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Citation for published version (APA):

Huijberts, H. J. C., Moog, C. H., & Pothin, R. (1998). *Input-output decoupling of nonlinear systems by static measurement feedback*. (RANA : reports on applied and numerical analysis; Vol. 9801). Technische Universiteit Eindhoven.

Document status and date:

Published: 01/01/1998

Document Version:

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
- The final author version and the galley proof are versions of the publication after peer review.
- The final published version features the final layout of the paper including the volume, issue and page numbers.

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RANA 98-01
January 1998

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Reports on Applied and Numerical Analysis
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P.O. Box 513
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ISSN: 0926-4507

Input-output decoupling of nonlinear systems by static measurement feedback

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Abstract

This paper gives necessary and sufficient conditions for solvability of the strong input-output decoupling problem by static measurement feedback for nonlinear control systems.

1 Introduction

Since the beginning of the 80's, a lot of progress has been made in the solution of nonlinear synthesis problems via static and dynamic *state* feedback (see the textbooks [9],[12] and the monograph [6] for an overview). In practice however, the usefulness of static state feedback is restricted, since often only a part of the state of the system can be measured. This calls for a theory on the solution of nonlinear synthesis problems by means of static *measurement* feedback.

To our best knowledge, however, there have hardly been any papers that tackle nonlinear synthesis problems via (static or dynamic) measurement feedback. We briefly mention the exceptions. In [10], conditions for controlled invariance of distributions via static output feedback (i.e., the outputs-to-be-controlled are the same as the measured outputs) are given. The

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paper [11] studies the strong input-output decoupling problem via structure preserving static state feedback for Hamiltonian systems (here, output feedback is necessary since a structure preserving state feedback for a Hamiltonian system is automatically an output feedback). In the monograph [1], some partial results on the strong input-output decoupling problem via static and dynamic output feedback are given, while in [2] the same author gives necessary and sufficient conditions for the solvability of the strong input-output decoupling problem via static measurement feedback under the assumptions that the system under consideration is strongly accessible and that certain distributions are involutive. Further, [7],[8] give necessary and sufficient conditions for solvability of the strong input-output decoupling problem via static output feedback. For linear systems, the input-output decoupling problem by static output feedback was solved using a geometric approach in [3], [5], and using a transfer function approach in [14].

The present paper extends the work done in [2],[7],[8] in that on the one hand it removes the assumptions that were made in [2], while on the other hand it allows for static *measurement* feedback where in [7],[8] only static *output* feedback was allowed. Further, in our opinion the present paper gives very concise and elegant conditions for solvability of a problem that has always been considered to be hard to solve. When specialized to static output feedback, this is particularly reflected in the fact that the conditions given in [7],[8] are considerably simplified.

The paper is organized as follows. In Section 2.1 we introduce some notations and analyze the structure of strongly input-output decoupled systems. The problem of strong input-output decoupling by static measurement feedback is stated and solved in Section 2.2. In Section 3, an example illustrates the main result. Concluding remarks may be found in Section 4.

2 Strong input-output decoupling of nonlinear systems

2.1 Structure of strongly input-output decoupled systems

We consider a nonlinear control system Σ of the form

$$\Sigma \begin{cases} \dot{x} &= f(x) + \sum_{i=1}^m u_i g_i(x) =: f(x) + g(x)u \\ y &= h(x) \\ z &= k(x) \end{cases} \quad (1)$$

where $x = \text{col}(x_1, \dots, x_n) \in \mathbb{R}^n$ are local coordinates for the state space manifold M , $u \in \mathbb{R}^m$ denotes the controls, $y \in \mathbb{R}^m$ denotes the outputs-to-be-controlled, and $z \in \mathbb{R}^q$ denotes the measured outputs. We will assume throughout that the vector fields f, g_1, \dots, g_m and the mappings $h : M \rightarrow \mathbb{R}^m$, $k : M \rightarrow \mathbb{R}^q$ are meromorphic.

Let \mathcal{K}_u denote the field of meromorphic functions of $\{x, \{u^{(k)} \mid k \geq 0\}\}$ and define the vector space $\mathcal{E} := \text{span}_{\mathcal{K}_u} \{d\xi \mid \xi \in \mathcal{K}_u\}$. For Σ , one defines in a natural way

$$\begin{aligned} \dot{y} &= \dot{y}(x, u) = \frac{\partial h}{\partial x} [f(x) + g(x)u] \\ y^{(k+1)} &= y^{(k+1)}(x, u, \dots, u^{(k)}) = \\ &\quad \frac{\partial y^{(k)}}{\partial x} [f(x) + g(x)u] + \sum_{i=0}^{k-1} \frac{\partial y^{(k)}}{\partial u^{(i)}} u^{(i+1)} \end{aligned} \quad (2)$$

Note that in this way we have that $dy^{(k)} \in \mathcal{E}$ ($k \in \mathbb{N}$). We define the *relative degrees* r_i ($i = 1, \dots, m$) of Σ by ([4])

$$r_i := \min\{k \in \mathbb{N} \mid dy_i^{(k)} \notin \text{span}_{\mathcal{K}_u} \{dx\}\} \quad (3)$$

If all relative degrees of Σ are finite, we define the *decoupling matrix* $B(x)$ of Σ to be the (m, m) -matrix with entries

$$b_{ij}(x) = \left(\frac{\partial y_i^{(r_i)}}{\partial u_j} \right) (x) \quad (i, j = 1, \dots, m) \quad (4)$$

The system Σ is said to be *input-output decoupled* if each of its inputs influences one and only one of its outputs-to-be-controlled. Σ is said to be *strongly input-output decoupled* if all relative degrees are finite, its decoupling matrix is an invertible diagonal matrix, and

$$\left(\frac{\partial y_i^{(k)}}{\partial u_j} \right) = 0 \quad (i, j = 1, \dots, m; j \neq i; k \geq r_i + 1) \quad (5)$$

Remark 2.1 Note that a strongly input-output decoupled system is input-output decoupled. However, the converse does not need to hold (see [12] for details).

In what follows, we will employ the following notation. Let $\omega = \sum_{i=1}^n \omega_i dx_i + \sum_{i=0}^N \sigma_i du^{(i)} \in \mathcal{E}$, where the functions $\omega_1, \dots, \omega_n, \sigma_0, \dots, \sigma_N$ are in \mathcal{K}_u , and depend on a finite number of time-derivatives of u . We then define

$$\dot{\omega} := \sum_{i=1}^n (\dot{\omega}_i dx_i + \omega_i d\dot{x}_i) + \sum_{i=0}^N (\dot{\sigma}_i du^{(i)} + \sigma_i du^{(i+1)})$$

where $\dot{\omega}_i$ and $\dot{\sigma}_i$ are defined analogously to (2). Further, we define inductively:

$$\omega^{(k)} := (\omega^{(k-1)})^{(1)}$$

From the definition of the relative degrees and (5) it follows that Σ is strongly input-output decoupled if and only if

$$\begin{aligned} dy_i^{(k)} \in \text{span}_{\mathcal{K}_u} \{dx, du_i, \dots, du_i^{(k-r_i)}\} = \\ \text{span}_{\mathcal{K}_u} \{dx, dy_i^{(r_i)}, \dots, dy_i^{(k-1)}\} \quad (i = 1, \dots, m; k \geq r_i) \end{aligned} \quad (6)$$

Based on this characterization, one obtains the following result, of which the proof may be found in [7],[8].

Proposition 2.2 Consider the nonlinear control system Σ in (1), and assume that all its relative degrees are finite. Define the following subspaces of \mathcal{E} :

$$\begin{aligned} \Omega_i &:= \{\omega \in \text{span}_{\mathcal{K}_u}\{dx\} \mid \\ &\quad \forall k \in \mathbb{N} : \omega^{(k)} \in \text{span}_{\mathcal{K}_u}\{dx, dy_i^{(r_i)}, \dots, dy_i^{(r_i+k-1)}\}\} \\ &\quad (i = 1, \dots, m) \end{aligned} \quad (7)$$

Then Σ is strongly input-output decoupled if and only if

$$dy_i^{(r_i)} \in \Omega_i + \text{span}_{\mathcal{K}_u}\{du_i\} \quad (i = 1, \dots, m) \quad (8)$$

■

The subspaces Ω_i defined in (7) may be calculated by means of the following algorithm (cf. [7],[8]):

$$\begin{cases} \Omega_i^0 &:= \text{span}_{\mathcal{K}_u}\{dx\} \\ \Omega_i^{k+1} &:= \{\omega \in \Omega_i^k \mid \dot{\omega} \in \Omega_i^k + \text{span}_{\mathcal{K}_u}\{dy_i^{(r_i)}\}\} \quad (k \in \mathbb{N}) \end{cases} \quad (9)$$

It may now be shown that in fact Ω_i may be identified with a codistribution on M , which is just the annihilator of the supremal controllability distribution contained in $\text{Ker } dy_i$ (cf. [7],[8]). This implies that Ω_i is integrable and invariant under regular static state feedback.

2.2 Strong input-output decoupling via regular static measurement feedback

We next investigate under what conditions there exists a regular static measurement feedback that renders Σ strongly input-output decoupled. We first give a formal definition of our problem.

Definition 2.3 Consider a nonlinear control system Σ of the form (1) and assume that all its relative degrees are finite. Let $x_0 \in M$ be given. Then the *strong input-output decoupling problem via regular static measurement feedback* (SIODPmf) is said to be solvable for Σ around x_0 if there exist a neighborhood $U \subset M$ of x_0 and mappings $\alpha : k(U) \rightarrow \mathbb{R}^m$, $\beta : k(U) \rightarrow \mathbb{R}^{m \times m}$ satisfying

$$|\beta \circ k| \neq 0 \quad \text{on } U \quad (10)$$

such that Σ , together with the measurement feedback

$$Q_{\text{mf}} : u = \alpha \circ k(x) + \beta \circ k(x)v \quad (11)$$

is strongly input-output decoupled on U .

Remark 2.4 To simplify notation in the sequel, we will simply write $u = \alpha(z) + \beta(z)v$ rather than (11) for a static measurement feedback.

In order to come up with conditions for solvability of the SIODPmf, we slightly reformulate the problem into more differential geometric terms. To this end, we define the extended manifold $M^e := M \times \mathbb{R}^m$, with local coordinates (x, u) . Then M is an immersed submanifold of M^e , with the natural immersion $\iota : M \rightarrow M^e$ defined by $\iota(x) = (x, 0)$. Define the following codistributions on M^e :

$$\begin{aligned}\mathcal{X} &:= \text{span}\{dx\} \\ \mathcal{Z} &:= \text{span}\{dz\} \\ \mathcal{U} &:= \text{span}\{du\}\end{aligned}\tag{12}$$

As was mentioned in the last paragraph of the previous subsection, the subspaces Ω_i may be identified with a codistribution on M . If this identification has been made, we have that $\Omega_i^e := \iota_*\Omega_i$ is a codistribution on M^e . A regular static measurement feedback $Q_{\text{mf}} : u = \alpha(z) + \beta(z)v$ may now be interpreted as a coordinate change for M^e of the form

$$\Phi(x, u) := \begin{pmatrix} x \\ \beta(z)^{-1}(u - \alpha(z)) \end{pmatrix}\tag{13}$$

Thus it follows from (8) that, in order to solve the SIODPmf, we need to find a coordinate change $(x, v) = \Phi(x, u)$, with Φ of the form (13), such that

$$\Phi_*dy_i^{(r_i)} \in \Phi_*\Omega_i^e + \text{span}\{dv_i\} \quad (i = 1, \dots, m)$$

The following proposition is the starting point for deriving necessary and sufficient conditions for solvability of the SIODPmf.

Proposition 2.5 *Consider a nonlinear control system Σ of the form (1), and let $x_0 \in M$ be given. Assume that all relative degrees of Σ are finite, and that the codistributions Ω_i ($i = 1, \dots, m$) have constant dimension around x_0 . Then the SIODPmf is solvable for Σ around x_0 if and only if there exists a neighborhood $U^e \subset M^e$ of $(x_0, 0)$ such that:*

$$(i) \quad \dim(\mathcal{X} + \text{span}\{dy_1^{(r_1)}, \dots, dy_m^{(r_m)}\}) = n + m \quad \text{on } U^e\tag{14}$$

(ii) *There exist exact one-forms $d\psi_i \in \mathcal{Z} + \mathcal{U}$ ($i = 1, \dots, m$) such that*

$$dy_i^{(r_i)} \in \Omega_i^e + \text{span}\{d\psi_i\} \quad \text{on } U^e \quad (i = 1, \dots, m)\tag{15}$$

Proof (necessity) It is straightforwardly checked that the decoupling matrix of Σ is invertible if and only if (14) holds. As is well known (see e.g. [12],[9]) this is already a necessary condition for solvability of the strong input-output decoupling problem by regular static state feedback. Thus, (14) is also a necessary condition for solvability of the SDIODPmf. Let $Q_{\text{mf}} : u = \alpha(z) + \beta(z)v$ be a regular static measurement feedback that solves the SDIODPmf

for Σ , and define the diffeomorphism $\Phi : M^e \rightarrow M^e$ by (13). It then follows from (8) that in the new coordinates $(x, v) := \Phi(x, u)$ of M^e we have

$$\Phi_* dy_i^{(r_i)} \in \Phi_* \Omega_i^e + \text{span}\{dv_i\} \quad (i = 1, \dots, m) \quad (16)$$

which is equivalent to

$$dy_i^{(r_i)} \in \Omega_i^e + \text{span}\{d(\Phi^* v_i)\} \quad (i = 1, \dots, m) \quad (17)$$

It now follows from (13) that $d(\Phi^* v_i) \in \mathcal{Z} + \mathcal{U}$, which establishes (15).

(*sufficiency*) Assume that (14),(15) hold. Since $y_i^{(r_i)}$ is affine in u , one may assume without loss of generality that

$$\psi(z, u) = \gamma(z) + \delta(z)u \quad (18)$$

where $\psi := \text{col}(\psi_1, \dots, \psi_m)$. It follows from (14),(15) that

$$\dim(\mathcal{X} + \text{span}\{d\psi_1, \dots, d\psi_m\}) = \dim(\mathcal{X} + \text{span}\{dy_1^{(r_1)}, \dots, dy_m^{(r_m)}\}) = n + m \quad (19)$$

which, together with the fact that $\mathcal{Z} \subset \mathcal{X}$, gives that δ in (18) is invertible. This implies that with the static measurement feedback $Q_{\text{mf}} : u = \delta(z)^{-1}(v - \gamma(z))$ we have for $\Sigma \circ Q_{\text{mf}}$ that $dy_i^{(r_i)} \in \Omega_i + \text{span}\{dv_i\}$ ($i = 1, \dots, m$). Thus, by Proposition 2.5 the SIODPmf is solvable for Σ . \blacksquare

For a codistribution Ω , we let Ω^* denote the maximal integrable codistribution contained in Ω . The following lemma gives a way to check (15).

Lemma 2.6 *Consider on M^e a codistribution $\Omega \subset \mathcal{X}$ and an exact one-form $d\phi \in \Omega + \mathcal{Z} + \mathcal{U}$ satisfying $d\phi \notin \mathcal{X}$. Let $\pi \in \mathcal{Z} + \mathcal{U}$ be such that*

$$d\phi - \pi \in \Omega \quad (20)$$

Then there locally exists an exact one-form $d\psi \in \mathcal{Z} + \mathcal{U}$ satisfying

$$d\phi \in \Omega + \text{span}\{d\psi\} \quad (21)$$

if and only if

$$\dim((\text{span}\{\pi\} + \Omega \cap \mathcal{Z})^*) \geq \dim((\Omega \cap \mathcal{Z})^*) + 1 \quad (22)$$

Proof (*sufficiency*) Assume that (22) holds. Then, by Poincaré's Lemma, there locally exist a function λ , an exact one-form $d\psi \in \mathcal{Z} + \mathcal{U}$, and a one-form $\omega \in \Omega \cap \mathcal{Z}$ such that

$$\pi = \lambda d\psi + \omega \quad (23)$$

By (20), this gives

$$\Omega \ni d\phi - \pi = d\phi - \lambda d\psi - \omega \quad (24)$$

which implies (21).

(necessity) Assume that there exists an exact one-form $d\psi \in \mathcal{Z} + \mathcal{U}$ satisfying (21). Then there exists a function λ such that

$$d\phi - \lambda d\psi \in \Omega \quad (25)$$

Note that $\pi \in \mathcal{Z} + \mathcal{U}$ satisfying (20) is unique modulo $\Omega \cap \mathcal{Z}$. It then follows from (20),(25) that

$$\pi - \lambda d\psi \in \Omega \cap \mathcal{Z} \quad (26)$$

This yields

$$\text{span}\{\pi\} + \Omega \cap \mathcal{Z} = \text{span}\{d\psi\} + \Omega \cap \mathcal{Z} \quad (27)$$

and thus

$$\text{span}\{d\psi\} \in (\text{span}\{\pi\} + \Omega \cap \mathcal{Z})^* \quad (28)$$

Since obviously also $(\Omega \cap \mathcal{Z})^* \subset (\text{span}\{\pi\} + \Omega \cap \mathcal{Z})^*$ and $\text{span}\{d\psi\} \cap \Omega \cap \mathcal{Z} = \{0\}$, this implies (22). \blacksquare

Combining Proposition 2.5 and Lemma 2.6, we obtain the following checkable conditions for solvability of the SIODPmf.

Theorem 2.7 Consider a nonlinear control system Σ of the form (1), and let $x_0 \in M$ be given. Assume that all relative degrees are finite, and that the codistributions Ω_i ($i = 1, \dots, m$) have constant dimension around x_0 . Then the SIODPmf is solvable for Σ around x_0 if and only if there exists a neighborhood $U^e \subset M^e$ of $(x_0, 0)$ such that

$$\dim(\mathcal{X} + \text{span}\{dy_1^{(r_1)}, \dots, dy_m^{(r_m)}\}) = n + m \quad \text{on } U^e \quad (29)$$

$$dy_i^{(r_i)} \in \Omega_i^e + \mathcal{Z} + \mathcal{U} \quad \text{on } U^e \quad (i = 1, \dots, m) \quad (30)$$

$$\dim((\text{span}\{\pi_i\} + \Omega_i \cap \mathcal{Z})^*) \geq \dim((\Omega_i \cap \mathcal{Z})^*) + 1 \quad \text{on } U \quad (i = 1, \dots, m) \quad (31)$$

where $\pi_i \in \mathcal{Z} + \mathcal{U}$ satisfies $dy_i^{(r_i)} - \pi_i \in \Omega_i^e$ ($i = 1, \dots, m$). \blacksquare

Remark 2.8 In [2] also necessary and sufficient conditions for solvability of the SIODPmf were given. In this paper however, the restrictive assumptions that Σ is strongly accessible and that the codistributions $\Omega_i^e \cap \mathcal{Z}$ are integrable were made. These assumptions are not present in Theorem 2.7. Note however that the conditions in [2] are *global*, while the conditions in Theorem 2.7 are *local*.

3 Example

We illustrate the theory developed in the previous section by means of an example. Consider the following system:

$$\Sigma \begin{cases} \dot{x}_1 = x_2 x_3 \\ \dot{x}_2 = x_1^2 \cos x_3 + \sin x_1 x_6 + x_2 x_6 + x_3 + x_6 u_1 \\ \dot{x}_3 = -x_3 \\ \dot{x}_4 = x_5^2 \cos x_4 + e^{x_2 x_6 + x_3} u_1 + u_2 \\ \dot{x}_5 = \sin x_4 \\ \dot{x}_6 = x_1 x_5 \\ y_1 = x_1 \\ y_2 = x_5 \\ z_1 = x_2 x_6 + x_3 \\ z_2 = \sin x_1 x_6 + x_3 \\ z_3 = x_6 \end{cases} \quad (32)$$

Using (9), we obtain

$$\begin{aligned} \Omega_1 &= \text{span}\{dx_1, dx_2, dx_3\} \\ \Omega_2 &= \text{span}\{dx_3, dx_4, dx_5\} \end{aligned} \quad (33)$$

Since $\Omega_1 + \Omega_2 \neq \mathcal{X}$, Σ is not strongly accessible. Further, we have $\Omega_1 \cap \mathcal{Z} = \text{span}\{x_6 dx_2 + dx_3, x_6 \cos x_1 x_6 dx_1 + dx_3\}$, $\Omega_2 \cap \mathcal{Z} = \text{span}\{\cos x_1 x_6 dx_5 + dx_3\}$. It is straightforwardly checked that $(\Omega_i \cap \mathcal{Z})^* = \{0\}$ ($i = 1, 2$), which implies in particular that these codistributions are not integrable. Thus, none of the extra assumptions in [2] are satisfied for Σ .

The relative degrees of Σ are given by $r_1 = r_2 = 2$, and we have

$$\begin{aligned} dy_1^{(2)} &= x_3(2x_1 \cos x_3 dx_1 + z_3 dx_2 - x_1^2 \sin x_3 dx_3 + x_2 dz_3 + d(z_2 + z_3 u_1)) + \\ &\quad (x_1^2 \cos x_3 + z_2 + x_2 z_3 + z_3 u_1 - x_2) dx_3 - x_3 dx_2 \\ dy_2^{(2)} &= -\sin x_4(2x_5^2 \cos x_4 + u_2 + u_1 e^{z_1}) dx_4 + 2x_5 \cos^2 x_4 dx_5 + u_1 \cos x_4 e^{z_1} dz_1 + \\ &\quad \cos x_4 e^{z_1} du_1 + \cos x_4 du_2 \end{aligned}$$

From this, it is readily seen that (29) and (30) hold. Further, we obtain

$$\begin{aligned} \pi_1 &= x_3(x_2 dz_3 + d(z_2 + z_3 u_1)) \\ \pi_2 &= \cos x_4 d(u_1 e^{z_1} + u_2) \end{aligned} \quad (34)$$

We then have

$$\begin{aligned} (\text{span}\{\pi_1\} + \Omega_1 \cap \mathcal{Z})^* &= (\text{span}\{x_2 dz_3 + d(z_2 + z_3 u_1), z_3 dx_2 + dx_3, \\ &\quad x_6 \cos x_1 x_6 dx_1 + dx_3\})^* = \\ (\text{span}\{d(z_2 + z_3 u_1 + x_2 z_3 + x_3), z_3 dx_2 + dx_3, x_6 \cos x_1 x_6 dx_1 + dx_3\})^* &= \\ \text{span}\{d(z_2 + z_3 u_1 + x_2 z_3 + x_3)\} \end{aligned}$$

which implies that (31) holds for $i = 1$. Further, we have

$$\text{span}\{\pi_2\} = \text{span}\{d(u_1 e^{z_1} + u_2)\}$$

which immediately gives that (31) holds for $i = 2$. Thus, the SIODPmf is solvable for Σ . A feedback solving the SIODPmf for Σ is given by

$$\begin{cases} u_1 &= \frac{1}{z_3}(v_1 - z_2 - z_1) \\ u_2 &= v_2 - \frac{e^{z_1}}{z_3}(v_1 - z_2 - z_1) \end{cases}$$

4 Conclusions

A complete solution was given to the strong input-output decoupling problem by static measurement feedback. Its extension to the case where dynamic measurement feedbacks are considered will of course weaken the conditions in Theorem 2.7. A solution of this problem remains a topic for further research, although some partial results are known ([2], [13]).

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