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## ***Incorporating temporal and spatial dependencies in a stochastic time-dependent multi-state supernetwork for individual activity-travel scheduling***

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***Abstract:*** Multi-state supernetworks have been expanded in space and time for modeling individual activity-travel scheduling (ATS) behavior. Any path in a time-expanded supernetwork represents an activity-travel pattern (ATP) at a high level of detail. To alleviate the limitation of a deterministic network representation, time uncertainty has been incorporated in multi-state supernetworks. However, the extension unrealistically assumed that all uncertain travel times are time-invariant and spatially independent. In this study, temporal and spatial dependencies among uncertain link travel times are considered in a stochastic time-dependent (STD) context using uncertain support points. ATS under uncertainty is formulated as a path finding problem in a stochastic multi-state supernetwork, given the individual's decision rule, which for illustrative purposes is assumed to be the minimization of expected disutility in this paper. Recursive dynamic programming is applied to find the non-dominated paths, subject to time window constraints at the activity locations.

***Keywords:*** Activity-travel scheduling, uncertainty, temporal and spatial dependencies, support point, multi-state supernetworks.

### **1. Introduction**

Reflecting the increasing interest in activity-based travel demand models that emerged in the late 1990s (Rasouli & Timmermans, 2014), Arentze and Timmermans (2004), and Liao et al. (2010, 2011, 2013) developed a multi-state supernetwork framework for modeling individual activity-travel scheduling (ATS) decisions by interconnecting integrated multimodal transport networks across different activity-travel stages. In a multi-state supernetwork, nodes denote physical locations in space and links connect different nodes at the same state or the same nodes at different states representing travel and state transitions respectively. A desirable feature of a multi-state supernetwork is that any path from a node in the first state to a node in the last state expresses a feasible way of conducting multiple activities possibly using multiple transport modes. Thus, the ATS process in multi-state supernetworks is captured as a path choice through networks of different states, according to some decision rule. Liao (2016) further expanded the spatial multi-state supernetwork representation as a time-dependent bipartite network to incorporate duration choices at the activity locations.

Since the deterministic representation of space and time is unrealistic, Liao et al. (2014) incorporated travel time uncertainty in multi-state supernetworks. However, it was assumed in their study that all uncertain variables are mutually independent and follow normal distributions for algorithmic convenience, ignoring empirical dependencies of uncertain link travel times in time and space. A number of studies have addressed travel time dependencies at the trip level in finding the shortest path in stochastic time-dependent (STD) networks. For example, limited temporal and spatial dependencies have been assumed for adjacent links or nodes (Waller & Ziliaskopoulos, 2002; Fan, et al., 2005). Chen et al. (2012) assumed that the travel time of a link is spatially correlated only with the neighboring links within an 'impact

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area', and that the correlation exists between every two links' stochastic travel speed distributions (Chen et al., 2013). Huang and Gao (2012, 2017) assumed complete dependencies of all link travel times and applied uncertain support points to capture the correlations of random link travel times, where a support point corresponds to a probability and a realization of link travel times of the entire transportation network.

However, these algorithms are not yet applicable to ATS in stochastic multi-state supernetworks. First, space-time constraints and duration choices are not considered, which are essential in ATS. Second, a more realistic and efficient representation of uncertain travel times is needed given that ATS usually has a much longer time frame than single trips. In addition, the principle of first-in-first-out (FIFO) in the STD context, which means that for every link, individuals entering the link earlier cannot arrive at the end of link later (Chen et al., 2013) is not satisfied in activity-travel behavior.

In view of the above, the aim of this study is to incorporate correlated link travel times in a STD multi-state supernetwork for ATS. Uncertain link travel times are represented using support points in multiple time zones to differentiate major travel time periods, extending Gao and Chabini (2006). Based on the state-of-the-art time-expanded multi-state supernetwork representation, duration choice, departure choice, waiting, and space-time constraints are considered in the STD context. The transfers between private vehicles and public transport represented by transition links are also extended in time and space. We apply the flow speed model (FSM) (Sung et al. 2000) to generate correlated link travel times. ATS under uncertainty is formulated as a path-finding problem, dependent on a specific decision rule. For illustrative purposes, in this paper we assume that individuals choose the ATP that has the minimum expected disutility. Dynamic recursive programming formulations are applied to find the non-dominated paths.

The remainder of this paper is organized as follows. The next section defines the STD multi-state supernetwork and the problem definition of ATS. Section 3 presents an improved ATS model incorporating activity duration and departure time/waiting choices in the presence of correlated link travel times. In section 4, a solution algorithm based on the recursive formulations is suggested. Finally, conclusions and plans for future work are given.

#### Notation List

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$G$	a stochastic transport network
$j, k$	nodes of $G$
$ \cdot $	an operator to calculate the number of elements in a set
$\mathbf{t}$	a set of equally divided time intervals within a day $\mathbf{t} = \{1, 2, \dots,  \mathbf{t} \}$
$t$	a discrete time, $t \in \mathbf{t}$
$s$	a state in multi-state supernetwork
$SNK$	a STD multi-state supernetwork
$m$	the $m$ -th element of a time zone set $\mathbf{m}$ , $m = 1, \dots,  \mathbf{m} $ .
$n$	the $n$ -th support point of link travel times/speeds in a time zone
$c_{jk}^{mn}(t)$	travel time of link $(j, k)$ at $t$ , the $n$ -th support point in time zone $m$ .
$\mathbf{r}$	the set of time zone-dependent support point combinations
$r$	a combination of time zone-dependent support points
$p_r$	the probability of the support point $r$ , $\sum_r p_r = 1, r \in \mathbf{r}$
$l_{jk}$	the physical length of travel link $(j, k)$
$v^{mn}(t)$	The time-dependent travel speed in the $n$ -th support point in time zone $m$
$H_0, H_1$	the origin and destination nodes of daily ATPs
$S_\lambda(j, t, r)$	the time expenses of path $\lambda$ departing from $H_0$ and arriving at node $j$ at time $t$ in support point $r$
$D_\lambda(j, t, r)$	the associated disutility of $S_\lambda(j, t, r)$

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$e_\lambda(j, t)$	the expected disutility of path $\lambda$ from the origin to $j$ with arrival time $t$
$a_{jk}^r(t)$	the arrival time at node $k$ by departing $j$ at $t$ in support point $r$
$d_{jk}^r(t)$	the disutility of a link $(j, k)$ in support point $r$
$[u_{o\alpha}, v_{o\alpha}]$	deterministic time window for conducting activity $\alpha$ at location $o$
$[u_H, v_H]$	time window for conducting out-of-home activities
$\beta_w$	the marginal disutility of waiting time
$\tau$	an activity duration choice
$t_w$	the waiting time or a waiting duration choice
$z_{so\alpha}(t, \tau)$	disutility of conducting $\alpha$ at $o$ with start time $t$ and duration $\tau$ at state $s$
$\mathcal{X}(j)$	the set of potential optimal paths from the origin to $j$
$\mathcal{L}_j$	the sub-paths from the origin to $j$
$\Gamma^{+1}(j)$	the set of successor node of node $j$

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## 2. Problem definition

### *Stochastic time-dependent multistate supernetwork*

Activity-travel scheduling (ATS) aims at finding an activity-travel pattern (ATP) and includes the choice of activity sequence, route, activity locations, transport modes and parking locations for conducting an daily activity program (AP) according to some decision-making principle. The multi-state supernetwork is constructed for representing the ATP space, traversing every possible combination of activity and vehicle states. In the multi-state supernetwork, the ATS process is considered as a path choice through networks of different states (Liao et al., 2013). Three states in a multi-state supernetwork are distinguished:

- (1) Activity state: which activities have already been conducted.
- (2) Vehicle state: where the private vehicles are being in used or parked somewhere.
- (3) Activity-vehicle state: the combination of activity and vehicle states.

The nodes in a multi-state supernetwork denote physical locations. Three types of links are defined:

- (1) Travel links: connecting different nodes representing the movement of an individual, staying in the same state.
- (2) Transition links: connecting the same nodes of different transport modes (i.e., parking/picking-up a PV, boarding/ alighting public transport).
- (3) Transaction links: connecting the same nodes of different activity states representing the implementation of activities.

A general supernetwork exclusive of time dimension is represented as Figure 1,  $A_1$  and  $A_2$  are two activities, car and bike are two private vehicles (PVs).  $P_1$  &  $P_2$  and  $P_3$  &  $P_4$  are the parking locations for car and bike respectively, and all PVs must be parked when conducting activities. In Figure 1, columns denote vehicle states and rows denotes activity states. The vertices of PVNs and PTNs are locations of home, activities and parking, and undirected links are bi-directed. Any path from  $H$  to  $H'$  represent a full ATP potentially including multi-modal multi-activity trip chaining. For example, the bold path represents the ATP that the individual leaves home by car with parking car at  $P_2$  and travels through PTN to conduct  $A_1$ , then returns home and rides bike which parking at  $P_4$  for conducting  $A_2$ , and finally returns home.

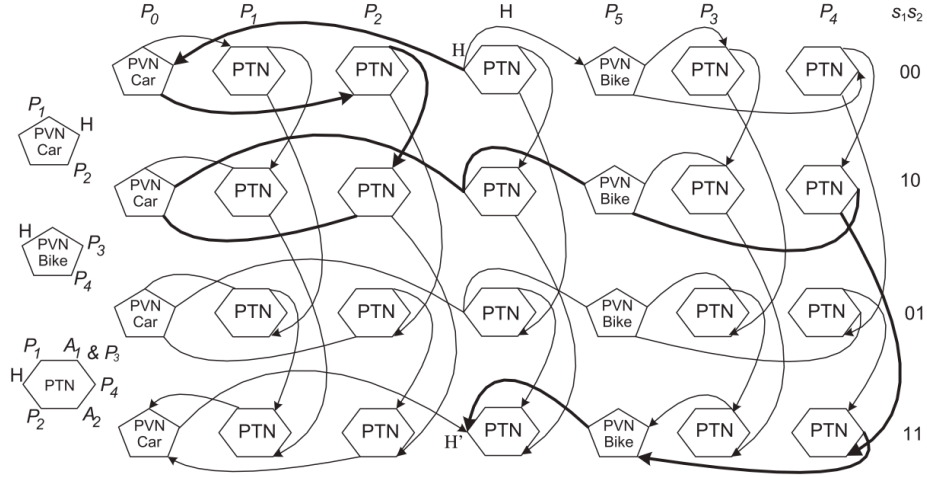


Figure 1 Multistate supernetwork representation (Liao et al., 2013)

In the deterministic representation of the space-time multi-state supernetwork, ATPs in the time-dependent context have deterministic link time expense and disutilities. The link disutility can be defined in a state- and time-dependent way as:

$$disU_{sml} = u(\boldsymbol{\beta}_{sml}, \boldsymbol{x}_{sml}(t)) \quad (1)$$

where  $disU_{sml}$  is the disutility of link  $l$  at state  $s$  with transport mode  $m$  at arrival time  $t$ .  $\boldsymbol{x}_{sml}(t)$  denotes the vector of static or time-dependent link travel times or durations. For the sake of simplicity, a linear additive utility function can be applied. In reality, however, non-linear utility functions better represent the (dis)utility of travel time, cost, activity duration and other attributes influencing activity-travel decisions. Thus, given this assumption, the individual's ATS is to find an ATP with the minimum disutility

$$\min\{disU_{sml}(p_{H_0 \rightarrow H_1})\}, p_{H_0 \rightarrow H_1} \in PATH \quad (2)$$

where  $p_{H_0 \rightarrow H_1}$  denotes a path from  $H_0$  to  $H_1$  in path space  $PATH$ .

Based on STD network  $G$ , a stochastic multi-state supernetwork is constructed for an AP to incorporate uncertainty in the link attributes. Let  $SNK = (\mathbf{v}, \mathbf{e}, \mathbf{t})$  denote the stochastic multi-state supernetwork which is expanded in space-time as a time-dependent network.  $\mathbf{v}$  is a finite set of nodes,  $\mathbf{e}$  is a finite set of links, and  $\mathbf{t}$  is a set of discrete times within a day  $\{1, 2, \dots, |\mathbf{t}|\}$ . The travel times on travel links in a stochastic network are time-dependent random variables. Any path from  $H_0$  to  $H_1$  represents a ATP with stochastic time expense and disutility subject to time window constraints at the locations. Similarly, any path  $\lambda$  encompasses the activity-travel choices and space-time constraints at a high level of detail. Let  $S_\lambda(j, t, r)$  be the time expenses of path  $\lambda$  departing from origin node  $H_0$  and arriving at node  $j$  at time  $t$  support point  $r$ , which covers the entire time frame  $\mathbf{t}$  in a day.  $D_\lambda(j, t, r)$  is the corresponding disutility. The paths departing from origin node  $H_0$  to  $j$  with the expected time expense and disutility, respectively denoted by  $s_\lambda(j, t)$  and  $e_\lambda(j, t)$ . To seek the minimum expected disutility, the ATS problem in the STD multi-state supernetwork is formulated as follows:

$$\min\{e_\lambda(H_1, t)\}, \lambda \in PATH \quad (3)$$

Note that minimum expected disutility is a commonly behavioral principle applied in the scheduling literature, but the principle has limited empirical support. More realistic behavioral principles can be applied in ATS modelling.

### 3. Modelling

In this section, we first represent the STD multi-state supernetwork considering correlated travel speeds/times in multiple time zones. Second, we discuss the incorporation of activity and waiting duration choices with time window constraints. Finally, we propose a solution algorithm based on dynamic programming to find the optimal ATPs.

#### *STD multi-state supernetwork*

Taking into account stochastic dependencies, Huang and Gao (2012) assumed complete dependencies between all travel times of all links at all time periods. They applied a set of support points which can be transformed into joint distributions of link travel times to capture the temporal and spatial dependencies of random variables. This solution is efficient on the trip level that usually has a short time span. When the time span is long, for example, covering a full-day, this assumption necessitates a large set of support points to represent the complete correlations. To avoid this, we suggest support points of multiple time zones of a long time frame, assuming that the support points of a time zone are independent of those in another, which is a realistic assumption (A1). For example, link travel times in the morning peak hours are largely independent of those in the non-peak hours. Formally, suppose a time frame is divided into  $|\mathbf{m}|$  time zones. The  $m$ -th ( $m = 1, \dots, |\mathbf{m}|$ ) time zone covers a time period and has distinct support points of uncertain time-dependent link travel times. Given time zone  $m$ , for travel link  $(j, k)$  in *SNK*, a deterministic value of link travel time  $c_{jk}^{mn}(t)$  is known in the  $n$ -th support point at time  $t \in m$  with probability  $p^{mn}$ ,  $\sum_n p^{mn} = 1$ .

**Assumption A1** The link travel times or travel speeds are mutually independent across multiple time zones.

With **A1**, it is possible to form a set of general support points that covers entire time frame  $\mathbf{t}$ . Let  $r$  be a combination of support points of multiple time zones with probability  $p_r$ ,  $\sum_r p_r = 1$ ,  $r \in \mathbf{r}$ .  $p_r$  can be obtained by multiplying the probabilities of time zone-dependent support points  $p^{mn}$ ,  $\forall m$ . Thus,  $|\mathbf{r}| = \prod_m |\mathbf{n}_m|$ , where  $\mathbf{n}_m$  is the set of support points in time zone  $m$ . In support point  $r$ , the travel times of all travel links at all discrete times are correlated in each time zone and captured by time zone-dependent support points. The flow speed model (FSM) (Sung et al. 2000) is applied to generate the time-dependent link travel times.

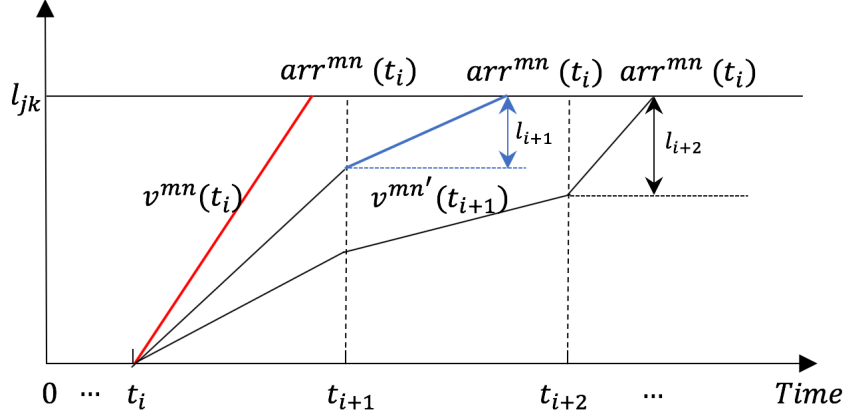
Suppose that the length of link  $(j, k)$  is  $l_{jk}$  and the travel speeds are STD variables taking the  $n$ -th support point in time zone  $m$ . Travel speeds in time intervals  $[t_i, t_{i+1})$  and  $[t_{i+1}, t_{i+2})$  are  $v^{mn}(t_i)$  and  $v^{mn}(t_{i+1})$  respectively, and they are correlated in support point  $n$  of  $m$ ,  $t_i, t_{i+1}, t_{i+2} \in m$ .  $t_i$  is the  $i$ -th discrete time in  $\mathbf{t}$ ,  $i = 0, 1, \dots, |\mathbf{t}| - 1$ .  $arr^{mn}(t_i)$  denotes the arrival time at node  $k$  departing from node  $j$  at time  $t_i$  in the  $n$ -th support point. If  $l_{jk} - v^{mn}(t_i)(t_{i+1} - t_i) < 0$  (see in Figure 2 the red line),  $arr^{mn}(t_i)$  is calculated under FSM as follows:

$$arr^{mn}(t_i) = t_i + \frac{l_{jk}}{v^{mn}(t_i)} \quad (4)$$

If  $arr^{mn}(t_i) \notin [t_i, t_{i+1})$ , the travel speed is updated such that if  $l_{jk} - v^{mn}(t_i)(t_{i+1} - t_i) > 0$ , and the remaining length of the link  $l_{jk} - v^{mn}(t_i)(t_{i+1} - t_i)$  that is traversed after  $t_{i+1}$  is denoted as  $l_{i+1}$ . If  $l_{i+1} - v^{mn}(t_{i+1})(t_{i+2} - t_{i+1}) < 0$  (see in Figure 2 the blue line),  $arr^{mn}(t_i)$  is calculated as:

$$arr^{mn}(t_i) = t_{i+1} + \frac{l_{i+1}}{v^{mn}(t_{i+1})} \quad (5)$$

Following the same logic, the time-dependent link travel times can be obtained.



**Figure 2** Travel along the link with time-varying speeds

The above calculation is under the assumption that  $t_i, t_{i+1}, t_{i+2} \in m$ , since a link may cross multiple time zones,  $t_{i+1}$  may be in the subsequent time zone and the time-dependent link travel speed can be taken at any support point  $n$  for  $\forall m$  and  $\forall t_i \in m$ . For a general representation, let  $m(t_i)$  denote the time zone which  $t_i$  is in, then  $v^{m(t_i)n}$  represents the time-dependent link travel speed at  $n$ -th support point in  $m(t_i)$  with time  $t_i$ . Assuming that  $(j, k)$  starts at  $t_i \in m$  at the  $n$ -th support point, the corresponding link travel time of  $(j, k)$  is  $arr^{mn}(t_i) - t_i$ . Given support points in  $\forall m$  and  $\forall t_i \in m$ , the stochastic time-dependent link travel times can be calculated as follows:

$$c_{jk}^{mn}(t_i) = \begin{cases} \frac{l_{jk}}{v^{mn}(t_i)}, & \text{if } l_{jk} - v^{mn}(t_i)(t_{i+1} - t_i) < 0 \\ t_{i+1} + \frac{l_{i+1}}{v^{m(t_{i+1})n}(t_{i+1})} - t_i, & \text{elseif } l_i - v^{m(t_i)n}(t_i)(t_{i+1} - t_i) < 0 \end{cases} \quad (6)$$

where  $l_{jk}$  is the initial  $l_i$ , for  $\forall m(t_i)$ ,  $l_{i+1} = l_i - v^{m(t_i)n}(t_i)(t_{i+1} - t_i)$ .

#### Activity duration choice

This study assumes that an individual's activity duration choices in the time-dependent context are subject to time window constraints (Liao et al., 2013; Liao, 2016). The duration of conducting activity  $\alpha$  at activity location  $o$  is represented as an individual's choice at arrival time  $t$  under activity state  $s$  by time-expanded transaction links in *SNK*.

Suppose  $[u_{o\alpha}, v_{o\alpha}]$  is the time window of activity location  $o$  for conducting activity  $\alpha$ . The individual has to wait for  $u_{o\alpha} - t$  if the arrival time  $t < u_{o\alpha}$  and is allowed to wait until a later time to start activity  $\alpha$  in the situation  $u_{o\alpha} \leq t < v_{o\alpha}$ . For the individual, waiting for a later departure time to traverse travel links after finishing activities may also cause less disutility. Waiting time choices are represented as start time choices for activity participation or departure time choice for travel.

As link travel times and travel speeds are time-dependent random variables, the arrival times at activity locations are also random variables. We assume (A2) that duration choice and time window constraints for conducting activities of the activity location are deterministic, which

implies that the later the individual arrives at the activity location, the fewer activity duration choices are left.

**Assumption A2** Activity duration choices are deterministic and the number of choice options depends on the arrival time relative to the time window at the activity locations.

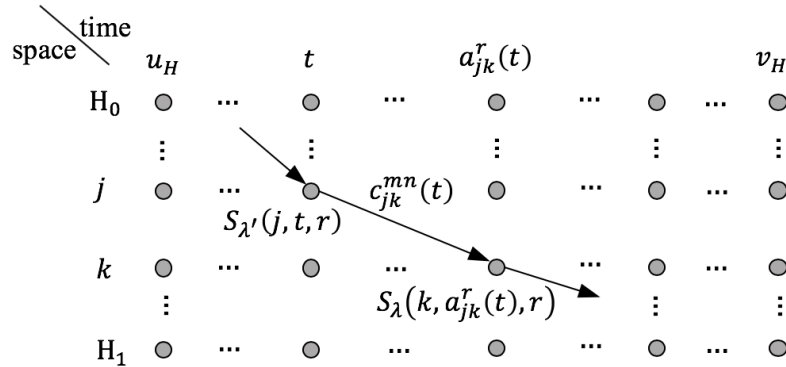
Suppose an individual arrives at activity location  $o$  for  $\alpha$  through travel link  $(j, k)$  with departure time  $t$ . With different support points of link travel times,  $c_{jk}^{mn}(t)$  and  $c_{jk}^{mn'}(t)$ ,  $c_{jk}^{mn}(t) < c_{jk}^{mn'}(t)$ , the arrival times at  $o$  are  $t + c_{jk}^{mn}(t)$  and  $t + c_{jk}^{mn'}(t)$ , where  $n'$  is another support point in time zone  $m$ . With the time window constraints at  $o$ , the activity duration choices may be less for arrival time  $t + c_{jk}^{mn'}(t)$ .

*Recursive formulation of travel links*

$a_{jk}^r(t)$  is the arrival time at node  $k$  in a general support point  $r$  and  $a_{jk}^r(t) = t + c_{jk}^{mn}(t)$ , where  $r$  corresponds to  $n$  and  $m$  at  $t$ . The relationship between the travel times of a path and of its sub-path in support point  $r$  is given by the following recursive formulation:

$$S_\lambda(k, a_{jk}^r(t), r) = S_{\lambda'}(j, t, r) + c_{jk}^{mn}(t) \quad (7)$$

where  $\lambda'$  is the sub-path path departing from origin  $H_0$  and arriving at  $j$  at time  $t \in m$  in support point  $r$ , path  $\lambda$  is constructed by taking sub-path  $\lambda'$  and the value  $c_{jk}^{mn}(t)$  as the link travel time departing from  $j$  to  $k$ . The representation of Equation (7) is shown in Figure 3 in the space-time network.



**Figure 3 Representation of traveling in the space-time network**

Denote  $D_\lambda(\cdot)$  as the travel disutility function of path  $\lambda$  and  $d(\cdot)$  as the disutility function of travel link,  $D_\lambda(k, a_{jk}^r(t), r) = D(S_\lambda(k, a_{jk}^r(t), r))$ . The travel disutility is represented in the form of recursive formulation as:

$$D_\lambda(k, a_{jk}^r(t), r) = D(S_{\lambda'}(j, t, r) + c_{jk}^{mn}(t)) \quad (8)$$

where the disutility travel time on link  $(j, k)$  with time zone-dependent support point  $mn$  in  $r$ , could be represented as  $d(c_{jk}^{mn}(t))$ .

*Recursive formulation of activity links*

Activity scheduling decisions are made under time window constraints that represent the opening hours of facilities and the start/end times of particular events. The disutility of conducting the activity  $\alpha$  with start time  $t$  and duration  $\tau$  at state  $s$  is denoted as  $z_{s\alpha}(t, \tau)$



under time window  $[u_{o\alpha}, v_{o\alpha}]$ . Activity duration choices can be incorporated in *SNK* in a discretized scheme and the duration can be chosen from the finite number of alternatives at discrete time within the feasible time window. Let  $\tau_\alpha$  denote the minimum duration for conducting activity  $\alpha$  and  $\Delta t$  denote the smallest time unit for activity scheduling, Given  $\tau_\alpha$  and  $\Delta t$ , there are  $\frac{v_{o\alpha}-t-\tau_\alpha}{\Delta t}$  time steps from  $t$  to  $v_{o\alpha}$  which defines the maximum number of activity duration alternatives, the corresponding duration choices at time  $t$  for the individual are a set of alternatives  $\{\tau_\alpha + \Delta t, \tau_\alpha + 2\Delta t, \dots, v_{o\alpha} - t\}$  subject to  $u_{o\alpha} \leq t < v_{o\alpha}$ .

As illustrated above, if the individual arrives at  $o$  when  $t < u_{o\alpha}$ , the earlier arrival leads to waiting time  $u_{o\alpha} - t$  until the service at  $o$  is open. The disutility of waiting duration  $t_w$  at location  $o$  is denoted as  $\gamma_{so\alpha}(t, t_w)$ . Assuming that the disutility function of waiting time is linear, the disutility of waiting equals  $\beta_w \times (u_{o\alpha} - t)$ . Since waiting until a later time to start the activity is allowed for an individual, the individual may wait longer even if  $u_{o\alpha} \leq t < v_{o\alpha}$ . The new activity start time is  $t + t_w$  with waiting time  $t_w$ , then there are  $\frac{v_{o\alpha}-t-\tau_\alpha-t_w}{\Delta t}$  activity duration choices to conduct. With the overall waiting time  $t_w$ ,  $\gamma_{so\alpha}(t, t_w)$  can be used to denote the disutility of overall waiting duration and  $z_{so\alpha}(t + t_w, \tau)$  the disutility of conducting the activity when the waiting time making the new start time for activity duration choice  $\tau$  to be  $t + t_w$ . If the arrival time  $t \geq v_{o\alpha}$ , time window constraints do not allow to conduct activity  $\alpha$  at  $o$  at time  $t$ .

By incorporating activity duration and waiting choices in the STD multi-state supernetwork, the disutility of a transaction link  $(j, k)$  at support point  $r$  in a recursive formulation equals:

$$D_\lambda(k, a_{jk}^r(t), r) = \begin{cases} D_\lambda'(j, t, r) + z_{so\alpha}(t, t_w, \tau_{jk}), & \text{if } a_{jk}^r(t) < v_{o\alpha} \\ +\infty, & \text{if } t \geq v_{o\alpha} \text{ or } a_{jk}^r(t) \geq v_{o\alpha} \end{cases} \quad (9)$$

where  $a_{jk}^r(t) = t + t_w + \tau_{jk}$  at support point  $r$ ,  $z_{so\alpha}(t, t_w, \tau_{jk})$  includes the disutility of waiting  $\gamma_{so\alpha}(t, t_w)$  and conducting the activity  $z_{so\alpha}(t + t_w, \tau_{jk})$ .

After activity  $\alpha$  is conducted, due to the uncertainty of link travel times, waiting at the activity location until a better departure time for travel may decrease the overall disutility and making the network non-FIFO, which is represented at the time-expanded nodes in *SNK*

Suppose that  $\alpha$  has been finished at time  $t' \in m$ , an individual has the choice to immediately start to traverse travel link  $(j, k)$  or wait and start at a later time  $t \in m$ . Link travel time is  $c_{jk}^{mn}(t')$  and  $c_{jk}^{mn'}(t)$  given the  $n$ -th and the  $n'$ -th support point in time zone  $m$ , since the link travel times are FIFO consistent,  $t' + c_{jk}^{mn}(t') \leq t + c_{jk}^{mn'}(t)$ . It can be proven that if the disutility function of waiting and travelling is assumed to be linear, waiting before traveling will cause equal or less disutility than departing immediately when the disutility of waiting times is equal or less than travel time, making the network non-FIFO.

Assuming that the disutility  $\gamma_{so\alpha}(t, \Delta t_w)$  of waiting or staying at activity location  $o$  at time  $t$  for duration  $\Delta t_w$  is additive, the waiting duration choices at  $o$  after activity  $\alpha$  is conducted can be represented in a recursive formulation as:

$$D_\lambda(k, t, r) = D_\lambda'(k, t - \Delta t_w, r) + \gamma_{so\alpha}(t - \Delta t_w, \Delta t_w) \quad (10)$$

Based on Liao (2016), activity and waiting duration choices at activity locations in an time-expanded bipartite network can be visualized as in Figure 4.

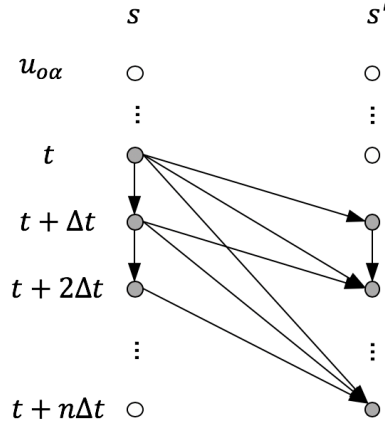


Figure 4 Activity duration and waiting time choices in a space-time network (Liao 2016)

Let  $t = a_{jk}^r(t')$ , Taking into account the travel and activity duration choices, given support point  $r$  and the time window of out-of-home activities is  $[u_H, v_H]$ , the recursive formulation is represented as:

$$D_\lambda(k, t, r) = \begin{cases} D_{\lambda'}(j, t', r) + d_{jk}^r(t'), & t \leq v_H \\ +\infty, & t > v_H \end{cases} \quad (11)$$

$d_{jk}^r(t')$  is the disutility of a travel or transaction link in support point  $r$ , if  $(j, k)$  is a travel link,  $d_{jk}^r(t') = d(c_{jk}^{mn}(t'))$ ,  $t' \in m$ ; if  $(j, k)$  is a transaction link,  $d_{jk}^r(t')$  is the disutility of duration and waiting choices within time window constraints,  $d_{jk}^r(t') = z_{so\alpha}(t', t_w, \tau_{jk})$ ; if waiting is considered for a better time to travel in transaction link,  $d_{jk}^r(t') = z_{so\alpha}(t', t_w, \tau_{jk}) + \gamma_{so\alpha}(t' + t_w + \tau_{jk}, \Delta t_w)$ .

Thus, in order to achieve the objective in Equation (3), the expected disutility is obtained by calculating the expectation over the support points  $r$  for path  $\lambda$  between the origin node and node  $j$  with arrival time  $t$ :

$$e_\lambda(j, t) = \sum_r^r p^r \cdot D_\lambda(j, t, r) \quad (12)$$

#### 4. Algorithm

The standard shortest path algorithms have been adapted for addressing the problems of finding the “shortest” paths in time-dependent or stochastic networks (e.g., Dean, 2004; Miller-Hooks & Mahmassani, 2000). In STD networks where the link travel times are STD variables, Bellman’s principle of optimality is violated and there may exist several non-dominated paths with the minimum expected disutility. By incorporating the activity and waiting duration choices in the time-dependent multi-state supernetwork, the network is non-FIFO. Thus, the standard label-setting algorithm is not applicable in the STD multi-state supernetwork. However, a label-correcting algorithm can be applied for finding the optimal activity-travel schedule in STD multi-state supernetworks with time window constraints.

Taking into account the complete dependency, Huang and Gao (2012) designed an exact label-correcting algorithm to find the optimal pure path with the minimum expected utility (MED). Under the assumption of stochastic dependencies, Huang and Gao (2012) indicated that if only considering the expected travel times for defining non-dominance, the generation of non-

dominated paths set is wrong in the STD network. Thus, the complete time-support point set  $\Omega$  is developed for checking the non-dominance of the paths,  $\Omega = \{(t, r) | t \in m, r \in \mathbf{r}\}, \forall m$ .

**Definition 1** (The dominance rule) A path  $\lambda$  from origin node  $H_0$  to destination node  $H_1$  is non-dominated with respect to subset  $\Omega'$  of  $\Omega$  iff  $\exists$  no other path  $\lambda'$  between the same OD pair such that

$$D_{\lambda'}(H_1, t, r) \leq D_{\lambda}(H_1, t, r), \forall (t, r) \in \Omega'$$

In this study, the uncertain travel speeds are assumed to be spatially and temporally correlated within each time zone and independent with any other zones. Activity duration and waiting duration choices are incorporated under time window constraints. For waiting at the same node in the same state, the labels are self-corrected. When finding the MED-Path for each node  $j$ , backtracking is implemented to check the non-dominated path set. The remaining paths set and the path with the MED can be found for each node  $j$  and each arrival time  $t$ . The departure time at origin node  $H_0$  can be obtained by backtracking the optimal path from  $H_1$ . The pseudo-code of the label-correcting algorithm based on the recursived formulation is given below. Particularly, calculation  $D_{\lambda}(k, t, r)$  involves exploring the full space of the temporal dimension and support points. In *SNK*, the algorithm has an exponential run-time complexity to record the subpaths to any node  $j$ .

- *The pseudo-code of the label-correcting algorithm:*

- 
1. input:  $\beta_w, SNK, [u_{\alpha}, v_{\alpha}], [u_H, v_H]$ , travel speeds at different support points.
  2. initialization:  $\text{ScanList} = \{(H_0, 1)\}$
  3.  $\mathcal{X}(j) \leftarrow +\infty, \mathcal{L}_j \leftarrow +\infty, S_{\lambda}(j, t, r) = +\infty, e_{\lambda}(j, t) = +\infty$  for  $\forall j \in V \setminus \{H_0\}, \forall t, \forall r, \forall \lambda$
  4.  $\mathcal{X}(H_0) = \{1\}, \mathcal{L}_{H_0} = \{1\}, S_{\lambda}(H_0, t, r) = 0, e_{\lambda}(H_0, t) = 0$  for  $H_0 \in V, \forall t, \forall r, \forall \lambda$
  5. scan node and update node labels:
  6. while  $\text{ScanList} \neq \emptyset$
  7.   select the first node-path pair  $(j, \lambda')$  from  $\text{ScanList}$ , and  $\text{ScanList} = \text{ScanList} - \{(j_0, \lambda_0)\}$
  8.   for each  $k \in \Gamma^{+1}(j)$
  9.      $\mathcal{L}_j \leftarrow \lambda'$ , construct a new path  $\lambda$  from  $H_0$  to  $k$
  10.    calculate  $D_{\lambda}(k, t, r), \forall t, \forall r$  in terms of link type of  $(j, k)$  by Eq. (11)
  11.    if  $t < v_H$
  12.     add  $\lambda$  to  $\mathcal{X}(j)$  and check dominance among the set. Remove dominated path.
  13.     if  $\lambda$  is non-dominated by any other path in  $\mathcal{X}(j)$
  14.     add node-path pair  $(k, \lambda)$  to the  $\text{ScanList}$ , and  $\text{ScanList} = \text{ScanList} + \{(k, \lambda)\}$
  15.     end if
  16.    end if
  17.   end for
  18. end while
  19. find the Path with Minimum Expected Disutility (MED)
  20. backtrack the optimal path from  $H_1$  to  $H_0$  to get departure time  $t_{H_0}$ .
- 

The uncertain and correlated link travel times, which are the input of label-correcting algorithms satisfy the FIFO property are generated by function *TravelTime* by applying FSM.

- *The pseudo-code of the FIFO link travel time generation function: TravelTime*

- 
1. Input:  $l_{kj}$  for each travel link  $(j, k) \in E$  in *SNK*, support points of time-dependent link travel speed  $v^{m(t_i)n}, [t_i, t_{i+1}), \forall m(t_i), \forall n, \forall i$ ,
-

- 
2. Arrival time and link travel time calculation:
  3.  $L = l_{kj}$
  4. if  $t(j) \in [t_i, t_{i+1})$
  5.     for each time-dependent  $v^{mn}$
  6.          $L = L - v^{m(t_i)n} \times (t_{i+1} - t(j))$
  7.         while  $L > 0$
  8.              $i = i + 1$
  9.              $L = L - v^{m(t_i)n} \times (t_{i+1} - t_i)$
  10.         end while
  11.          $arr^{mn}(t(j)) = t_{i+1} + L/v^{m(t_i)n}$
  12.          $c_{jk}^{mn}(t(j)) = arr^{mn}(t(j)) - t(j)$
  13.     end for
  14. end if
  15. Return
- 

## 5. Conclusions and future work

This paper addressed the problem of finding the optimal ATP in a STD multi-state supernetwork, subject to time window constraints, in which the uncertain link travel times are temporally and spatially correlated within each time zone and assumed to be independent between time zones. The stochastic dependencies of link travel times are captured by using the FSM model. It generates link travel times given support points of stochastic travel speeds. Dynamic recursive formulations are applied to find the non-dominated paths, from which the ATP with the minimum expected disutility is chosen.

It should be emphasized that this paper reports work in progress. Many aspects need to be elaborated and some principles assumed for simplicity in the current paper will be replaced with empirically more realistic ones. For example, empirical research on decision-making under uncertainty has shown the relative poor empirical performance of the minimization of expected disutility principle (see Rasouli & Timmermans, 2014 for an overview). Thus, the final version of the suggested approach will include a more realistic mechanism for the analysis of individuals' ATS behavior under uncertainty. In addition, we plan to include dependencies between the travel times and activity duration choices, allow for time-varying correlations, and include representations of individuals' ATS behavior under information provision.

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