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Citation for published version (APA):

DOI:
10.1109/JPHOT.2019.2915163

Document status and date:
Published: 06/05/2019

Document Version:
Publisher’s PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

• A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
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Volume 11, Number 3, June 2019

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DOI: 10.1109/JPHOT.2019.2915163
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Abstract: Residual amplitude modulation of the phase modulator deployed in Pound–Drever–Hall frequency stabilization is an effect known to cause instabilities to the absolute wavelength of the stabilized laser. We present measurements and analysis of the residual amplitude modulation in an InP-based waveguide electro-optic phase modulator. The modulator is monolithically integrated in the output waveguide of a tuneable laser. The effects on the frequency stabilization of such a laser system to a reference Fabry–Perot etalon using a Pound–Drever–Hall frequency stabilization scheme are quantified. Frequency offset values in the stabilization point from the reference Fabry–Perot etalon resonance caused by the amplitude modulation are predicted and optimum operating points to minimize residual amplitude modulation are discussed. By operating an electro-refractive phase modulator at the proper bias point, we show that frequency offsets corresponding to less than $3 \times 10^{-3}$ of the reference cavity full-width half-maximum can be achieved.


1. Introduction

Lasers with high frequency stability and low linewidth are at the core of numerous systems deployed in sensing, spectroscopy and metrology. Due to mechanisms such as spontaneous emission [1], shot noise and environmental disturbances (temperature, humidity and pressure fluctuations, flicker noise from current sources, mechanical vibrations), free running lasers often have insufficient long- and short-term stability. Laser output frequency stabilization techniques are widely used to suppress frequency noise and mitigate absolute frequency deviations. One of the most powerful and widespread schemes is the Pound–Drever-Hall (PDH) [2] frequency locking technique, originally developed by R.V. Pound for microwave oscillators [3]. In the PDH technique, a laser is locked to an absolute reference (high finesse cavity, $\pi$-shifted fibre Bragg grating etc.). The reference in
combination with the phase modulation induced sidebands generates a frequency discriminator-
like error signal which drives the laser through a proportional-integral-differential (PID) controller. Extremely low linewidths, down to mHz level, and high absolute frequency stabilities [4]–[9] have been reported using this technique.

The stability of a laser locked with PDH locking can be compromised by the residual amplitude modulation (RAM) of the phase modulated light used in the scheme. RAM causes a well-known problem in PDH locking [10]–[12]. It deteriorates the error signal generated by the PDH feedback system by introducing undesired frequency offsets and spurious signals which may be varying over time. These result in unpredictable variations in the nominal locking point thus reducing the stability. The locking point in the absence of any RAM is fixed at the zero-crossing point in the error signal, however in the presence of RAM this crossing point drifts away from the resonance’s maximum. In practice the acceptable amount of RAM depends on the required absolute frequency stability.

The most common choice for the phase modulator in PDH locking are lithium niobate (LiNbO3) bulk phase modulators which utilize the Pockels effect to induce the desired phase shift. Such modulators can achieve RAM as low as $10^{-4}$ [13] and further reduction down to $10^{-6}$ [14] has been reported with active RAM reduction schemes [12].

The PDH frequency locking technique has been successfully applied on semiconductor lasers [15]–[18]. The phase modulation of PDH locking in these examples was implemented using a bulk lithium niobate based or acousto-optic phase modulators. Leveraging InP-based active-passive integration technology [19], [20] however, one or more semiconductor lasers can be monolithically integrated with its own electro-refractive phase modulator (ERM) on a single chip thus eliminating the necessity for the bulk phase modulators. This configuration reduces the number of discrete components and volume of the system. Integration of more components used in a PDH system e.g., on-chip references and photodetectors (PD) is possible. A laser with an ERM at its output for use in a PDH locking scheme can thus be realized.

Semiconductor ERMs however can suffer from pronounced residual amplitude modulation. While for instance a $10^{-3}$ RAM is considered a rather poor performance for a LiNbO3 modulator, it can be a very difficult to achieve such a number for a semiconductor integrated phase modulator. Such inferior RAM performance arises due to the relatively strong link between phase shift and losses in semiconductors leading to voltage/current dependent losses.

The goal of the present work is to investigate and establish the limits of the accuracy of the locking frequency using PDH technique, determined solely by the RAM of the InP-based ERM monolithically integrated with a semiconductor laser. Only the RAM due to the voltage dependent losses is considered. Other sources of residual amplitude modulation commonly (polarization misalignment to the modulation axis or spurious cavities) discussed in literature in the context of lithium niobate phase modulators are not treated.

In the first part, we briefly describe the integrated InP-based ERM structure and present the measurement of the voltage dependent losses that cause the RAM. At the end of this section we analyze the experimentally observed behavior using a steady state 2D simulation model. The model accounts for the main physical mechanisms from which the voltage dependent behavior of the ERM originates. In the second section, we derive a general expression for the PDH error signal in the presence of a small amplitude modulation at the phase modulator. The frequency offset in the locking point as a function of the amount of amplitude modulation and the spectral width of the reference resonance used for the locking is presented. A link between the measurement results and the parameters used in the general expression is made. In the third section, we assess and discuss the results for determining the limits of using integrated ERMs fabricated by the current technology for an integrated PDH stabilization scheme. More specifically, minimization of frequency offset in the locking point by optimally biasing the ERM is addressed. It should be noted that although we focus on the effect of RAM originating from the phase modulator on PDH stabilization, there are more applications such as frequency modulation spectroscopy [21], [22], cavity ring down spectroscopy [23] and modulation transfer spectroscopy [24] for which pure phase modulation is important.
2. Characterization of Integrated Electro-Refractive Phase Modulators With an On-Chip Laser Source

In this section, we first describe the structure of InP-based ERMs and explain the governing physical processes. Then the characterization method used to measure the RAM is discussed. This is followed by the results for the amplitude modulation and the fitting of the observed behavior to a physical model.

2.1 Electro-Refractive Phase Modulators

In the Smart Photonics [19] InP photonic integration technology platform, an ERM is a ridge waveguide with a horizontal PIN junction and an InGaAs layer and a metallization layer on top. The cross section is illustrated in Fig. 1 where the InGaAsP (Q1.25), p- and n-doped InP layers are indicated along with their corresponding doping levels. The substrate is heavily n-doped \( 10^{19} \) [25]. Multiple n-doped InP layers with constant doping follow. Each layer has a decreasing doping level down to \( 10^{17} \). The guiding Q-layer is not intentionally doped (n.i.d.). Then the p-doping of multiple InP layers increases from \( 10^{17} \) up to \( 10^{19} \). More details about the doping profile can be found in [25]. The intrinsic carrier concentration of the quaternary material in room temperature lattice matched to InP is \( \sim 10^{10} \) [26]. This carrier concentration is about six orders of magnitude lower than the doping concentration of the p- and n-doped layers around the guide layer. We therefore do not expect significant changes of the losses with temperature. The optical mode is mainly confined in the waveguiding Q1.25 film. The electro-optic efficiency at reverse bias voltage operation is \( 12^\circ-15^\circ/V\text{mm} \). This efficiency has been both measured and simulated [19], [25] for TE polarized light. Furthermore, we should stress that only TE polarized light is relevant here. This is due to active quantum well material used for light generation which emits and amplifies mainly polarization mode [27].

The change in effective index and the subsequent phase shift in ERM's is a combination of different field and carrier effects. The strength and relative importance of these effects depend on the applied voltage and the resulting electric field [25]. The field effects are the Pockels and Kerr electro-optic effects and the carrier effects are the plasma and band-filling effects. Propagation losses increase with increasing field strength due to mainly two effects. The first is electro-absorption (Franz-Keldysh effect [28]) because the applied electric field bends the conduction and valence bands thus leading to decreasing the photon-assisted tunneling distance between the valence and conduction band. The second effect is the change in free carrier concentration. The carrier concentration reduces when the reversely biased voltage is applied to the PIN heterostucture structure due to the extension of the carrier depletion zone which mainly lies in the waveguiding layer which is non-intentionally doped. The two effects are a function of wavelength, applied electric field strength, optical power and polarization.
### 2.2 Voltage Dependent Amplitude Modulation Measurement Method

In order to measure the voltage dependent amplitude modulation $\Delta_{AM}(V)$ of the InP-based ERMs we use a gated detection method. The principle of this method is presented in Fig. 2. The laser and ERM are integrated on the same chip as indicated by the dashed rectangle. A short voltage pulse is applied on an ERM while output light from a laser passes through. At the output of the ERM light goes to a photodetector. Optical power variations due to the applied voltage pulse can then be seen as voltage variations in a time trace recorded by a real-time oscilloscope (Fig. 2, blue trace). The $\Delta_{AM}$ is then defined as

$$\Delta_{AM} = \frac{S_{on} - S_{off}}{S_{off}}$$  \hspace{1cm} (1)$$

where $S_{on}$ and $S_{off}$ are the signal levels of the photodetector at the output of the ERM when the applied voltage on the ERM is non-zero and zero respectively.

In (1) we have implicitly considered the optical power that enters the ERM to be stable. In practice however, power variations from the integrated free-running laser can occur. These power fluctuations can be uncorrelated power fluctuations from the laser or variations correlated with the ERM control signal. These correlations can occur due to electrical crosstalk between the biased ERM and the laser due to ERM voltage control since the laser and ERM are integrated on the same chip, and in particular share a common ground on the chip.

To correct for these power fluctuations, the output of the laser that enters the ERM is monitored with a second photodetector as shown in Fig. 2 (orange rectangle). The corrected $\Delta_{AM}$ modulation depth can then be calculated as

$$\Delta_{AM} = \frac{S_{in,on}}{S_{in,off}} - 1$$  \hspace{1cm} (2)$$

where $S_{in,on}$ and $S_{in,off}$ are the output signal levels of the second photodetector (Fig. 2, orange) that detects the input power of the ERM for non-zero and zero voltage applied on the ERM respectively.
2.3 Characterization Setup

For the $\Delta_{\text{AM}}$ measurements we have used an integrated test structure on an InP chip fabricated by Smart Photonics [19] in a multi-project wafer run. The test structure includes a monolithically integrated single mode continuous wave tuneable laser source [29], a 2-by-1 3 dB multimode interference coupler and an ERM is depicted in Fig. 3(a). The laser output is split by the 3-dB splitter. The first part goes through ERM before it is led to the chip facet and the second, used for power fluctuation corrections, is directly coupled out of the chip. This configuration with an ERM at the output of a laser is one that can be used for PDH locking and which eliminates the need for an external bulk phase modulator.

The chip on which the laser and ERM are integrated is mounted on an aluminum sub-mount using a thermally cured electrically conductive adhesive and kept thermally stable at 18 °C using a water-cooler. The emitted laser light is TE polarized due to the gain material. The laser emitted wavelength can be tuned across the C-band in order to observe any wavelength dependent behavior. Tuning is performed by changing the reverse bias voltage of intra-cavity ERMs and keeping the SOA current fixed. The single mode operation of the laser is confirmed using a high resolution optical spectrum analyzer and ensuring good side-mode suppression ratio (>40 dB). The design is the same as in [29] where more information regarding the design and performance can be found. The ERM at the output of the laser is 2 mm long. The ERM length is chosen such that with a relatively low voltage swing the optimum modulation depth (60°) for the PDH locking can be achieved. For a typical ERM efficiency of 15°/Vmm a 2 mm long ERM needs a voltage swing of 2 V. This is well within the specifications of commercially available PDH locking electronics. As the setup schematic in Fig. 3(b) shows light was collected from the two sides of the chip and isolators were used to avoid reflections to the integrated laser. The output waveguides were angled with respect to the anti-reflection coated facets to further suppress reflections and the light was collected using single-mode lensed fibers. The photodetectors used are connected to variable trans-impedance gain required to amplify low voltages.

A 0.5 ms electrical square pulse provided by a waveform generator, with a negative peak voltage and 0.5% duty cycle is applied on the ERM. The ERM remains biased at zero volt for the rest of the time. The light from the two optical outputs of the circuit is led to the two photodetectors. The electrical output of the two photodetectors are both split and led to a real time oscilloscope where the two signals are each recorded using both DC- and AC-coupled ($f_{\text{cut-off}} = 10 \text{ Hz}$) oscilloscope.
channels. This allows for the measurement of the voltage levels with maximum resolution for the ON and OFF states.

An example of the outputs of the two photodetectors for both AC and DC coupled channels is shown in Fig. 4(a) and (b) respectively. The peak-voltage of the applied pulse is 2.5 V. The traces of the direct laser output reveal why the correction with the input power is necessary. There is significant electrical crosstalk between the laser and the ERM at the laser output. To apply (2) and calculate $\Delta AM$ we calculate first the signals $S_{\text{on}}$, $S_{\text{off}}$, $S_{\text{in, on}}$, and $S_{\text{in, off}}$. This is done by reconstructing the voltage at the output of the two photodetectors with maximum resolution from the four oscilloscope channels. Signal $S_{\text{on}}$ is the sum of the voltage level of the DC channel (Fig. 4(b)) while the pulse is OFF and the AC channel (Fig. 4(a)) while the pulse is ON, both for the modulated output (Fig. 4, blue traces). On the other hand $S_{\text{off}}$ is just the voltage level of the DC channel from the modulated output when the pulse is OFF. $S_{\text{in, on}}$ and $S_{\text{in, off}}$ are calculated in the same way but for the direct laser output (Fig. 4, orange traces).

### 2.4 Voltage Dependent Amplitude Modulation of InP-Based Integrated ERM

The $\Delta AM$ as a function of bias voltage for five wavelengths is presented in Fig. 5. All curves follow the same trend. At first, an increase in the reverse bias voltage results in a loss decrease through the ERM. This behavior is attributed to the reduction of photon absorption. Reverse bias operation (below 4–5 V) results in carrier depletion and in turn decrease in concentration of free carriers. The resulting decrease in loss does not show a clear apparent wavelength dependent behavior. The maximum amount of reduction in transmission loss per unit length is about 0.5 dB/cm corresponding to about 2.5% of $\Delta AM$ on a 2 mm long ERM.

At voltages from 3–5 V depending on the wavelength a minimum for the optical losses is observed. At this point electro-absorption starts dominating. Electro-absorption is a wavelength dependent process because the photon-assisted tunneling distance of electrons from valence to conduction band depends on the photon energy. This is most pronounced at the highest voltage (10 V) where the amplitude modulation is higher for shorter wavelengths, indicating increased electro-absorption.
The maximum amplitude modulation is observed at the shortest wavelength (1525 nm) and it is 2.2 dB/cm.

2.5 ERM Voltage Dependent Losses Simulation

In order to verify our explanation of the origin of the ERM voltage dependent losses we have constructed a cross-sectional steady state 2D simulation model using the commercial software Device and Mode by Lumerical, for electrical and optical simulations respectively. The layers’ thicknesses, compositions and doping levels (Fig. 1) for building the phase modulator cross section are chosen initially according to the nominal values from [25]. First the electric field and carrier concentration are calculated along the cross section for different reverse bias voltages.

The electro-absorption losses are then calculated for every point on the discretization grid according to the theoretical electro-absorption model from [30]

\[
a(\omega, E) = \frac{C}{\omega} \frac{E}{E_g - \hbar \omega} e^{-\frac{\eta}{2}}
\]

where \(E\) is the electric field, \(E_g\) is the material bandgap, \(\omega\) is the angular frequency of the light, \(\hbar\) is the reduced Plank’s constant and \(C\) is a scaling parameter of the loss. The parameter \(\eta\) is defined as

\[
\eta = \frac{E_g - \hbar \omega}{\hbar \varphi}
\]

where

\[
\varphi^3 = \frac{e^2 |E|^2}{2\mu_e \hbar}
\]

where \(\mu_e\) is the electron’s effective mass and \(e\) is electron’s charge. All of the above quantities are known or calculated except the scaling parameter \(C\) in (3).

Once the discretization grid contains the electro-absorption losses and carrier concentration for each point, they are imported to the optical solver in which the total losses for the guided mode are calculated. In Fig. 6, the effects from the electro-absorption and the carrier concentration changes are presented both separately (orange and blue dashed lines respectively) and in combination (purple solid line). The measurement points for 1557 nm are the purple markers. Good agreement is obtained. The discrepancy between the simulation and measurements is attributed to uncertainties in the deviation of the doping profile, layer thicknesses and compositions, especially of the guiding
Fig. 6. Change of loss per unit length for 1557 nm. The blue and orange dashed lines are the simulated carrier concentration changes and electro-absorption effects. The purple solid line is the combination of the two effects and the purple markers are the measurement points.

Fig. 7. Pound–Drever–Hall frequency stabilization schematic.

Q-layer and the resulting bandgap, waveguide etching depth and width from the values used in the modelling.

3. Theoretical Investigation and Quantification of the RAM Effect on the PDH Technique

3.1 Expression for Error Signal in the Presence of Residual Amplitude Modulation

A schematic for the PDH locking scheme is shown in Fig. 7. The laser output light from a single mode laser is first phase modulated and then guided to a linear Fabry-Perot cavity. A circulator is used to guide the reflected optical power, of which the largest part is in the phase modulation sidebands, to a photodetector. The electrical signal is then down-converted to baseband by mixing it with the signal from the oscillator driving the phase modulator with the appropriate phase to ensure the maximum amplitude of the down-converted signal. This signal is then fed to the PID controller, the output of which forms the electrical feedback to the frequency control of the laser. The electric field of the ideally purely phase modulated light signal onto the etalon can be written as

\[ E_{\text{mod,ideal}} = E_0 e^{i(\omega t + \beta \sin \Omega t)} \]
where \( E_0 \) is the amplitude of the electric field, \( \omega \) the optical angular frequency of light, \( \beta \) the modulation depth and \( \Omega \) the PDH modulation frequency. Equation (6) can then be written in the form of Bessel functions of the 1st kind as

\[
E_{\text{mod, ideal}} = E_0 \left[ J_0 (\beta) e^{i\omega t} + J_1 (\beta) e^{i(\omega + \Omega) t} - J_1 (\beta) e^{i(\omega - \Omega) t} \right]
\]  

(7)

Here only the first order sidebands are taken into account because for the optimum modulation index for PDH locking \( (\beta = 60^\circ) \) only 4% of the power is located in higher order sidebands. The reflected modulated signal is

\[
E_{r, \text{ideal}} = E_{\text{mod, ideal}} F(\omega)
\]  

(8)

where \( F(\omega) \) is the etalon electric field reflectivity as a function of angular frequency. The electrical output of the PD will then be proportional to \( P_{\text{out}} = E_{\text{in}} E_{\text{in}}^* = |E_{\text{in}}|^2 \) where \( E_{\text{in}} \) is the input electric field of the reflected optical signal \( E_{r, \text{ideal}} \) and the asterisk is used for the complex conjugate. In the ideal case the signal \( \varepsilon_{\text{ideal}} \) that drives the PID controller after the down-conversion is [31]

\[
\varepsilon_{\text{ideal}} = 2\sqrt{(P_c P_s)} K
\]  

(9)

where \( P_c \) and \( P_s \) are the optical power levels at the carrier frequency and first order sideband respectively and \( K \) is defined as

\[
K = \text{Im} \left[ F(\omega) F^*(\omega + \Omega) - F^*(\omega) F(\omega - \Omega) \right]
\]  

(10)

This is the term that shapes the characteristic PDH frequency discriminator with the large slope around the operating point.

In practice however, the phase modulation is not pure. The phase modulated optical field is amplitude modulated as well at the same frequency as the phase modulation. We define \( R \) as the relative depth of the residual optical field amplitude modulation. It is a small dithering of the amplitude of the electric field. Because \( R \) is small the modulated optical field can be written as

\[
E_{\text{mod}} = E_{\text{mod, ideal}} \left[ 1 + R \sin (\Omega t + \Psi) \right]
\]  

(11)

where \( \Psi \) is the phase relative to the phase modulation. By combining (7) and (11) we get

\[
E_{\text{mod}} = E_0 \left[ J_0 e^{i\omega t} + J_1 e^{i(\omega + \Omega) t} - J_1 e^{i(\omega - \Omega) t} + \frac{R J_0}{2j} e^{i(\omega + \Omega + \Psi) t} - \frac{R J_0}{2j} e^{i(\omega - \Omega - \Psi) t} + \frac{R J_1}{2j} e^{i(\omega + \Psi) t} - \frac{R J_1}{2j} e^{i(\omega - \Psi) t} \right]
\]  

(12)

From (12), eight terms are generated in both \( \omega \pm \Omega \) and \( \omega \pm 2\Omega \) frequencies, four in each. After some algebra we can express the electrical signal that is fed to the PID controller as

\[
\varepsilon = \varepsilon_{\text{ideal}} + R (P_c + 2P_s) \Re(L) + R P_s (\Re(M) + \Re(N))
\]  

(13)

where \( L, M \) and \( N \) are defined as

\[
L = F(\omega) F^*(\omega + \Omega) + F^*(\omega) F(\omega - \Omega)
\]  

(14)

\[
M = F(\omega + \Omega) F^*(\omega + 2\Omega)
\]  

(15)

\[
N = F(\omega - \Omega) F^*(\omega - 2\Omega)
\]  

(16)

are terms similar to \( K \) but \( M \) and \( N \) contain 2 \( \Omega \) terms as well. To reach the above expression, \( R^2 \) terms were considered negligible because for small values of \( R \), the assumption \( R^2 \ll R \) is valid. Moreover, similar to (7), only the first order sidebands were considered. Finally, the phase difference \( \Psi \) between phase and amplitude modulation is considered to be zero. This assumption is valid because the electro-optic and carrier effects taking place in the ERM are much faster (<100 ps) [27] than the modulation period (>10 ns). In (13) the error signal is finally expressed as the sum of the error signal in the ideal case (eq. (9)) plus some additional terms which are proportional to \( R \).
Fig. 8. Error signal calculated for different values of residual amplitude modulation depth $R$ and the FWHM of the reference cavity resonance. The blue, orange, yellow and purple curves correspond to $R$ values of 0.0001, 0.001, 0.1 and 0.2 respectively. The solid and dashed curves indicate the two values of 10 (solid) and 1 MHz (dashed). For increasing amount of $R$ the error signal starts to shift thus a DC offset in the locking point is generated.

In Fig. 9 the laser frequency at which the error signal becomes zero and at which the PID controller will lock to, is plotted as a function of the residual amplitude modulation depth $R$ for different FWHM of a resonance while the phase modulation frequency is kept constant (40 MHz). The amount of frequency offset that is introduced is proportional to $R$ but also depends on the ratio of the resonance FWHM and the modulation frequency. As the resonance for the locking becomes more narrow, the effect of the RAM is reduced proportionally.

3.2 Frequency Offset Induced by the Residual Amplitude Modulation

The error signal from (13) is plotted in Fig. 8 as a function of frequency for different values of amplitude modulation depth $R$ (0.001, 0.01, 0.1 and 0.2) for a specific value of $\Omega$ and two specific reference etalons: the modulation frequency is 40 MHz and the full-width half-maximum (FWHM) of the resonance of the cavity $\Delta f_{\text{FWHM}}$ is 10 MHz (solid lines) and 1 MHz (dashed lines). Increasing $R$ causes the error signal to shift on the vertical axis thus creating a DC offset and changing the zero-crossing point. This will cause a lock at a frequency that is different from the center of the resonance peak.

As the resonance for the locking becomes more narrow, the effect of the RAM is reduced proportionally.
The plot in Fig. 9 can be used to determine the acceptable amount of RAM linked to the phase modulation. As an example on how to use this plot, consider a stabilization system which uses a 1 MHz FWHM etalon and a 40 MHz modulation frequency and one has a specification that one cannot accept long term frequency drifts higher than 10 kHz. According to Fig. 9, this frequency offset corresponds to an $R$ of 3%. If this $R$ is exceeded higher frequency offsets will occur. In this case, additional measures should be taken, such as active RAM suppression.

### 3.3 Link Between Voltage Dependent $\Delta_{AM}$ and Relative Depth of the Residual Optical Field Amplitude Modulation

In the above derivation an expression for the error signal as a function of $R$ was derived. The term $R$ however is a perturbation of the propagating electric field which is not directly measurable. For this reason, amplitude modulation thus power fluctuations were measured in Section 2. A direct comparison between theory and measurement of RAM as presented in the previous part cannot be done. An expression that links the two must be derived.

The transfer function of the electric field as a function of voltage travelling through an ERM is

$$T(V(t)) = \sqrt{\frac{P_{out}(V(t))}{P_{out}(V(t))_{V=0}}} = \sqrt{1 + \Delta_{AM}(V(t))} \tag{17}$$

where $V(t)$ is the applied voltage which is a function of time and $P_{out}(V(t))$ is the output optical power which we consider a function of $V(t)$ and $\Delta_{AM}(V(t))$ is a relative change in amplitude due to the voltage dependent losses of the ERM. In fact $\Delta_{AM}(V)$ is exactly what was measured in Section 2. Assuming small values for the $\Delta_{AM}(V(t))$, we can make an approximation using a Taylor expansion in order to simplify calculations. Eq. (17) then becomes

$$T(V(t)) = 1 + \frac{1}{2} \Delta_{AM} (V(t)) \tag{18}$$

The amplitude modulation term $\Delta_{AM}(V(t))$ can be in general an arbitrary function of the applied voltage. More specifically we have fitted the $\Delta_{AM}(V)$ curves from Fig. 5 to a second order polynomial. The time dependent voltage signal applied on the ERM can be expressed as

$$V(t) = V_0 + \Delta V \sin(\Omega t) \tag{19}$$

Since $\Delta_{AM}(V(t))$ has been measured to be small (a couple of percent), we can linearize it around the bias $V_0$ of the ERM and therefore neglecting the harmonics of the sinusoidal signal in (19). The $\Delta_{AM}(V(t))$ change in amplitude then becomes $\Delta_{AM}(V_0 + \Delta V(t))$ and it can be approximated with

$$\Delta_{AM} (V(t)) \approx \Delta_{AM} (V_0) + \frac{1}{2} \left( \frac{\partial \Delta_{AM}}{\partial V} \right)_{V=V_0} \Delta V \sin(\Omega t) \tag{20}$$

The term $\Delta_{AM}(V_0)$ can be disregarded because it is only a DC offset. Eq. (20) is then simplified to

$$\Delta_{AM} (V(t)) \approx \frac{1}{2} \left( \frac{\partial \Delta_{AM}}{\partial V} \right)_{V=V_0} \Delta V \sin(\Omega t) \tag{21}$$

This approximation is only valid for small signal regime. Combining (18) and (21) and comparing with (11) we finally conclude that

$$R = \frac{1}{4} \left( \frac{\partial \Delta_{AM}}{\partial V} \right)_{V=V_0} \Delta V \tag{22}$$

$R$ is calculated using (22) from the measured $\Delta_{AM}$ for the different wavelengths. We calculate and present $R$ as a function of the ERM DC bias for a fixed voltage swing of 2 V which yields the optimum modulation for the 2 mm ERM length. The ERM bias is wavelength dependent as the maxima of the transmission curves in Fig. 5 are also wavelength dependent. The precise DC bias is chosen as the maximum of the fitted second order polynomial for each wavelength. $R$ is presented...
in Fig. 10. The DC bias starts from 1 V in order to keep the ERM reversed biased. $R$ exhibits a monotonic behavior and the zero-crossing points vary for the five wavelengths between 3 V and 4.5 V. The change in sign can be interpreted as a phase change of the sinusoidal signal from (11). The absolute value of $R$ vary from about 1% to 2% depending on the bias. According to Fig. 9 this amount of $R$ corresponds to a frequency offset between 1.5 to 3 kHz for 1 MHz cavity FWHM and modulation frequency of 40 MHz. The bias point can be chosen at the zero-crossing point however therefore $R$ and the consequent frequency offset can be minimized.

4. Estimated Frequency Offsets and Optimum Operation Points

In Section 3, the frequency shift that occurs for a specific amount of $R$ in the PDH stabilization has been quantified and linked directly to the amplitude modulation $\Delta_{AM}$ of the integrated InP-based ERM's measured in Section 2. The residual amplitude modulation may or may not have a significant impact on the absolute stability of the stabilized laser depending on the required absolute stability, the FWHM of the cavity used as reference, the modulation frequency and the operation point (DC bias and voltage swing) of the ERM. Here we will discuss how to utilize the findings for the voltage dependent losses and theoretical investigation for the frequency offsets induced by the RAM depending on the ERM operation.

As presented in Fig. 10, $R$ is wavelength dependent. Therefore the DC bias should be chosen based on the operating wavelength and such that the $R$ is minimized. The choices for the ERM length and voltage swing are linked. It is important that the ERM length should be chosen long enough such that the voltage swing is not much larger than the voltage span in which $\Delta_{AM}$ in Fig. 5 can be considered linear. This is related to the assumptions made for the derivation of (22) and the small signal analysis. In case the voltage swing is not sufficiently low, higher order harmonics which are not treated in our analysis will be generated. In general the ERM length is a design parameter which can vary between tens of microns (technological limitations) to several millimeters (photonic integrated chip size available in a multi-project wafer run). It is also worth noting that due to the relatively low frequencies used the bandwidth of the electro-optic response of the longer ERMs is not an issue. For example, the voltage swing needed for $60^\circ$ with a 2 mm long ERM, for an electro-optic efficiency of $15^\circ$/Vmm, is 2 V. According to the plot in Fig. 10 which is calculated for
this length, the bias point should be \( \sim 4.3 \text{ V} \) for 1567 nm. The frequency offset is then be minimized since \( R \) is also minimized.

5. Conclusion
The voltage dependent losses of InP-based ERM integrated on the same chip with a laser source have been characterized using a gated detection method. The ERM voltage dependent behavior is verified by electro-optical 2D simulation which is found in good agreement. We have theoretically quantified the frequency offset in the PDH locking induced by the RAM of the phase modulator. Using the measurements and theoretical model the amount of RAM induced frequency offset was predicted. E.g. frequency offsets below 3 kHz can be achieved using a reference etalon with 1 MHz FWHM resonance width, 40 MHz modulation frequency, and a 2 mm long ERM operated with 2 V voltage swing of the sinusoidal signal and 2 V DC reverse bias. Therefore a frequency offset of less than \( 3 \times 10^{-3} \) of the reference cavity FWHM should be achievable. By optimizing the ERM bias point to the point of maximum transmission where \( R \) becomes 0, frequency offsets from the required locking point can be reduced even further. The frequency offset in this case is limited by the generation of higher order harmonics due to the non-perfectly symmetrical parabola of \( \Delta AM(V) \). This case is not treated here. This effect however can be reduced by choosing a longer ERM in order to reduce the necessary voltage swing.

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