Physical parameter estimation of an unbalanced disc system

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P.J.W. Koelewijn and R. Tóth

I. Introduction

For the development of high performing controllers, an accurate model of the to-be-controlled system is required. In this brief technical report, the estimation of the physical parameters of the unbalanced disc system is described in order to obtain an accurate model of system. The unbalanced disc system is a DC motor connected to a disc with an added (unbalanced) weight, see Figure 1, which functions as a rotational pendulum. The dynamics of this setup can be modeled using the following nonlinear (NL) state-space equation (neglecting the fast electronic subsystem) [1]

$$\begin{aligned}\dot{\theta}(t) &= \omega(t),\\ \dot{\omega}(t) &= \frac{Mgl}{J}\sin(\theta(t)) - \frac{1}{\tau}\omega(t) + \frac{K_\mathrm{m}}{\tau}u(t),\end{aligned} \tag{1}$$

where θ is the angle of the disk in rad (where θ is considered zero when the mass is at the top), ω the angular velocity of the disk in rad/s and u the input to the system in V. The physical parameters of the system are given in [1], but based on experimental data were found incompatible with the setup available to us. Therefore for the accurate modeling of the system, the physical parameters need to be estimated. In the subsequent section the estimation of the physical parameters, i.e. M, g, l, J, K_m and τ, based on measurement data is explained.

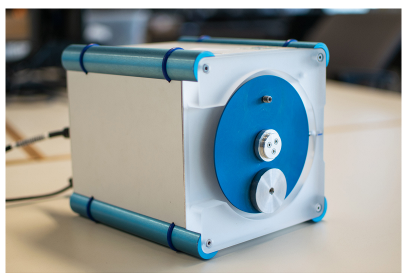

Fig. 1: Unbalanced Disc Setup.

II. Main Results

In order to develop high performing controllers for the unbalanced disc setup, an accurate model is required. It was found that the physical parameters of the system given in [1], did not correspond with the setup used available to us. Due to this setup being used for a variety of controller synthesis techniques, an accurate nonlinear model of the system is required. Therefore, instead of using system identification techniques from which the physical parameters could be inferred, it is opted to directly estimate the physical parameters of the NL model (1). To estimate the physical parameters of the system, the Parameter Estimation Tool from Simulink™ is used [2]. The Parameter Estimation tool formulates the problem as an optimization problem, where the cost, in this case, is given by the sum squared of the difference between the output of the experimental and the simulation data, i.e.

$$C(p) = \sum_{t=0}^{T} \left(y_\mathrm{exp}(t) - y_\mathrm{sim}(t)\right)^2, \tag{2}$$

where y_exp is the experimental output data, y_sim the simulation output data and p denotes the vector of physical parameters. The cost is then optimized using a Nonlinear Least Squares Method, i.e.

$$\begin{aligned}&\min_{p} \|C(p)\|_2^2,\\ &\text{s.t.} \quad \underline{p} \le p \le \bar{p},\end{aligned} \tag{3}$$

where $\underline{p}$ and $\bar{p}$ are the lower and upper-bounds of the parameter vector respectively, see also [2] and the links therein for more details. For the estimation of the parameters of the unbalanced disc, the angle was chosen as output of the system. Various input signals were chosen in order to excite the experimental setup and the corresponding outputs are used for estimation and validation. The control input signals and the corresponding outputs of the simulations and experimental setup can be found in

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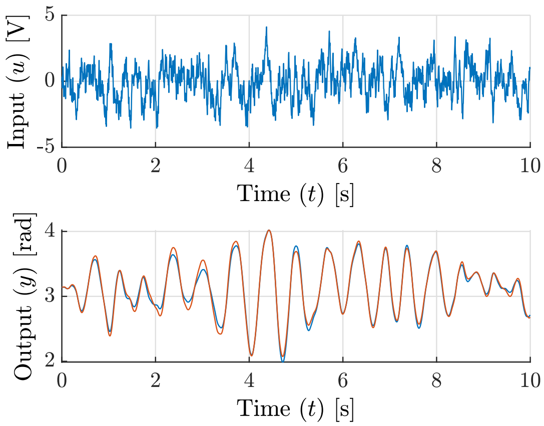
TABLE I: Estimated physical parameters unbalanced disc.

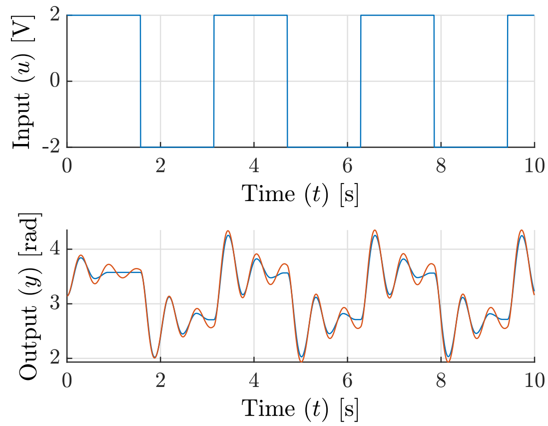
g [m/s^2]	J [kg $\cdot$ m^2]	K_m [rad/Vs^2]	l [m]	M [kg]	τ [1/s]
9.8	$2.4 \cdot 10^{-4}$	11	0.041	0.076	0.40

TABLE II: BFR corresponding to the data in Figure 2.

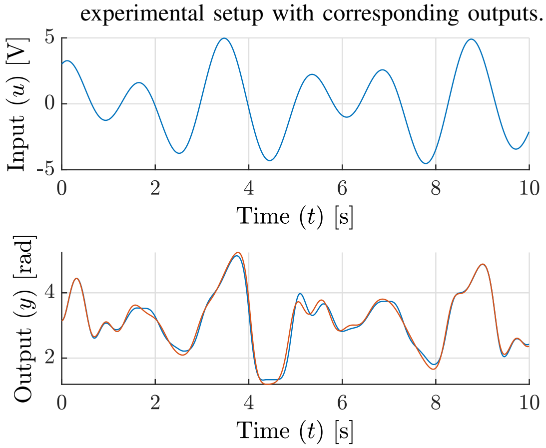
(a)	(b)	(c)	(d)
98.4%	98.0%	98.4%	98.5%



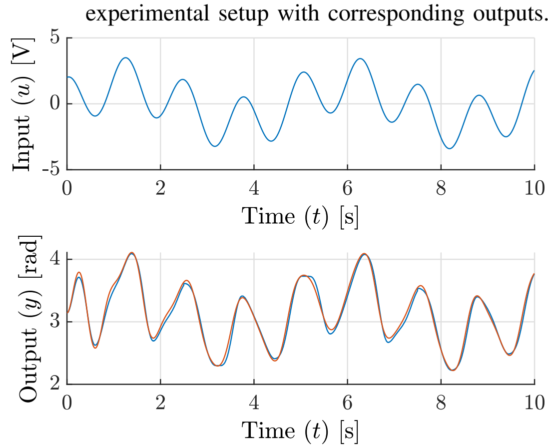
(a) Filtered noise control input given to simulation and experimental setup with corresponding outputs.



(b) Square wave control input given to simulation and experimental setup with corresponding outputs.



(c) Multisine control input given to simulation and experimental setup with corresponding outputs.



(d) Second multisine control input given to simulation and experimental setup with corresponding outputs.

Fig. 2: Experimental (—) and simulation data (—) used for parameter estimation.

Figure 2. The estimated parameters of the system using the described method can be found in Talbe I. In Table II, the error between experimental and simulation data is expressed in terms of a *best fit rate* (BFR), which is defined as follows:

$$\mathrm{BFR}(y, \hat{y}) := \max\left(1 - \frac{\sum_{k=1}^{N}(y(k) - \hat{y}(k))^2}{\sum_{k=1}^{N}(y(k) - \mathrm{mean}(y))^2}, 0\right), \tag{4}$$

where $y(k)$ (N data points) are the data samples which are compared w.r.t an approximation $\hat{y}(k)$. In this report, y is the measurement data and $\hat{y}$ the simulation data. It is apparent from Figure 2 and Table II that using the estimated parameters the output data of the simulation and experiment closely match. The mismatches between simulation and experimental data are likely due to the omission of static friction from the model. This can be seen especially well in Figure 2b, where for the experimental data the oscillations decay quicker.

References

[1] B. Kulcsár, J. Dong, J. W. Van Wingerden, and M. Verhaegen, "LPV subspace identification of a DC motor with unbalanced disc," *Proc. of the 15th IFAC Symposium on System Identification*, 2009.

[2] MathWorks$^{\mathrm{TM}}$, "Parameter Estimation (R2018a)." [Online]. Available: https://nl.mathworks.com/help/sldo/parameter-estimation.html