Combining experiments for linear dynamic network identification in the presence of nonlinearities

Citation for published version (APA):

DOI:
10.1088/1742-6596/1065/21/212026

Document status and date:
Published: 13/11/2018

Document Version:
Accepted manuscript including changes made at the peer-review stage

Please check the document version of this publication:
• A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
• The final author version and the galley proof are versions of the publication after peer review.
• The final published version features the final layout of the paper including the volume, issue and page numbers.

Link to publication

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

• Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
• You may not further distribute the material or use it for any profit-making activity or commercial gain
• You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the “Taverne” license above, please follow below link for the End User Agreement:
www.tue.nl/taverne

Take down policy
If you believe that this document breaches copyright please contact us at:
openaccess@tue.nl
providing details and we will investigate your claim.
Combining Experiments for Linear Dynamic Network Identification in the Presence of Nonlinearities

M. Schoukens\textsuperscript{1}, J.P. Noël\textsuperscript{2}, P.M.J. Van den Hof\textsuperscript{1}

\textsuperscript{1} Control Systems, Eindhoven University of Technology, Eindhoven, The Netherlands.
\textsuperscript{2} Université de Liège, Quartier Polytech 1, Allée de la Découverte 9, 4000 Liège, Belgium

E-mail: m.schoukens@tue.nl

Abstract. In many practical applications it might be desirable to excite only point at a time in an interconnection of multiple dynamic subsystems (e.g. large-scale system). Therefore multiple experiments need to be combined to successfully identify one or more subsystems in the network of subsystems. This papers illustrates how the identification of a linear subsystem of a dynamical network containing one or more nonlinear subsystems can result in biased estimates when multiple experiments are combined using the Best Linear Approximation (BLA) based approach.

1. Introduction

Large scale mechanical systems consisting of many components, the electrical grid, biological systems or industrial plant can be interpreted as the interconnection of multiple subsystems, i.e. a dynamic network setting.

The identification of linear dynamical networks has received quite some attention over the last years focusing on e.g. network structure detection [1, 2, 3], identification of one or more subsystems in the network [3, 4, 5, 6], input selection [7], and multiple noise frameworks [8, 6]. However, the identification of systems operating in a nonlinear dynamic network has received considerably less attention [9, 10, 11, 12].

In many practical applications it might be desirable to excite only one node at a time in a dynamic network, e.g. for safety reasons, limited actuation capabilities, or due to a geographical spread of the different nodes. Therefore multiple experiments need to be combined to successfully identify of one or more subsystems in the network. This papers illustrates how the identification of a linear subsystem of a dynamical network containing one or more nonlinear subsystems can result in biased estimates when multiple experiments are combined using the Best Linear Approximation (BLA) based approach presented in [12].

A short introduction to nonlinear dynamic networks is given in Section 2. The BLA framework is discussed next in Section 3. Section 4 discusses how multiple experiments can be combined to estimate the subsystems in a dynamic network. A simulation example illustrates that effect of the presence of one or more nonlinearities in the dynamic network on the obtained estimates in Section 5.

2. Dynamic Networks

The dynamic networks considered here follow the same definitions and visualization as in [4, 13]. A dynamic network (see Figure 1) consists of a total of \( L \) nodes, representing internal variables of the network, which are interconnected with other nodes by (nonlinear) dynamic systems. A node signal, denoted \( w_i(t) \), is obtained as the sum of the outputs of the incoming (nonlinear)
dynamic subsystems \( y_{ij}(t) \) denotes the output of the subsystem connecting node \( j \) to node \( i \), an external reference signal \( r_i(t) \), and a noise signal \( v_i(t) \): 
\[
w_i(t) = \sum_{j=1}^{L} y_{ij}(t) + r_i(t) + v_i(t).
\]
Only the node signals \( w_i(t) \) and the reference signals \( r_i(t) \) are known.

The node noise signal \( v_i(t) \) is assumed to be zero-mean and to have a finite variance \( \sigma^2_{v_i} \).
Note that only noise at the network nodes are considered. No measurement noise is present in 
the networked system.

3. Best Linear Approximation

The BLA model of a nonlinear system is a linear time-invariant (LTI) approximation of the behavior of that system, best in least squares sense (Figure 2). For the open-loop, single-input single-output case, the BLA is defined as \([14, 15, 16, 17] \): 
\[
G_{bla}(q) = \arg \min_{G(q)} \left\{ |\tilde{y}(t) - G(q)\tilde{u}(t)|^2 \right\},
\]
where, \( \tilde{u}(t) = u(t) - E_u \{u(t)\} \), \( \tilde{y}(t) = y(t) - E_{u,n_y} \{y(t)\} \), and 
\( E_{u,n_y} \{\cdot\} \) denotes the expected value operator taken w.r.t. the random variations due to the 
input \( u(t) \) and the output noise \( n_y(t) \) and \( G(q) \) belongs to the set of all possible LTI systems.
The extension of the BLA framework to the dynamical network setting is presented in \([12] \).

4. Combining Multiple Experiments

The indirect networked identification approach in the frequency domain, using multiple experiments \( M \), is given in practice as follows. First, the FRF from the reference to the node is obtained: 
\[
\hat{S}_{bla}(j\omega) = [R^H(j\omega)R(j\omega)]^{-1}R^H(j\omega)W(j\omega),
\]
where \( R(j\omega) \) denotes the Hermitian operator, and \( R(j\omega), W(j\omega) \) are given by (for a 3-node network):
\[
R(j\omega) = \begin{bmatrix}
R_{11}^{(i)}(j\omega) & 0 & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & R_{33}^{(i,M+1)}(j\omega)
\end{bmatrix}, \quad W(j\omega) = \begin{bmatrix}
W_{11}^{(i)}(j\omega) & W_{12}^{[1]}(j\omega) & W_{13}^{[1]}(j\omega) \\
W_{21}^{[2]}(j\omega) & W_{22}^{[2]}(j\omega) & W_{23}^{[2]}(j\omega) \\
\vdots & \vdots & \vdots \\
W_{M1}^{[M]}(j\omega) & W_{M2}^{[M]}(j\omega) & W_{M3}^{[M]}(j\omega)
\end{bmatrix}. \quad (1)
\]

Note that in the considered setting, only one reference signal is active simultaneously in each experiment, resulting in a sparse matrix \( R(j\omega) \). \( W(j\omega) \), unlike \( R(j\omega) \) is typically not sparse, since, depending on the interconnections that are present in the network, each reference can 
evoke a response at each node in the network. In the purely linear case one could also combine 
the experiments by using the superposition principle.
The nonlinear subsystem $F$ discussed below in Figure 5, this reduces the bias on the estimate of module used to ensure that the nonlinearity is in a similar setpoint for all experiments, as can be observed while the nonlinear dynamic network is excited by reference signals with different power:

Case 3: subsystems connecting to the same node as the nonlinear subsystem, in this case node 3. The estimate (blue) coincides.

A noiseless estimate of the node signals is obtained: $\hat{W}_j^{[i]}(j\omega) = \sum_{k=1}^{n_\omega} \hat{S}_{bla,i,r_j}(q) R_k^{[i]}(j\omega)$. The networked BLA is now obtained in practice as: $\hat{G}_{bla,i}(j\omega) = [K_i^H(j\omega)K_i(j\omega)]^{-1} K_i^H(j\omega)W_i(j\omega)$, where $K_i(j\omega)$, $W_i(j\omega)$, $\hat{G}_{bla,i}(j\omega)$ are given by (the $j\omega$ notation is dropped for compactness):

$K_i(j\omega) = \begin{bmatrix} W_1^{[1]} & \ldots & W_1^{[3]} \\ W_2^{[1]} & \ldots & W_2^{[3]} \\ \vdots & \ddots & \vdots \\ W_{M}^{[1]} & \ldots & W_{M}^{[3]} \end{bmatrix}$. $W_i(j\omega) = \begin{bmatrix} W_i^{[1]} \\ \vdots \\ W_i^{[M]} \end{bmatrix}$. $\hat{G}_{bla,i}(j\omega) = \begin{bmatrix} \hat{G}_{bla,i,1} \\ \vdots \\ \hat{G}_{bla,i,M} \end{bmatrix}$. \hspace{1cm} (2)

5. Simulation Example
The structure of the simulated system is visualized in Figure 1. The linear subsystems $G_{21}$, $G_{32}$ and $G_{13}$ are first order systems of the form:

$$
x_{ij}(t+1) = A_{ij}x_{ij}(t) + B_{ij}w_j(t) \\
w_i(t) = C_{ij}x_{ij}(t),
$$

$A_{21} = 0.9, B_{21} = 1.0, C_{21} = 0.5$
$A_{32} = 0.8, B_{32} = 0.1, C_{32} = 1.0$
$A_{13} = 0.3, B_{13} = 1.0, C_{13} = -0.9$

The nonlinear subsystem $F_{31}$ is given by $w_3(t) = \tanh(w_1(t-1))$, in the linear simulation case discussed below $F_{31}$ is replaced by a unit delay: $w_3(t) = w_1(t-1)$.

The system is excited by three reference signals $r_1(t)$, $r_2(t)$ and $r_3(t)$. These signals are all three random phase multisine signals [17] exciting all the frequencies $|f|/2$ with a flat amplitude spectrum. The random phases are uniformly distributed between 0 and $2\pi$. Only one reference signal is active simultaneously. $M = 20$ realizations of the multisines are applied to the system for each reference signal, each realization contains $P = 2$ steady state periods of $N = 4096$ points per period. The reference signals have each a standard deviation of 0.5. No noise is present in the presented simulation example.

Case 1: the proposed framework is tested on a linear dynamic network. As can be expected from [13, 17], an unbiased estimate is obtained, see Figure 3.

Case 2: the BLA framework is applied on a nonlinear dynamic network. Figure 4 indicates the clear presence of a bias on the estimate of the linear network module $G_{32}$. The estimate of the network module $G_{31}$ represents the linear approximation of the nonlinear system. The observed bias can be explained due to the different setpoints of the nonlinearity for the different experiments: each reference signal excites a different range of the nonlinear module of the network. Combining these measurements can lead to a bias on the estimates of the linear subsystems connecting to the same node as the nonlinear subsystem, in this case node 3.

Case 3: the nonlinear dynamic network is excited by reference signals with different power: while $r_1, r_3$ have a std = 0.5, $r_2$ has a std of 4.5 in this setup. The different reference amplitudes used ensure that the nonlinearity is in a similar setpoint for all experiments, as can be observed in Figure 5, this reduces the bias on the estimate of module $G_{32}$ significantly.
6. Conclusion

While combining experiments for the estimation of dynamic modules in linear dynamic networks still leads to consistent estimates using the BLA-framework, a bias can be introduced on the BLA-framework estimates in case one or more of the network modules behaves nonlinear. This nonlinearity is caused by combining measurements where the nonlinearity operates in a different setpoint, leading to different approximations of this nonlinear behavior.

Acknowledgment

Maarten Schoukens is the holder of a H2020 Marie Skłodowska-Curie European Fellowship: this work has received funding from the European Union’s Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 798627. Furthermore, this work has received funding from the European Research Council (ERC), Advanced Research Grant SYSDYNET, under the European Unions Horizon 2020 research and innovation programme (grant agreement No 694504).

References