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Achievable Information Rate Losses for High Order Modulation and Hard-Decision Forward Error Correction

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Abstract *We study the combination of high-order modulation formats and hard-decision FEC from an information rate point of view. It is shown that the relative rate losses are approximately constant for the same FEC overhead when the constellation size increases.*

Introduction

Achievable information rates (AIRs) have emerged as a practical tool to design fiber optical communication systems[1,2]. AIRs can be used for example to design modulations formats and to predict the performance of forward error correction (FEC)[3–5]. Binary FEC typically comes in two flavors: hard-decision (HD) and soft-decision (SD) FEC. HD-FEC decoders are fed with bits, while SD-FEC decoders are fed with logarithmic likelihood ratios (known as “soft” bits).

To increase the spectral efficiency, high order constellations such as QAM are required. The combination of these modulation formats and FEC is called coded modulation (CM), which dates back to G. Ungerboeck’s trellis coded modulation[6]. CM is a key technique for designing high rate coherent receivers[7], or to achieve gains via probabilistic and geometric shaping[8,9].

While earlier generations of fiber optical communication systems used HD-FEC, the increased gain of modern SD-FEC such as low-density parity-check codes led to their widespread adoption. These SD-FEC codes offer an SNR improvement in the region of 1–2 dB compared with HD-FEC codes of the same rate, at the cost of increased power consumption and latency. The development of digital coherent systems for use in short reach applications, combined with the exponential increase in data-rates has recently led to a renewed interest in HD-FEC. One particularly popular family of HD-FEC codes is staircase codes (SCCs)[10], whose combination with probabilistic shaping has been recently studied[11].

In this paper, we analyze CM based on HD-FEC from an achievable information rate point of view. We quantify the penalties caused by using a HD-FEC both from information-theoretic and coding points of view. We first show that as the constellation cardinality increases, the theoretical rate loss caused by using HD-FEC instead of SD-FEC increases. We then argue that a better metric to study is the <u>relative</u> rate loss. This rate loss is then shown to remain approximately constant as the cardinality increases. Finally, we show that the same effect is observed if SCCs are used and compared to HD-FEC AIRs.

System Model, AIRs and Staircase Codes

The most popular AIR for (binary) SD-FEC is the generalized mutual information (GMI)[1,12]. The GMI is defined as

$$\mathsf{I}^{\mathsf{SD}} = \mathsf{GMI} \triangleq \sum_{k=1}^{m} I(B_k;Y), \tag{1}$$

where $I(B_k;Y)$ is the mutual information between the code bits and the received symbols, and $M = 2^m$ is the number of constellation points. When the FEC is HD, an AIR is given by

$$\mathsf{I}^{\mathsf{HD}} = m(1 - H_{\mathsf{b}}(\mathsf{BER})), \tag{2}$$

where BER is the average bit error rate (across m bits) and $H_{\mathsf{b}}(\cdot)$ is the binary entropy function.

Throughout this paper we consider a four-dimensional (4D) real additive white Gaussian noise (AWGN) channel and pulse amplitude modulation (PAM) labeled by the binary reflected Gray code. The 4D constellation with cardinality M is formed by the product of four PAM constellations with $\sqrt[4]{M}$ constellation points. The constellation sizes under consideration are $M = 16, 256, 4096, 65536$, which correspond to constellations with $1, 2, 3$ and 4 bits per real dimension. This is equivalent to consider 4 dual-polarization (DP) constellations: DP-QPSK, DP-16QAM, DP-64QAM, DP-256QAM.

To make the results in this paper more practically relevant, we also consider SCCs with dif-

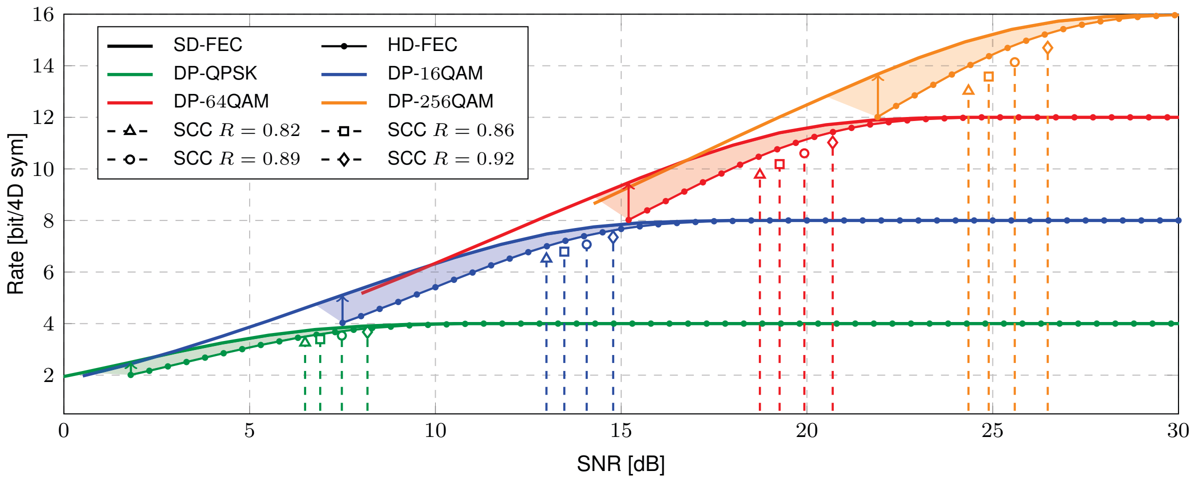

Fig. 1: AIRs for SD (thick) and HD (thin with filled circles) for different constellations (colors). Markers show results for SCCs.

ferent FEC overheads (OHs). Table 1 shows the parameters of the SCCs under consideration. The component codes of these SCCs are rate $R_c = k_c/n_c$ binary extended Bose-Chaudhuri-Hocquenghem (BCH) codes with 1 extra parity bit, $t_c = 2, 3, 4, 5$ and $m_c = 9$. The codeword length is $n_c = 2^{m_c}$, which contains $k_c = n_c - m_c t_c - 1$ information bits. As shown in Table 1, the resulting SCCs code rates are $R = 0.82, 0.86, 0.89, 0.93$, which are obtained as $R = 2k_c/n_c - 1$. Decoding is done using iterative bounded-distance decoding with a window length of 9 and 7 iterations per window. The SCC parameters were chosen because they lead to multiple FEC code rates and constant block length ($65,536$ code bits). We decided to have this parameter fixed as it gives a rough indication of decoding complexity and decoding delay.

t_c	m_c	n_c	k_c	R_c	R	OH%
2	9	512	493	0.96	0.93	8.02
3	9	512	484	0.95	0.89	12.28
4	9	512	475	0.93	0.86	16.89
5	9	512	466	0.91	0.82	21.90

Tab. 1: Parameters for SCCs considered in this paper

Numerical Results: AIRs

The SD- and HD-AIRs in (1) and (2) are shown in Fig. 1 (thick and thin lines with filled circles, resp.). These results show the theoretical loss caused by using HD-FEC instead of SD-FEC. These results also indicate that as the constellation size increases, the rate losses increase. Equivalently, the SNR penalty caused by using HD-FEC instead of SD-FEC increases as M increases. This effect is schematically shown in Fig. 1 with shaded areas. The rate losses of (ideal) HD-FEC codes with 100% OH are shown with four vertical arrows.

Fig. 1 also shows (with markers) the SNR required for SCCs to achieve a post-FEC BER of $5 \cdot 10^{-5}$. Here we assume a concatenation with an outer BCH code with rate $R_o = 0.9922$, which will bring the post-SCC BER down to 10^{-15}.[7] The "heights" of these markers correspond to the achievable throughput of the specific code and modulation under consideration. This rate is calculated as $I^{SC} = R_o R m$ [bit/4D sym], where R is the rate of the SCC and m is the number of bits per symbol of the constellation under consideration.

Numerical Results: Absolute Losses

Fig. 2 shows the absolute AIR losses (AL) caused by HD-FEC with respect to SD-FEC (solid lines), which is defined as $\mathrm{AL}^{\text{SD-HD}} = \mathrm{I}^{\text{SD}} - \mathrm{I}^{\text{HD}}$. The SNRs corresponding to HD-FEC codes with 100% OH are shown with four vertical arrows. This figure shows that indeed the rate losses increase as the modulation format increases.

Fig. 2 also shows the results obtained by SCCs (markers). In this case, the AIR loss is given by $\mathrm{AL}^{\text{HD-SC}} = \mathrm{I}^{\text{HD}} - \mathrm{I}^{\text{SC}}$. This loss is shown for the SNR values where the SCC under consideration achieves a post-FEC BER of $5 \cdot 10^{-5}$ (see thresholds in Fig. 1). The results in Fig. 2 show that this absolute loss caused by using the (suboptimal) SCC instead of considering I^{HD} is also increasing as the constellation size increasing. These results highlight the fact that for a given SCC (a given marker), the AIR loss grows approximately linearly with constellation size. The slope of this increasing AIR loss decreases as the code rate of the SCC decreases. This is shown in Fig. 2 with dashed lines. Note that solid lines and markers in Fig. 2 represent two different losses: SD vs. HD and HD vs. SCCs. Nevertheless, the markers

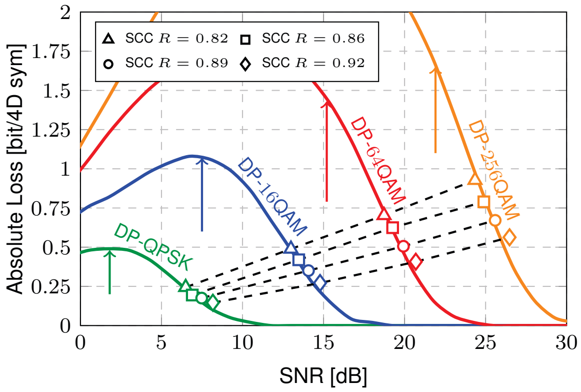

Fig. 2: Absolute rate losses: SD-FEC vs. HD-FEC (solid lines) and HD-FEC vs. SCCs (markers).

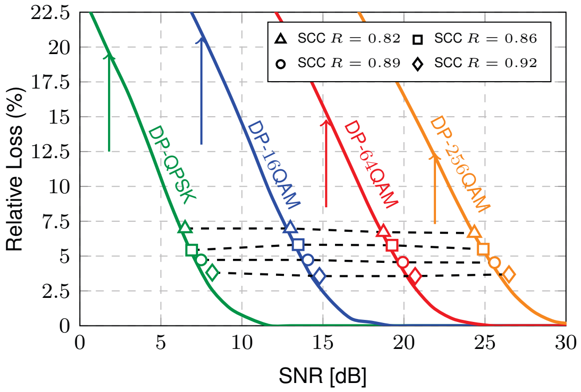

Fig. 3: Relative rate losses: SD-FEC vs. HD-FEC (solid lines) and HD-FEC vs. SCCs (markers).

closely follow the lines, indicating that the losses caused by HD-FEC w.r.t. SD-FEC are comparable to the losses caused by using SCCs instead of an ideal HD-FEC.

Numerical Results: Relative Losses

The absolute AIR loss results in Fig. 2 could be misleading. They indeed show that, for a given code rate, the AIR loss increases as the constellation size increases. However, for large constellation sizes, large rates are being considered. For a more fair comparison, we propose here to study relative AIR losses (RL): $\mathrm{RL}^{\text{SD-HD}} = \mathrm{AL}^{\text{SD-HD}}/\mathrm{I}^{\text{SD}}$ and $\mathrm{RL}^{\text{HD-SC}} = \mathrm{AL}^{\text{HD-SC}}/\mathrm{I}^{\text{HD}}$. The RL are shown in Fig. 3. The results in this figure shows that the theoretical relative losses remain below 20% for HD-FEC with 100% OH (solid lines). For the SCC results (markers)—unlike the results in Fig. 2 where for a given SCC the loss increases as the constellation size increases—the losses are approximately constant. For the codes under consideration, this relative loss is at most 7.5%. This happens for the FEC rate $R = 0.82$, which is a relatively low FEC rate for HD-FEC. For FEC rate $R = 0.93$, the relative loss is only around 3.5%, regardless of the constellation size under consideration.

The results in Fig. 3 show that even if the constellation size increases, the relative loss caused by the demapper making a hard decision on the noisy symbols is constant. This result is somewhat counter-intuitive as one would expect larger losses when the constellation size increases.

Conclusions

In this paper we studied two rate losses for coded modulation. The first one is the one caused by the use of hard-decision instead of soft-decision FEC. This first analysis was made based on information theoretic quantities and applies to ideal codes. The second rate loss we studied was the one caused by the use of practical hard-decision FEC. We focused on staircase codes and their loss with respect to the information theoretic maximum. In both analyses, it was shown that the absolute rate loss increases as the constellation size increases. However, the relative rate losses were shown to be approximately constant, regardless of the modulation format. The results of this paper give a strong theoretical and practical support for combining high-rate hard-decision FEC and high order modulation formats for future spectrally-efficient fiber optical communications.

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