, van de Grift, and Creemers 2004; Seidel and Shavelson 2007; Kyriakides, Christoforou, and Charalambous 2013). In this model, a lesson is structured around four phases: orienting and instruction about new learning content, guided practice, working individually or in small groups and discussion. Fourth, teachers that communicate high, yet realistic expectations to their students seem more effective (Sammons, Hillman, and Mortimore 1995; Reynolds and Muijs 2000; Hiebert and Grouws 2007). By challenging students, they were able to make much progress. Fifth, the extent to which teachers (literally) use lesson suggestions and learning content of the textbooks influences the effectiveness of instruction (Hill and Charalambous 2012b).
Mathematical quality of instruction

Mathematical quality of instruction (MQI) is a constellation of dimensions that describe ‘the rigour and richness of the mathematics of the lesson’ (Hill, Ball, and Schilling 2008, 431), and – as such – the quality of domain-specific aspects of instruction. As a whole, the MQI dimensions consider how teacher, students and the content of the lesson interact. MQI consists of four main dimensions. First, the richness of the mathematics in a lesson is important, which concerns how teachers present the content of the lesson to their students (see also Houtveen, van de Grift, and Creemers 2004). This is related, for example, to linking or connections between learning content and developing mathematical generalisations in instruction. Richness also relates to connections between visual representations and mathematical concepts. Within the domain of fractions this pertains, for example, to drawings of several pizza’s and the concept of unit or the use of Cuisenaire strips as an aid to compare fractions with different denominators. The second dimension pertains to working with students and mathematics, which concerns how teachers and students work together and interact about the content of lesson (see also Houtveen, van de Grift, and Creemers 2004). This includes thorough and conceptual remediation of student errors and if and how a teacher uses student input in instruction. It can be expected that remediation is particularly relevant when students experience difficulties in understanding complicated learning content, such as fractions. The third dimension pertains to errors and imprecision, which deals with errors made by teachers in for instance, language use and notation (see also Kyriakides, Campbell, and Gagatsis 2010). The fourth dimension pertains to student participation in meaning-making and reasoning, which deals with how students interact with the lesson content (see also Slavin and Lake 2008). This includes, for example, the degree to which students are active in explaining learning content and student mathematical reasoning. Hill and Charalambous (2012a) showed that when teaching fractions, teachers differed greatly in the way in which they succeeded in activating students.

Hill, Kapitula, and Umland (2011) showed that student outcomes in mathematics were related to MQI. Kersting et al. (2012) also demonstrated that mathematics instruction and student achievement were related. Siegler et al. (2010) reported effective teaching strategies for teaching fractions that align with the MQI dimensions, such explaining why procedures work and using representational tools such as number lines. Some studies reported positive relations between the amount of MKT of teachers and their MQI (Hill and Charalambous 2012a; Hill et al. 2012). Finally, high MQI is assumed to be closely related to connectionist conceptions about teaching and learning mathematics.

Method

Participants

Twenty-four Dutch teachers and 538 of their students participated in our study between September 2012 and June 2013. The teachers were from 23 schools. The schools responded to an e-mail about the research that was send to approximately 2000 schools for primary education and volunteered to participate. The schools ranged from more traditional schools in which textbooks are predominantly used to teach mathematics to schools in which teachers take more freedom to create their own curriculum materials, thus representing how mathematics is taught in the Netherlands. The schools were situated in urban and rural areas.
and had between 150 and 380 students, which is also representative for the Netherlands. Eight of the teachers were male; 16 female. Their mean age was 43 years (SD = 10.34 years) and most of them (40%) had been teaching for 8 to 15 years, 32% had been teaching for four to seven years, 20% more than 16 years; and 4% for zero to three years. Their classes had between 20 and 31 students, which is customary in the Netherlands (50% girls and 50% boys). Four hundred and seventy-five of these students were in Grade 5. In some classes, Grade 5 students were combined with Grade 4 students (n = 17 students) and Grade 6 students (n = 46 students). Grade 5 is the penultimate grade in Dutch primary schools. Grade 5 was chosen because fractions play a prominent role in its mathematics curriculum.

Measures

**Independent variable: student fraction proficiency**

Students’ proficiency development in the fraction domain was tested by administering the Development of Fraction Proficiency (DoFP; Bruin-Muurling 2010) test twice: once at the beginning of the school year and once at the end. This proficiency test is an open-ended, paper and pencil test. The test was constructed such that it encompassed different aspects of fractions learning relevant for Grade 5 students, such as addition and multiplication or reducing and complicating. The test contained items that were supposedly fairly easy (e.g. addition of proper fractions with common denominators, such as 2/7 + 4/7) to items that were supposedly difficult (e.g. subtraction or multiplication of unlike denominators, such as 4 1/4 – 3 5/6 or 2 1/3 x 1 2/5; see Figure 2 for sample items). The proficiency test consisted of 39 items and could be filled in throughout a regular lesson of about 45 min. The 1213 pre- and post-test tests were analysed by means of Rasch analyses (Bond and Fox 2001).

A scale was created arranging students on their ability and items on their difficulty. The fit of the model to our data were checked in four steps. First, item polarity was checked: no items with negative correlations of responses to the latent variable were found. Second, outfit and infit measures were checked: person infit t-value was 0.0 (SD = 1.1); person outfit t value was 0.1 (SD = 1.1); item infit t-value was −0.4 (SD = 4.2); item outfit t-value was −0.2

---

**Part-whole (easy item)**

Which part of the figure is coloured grey?

**Addition and subtraction (easy item)**

You ate 2/3 of a bag of candy. Which part is left?

**Division (difficult item)**

Two friends share ¾ pizza. Which part of the pizza does each person get?

**Application (difficult item)**

This necklace consists of 60 beads. You cannot see the entire necklace. How many white beads does the entire necklace have?

---

*Figure 2. Sample items of DoFP.*
(SD = 4.1); no patterns in the answers of students with high outfit scores were found and these students were kept in the analysis. Third, we checked the person reliability (indicating consistent behaviour of persons), which at 0.91 could be considered satisfactory. Fourth, we investigated the model reliability, which was 1.00. Based on the findings of these steps, we concluded that all measures indicated a good fit of the data to the Rasch model. In Figure 3, the Rasch scale with all items and students, ordered by subdomain is presented.

**Teacher background variables**

Information about teachers’ age and gender (male = 0; female = 1) was collected. Information about the teachers’ own schooling was gathered (both their secondary and post-secondary education). The number of years of experience they had was measured by means of a scale (0 = 0–3 years of experience; 1 = 4–7 years; 2 = 8–15 years; and 3 = more than 16 years of experience). The same scale was used to measure their specific experience in the upper classes of primary education.

**Teachers’ knowledge and conceptions**

**MKT test.** Teachers’ MKT regarding fractions was tested using a proficiency test that was built upon the MKT framework (Hill, Ball, and Schilling 2008; Loewenberg Ball, Thames, and Phelps 2008). The proficiency test held 36 items that concerned SMK and 38 items pertained to PCK (see Figure 4). The SMK items included fraction problems as present in the primary education curriculum in the Netherlands and were comparable to the student DoFP test, as well as more difficult items demanding algebraic skills as present in the curriculum of the first years of secondary education. The PCK items dealt with teaching fractions and related topics, such as ratio and strategies students use for multiplication or division. These items stemmed from the MKT database (see e.g. Loewenberg Ball, Thames, and Phelps 2008) or were adapted to the content of fractions teaching. The nature of these items could be related to the Grade 5 curriculum. The test could be filled out in about 90 min. The number of correct answers was counted. The correlation between the scales was 0.45 ($p = 0.02$), suggesting a limited degree of overlap between the scales.

![Figure 3. Rasch scale.](image-url)
The teachers made a concept map in which they displayed their knowledge about ‘learning fractions’. In contrast to the measurement of knowledge using traditional tests, concept mapping can be used to visualise the structure of knowledge in terms of both organisation of the individual’s conceptual knowledge and the elaborateness of this knowledge (cf. Koopman, Teune, and Beijaard 2011). The teachers were asked: (a) to note concepts they found important related to ‘learning fractions’, (b) to order these concepts on a blank sheet of paper in a manner in which they considered logical and (c) to link the concepts in the map and to describe these links.

To analyse these concept maps, a procedure was followed in which each concept map was compared to an expert’s concept map and to the other concept maps. In the first phase of the analysis, the characteristics of the concept maps were scored by hand by means of a coding scheme using as guidelines the indicators (see Table 1; Koopman, Teune, and Beijaard 2011). To judge the relevance of the concept maps, all concepts used by the teachers were

<table>
<thead>
<tr>
<th>SMK</th>
<th>PCK</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gerald has a piece of rope of 6 3/4 meters. How many pieces of 3/4 meters can he cut?</strong></td>
<td><strong>Nick, one of your students, comes to your desk with following. He typed 0.2 x 6 on this calculator and is surprised to find an answer smaller than 6. Next, he typed 6 : 0.2 and got an answer bigger than 6. He thinks the calculator is broken and asks for a new one.</strong></td>
</tr>
<tr>
<td><strong>2 1/3 x 1 2/5 = ...</strong></td>
<td><strong>a) What might be Nick’s misconception?</strong></td>
</tr>
<tr>
<td></td>
<td><strong>b) Which model can be used to explain 0.2 x 6 to Nick?</strong></td>
</tr>
<tr>
<td><strong>1/3 of the visitors to a zoo brought their own drinks. These are 90 persons. How many persons are visiting the zoo?</strong></td>
<td><strong>Mrs. Johnson thinks it is important to vary the whole when she teaches fractions. For example, she might use five dollars to be the whole, or 10 students, or a single rectangle. Today she uses as the whole a picture of two pizzas. What fraction of the two pizzas is she illustrating below?</strong></td>
</tr>
<tr>
<td></td>
<td><strong>a. 5/4</strong></td>
</tr>
<tr>
<td></td>
<td><strong>b. 5/3</strong></td>
</tr>
<tr>
<td></td>
<td><strong>c. 5/8</strong></td>
</tr>
<tr>
<td></td>
<td><strong>d. 1/4</strong></td>
</tr>
</tbody>
</table>

Figure 4. Sample items of the MKT test.

**Concept maps.** The teachers made a concept map in which they displayed their knowledge about ‘learning fractions’. In contrast to the measurement of knowledge using traditional tests, concept mapping can be used to visualise the structure of knowledge in terms of both organisation of the individual’s conceptual knowledge and the elaborateness of this knowledge (cf. Koopman, Teune, and Beijaard 2011). The teachers were asked: (a) to note concepts they found important related to ‘learning fractions’, (b) to order these concepts on a blank sheet of paper in a manner in which they considered logical and (c) to link the concepts in the map and to describe these links.

To analyse these concept maps, a procedure was followed in which each concept map was compared to an expert’s concept map and to the other concept maps. In the first phase of the analysis, the characteristics of the concept maps were scored by hand by means of a coding scheme using as guidelines the indicators (see Table 1; Koopman, Teune, and Beijaard 2011). To judge the relevance of the concept maps, all concepts used by the teachers were
listed and ordered, using the expert concept map as a frame of reference. In the second phase, the overall quality of the concept maps was determined by means of combining the findings for the separate indicators. An overall rating was assigned using a five-point Likert scale (1 = very poor quality of knowledge; 2 = poor quality; 3 = neutral; 4 = good quality; 5 = very good quality of knowledge). An audit procedure was followed to establish that the conclusions the coder had drawn were acceptable and justifiable (Akkerman et al. 2006). The correlations between the quality of the concept maps and teachers’ MKT were 0.21 ($p = 0.32$) for SMK and 0.52 ($p = 0.00$) for PCK. This was in line with our expectations as the concept maps were about 'learning fractions', which is more closely related to PCK than to SMK.

**Teacher conceptions questionnaire.** Teachers’ conceptions about mathematics, mathematics teaching and learning mathematics were investigated by means of a questionnaire that was inspired by the work of Askew et al. (1997), Raymond (1997), and Nisbet and Warren (2000). The questionnaire consisted of 56 items pertaining to preferences for either more social-constructivist, traditional, or application-oriented preferences (see Table 2). Teachers had to rank to which extent they agreed with the statements on a five-point Likert scale. Missing values were estimated using the SPSS missing value analysis command. To assess the construct validity of the questionnaire, a confirmatory factor analysis was conducted on the scales of the questionnaire to test its structure (with MPlus; Muthén and Muthén 1999). The model fit was improved by removing five items that did not load significantly. A reasonable model fit was found ($\chi^2 = 82.85$ with $df = 65$ ($p = 0.067$); CFI = 0.88; TLI = 0.86; RMSEA = 0.07). Factor loadings ranged between 0.19 and 0.81 and the factors explained a considerable amount of variance in items (between 0.04 and 0.67% with most items

### Table 1. Indicators for the quality of knowledge.

<table>
<thead>
<tr>
<th>Indicators</th>
<th>Visible in</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elaborateness</td>
<td>Number of concepts and links</td>
</tr>
<tr>
<td>Relevance</td>
<td>Ratio between relevant and irrelevant concepts</td>
</tr>
<tr>
<td>Relative importance</td>
<td>Position of concepts relative to the core concept (qualitative analysis using a scale: illogical – tolerably logical – logical arrangement)</td>
</tr>
<tr>
<td>Type of connections</td>
<td>Categorisation: unconnected, linear, one-centred, several-centred, network</td>
</tr>
<tr>
<td>Stratification</td>
<td>Number of layers</td>
</tr>
<tr>
<td>Clusters of concepts</td>
<td>Number of clusters with different topics distinguished in the concept map</td>
</tr>
</tbody>
</table>

### Table 2. Scales and sample items of the teacher conceptions questionnaire.

<table>
<thead>
<tr>
<th>Scale</th>
<th>n items (Cronbach's alpha)</th>
<th>Sample items</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social-constructivist</td>
<td>22 (0.85)</td>
<td>• Mathematics is mainly about seeing links and connections</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Good mathematics education is based on a dialogue between students and teachers, paying attention to each other’s strategies</td>
</tr>
<tr>
<td>Traditional</td>
<td>13 (0.80)</td>
<td>• Being proficient in mathematics has to do with solving mathematical problems using a standard procedure</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Becoming proficient in mathematics is an individual activity in which instructions have to be followed</td>
</tr>
<tr>
<td>Application-oriented</td>
<td>16 (0.79)</td>
<td>• Being proficient in mathematics has to do with applying and using mathematics in daily practice</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Good mathematics education is based on practical activities, so students can discover their own solution methods</td>
</tr>
</tbody>
</table>
having a percentage of explained variance from approximately 0.25%). Correlations between the scales were −0.10 (\( p = 0.47 \)) for social constructivist and traditional; 0.50 (\( p = 0.00 \)) for social-constructivist and application-oriented and −0.01 (\( p = 0.96 \)) for traditional and application-oriented.

**Instructional characteristics**

Three fraction lesson observations per teacher took place during one school year. The observed lessons were videotaped and analysed using the MQI observation scheme (Learning Mathematics for Teaching Project 2010). The scheme was developed to measure the four dimensions described earlier. Each dimension consists of several characteristics or codes that are scored on fragments of about seven minutes. Each code can be given a score of low (i.e. did not occur or was of poor quality), mid (i.e. did occur and was of reasonable quality) or high (i.e. did occur and was of good or exceptional quality). Because we wanted to get an overall perspective on the MQI per lesson, the scoring was aggregated from the fragment scores to the level of the four MQI dimensions per lesson. To do so, a five-point scale was developed (1 = well below mid, 2 = below mid, 3 = mid, 4 = above mid and 5 = well above mid).

In addition to the four dimensions, the MQI also encompasses some binary codes that align with general pedagogical strategies we described earlier. One of these focuses on whether the classroom work is connected to mathematics (rated on a five-point scale ranging from 1 = no connection to mathematics in entire lesson to 5 = connection to mathematics in entire lesson), thus representing time on task. Others focus on the teaching strategies being present or not (orienting, guided instruction, checking for understanding, and summarising; 0 = not present and 1 = present).

Three general pedagogical strategies were added to the original instrument. First, the phases of the direct instruction model were investigated. We checked if the phases (i.e. orienting, guided practice, working individually or in small groups, and discussion) occurred during the whole lesson. A three-point scale was then used to describe alignment in the lesson phasing with the direct instruction model (0 = (almost) no characteristics of direct instruction; 1 = some alignment with direct instruction; 2 = complete use of all phases). Second, we added the dimension ‘teachers communicate high expectations to their students’, which was scored on a five-point Likert scale (1 = completely invisible; 5 = clearly visible). Third, we scored the degree to which textbooks were used (0 = no use of textbook; 1 = textbook used in part of lesson; 2 = textbook used in entire lesson).

Regarding the reliability of the scores per fragment, the percentage of agreement between two coders was calculated. Two researchers independently scored 48 fragments of six lessons and reached an overall agreement of 79.6%. Correlations between the four MQI dimensions were statistically significant and ranged between 0.40 and 0.92. These correlations can be considered moderate to very strong, which may be expected as all dimensions are part of the MQI construct. Correlations between the general pedagogical strategies were moderate and ranged between −0.03 and 0.51 (see Appendix 1).

**Data analyses**

To analyse the data, first missing values were estimated and imputed using EM command in SPSS; missing values were random and no patterns in missing values were found (0.26%
of the data were missing). Descriptive statistics for all the measurements were calculated (see Table 3).

Because of the nested data structure, multilevel analysis were conducted in SPSS to estimate effects on student fraction proficiency. Two levels were distinguished: a student and a class/teacher level. The final model was build up in step-wise manner. As an overall approach for doing the multilevel analysis, we used a sequence of strategies that were consistent with the theoretical framework and purposefully ordered to address our research question. First, an empty model was made, which made it possible to distinguish the separate variance components at both levels of the model. Second, a prior achievement model was created in which students' fraction proficiency at pre-test was included as covariate, in order to control for prior knowledge. Third, we estimated the effect of teacher background variables, knowledge and conceptions, and instructional characteristics on student achievement, thus creating the explanatory model. We strived for a balance between parsimony and completeness (Palardy and Rumberger 2008). We started based on our theoretical model and added variables in theoretically cohesive and sequential sets: first we added all instructional characteristics because we expected these to have the largest effects, second we added all variables related to teacher knowledge and conceptions, and third the variables related to teacher background variables. This, however, did not result in a parsimonious picture in which all variables contributed significantly to the post-test score. We then systematically added covariates to the model based on

<table>
<thead>
<tr>
<th>Variable</th>
<th>Scale</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student proficiency development</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre-test</td>
<td>−6.02 to 5.92</td>
<td>−6.02</td>
<td>4.65</td>
<td>−0.41</td>
<td>1.34</td>
</tr>
<tr>
<td>Post-test</td>
<td>−3.96</td>
<td>5.92</td>
<td>0.56</td>
<td>1.40</td>
<td></td>
</tr>
<tr>
<td>Teacher background characteristics</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>27–61</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Secondary education</td>
<td>0–2</td>
<td>0</td>
<td>2</td>
<td>1.00</td>
<td>0.72</td>
</tr>
<tr>
<td>Post-secondary education</td>
<td>0–2</td>
<td>0</td>
<td>2</td>
<td>1.08</td>
<td>0.70</td>
</tr>
<tr>
<td>Gender</td>
<td>0–1</td>
<td>0</td>
<td>1</td>
<td>0.68</td>
<td>0.47</td>
</tr>
<tr>
<td>General years of experience</td>
<td>0–3</td>
<td>0</td>
<td>3</td>
<td>1.76</td>
<td>0.81</td>
</tr>
<tr>
<td>Experience in upper grades</td>
<td>0–3</td>
<td>0</td>
<td>3</td>
<td>1.52</td>
<td>0.94</td>
</tr>
<tr>
<td>Knowledge</td>
<td>0–36</td>
<td>23</td>
<td>36</td>
<td>30.68</td>
<td>3.02</td>
</tr>
<tr>
<td>MKT subject matter knowledge</td>
<td>0–38</td>
<td>15</td>
<td>27</td>
<td>22.07</td>
<td>3.34</td>
</tr>
<tr>
<td>Quality of concept map</td>
<td>1–5</td>
<td>1</td>
<td>5</td>
<td>2.88</td>
<td>1.26</td>
</tr>
<tr>
<td>Social-constructivist conception</td>
<td>1–5</td>
<td>1.86</td>
<td>4.59</td>
<td>3.90</td>
<td>0.59</td>
</tr>
<tr>
<td>Traditional conception</td>
<td>1–5</td>
<td>1.77</td>
<td>3.77</td>
<td>2.38</td>
<td>0.55</td>
</tr>
<tr>
<td>Application-oriented conception</td>
<td>1–5</td>
<td>2.13</td>
<td>4.38</td>
<td>3.52</td>
<td>0.69</td>
</tr>
<tr>
<td>Conceptions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Richness of the mathematics</td>
<td>1–5</td>
<td>1</td>
<td>4.33</td>
<td>2.78</td>
<td>0.95</td>
</tr>
<tr>
<td>Working with students and maths</td>
<td>1–5</td>
<td>1</td>
<td>4.67</td>
<td>3.13</td>
<td>0.96</td>
</tr>
<tr>
<td>Errors and imprecision</td>
<td>1–5</td>
<td>2.67</td>
<td>5</td>
<td>4.27</td>
<td>0.77</td>
</tr>
<tr>
<td>Student participation</td>
<td>1–5</td>
<td>1</td>
<td>5</td>
<td>2.85</td>
<td>1.08</td>
</tr>
<tr>
<td>Time on task</td>
<td>1–5</td>
<td>2.67</td>
<td>5</td>
<td>4.41</td>
<td>0.63</td>
</tr>
<tr>
<td>Strategy orienting</td>
<td>0–1</td>
<td>0.67</td>
<td>1</td>
<td>0.93</td>
<td>0.14</td>
</tr>
<tr>
<td>Strategy guided instruction</td>
<td>0–1</td>
<td>0</td>
<td>1</td>
<td>0.81</td>
<td>0.29</td>
</tr>
<tr>
<td>Strategy checking for understanding</td>
<td>0–1</td>
<td>0</td>
<td>1</td>
<td>0.79</td>
<td>0.32</td>
</tr>
<tr>
<td>MQI</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Richness of the mathematics</td>
<td>1–5</td>
<td>1</td>
<td>4.33</td>
<td>2.78</td>
<td>0.95</td>
</tr>
<tr>
<td>Working with students and maths</td>
<td>1–5</td>
<td>1</td>
<td>4.67</td>
<td>3.13</td>
<td>0.96</td>
</tr>
<tr>
<td>High expectations</td>
<td>1–5</td>
<td>1.33</td>
<td>5</td>
<td>3.89</td>
<td>1.10</td>
</tr>
<tr>
<td>Use of textbooks</td>
<td>0–2</td>
<td>0</td>
<td>2</td>
<td>1.34</td>
<td>0.64</td>
</tr>
</tbody>
</table>
the magnitude of their correlations with the post-test scores. Thus, first, covariates with a strong correlation were added, followed by covariates with weaker correlation. Several models were tested accordingly. Those variables with a nonsignificant coefficient were continually omitted. This resulted in our final model, which was a model that (a) was a significant improvement of the prior achievement model, (b) consisted only of statistically significant variables and (c) explained as much variance as possible. Fourth, interactions between the different teaching variables, both within and across the three perspectives, were tested. None of these interactions were found to be statistically significant. Fifth and finally, a random slope (pre-test) was added to assess if teacher characteristics had different effects per student given their pre-test score. Per covariate, effect sizes were computed (Snijders and Bosker 1999), as well as the amount of variance explained at the two levels of the model. Also, it was checked if the fit of the models we created could be considered a significant improvement compared to the empty model (using the $-2 \times \text{log-likelihood}$ ratio). Finally, in order to gain more insight into the nature of the effects, we examined patterns of scores on the final model’s variables of the top three and bottom three teachers based on their average class post-test scores. The results of these analyses are used to illustrate the effects found.

**Results**

In the following sections, we will respectively report on the results found for (a) the empty model, (b) the prior achievement model and (c) the explanatory model.

**Empty model**

We created an empty model, including students’ post-test proficiency scores only, in order to calculate the intraclass correlation coefficient (Snijders and Bosker 1999) and to see how much variance could be found on the student level and on the class/teacher level. The amount of variance at the class/teacher level was 22.5% and thus, 77.5% of the variance was at the student level (see Table 4), showing that there were considerable differences between classes/teachers in performance.

**Effects of prior achievement**

In the next model, students’ pre-test proficiency scores were incorporated, in order to control for prior knowledge (see Table 4). This model was a significant improvement compared to the empty model (difference in $-2 \times \text{log-like}$ = 494.4; df = 1). Prior achievement turned out to explain a large amount of variance that was distinguished in the empty model at both levels: 69.6% at the class/teacher level and 59.9% at the student level. In total, 62.1% of the variance of the empty model was explained.

**Explanatory model: effects of teacher characteristics**

Finally, we investigated the effects of the three perspectives, that is teacher background characteristics, teacher knowledge and conceptions and the instructional characteristics of their lessons. A model was established, in which teachers’ age and experience in the upper
grades, their SMK, PCK, concept maps, the richness of the mathematics in their lessons and the degree of student participation in their lessons played a role (see Table 4). This model was significant improvement compared to the prior achievement model (difference in $-2 \log$-likelihood $= 77.72$; df $= 11$). Teacher covariates explained variance at both levels: of the variance of found in the empty model 98.3% was explained on the class/teacher level and 59.9% on the student level. Thus, almost all variance on the class/teacher level was eventually explained.

The effects we found were statistically significant and ranged from large (DoFP pre-test) to small (other variables). Most effects were positive: students’ pre-test score, teachers’ age and experience in the upper grades, teachers’ PCK and student participation in reasoning and meaning-making contributed in a positive manner to students’ post-test fraction proficiency. Some of the effects found were negative: teachers’ SMK, the quality of their concept maps and the richness of their lessons did not contribute positively to students’ post-test scores. Also, some of the variables that were investigated in this study did not have an influence on students’ post-test scores. Regarding teacher background variables, their gender, schooling and total experience as a teacher (combination of upper- and lower grade experience) did not have an effect on student fraction achievement. Regarding teacher knowledge and conceptions, the three types of conceptions about mathematics and mathematics teaching and learning teachers might prefer did not have an influence on students’

### Table 4. Student fraction proficiency development: regression coefficients (significant at 0.05 or 0.10 level; standard errors in parentheses), variance components and model summary.

<table>
<thead>
<tr>
<th>Perspective</th>
<th>Empty model</th>
<th>Prior achievement model</th>
<th>Explanatory model with random slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.63** (0.14)</td>
<td>0.90** (0.08)</td>
<td>3.20** (0.61)</td>
</tr>
<tr>
<td>DoFP pre-test</td>
<td>0.79** (0.03)</td>
<td>0.76</td>
<td>0.80** (0.04)</td>
</tr>
<tr>
<td>Teacher background characteristics</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>0.01** (0.01)</td>
<td>0.09</td>
<td>0.12* (0.06)</td>
</tr>
<tr>
<td>Experience in upper grades</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Teacher knowledge and conceptions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SMK</td>
<td>−0.12** (0.02)</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>PCK</td>
<td>0.04* (0.02)</td>
<td>0.09</td>
<td>0.16</td>
</tr>
<tr>
<td>Concept map</td>
<td>−0.18** (0.04)</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>Instructional characteristics</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Richness of mathematics</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student participation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variance Class/teacher</td>
<td>22.5%</td>
<td>6.81%</td>
<td>0.39%</td>
</tr>
<tr>
<td>Var Student Random part</td>
<td>77.5%</td>
<td>31.08%</td>
<td>33.11%</td>
</tr>
<tr>
<td>Explained</td>
<td>–</td>
<td>–</td>
<td>0.76%</td>
</tr>
<tr>
<td>$-2 \times \log$-likelihood (df)</td>
<td>1819.21 (3)</td>
<td>1324.81 (4)</td>
<td>1247.09 (12)</td>
</tr>
<tr>
<td>Difference log (df)</td>
<td>494.4 (1)</td>
<td>77.72 (11)</td>
<td></td>
</tr>
</tbody>
</table>
fraction proficiency. Regarding teachers’ instructional quality, all general pedagogical characteristics we investigated and the MQI dimensions working with students and mathematics and errors and imprecision did not affect students’ fraction proficiency.

In the following, we discuss the size and nature of the effects. It is important to realise that all effects reported here exist only in combination with the other effects in the model. Students’ pre-test scores on the fraction proficiency test had the largest effect on their post-test scores, compared to the other variables we investigated (effect size = 0.77; which is approximately three times as large as the second largest effect). As stated earlier, the effect of the pre-test was positive, and it had a random component. Analyses of the top and bottom three teachers provided some more insights. More precisely and in relation with all other covariates, in classes in which students generally had good pre-test scores, teacher characteristics appeared to matter less than in classes in which students generally had less sufficient pre-test scores. In other words, in the classes with good pre-test scores, students’ proficiency seemed to grow no matter what teachers knew, thought, or did. Conversely, in classes with lower pre-test scores, students’ proficiency seems to grow less, even if their teachers had, for example, relatively much MKT.

Additionally, the teacher background characteristics age and years of experience in teaching in the upper grades of primary education both had a small but positive effect on students’ post-test proficiency scores. Their respective effect sizes were 0.09 for age and 0.08 for experience in the upper grades. A certain amount of experience in the grades in which fractions are taught, which is generally correlated with a teacher’s age ($r = 0.30; p = 0.00$), seems important to improve students’ fraction proficiency in Grade 5.

Furthermore, the relation of different elements of teacher knowledge and student fraction proficiency seems less straightforward. While the amount of PCK a teacher had positively affected students’ post-test fraction proficiency (effect size = 0.09), the amount of SMK they had and the quality of the concept maps they made were negatively related to students’ post-test scores (effect sizes were 0.25 and 0.16, respectively). Analyses of the top and bottom three teachers showed that the positive role of PCK, especially played a role in classes with students generally having good pre-test proficiency: when their teacher also displayed much PCK, students’ proficiency appeared to grow strongly. Regarding SMK, the effects found were related to a curvilinear relation between students’ post-test scores and teachers’ SMK: a certain basic level of SMK seemed important, but a very large amount of SMK might be an obstruction for reaching high-post-test scores ($r = -0.19; p = 0.00$). Analyses of the top and bottom three teachers revealed that this is particularly visible in classes with students having generally low pre-test scores. Regarding the effect of the quality of teachers’ concept maps on students’ post-test fraction proficiency, also a curvilinear relation was found ($r = -0.12; p = 0.01$). Here, an average score representing a threshold level of conceptual knowledge seemed to be most effective. Moreover, in a class in which students averagely had good pre-test scores, teachers appeared to get away with less conceptual knowledge, while in a class with less sufficient pre-test scores students did not seem to benefit from greater conceptual knowledge of their teachers.

Finally, concerning instructional quality, while student participation in reasoning and meaning-making had a positive effect on students’ post-test proficiency (effect size = 0.27; the second largest effect), the richness of the mathematics in the lessons had a negative effect (effect size = 0.20). Analyses of the top and bottom three teachers showed that especially in classes where many students had poorer pre-test scores, students’ post-tests did not seem
to benefit from much richness, in terms of for example, explanations, multiple solution methods or links and connections. Perhaps, the links between visual representations and concepts or the explanations present in the lessons of these teachers were too complicated given the level of understanding of these students. Moreover, the bottom three teachers did use links between visual representations and fraction problems, but they used a narrow repertoire of representations (only pizza’s, for example) which might not have been helpful for all students. Getting students to think actively and deeply about the fraction learning content and having them explain about fractions, as expressed in the student participation score, seemed to be beneficial for students’ post-test scores in general, and for students in classes with poorer pre-test scores in particular.

**Conclusions and discussion**

This study focused on which characteristics of Grade 5 primary school teachers and their teaching regarding fractions influenced their students’ proficiency development in the fraction domain. Three perspectives on teacher characteristics were taken into account, that is teacher background characteristics, teacher knowledge and conceptions and teachers’ instructional quality. Multilevel analyses showed that, concerning teacher background characteristics, their age and the number of years of experience they had in the upper grades of primary education had an influence on students’ fraction proficiency development. Though other studies have shown mixed results, our findings are in line with studies of Boonen, Van Damme, and Onghena (2014) and Hill, Rowan, and Ball (2005). In contrast, the type and level of schooling the participating teachers had and their gender did not affect their students’ fraction achievement, which is in line with results of several other effectiveness studies within the domain of mathematics education (see e.g. Ehrenberg, Goldhaber, and Brewer 1995; Lamb and Fullarton 2002; Palardy and Rumberger 2008).

The analyses also revealed that the characteristics of teacher knowledge we investigated all had an influence on students’ fraction proficiency development. More precisely, teachers’ SMK and the quality of the concept maps they made – representing their conceptual knowledge – had a negative influence and the amount of PCK had a positive influence on students’ fraction proficiency. The relation we found with PCK is in line with what other studies found to be effective (see e.g. Hill, Rowan, and Ball 2005; Baumert et al. 2009; Shechtman et al. 2010; Hill, Kapitula, and Umland 2011; Campbell et al. 2014). The negative effect of the other two aspects of teacher knowledge is a result that is less usually found (see e.g. Hill et al. 2012). Analyses of the nature of the relation between SMK or concept map and post-test scores revealed curvilinear relations, suggesting that a certain threshold level of knowledge is necessary. In contrast with for example Askew et al. (1997), we found that teachers’ conceptions about mathematics and mathematics teaching and learning did not affect student outcomes regarding fractions.

Finally, the multilevel analyses showed that some of the MQI characteristics, that is richness and student participation, had an effect on student fraction proficiency development and that these effects were relatively large, which was in line with our expectations. Interestingly, none of the general pedagogical strategies had an effect.

Comparable to what is generally found in school effectiveness studies, we found that roughly about 20% of the variance could be explained by factors related to the level of teachers (see e.g. Kyriakides, Christoforou, and Charalambous 2013). By means of the
variables in the model, moreover, we were able to explain almost all variance at the level of the teachers (98.3%), whereas 59.9% of the variance at the student level was explained. This remaining amount of variance at the student level was not very surprising as we did not include any student variables such as their gender, socio-economic background, intelligence or motivation for mathematics. Strikingly, effects were found pertaining to all three perspectives on teachers and their teaching we included in our study. The results showed a complex picture in which some characteristics have a positive effect and other a negative effect on student fraction proficiency. It appeared that teachers need to have balanced knowledge for effective fraction teaching, for example, possessing a threshold (yet not too high) level of subject matter and conceptual knowledge about fractions and as much PCK as possible regarding fractions. It also seemed that teachers have to balance between providing rich lessons, which in our study was found to be less effective, and activating students regarding reasoning and meaning-making (see e.g. Muijs and Reynolds 2000; Houtveen, van de Grift, and Creemers 2004; Slavin and Lake 2008; Ing et al. 2015). Perhaps richness has a downside: putting all elements of the richness dimension – which obviously all have their merits – time and again in each lesson might be too complicated for some students. The complexity of the relations found was particularly important for classes in which the students on average had low pre-test scores regarding fraction proficiency. In these classes it seemed as if much SMK about fractions, conceptual fraction knowledge and richness of instruction complicate student learning, withholding them to perform well on the fraction post-test, while these classes particularly seemed to benefit from much student participation. In classes in which the students generally scored well on the pre-test, it appeared to matter less how much knowledge their teacher had and what their instruction looked like, although these classes did seem to benefit strongly from their teachers having much PCK.

As can be concluded from the above, some of the characteristics that were assumed to have an influence on students’ achievement beforehand did not have the expected effect. This was particularly striking for the general pedagogical strategies that are often argued to be effective in other studies in primary mathematics education (see e.g. Reynolds and Muijs 2000; Muijs and Reynolds 2000; Seidel and Shavelson 2007; Palardy and Rumberger 2008; Kyriakides, Christoforou, and Charalambous 2013; Boonen, Van Damme, and Onghena 2014). Maybe this can be explained by the particular focus of our study on the topic of fractions and our decision to collect data in fraction lessons and on students’ fraction proficiency, resulting in mainly mathematics specific characteristics having an effect on student outcomes. The results found in our study plead for an approach in effectiveness studies in which domain-specific and more general pedagogical aspects of teaching are both incorporated.

While all three perspectives seemed important, compared to the other two perspectives the largest effect sizes were found for teacher behaviour. This is in line with assumptions of for example Brophy (1986) and Kyriakides, Christoforou, and Charalambous (2013), who suggested that classroom level variance in students’ mathematics achievement can mainly be explained by conglomerates of teaching behaviours, rather than their beliefs or personal characteristics. In the current study, variables from the three perspective were investigated in combination and the reported effects exist only in combination with other effects. Some analyses of the data creating multilevel models per perspective showed that some of the effects reverse (e.g. become negative instead of positive) when, for example, only teacher behaviour is taken into account. That is, in a multilevel model in which only
the effects of richness of the mathematics and student participation were included (plus a correction for students’ pre-test scores), richness had a positive effect and student participation has a negative effect on students’ post-test scores. Only in combination with relevant variables from the other perspectives these effects reversed, suggesting that the interplay between teacher knowledge and behaviour diminishes the positive effect of richness per se. Therefore besides multilevel analysis, the complex picture we found demands different statistical techniques to better grasp possible patterns in teacher knowledge and behaviour or causality of possible indirect relations and/or interactions between variables. We suspect, for example, that a teacher’s experience in the upper grades might have had both a direct and an indirect effect on student proficiency as it may overlap with their knowledge and behaviour. Also, PCK might serve as a precondition for effective fraction teaching. A larger sample size would be necessary to investigate latent profiles or create structural models in which such relations and interactions could be included.

Some other limitations to this study need to be addressed here. The sampling of this study, for example, did not allow us to include a separate school level, as in almost all cases only one teacher per school participated. As a result, it was not possible to investigate possible effects of specialised programmes, such as Montessori or Steiner school systems and the particular manners of teaching fractions in these programmes. The unexpected negative effects of richness are also worth further investigations. Also, and as stated earlier, we did not include variables at the student level, except from their fraction proficiency. This was the result of the focus on the role of teachers that was chosen for this study, but including more student variables, such as their gender or the sizes of the classes they were in, might contribute to explaining more variance at the student level. Finally, only fraction proficiency was included as an outcome measure in this study. We did not investigate which teacher background characteristics, knowledge and conceptions and behaviours contribute to other student outcome measures, such as motivation for mathematics or (reduction of) mathematics anxiety. Effective teaching may be related to more factors than striving for cognitive achievement only. In future research, it might be interesting to investigate teacher factors that contribute to more affective outcomes as well.

The results of this study have some implications. The study emphasises the importance of PCK for fraction teaching. Combined with the effects of teacher age and experience, our results suggest that it is important to develop PCK early on in teachers’ careers. Therefore, in teacher training attention should be paid to developing this particular type of knowledge. Of course, teachers also need to have a certain degree of SMK about fractions, but we suspect that having much SMK might have an influence on the amount of instruction a teacher provides and, as a result, on the richness of the mathematics that is visible in a lesson. That is, teachers having much SMK might incorporate more of their knowledge in their instruction – for example, in terms of linking concepts or explaining multiple strategies – as such providing rich instruction. This type of rich instruction might make lessons too teacher-centred and complicated for some of the students, particularly in weaker classes. More research is needed to provide evidence for this claim, but our results do suggest that teachers need to find a balance in using the mathematical quality for teaching dimensions in their instruction. Therefore, in teacher training attention should be paid to how student teachers can enrich their lessons in a comprehensible manner, for example, using models to explain fractions to their students. Also, student teachers need to learn about how to deeply activate their students in mathematics classes.
Notes

1. Secondary education: 0 = pre-vocational secondary education; 1 = higher general secondary education; 2 = pre-university education. Post-secondary education: 0 = senior secondary vocational education; 1 = university of applied sciences, bachelor level; 3 = university of applied sciences, professional master; 4 = university, master level.

2. The researchers followed an online MQI training for accurate scoring (http://isites.harvard.edu/icb/icb.do?keyword=mqi_training). The percentages of agreement per MQI dimension were: Richness of mathematics: 78.33%, Working with students and mathematics: 85.42%, Errors and imprecision: 94.44%, and Student-participating in meaning making and reasoning: 88.19%.

Disclosure statement

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References


Appendix 1. Correlations between instructional characteristics

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*p < 0.05; **p < 0.01.