Motion signals with velocity jumps

Motion Signals with Velocity Jumps: Velocity Estimation Employing Only Quantized Position Data

Mark Rijnen, Alessandro Saccon, and Henk Nijmeijer

***Abstract*—This paper addresses the problem of causal velocity estimation of motion signals exhibiting sudden velocity changes. It is explicitly assumed that only sampled quantized position data is available, and that no dynamic system model is at hand, to allow for an effective solution with minimal a priori information. Our solution consists of a finite-response filter with adaptive window length and a velocity jump detection algorithm. A velocity jump is detected when the difference between sampled position data and predicted position is larger than a parametric threshold. The specific value for the threshold is obtained via an analytical formula that depends on sampling time, maximum allowed variation in acceleration (away from velocity jumps), and encoder resolution. The developed method is first demonstrated on simulated quantized data corresponding to a bouncing motion and its performance compared to that of other existing non-model-based methods. Its effectiveness is furthermore illustrated by means of trajectory tracking results on an experimental setup experiencing impacts.**

***Index Terms*—Dynamics, Motion control, Velocity estimation, Impact**

I. INTRODUCTION

MANY controlled motion systems use position measurements to estimate their velocity and determine the control input required to achieve a desired closed-loop response. Optical incremental encoders are typically employed to obtain these position signals. Obtaining an accurate estimation of the velocity signal is important in controlled motion systems that experience (desired or unexpected) hard impacts [1]. The added complication for such applications is that the velocity shows abrupt changes at every impact time. These changes can be effectively modeled as velocity discontinuities, taking zero time. Such a simplified modeling of the impact behavior is supported by its match with observed experimental results [2] and it is a common and well established assumption [3], [4]. Motions with these velocity jump characteristics naturally occur in, for example, dynamic robot locomotion and manipulation. The modeling and control of systems performing such trajectories is an active field of research in the robotics and control communities.

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A possible solution to tracking time-varying reference trajectories of impacting mechanical systems is provided in [1], [2], [5]: this control framework goes under the name of reference spreading hybrid control. In [2], it is shown that if traditional methods for filtering and estimation are employed for retrieving position and velocity information from the quantized encoder data, the resulting control input might present spiking behavior about the times of impacts solely due to incorrect velocity estimation, in particular when feedback gains are increased for better tracking performance. This motivates the development of accurate online estimation methods to deal with a signal with piecewise continuous velocity and whose measurement is available only in the form of sampled quantized position data.

There is a vast literature on accurate position and velocity estimation from quantized position data. Whereas many methods exist to deduce accurate estimations of these quantities for smooth motions (e.g. least-squares filtering, Kalman filtering/observers, and low pass filtering, see [6]–[8] and references therein), the number of techniques tailored to signals with velocity jumps is limited. General approaches are not sufficiently accurate when applied to signals with jumping velocities as they act as low pass filters with a fixed bandwidth whereas the velocity jumps can be considered as high-frequency artefacts occurring at a-priori-unknown times. Techniques exist to retain part of the spectrum at the higher frequency range, as for example the mixed-bandwidth filter presented in [9]. Other approaches make use of additional information other than position data (e.g., the acceleration provided by an accelerometer) or an observer that encapsulates a hybrid model of the impact process (including the knowledge of the coefficient of restitution) done, e.g., in [10], [11]. All these approaches however require additional sensors or a hybrid model of the system (and possibly its force/acceleration input) and its interaction with the environment. While sensor fusion and hybrid observer techniques are certainly relevant to improve the velocity estimate in the presence of impacts, this paper is concerned with the case where only quantized position information and some basic knowledge of maximum variation in acceleration (away from impact events) is available, allowing to perform tracking of impact motions with limited system information.

When the only available information is a sampled position signal, an effective strategy to obtain accurate velocity estimations is to suitably select at each instant of time how many past measurement samples to consider. This approach is referred to as length adaptation of the moving window. Although not specific for velocity jump situations, in [12], a window length adaptation is proposed to improve velocity estimation. The key idea is that velocity estimation accuracy can be improved by enlarging the window when the acceleration is low and by shrinking the window when the acceleration is high. In [13], the approach is extended to acceleration estimation.

Motivated by the need of experimenting with reference spreading control [1], [2], [5], in this paper we propose a new window length adaptation method that is also combined with an impact detector. Impact detection was not considered in [12] as velocity therein was not explicitly supposed to be discontinuous. Existing techniques for just detecting impacts and estimating the corresponding times can be found in e.g. [14] and [15]. These works, however, require the use of a model of the impacting system as well as the input signal and do not focus on instantaneous detection of impact events, contrary to our assumptions and needs.

The contribution of this paper is therefore twofold: firstly, we detail an adaptive-length moving-window filter and secondly, an impact detection method. The two are combined for accurate velocity estimation in the presence of velocity jumps. The strategy behind the window length adaptation is based on selecting only those data points that are taken after the previous velocity jump. This inherently requires knowledge on when the previous jump event occurred which is obtained from the detection algorithm. The resulting approach will be referred to in this paper as jump aware (JA) filtering. Other than quantized position data, the only other assumptions we make is to possess a bound on the maximum change in acceleration within the window, away from velocity jumps, and the (trivial) requirement that the encoder resolution and sampling time are known. The threshold used in the jump detection algorithm can be obtained via an easy-to-compute analytical formula. The accuracy of the JA filter is investigated both in simulation and using experiments.

Whereas methods for velocity estimation and impact detection exist in literature, to the best of the authors' knowledge, no combined approach is readily available with such a minimalistic set of requirements. This paper aims at filling this gap in the literature.

This paper is organized as follows. In Section II, the considered problem is stated and the relevant quantities are defined. Subsequently, in Section III, we discuss the developed filtering method. An approach for computing the event detection bound associated to the varying-window filter is elaborated on in Section IV and in Section V we present a comparative simulation example. In Section VI experimental results are discussed. The conclusions of this work can be found in Section VII.

II. Problem description

Consider a piecewise continuously differentiable signal $q(t) \in \mathbb{R}$ representing an angular or longitudinal displacement. Wherever defined, we indicate with $v(t)$ and $a(t)$ its first and second time derivatives. We assume that for every time interval of finite length, only a finite number of velocity jumps can occur (in terms of hybrid systems literature [16], this corresponds to excluding Zeno type behaviors). The acceleration $a(t)$ is assumed to be bounded for all times, away from these jump times, while it is allowed to be impulsive at every velocity jump. More details on this acceleration bound are detailed in Section IV.

The signal $q(t)$ is sampled at a constant rate with sampling time h and the k-th (ideal) sample is denoted $q_k := q(t_k)$, with $t_k = kh$, $k = \{0, 1, 2, \dots\}$. Similarly, the velocity and acceleration at the time t_k are denoted $v_k := v(t_k)$ and $a_k := a(t_k)$, respectively. If t_k corresponds exactly to a velocity jump, then by $v(t_k)$ we mean the right limit (the post impact velocity) while $a(t_k)$ is undefined.

We have indirect access to the signal q_k via its quantized version, that we denote $\tilde{q}_k$, and that results from uniform quantization of the original signal q with resolution r.

The problem considered in this work is that of finding causal estimates of the position and velocity, respectively denoted $\hat{q}_k$ and $\hat{v}_k$, with particular attention on obtaining accurate estimations in a neighborhood of a velocity jump. Moreover, we are interested in detecting whenever a velocity jump took place, in as few samples after its occurrence as possible. In solving this composite problem, only position measurements are assumed to be available and, since we seek for a causal estimation, at time t_k, only the quantized samples $\tilde{q}_j$ with $j \leq k$ can be employed.

III. Jump Aware (JA) filtering

Conventional filters. Several fixed sampling time methods exist to filter a quantized signal and estimate its first (and possibly higher) derivative(s). Methods not relying on a dynamic observer (i.e. not 'model-based'), consider both the current sample and a constant finite maximum number M of data points before it, forming a moving window of length $M+1$. The window length is tuned to provide a sufficient filter bandwidth, i.e. M is a design variable. The position filter and velocity estimator can be written as

$$\hat{q}_k = f_{\mathrm{q}}(\underline{\tilde{q}_k}) \quad \text{and} \quad \hat{v}_k = f_{\mathrm{v}}(\underline{\tilde{q}_k}), \tag{1}$$

respectively, where f_{q} and $f_{\mathrm{v}} : \mathbb{R}^{M+1} \to \mathbb{R}$ describe the filter and first derivative estimator (e.g. LSF filters [6]) and

$$\underline{\tilde{q}_k} := \begin{bmatrix} \tilde{q}_{k-M} & \dots & \tilde{q}_{k-1} & \tilde{q}_k \end{bmatrix}^{\mathrm{T}}$$

denotes the moving window at time t_k.

As mentioned in the introduction, since the filters described above generally only maintain the signal's spectrum within a desired low-frequency pass-band to reduce noise, the distinct velocity jumps in the motion are removed. These filters consequently do not perform well about the impact times as the velocity jumps, that are high frequency artefacts, are part of the to-be-reconstructed underlying signal $q(t)$. An illustration of this fact is provided in Section V.

Adaptive window. We improve estimation accuracy by introducing the time-varying window length variable m_k corresponding to the time t_k and filtering position and estimating velocity only on the basis of the $m_k + 1 \leq M + 1$ samples guaranteed to be taken after the previous velocity jump. As the position signal after that particular time and up to the next velocity jump is assumed to be continuously differentiable, estimation functions in the form of (1) will perform similarly in that domain as for standard smooth signals. Algorithmically, this entails that upon the detection of a jump event, the moving window length at current time t_k is reset to one (i.e. $m_k = 0$), allowing the filter to only use the current sample because previous data cannot be assumed to be part of the same continuously differentiable portion of the position signal. Subsequently, the window length is incremented gradually, up to the next jump event or until the window reaches the designed maximum length $M+1$, to regain the desired smoothing properties away from impacts. Therefore, the moving-window length m_k+1 is allowed to change at each instant of time, and equals $M+1$ if the previous impact occurred sufficiently long ago.

Jump detection. As anticipated in the introduction, in this work we propose a specific jump detection strategy. This strategy is illustrated by Fig. 1 and its main idea is described in the following — mathematical detail will be presented in Section IV. Jump detection is based on comparing a prediction $\overset{\triangle}{q}_{k+1}$, that is a function of the previous data points, to the current sample $\tilde{q}_{k+1}$. The prediction is formally written as

$$\overset{\triangle}{q}_{k+1} := p(\underline{\tilde{q}_k}, m_k), \tag{2}$$

with $p : \mathbb{R}^{M+1} \times \mathbb{N} \to \mathbb{R}$. Whenever the difference between prediction and measurement is larger than a bound $b(m_k) : \mathbb{N} \to \mathbb{R}$ detailed in the next Section, a jump event is assumed to have happened. The algorithm exploits the assumption that during the phases where $v(t)$ is continuous, its derivative $a(t)$ is limited (see Section II), therewith limiting the possible change between v_{k-1} and v_k. This limitation does not apply in the case of ideal velocity jumps though, where the acceleration is impulsive. Note that the requirement for additional signal knowledge (other than position measurements) is inevitable to identify velocity jumps from the measurement data. We found that the minimalistic knowledge of maximum acceleration variation is sufficient to do so.

JA filter. The jump-aware filter is the combination of adaptive window filter with jump detection algorithm and can be formulated as

$$\begin{aligned} \hat{q}_k &= f_{\mathrm{q}}(\underline{\tilde{q}_k}, m_k), \\ \hat{v}_k &= f_{\mathrm{v}}(\underline{\tilde{q}_k}, m_k), \qquad \text{with} \\ m_k &= \begin{cases} \min(M, m_{k-1}+1), & |\tilde{q}_k - \overset{\triangle}{q}_k| \le b(m_{k-1}) \\ 0, & |\tilde{q}_k - \overset{\triangle}{q}_k| > b(m_{k-1}) \end{cases} \end{aligned} \tag{3}$$

where $f_{\mathrm{q}} : \mathbb{R}^{M+1} \times \mathbb{N} \to \mathbb{R}$ and $f_{\mathrm{v}} : \mathbb{R}^{M+1} \times \mathbb{N} \to \mathbb{R}$ make use of the adaptive window length variable m_k (with initial value $m_0 = 0$). For $m_k = 0$, it is not possible to estimate velocity since only one data sample is available and past samples are not indicative of post-impact velocity. This means that $f_{\mathrm{v}}(\underline{\tilde{q}_k}, 0)$ cannot be defined and that $f_{\mathrm{q}}(\underline{\tilde{q}_k}, 0)$ can be taken to be equal to $\tilde{q}_k$. The estimation function $f_{\mathrm{v}}(\underline{\tilde{q}_k}, 1)$ is constructed to perform finite differencing using the last two samples. The function $f_{\mathrm{v}}(\underline{\tilde{q}_k}, m_k)$, with $m_k > 1$, may be chosen along the line of other finite-response filters. In Section V, least-squares fit filters of different order will be used.

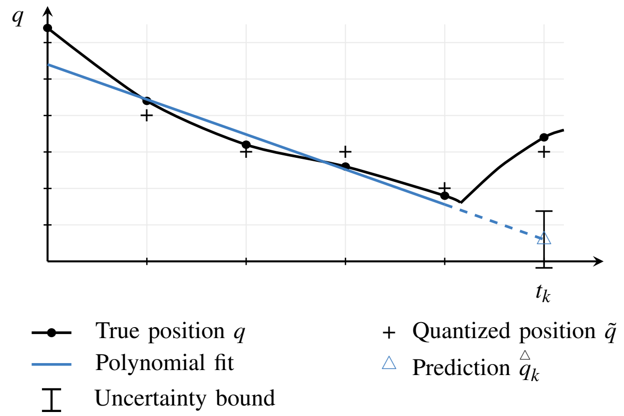

Fig. 1. Example illustrating the velocity jump detection strategy. A prediction for the current time t_k is used, together with an uncertainty bound, to check whether or not a jump has occurred since the previous sample.

IV. Prediction Function and Event Detection Bound Selection

This section proposes a specific prediction function, in the form of (2), that is based on first principles and we found to work well in practice. Furthermore, we introduce a corresponding jump detection bound guaranteeing that no velocity jump will be detected where there is none. The analytical bound is explicit and easy-to-compute, depending on the quantizer resolution, number of samples, and acceleration variation limits away from a jump.

Consider the time t_k with moving window $\underline{q_k} \in \mathbb{R}^{M+1}$ of which the last m_k+1 elements ($m_k \le M$) can be assumed to be drawn from a C^1 position signal. To predict the position at the next time step t_{k+1}, one can fit a second order polynomial to the m_k+1 last measurements and extrapolate at $t = t_{k+1}$ (cf. [17]). Given $y(s) = \xi_0 + \xi_1 s + \frac{1}{2}\xi_2 s^2$, with $s := t - t_k$ and ξ_0, ξ_1, and ξ_2 are to be determined coefficients. These coefficients can be found by solving $A\xi = S\underline{\tilde{q}_k}$ in a least-squares sense, where

$$A := \begin{bmatrix} 1 & -m_k h & \frac{1}{2}m_k^2 h^2 \\ 1 & (-m_k+1)h & \frac{1}{2}(-m_k+1)^2 h^2 \\ \vdots & \vdots & \vdots \\ 1 & 0 & 0 \end{bmatrix}, \tag{4}$$

$\xi := \begin{bmatrix} \xi_0 & \xi_1 & \xi_2 \end{bmatrix}^{\mathrm{T}}$, and $S := \begin{bmatrix} 0_{(m_k+1)\times(M-m_k)} & I_{m_k+1} \end{bmatrix}$ selects the last m_k+1 elements of $\underline{\tilde{q}_k}$, with 0 and I denoting the zero and identity matrices of indicated dimensions. The least-squares solution is $\xi = A^{\dagger} S \underline{\tilde{q}_k}$, where $A^{\dagger} := (A^{\mathrm{T}}A)^{-1}A^{\mathrm{T}}$ is the Moore-Penrose pseudo-inverse of A. Extrapolating the polynomial one time step gives the position prediction $\overset{\triangle}{q}_{k+1} = \begin{bmatrix} 1 & h & \frac{1}{2}h^2 \end{bmatrix} \xi = P^{\mathrm{T}}(m_k)\underline{\tilde{q}_k}$ where

$$P^{\mathrm{T}}(m_k) := \begin{bmatrix} 1 & h & \frac{1}{2}h^2 \end{bmatrix} A^{\dagger} S \in \mathbb{R}^{M+1}. \tag{5}$$

Consequently, the prediction function (2) specializes to

$$p(\underline{\tilde{q}_k}, m_k) = P^{\mathrm{T}}(m_k)\underline{\tilde{q}_k}. \tag{6}$$

That the vector P depends on m_k follows from A and S depending on the window length.

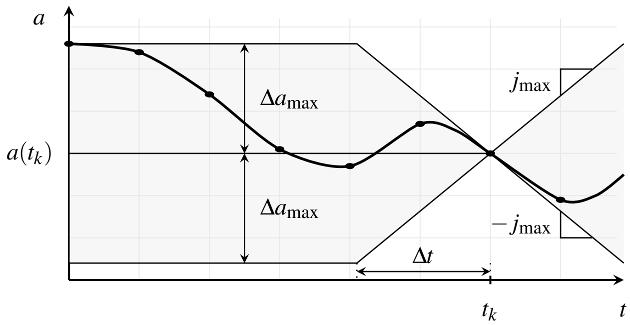

Fig. 2. Illustration of the assumption on the allowed variation of the acceleration over time (sector condition).

Remark 1: The prediction function (6) *requires* m_k *to be larger than a certain minimal value for it to be applicable, i.e* $m_k \geq 2$. *The proposed detection method thus implicitly requires the velocity jumps to be more than two time steps apart. In implementing the JA filter, one can choose to consider* $p(\underline{\tilde{q}_k}, m_k) = \tilde{q}_k$ *and bounds* $b(m_k) = \infty$ *(i.e. turn off the triggering) whenever* $m_k < 2$. *Alternatively, for* $m_k = 1$, *the prediction function can be chosen as an extrapolation of a first order polynomial fit and a similar reasoning as given in the Appendix can be taken to find a suitable bound* $b(1)$. *In any case, there is no way of identifying if a velocity jump has occurred between* t_k *and* t_{k+1} *in the case* $m_k = 0$ *using position measurements only.* ■

In the following, we provide the event detection bound b associated to the specific prediction function (6).

Consider the time t_{k+1} and recall that the velocity during the time interval $[t_{k-m_k}, t_{k+1}]$ is assumed to have experienced no jump. A reasonable assumption on the acceleration on $[t_{k-m_k}, t_{k+1}]$ is that its variation is bounded. We assume that there exist constants $\Delta a_{\max} > 0$ and $j_{\max} > 0$, such that for all $t \in [t_{k-m_k}, t_{k+1}]$

$$|a(t) - a(t_k)| \leq \min(\Delta a_{\max}, |t - t_k| j_{\max}). \tag{7}$$

See Fig. 2 for an illustration of (7). The prediction error $\tilde{q}_{k+1} - \overset{\triangle}{q}_{k+1}$ is then bounded according to $|\tilde{q}_{k+1} - \overset{\triangle}{q}_{k+1}| \leq b(m_k)$ with

$$\begin{aligned} b(m_k) =& (1 + \|P(m_k)\|_1)\frac{r}{2} + Q(h) + \dots \\ & \dots + \sum_{j=1}^{M+1} |P^j(m_k) Q((M - j + 1)h)|, \end{aligned} \tag{8}$$

where $P^j(m_k)$ denotes the j-th element of $P(m_k)$ defined in (5) and where, introducing $\Delta t := \Delta a_{\max}/j_{\max}$,

$$Q(x) := \begin{cases} \frac{j_{\max}}{6} x^3 & x \leq \Delta t \\ \frac{\Delta a_{\max}}{2}(x - \Delta t)^2 + \frac{j_{\max}}{2}\Delta t^2 (x - \frac{2}{3}\Delta t) & x > \Delta t \end{cases} \tag{9}$$

The derivation of this bound is given in the Appendix. Whenever the prediction error exceeds (8), then (7) is violated and a velocity jump can be assumed to have occurred.

Remark 2: The condition (7) *is chosen for its generality. It can handle cases where the considered signal simply has bounded jerk (away from jumps) as well as those where the acceleration can change almost instantaneously* ($j_{max} \to \infty$). *The latter is seen in mechanical systems controlled, e.g., using a zero-order-hold approach.*

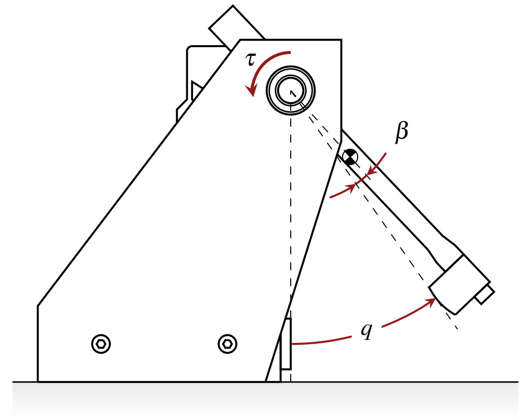

Fig. 3. Schematic of a one degree-of-freedom system consisting of an actuated pendulum with moving mass $m = 0.141$ kg, and inertia $I = 4.7 \cdot 10^{-4}$ kg m^2, impacting an object with coefficient of restitution $e = 0.56$ and contact angle $\beta = 0.116$ rad.

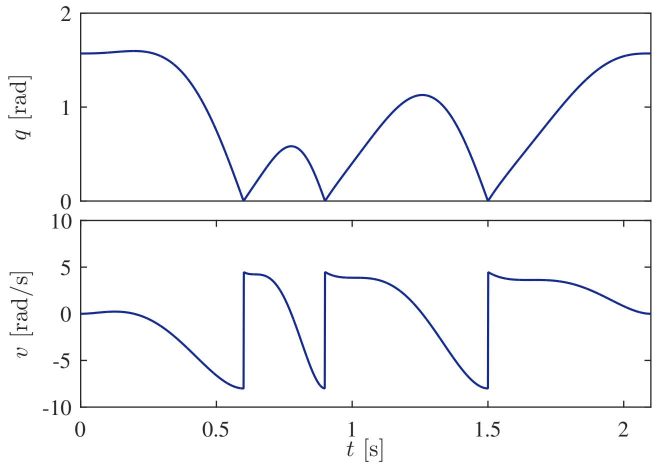

Fig. 4. Real position and velocity as a function of time.

V. NUMERICAL SIMULATION RESULTS

In this section, we will apply the developed filtering approach to estimate position and velocity from synthetic measurements of a motion with impacts. Synthetic data is used to show the ability of the filter to accurately reconstruct the known velocity signal with jumps. In Section VI, we will show the results on the experimental setup depicted in Fig. 3. Consider the motion with bounces shown in Fig. 4.

The position $q(t)$ is sampled using a sampling time of $h = 1$ ms and quantization errors corresponding to a resolution $r = 2\pi/2000$ rad are introduced to form the data $\tilde{q}_k$, $k = \{0, 1, 2, \dots\}$. Conventional filters as well as the proposed method are applied to the synthetic measurement data to estimate position and velocity. First, we will compare the JA filter to conventional fixed window filters and, later on in this section, we will compare it to the adaptive window method introduced in [12]. The estimation results are depicted in Fig. 5 and the mean-absolute norms of the position estimation error

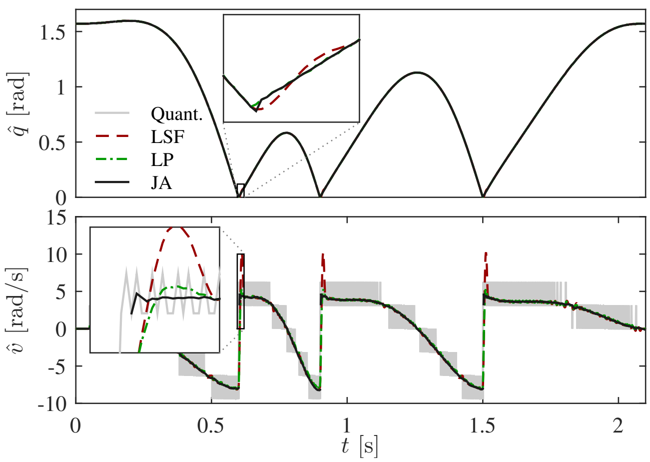

Fig. 5. Estimated position and velocity as a function of time for four different estimation methods.

$e_q := \hat{q} - q$ and velocity estimation error $e_v := \hat{v} - v$ are given in Table I. Since we will compare the effectiveness of the filters on the basis of $\|e_v\|_{MA}$, for the sake of fair comparison, in this work we used numerical optimization in designing the different filters, considering the signal in Fig. 4. In general however, design of the JA filter follows a similar strategy as common for existing fixed window filters in that the design variables should be chosen to get a desired pass-band. The main difference now is that this needs to be done for every $1 \le m_k \le M$, where for choosing M also a conventional design approach can be followed.

Fig. 5 shows the quantized position data and corresponding velocity from finite differencing. It furthermore depicts the estimation results from applying a least-squares fit filter (LSF [6], also known in literature as a Savitzky-Golay filter) using polynomials of order two and considering the last 20 points (i.e. $M = 19$). The outcome of feeding the finite differencing data through a second order low pass (LP) filter with a cut-off frequency of 387.6 rad/s and damping ratio 0.53 is also included in Fig. 5. The low pass filter is not used for estimating position, since that, in this case, would decrease accuracy. The considered JA filter also uses polynomial fitting (i.e. least-squares fit filters), but of different window lengths and of different order. The filter uses polynomials of order one for $m_k < 14$ and fits second order polynomials whenever $43 = M \ge m_k \ge 14$. For $m_k = 0$, no filtering or estimation is performed. The event detection bound is chosen according to (8) for $\Delta a_{\max} = 65$ rad/s^2 and $j_{\max} = 2130$ rad/s^3, with $b(m_k) = \infty$ for $m_k < 2$ (see Remark 1). The value for $j_{\max}$ is obtained by taking the maximum jerk for the signal in Fig. 4 and $\Delta a_{\max}$ derives from moving an adaptive window along the true acceleration and, for each time t_k, finding the maximum of $|a(t) - a(t_k)|$ for $t \in [t_{k-m_k}, t_{k+1}]$. The prediction error $|\tilde{q}_k - \overset{\triangle}{q}_k|$ and the bound $b(m_{k-1})$ are shown in Fig. 6.

Fig. 5 clearly illustrates the effect quantization has on predicting the velocity via finite differencing (grey). The errors are amplified significantly. The low pass filter (green) effectively reduces this quantization noise at the cost of introducing an undesired lag. Low pass filtering significantly smoothens the velocity jumps, requires a significant amount of time to recover, and presents overshoot. The lag can be reduced by increasing the filters cross-over frequency at the cost of amplifying the quantization errors in the continuously differentiable phases. In fact, as the filter's cross-over frequency and damping ratio were chosen to minimize the velocity estimation error, the filtering performance for the smooth motion phases has already been compromised.

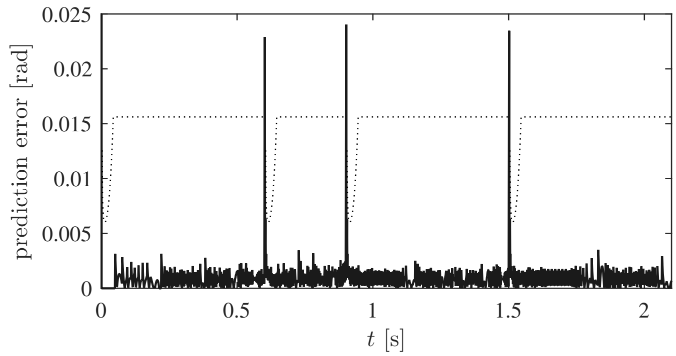

Fig. 6. The prediction error $|\tilde{q}_k - \overset{\triangle}{q}_k|$ as a function of time (solid) and the time-varying bound b (dotted).

TABLE I
MEAN ABSOLUTE NORM OF THE POSITION AND VELOCITY ESTIMATION ERRORS FOR DIFFERENT FILTERS

Estimation method	$\|e_q\|_{\mathrm{MA}}$ [rad]	$\|e_v\|_{\mathrm{MA}}$ [rad/s]
Quantized data	$7.73 \cdot 10^{-4}$	0.943
Least-squares fit filter	$6.24 \cdot 10^{-4}$	0.232
Second order low pass	–	0.185
Best-fit FOAW	$4.66 \cdot 10^{-4}$	0.156
JA filtering	$3.32 \cdot 10^{-4}$	0.059

The LSF (red) performs similarly as the LP filter away from the velocity jumps. The results for the LSF clearly illustrate that it is not desirable to use the data before the jump for estimating position and velocity after. Typically, a 'riple' is observed in the filtered position and a large overshoot is encountered in the estimated velocity. Furthermore, as the found optimal window length for this filter is much lower than that of the tuned JA filter, it is apparent that it too suffers from the trade-off between desiring a fast response at velocity jumps, while at the same time aiming at significant smoothing away from those jumps. It can be concluded that the traditional filters do not work well for signals with velocity jumps as can also be clearly deduced from Table I. The JA filter (black) instead gives accurate estimates of position and velocity and leaves the jumps almost intact. A reduction in $||e_v||_{\mathrm{MA}}$ by a factor of over three is seen in comparing the JA filter's performance to that of the others.

From Fig. 6 appears that the bound (8) is conservative. The velocity jumps are detected when the prediction error exceeds the bound. Subsequently, the window length is reset and gradually built up again. The three velocity jumps are detected just two samples later than their occurrences.

Since the JA filter has an adaptive window length and the filters to which its performance is compared so far are not,

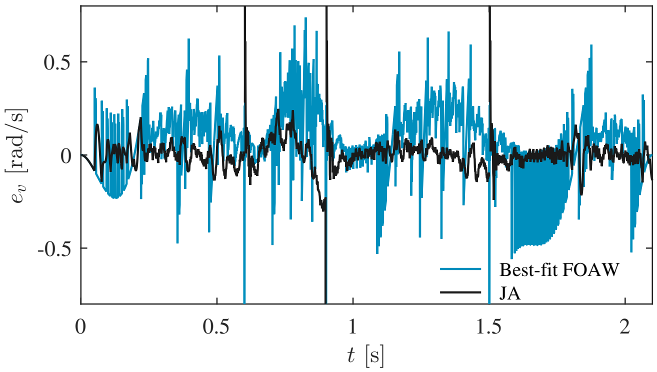

Fig. 7. The velocity estimation error $e_v := \hat{v} - v$ as a function of time for the Best-fit FOAW method [12] and from using a JA filter.

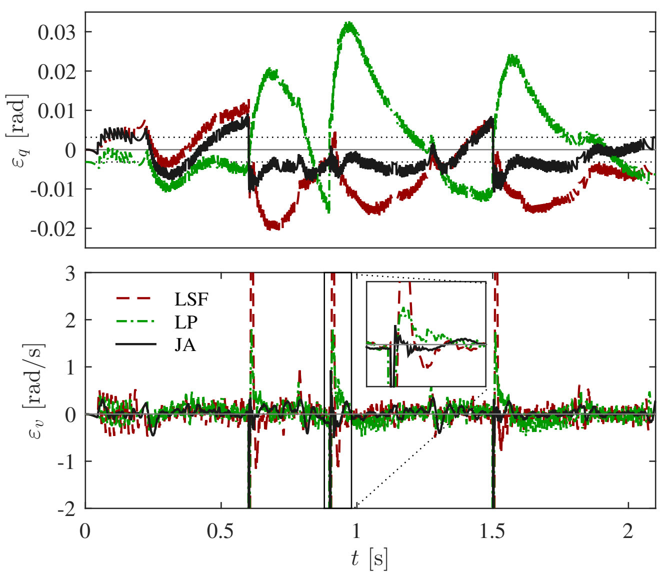

Fig. 8. Tracking errors as a function of time for three different velocity estimation methods. The horizontal dotted lines indicate encoder resolution.

we also considered the adaptive window technique in [12] for comparison. To be more precise, the best-fit first order adaptive window (best-fit FOAW) method is selected and applied to the signal to estimate velocity. The reader is referred to [12] for a description of the method. The velocity estimation error e_v for the best-fit FOAW and the JA filter described above are shown in Fig. 7. The norm of the position and velocity errors are given in Table I. It can be concluded from both the figure and the table that both methods handle the velocity jumps well and only give a relatively large error for the one sample after the jumps, but that the JA filter outperforms the best-fit FOAW approach. As a general remark we would like to add that we found the best-fit FOAW filter more difficult to implement and significantly more computationally costly than JA filtering.

VI. Experimental results

In this section, we apply reference spreading control (see [1], [2], [5]) on an experimental setup to track the motion in Fig. 4. The setup consists of a pendulum that is actuated at the axle and of which the operating space is confined by a metal block (see Fig. 3 and [2]). The LSF, LP, and JA filters discussed in the previous section with the same parameter settings are employed to estimate velocity (and position) and trajectory tracking experiments with identical controller settings are performed. For the JA filter, whenever $m_k = 0$, the velocity-dependent part of the feedback control is switched off. To improve robustness of the filter, considering the fact that the filter is now incorporated in a closed-loop control with imperfect encoder, we only apply the event detection (triggering) when $m_{k-1} \geq 5$. This is also done since there likely are unknown vibrational transients introduced by the impacts. Note that since we are interested in applying the JA filter in a tracking control framework, approximate values for $j_{\max}$ and $\Delta a_{\max}$ can be found from the reference trajectory. The tracking results are depicted in Fig. 8, illustrating the position error $\varepsilon_q := \tilde{q} - q^d$ and velocity error $\varepsilon_v := \hat{v} - v^d$. In this, q^d and v^d are the desired position and velocity, respectively. Tracking and velocity estimation results can also be found in this paper's accompanying video, available at http://ieeexplore.ieee.org. The evolutions of estimated position and velocity are not depicted here as such a figure is extremely similar to Fig. 5.

Upon close inspection, Fig. 8 shows that the velocity error ε_v is larger for a longer period of time after the impacts both for the LSF and LP filters when compared with the JA filter. This is due to their lagged response to the velocity jumps. Moreover, the error is significantly noisier. These two aspects lead to larger (peaking) and erroneous feedback efforts that in turn lead to larger tracking errors ε_q. The position errors for the LSF and LP estimators clearly show an unwanted response to the impacts. From Fig. 8 can be concluded that, just by using the developed estimation method instead of conventional filters, the maximal tracking error is reduced by a factor of over two. In this case, the remaining error is in the order of magnitude of the encoder resolution $r = 2\pi/2000 \approx 0.0031$ rad.

VII. Conclusions & discussion

In this work a novel filtering method has been introduced for estimating position and velocity from a quantized position signal sampled at a constant rate under the assumptions that the underlying signal exhibits non-accumulating velocity jumps and that an a-priori bound on acceleration away from these jumps is available. The proposed approach consists of an adaptive window filter and a jump event detection algorithm. The detection is based on comparing predictions to measurements. When the prediction error exceeds a specified threshold, the window length is reset, as a velocity jump must have occurred. Subsequently, the window is gradually expanded again. An acceleration bound can be specified in terms of maximum variations within the moving window, and a corresponding analytical bound on the prediction error, that takes also into account the sampling frequency and encoder resolution, has been proposed.

The approach has been demonstrated by means of numerical simulations and physical experiments for filtering of a quantized position signal and estimation of the discontinuous velocity profile, showing excellent performance.

It was furthermore shown that existing filtering techniques have significantly worse performance in terms of position and velocity reconstruction from quantized data in proximity of velocity jumps. The presented filter has the added benefit of having an impact detection feature, which is a key asset in the reference spreading control framework.

In this work we focused on estimating scalar velocity for mechanical systems with non-accumulating transversal (partially) elastic impacts. The developed estimation method may however also be applied to signals with jumps in applications other than mechanics as well as in motions with inelastic impacts. The multidimensional case can be tackled as a collection of scalar signals, each with their own JA filter. A possible adaptation of the detection strategy for multidimensional signals, e.g. to improve estimation accuracy and efficiency, is however left for further research. Moreover, the effects of almost tangential impact and of accumulating impacts remain to be investigated. As can easily be expected, preliminary results suggest that, as the velocity jumps get relatively small, the JA filter will not detect them. However, since the velocity jumps will become negligible (both for almost tangential impact and in the Zeno case), a conventional low-pass filter will work sufficiently well in providing a reasonable velocity estimate and note that, since the window will not be reset due to the small velocity jumps, the JA filter will actually behave as a fixed window filter in such scenario.

Appendix

In this appendix, we provide the proof of the analytical bound (8), associated to assumption (7) on the underlying acceleration away from velocity jumps. We start by observing that the position q at the time t_{k+1} can be approximated, similarly as done for (6), from the position, velocity, and acceleration at the previous time step as

$$q_{k+1} - \varepsilon_{k+1} = q_k + v_k h + \frac{1}{2} a_k h^2, \tag{10}$$

where ε_{k+1} is the approximation/extrapolation error at the time t_{k+1}. Furthermore, the measurement noise is uniformly bounded and finite so that

$$\tilde{q}_{k+1} = q_{k+1} + \sigma_{k+1}, \tag{11}$$

with $|\sigma_{k+1}| \leq r/2$ the measurement error at $t = t_{k+1}$. Using (10), (11), and triangular inequality, it follows that the prediction error satisfies the bound

$$\begin{aligned} |\tilde{q}_{k+1} - \overset{\triangle}{q}_{k+1}| &= |q_{k+1} + \sigma_{k+1} - \overset{\triangle}{q}_{k+1}|, && (12)\\ &= |e_{k+1} + \sigma_{k+1} + \varepsilon_{k+1}|, && (13)\\ &\leq |e_{k+1}| + \frac{r}{2} + |\varepsilon_{k+1}| && (14) \end{aligned}$$

where $e_{k+1} := q_k + v_k h + \frac{1}{2} a_k h^2 - P^{\mathrm{T}} \underline{\tilde{q}_k}$ is the error contribution from the data not exactly forming a parabola.

We look for an upper bound for $|e_{k+1}|$ under the assumption that no velocity jump has occurred in the domain $[t_{k-m_k}, t_{k+1}]$. In order to do so, the vector $\underline{\tilde{q}_k}$ containing the measured position for the sample times in the domain $[t_{k-M}, t_k]$ is split in three components, namely

$$\underline{\tilde{q}_k} = \underline{q_k^o} + \underline{\sigma_k} + \underline{\Delta q_k}. \tag{15}$$

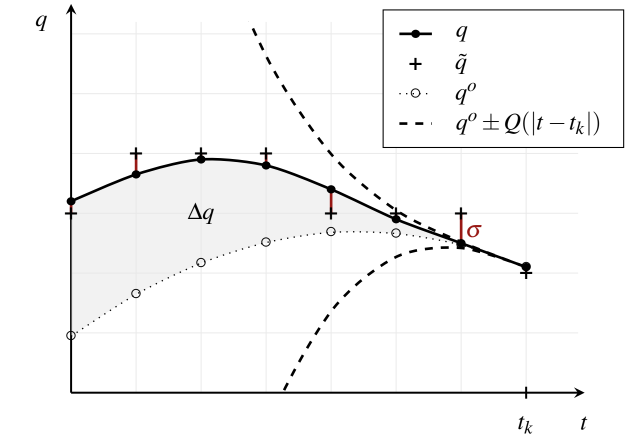

Fig. 9. Illustration of bound (19) and the different components of $\underline{\tilde{q}_k}$, i.e. a parabola aligning with q at the time t_k, an approximation error $\underline{\Delta q_k}$ (together forming $\underline{q_k}$), and the quantization error $\underline{\sigma_k}$.

In (15),

$$\underline{q_k^o} := \begin{bmatrix} 1 & -Mh & \frac{1}{2}M^2h^2 \\ 1 & (-M+1)h & \frac{1}{2}(-M+1)^2h^2 \\ \vdots & \vdots & \vdots \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_k \\ v_k \\ a_k \end{bmatrix}, \tag{16}$$

$\underline{\sigma_k} := \begin{bmatrix} \sigma_{k-M} & \dots & \sigma_{k-1} & \sigma_k \end{bmatrix}^{\mathrm{T}}$, and $\underline{\Delta q_k} := \underline{\tilde{q}_k} - \underline{q_k^o} - \underline{\sigma_k}$. In other words, $\underline{q_k^o}$ is a vector containing points on a parabola (see Fig. 9), $\underline{\sigma_k}$ is a vector containing the quantization errors for the samples in the considered domain, and $\underline{\Delta q_k}$ is what remains. Note that $\underline{q_k^o}$ satisfies $P^{\mathrm{T}} \underline{q_k^o} = q_k + v_k h + \frac{1}{2} a_k h^2$ and that $\underline{\Delta q_k}$ can be thought of as the contribution to the data from non-constant acceleration. Using (15), Hölder's inequality, and the fact that $|\sigma_j| \leq r/2$ for every $j \in \mathbb{N}$, it follows that

$$\begin{aligned} |e_{k+1}| &= |q_k + v_k h + \frac{1}{2} a_k h^2 - P^{\mathrm{T}} \left(\underline{q_k^o} + \underline{\sigma_k} + \underline{\Delta q_k} \right) | \\ &= |P^{\mathrm{T}} \left(\underline{\sigma_k} + \underline{\Delta q_k} \right) | && (17)\\ &\leq \|P\|_1 \frac{r}{2} + \sum_{j=1}^{M+1} |P^j| |\underline{\Delta q_k}^j|. && (18) \end{aligned}$$

Since $\underline{\Delta q_k}$ describes the difference between the real position as a function of time and a parabola with the same value, first derivative, and second derivative at the time t_k, a bound on the deviation of the acceleration $a(t)$ from the polynomials second derivative can be translated to a bound on the elements of $\underline{\Delta q_k}$ (at least to those inside the interval $[t_{k-m_k}, t_k]$). Assume that (7) holds for all $t \in [t_{k-m_k}, t_k]$. From integrating (7) twice, it follows that $|q(t) - (q_k + v_k(t - t_k) + \frac{1}{2} a_k (t - t_k)^2| \leq Q(|t - t_k|)$, where Q is given by (9) and consequently that

$$|\underline{\Delta q_k^j}| \leq Q((M - j + 1)h) \tag{19}$$

for all $j \in \{M - m_k + 1, M - m_k + 2, \dots, M + 1\}$. Incorporating (19) in (18) (and considering that the first $M - m_k$ elements of P are zero) now gives a bound on $|e_{k+1}|$. In finding a bound for $|\varepsilon_{k+1}|$ in (14) a similar reasoning as followed in deriving (19) is taken. If (7) holds for all $t \in [t_k, t_{k+1}]$, then (from integrating twice) it follows that

$$|\varepsilon_{k+1}| := |q_{k+1} - (q_k + v_k h + \frac{1}{2} a_k h^2)| \leq Q(h). \qquad (20)$$

Combining (14), (18), (19), and (20) gives exactly the expression for the bound b in (8) and concludes the derivation.

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