System inversion for sampled-data feedforward control

Citation for published version (APA):

Document status and date:
Published: 01/07/2019

Document Version:
Accepted manuscript including changes made at the peer-review stage

Please check the document version of this publication:

• A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher’s website.
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Download date: 23. Aug. 2019
System Inversion for Sampled-Data Feedforward Control: Balancing On-Sample and Intersample Behavior

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Abstract—Discrete-time system inversion for perfect tracking goes at the expense of intersample behavior. The aim of this paper is the development of a discrete-time inversion approach that improves continuous-time performance by also addressing the intersample behavior. The proposed approach balances the on-sample and intersample behavior and provides a whole range of new solutions, with stable inversion and multirate inversion as special cases. The approach is successfully applied to a motion system. The proposed approach improves the intersample behavior through discrete-time system inversion.

I. INTRODUCTION

Physical systems evolve in continuous time and hence their performance is naturally defined in continuous time. Many approaches for tracking control, including inverse model feedforward and iterative learning control (ILC), are based on system inversion, see, for example [1], [2]. For continuous-time systems, system inversion approaches such as, for example, [3] can be used. However, since controllers are often implemented in a digital environment [4], discrete-time control is often used.

One of the main challenges in system inversion is non-minimum-phase behavior. Causal inversion of nonminimum-phase systems yields unbounded signals. To avoid unbounded signals, many discrete-time inversion approaches have been proposed, see, e.g., [5] for a recent overview. Approximate inversion approaches such as ZPETC, ZMETC, and NPZ-Ignose [1] are well-known, but yield limited performance due to the approximation. Optimal approaches, such as norm-optimal feedforward, $H_2$-preview control, and $H_{\infty}$-preview control [5, Sections 4.3 and 4.4], yield high performance in discrete time. Discrete-time stable inversion [5, Section 4.2] yields exact tracking at the discrete-time samples.

Discrete-time inversion approaches focus on the on-sample performance, i.e., at the discrete-time samples, occasionally resulting in poor intersample behavior, i.e., in between the samples. This holds in particular for zeros close to $z = -1$ [6], see, e.g., [7], [8]. As a consequence, the continuous-time behavior is poor. Indeed, the best on-sample performance does not always give the best continuous-time performance.

Multirate inversion [9], [10], not to be confused with multirate feedforward control [11], provides an interesting alternative to improve intersample behavior by sacrificing on-sample performance. However, the approach does not take into account the system dynamics when balancing the intersample and on-sample performance, i.e., the solution is not optimized for the system at hand. As a consequence, the continuous-time performance is generally sub-optimal.

Although many discrete-time inversion approaches exist, the balance between on-sample performance and intersample behavior is not optimized. The main contribution of this paper is a discrete-time inversion approach that finds the optimal balance between on-sample performance and intersample behavior for the purpose of continuous-time performance. The stable inversion and multirate inversion approaches are recovered as special cases. Related work includes [4], [12], [13] where synthesis-based approaches are presented. In contrast, the approach presented in this paper does not require synthesis.

The outline of this paper is as follows. In Section II, the control objective is formulated. The concept of the proposed approach is presented in Section III. Key ingredients to the approach are presented in Section IV and Section V. Based on these results, the approach is presented in Section VI. The advantages of the approach are demonstrated by application to a motion system application in Section VII. Conclusions are presented in Section VIII.

Notation. For notational convenience, single-input, single-output (SISO) systems are considered. The results can directly be generalized to square multivariable systems. Let $s^{\langle i \rangle} = \frac{d^i}{dt^i}$ denote the $i$th time-derivative of $s$, $\rho$ the Heaviside operator, $B(\cdot)$ a bilinear transformation, and $R_{\geq a} = \{ x \in R^n \mid x[k] > a \text{ for all } k = 0, 1, \ldots, b - 1 \}$. Let $\Sigma = (A, B, C, D)$ be a state-space model and define $T(\Sigma, T) = (TAT^{-1}, TB, CT^{-1}, D)$.

II. PROBLEM FORMULATION

In this section, the control problem is formulated. The considered tracking control configuration is shown in Fig. 1, with reference trajectory $r(t) \in \mathbb{R}$, control input $u(t) \in \mathbb{R}$, output $y(t) \in \mathbb{R}$, digital controller $F$, sampler $S$, and zero-order hold $H$. The continuous-time, linear time-invariant (LTI) system $H_c$ is given by

$$\dot{x}(t) = A_c x(t) + B_c u(t), \quad (1a)$$

$$y(t) = C_c x(t), \quad (1b)$$

with $x(t) \in \mathbb{R}^n$, $n \in \mathbb{N}$ and can be either an open-loop or closed-loop system.

Conventional discrete-time control focuses on the on-sample performance. The discrete-time system $H = SH_c H$
with $H_c$ in (1) and sampling time $\delta$ is given by

$$x[k + 1] = Ax[k] + Bu[k], \quad (2a)$$

$$y[k] = Cx[k], \quad (2b)$$

with $A = e^{A\delta}$, $B = \int_0^\delta e^{A\tau}B\,d\tau$, and $C = C_c$. In this setting, perfect on-sample tracking, i.e., $e[k] = 0$, for all $k$, is achieved for $F = H^{-1}$. However, this does not provide any guarantees for the intersample performance $e(t)$, $t \neq k\delta$. Hence, the continuous-time performance in terms of $e(t)$, for all $t$, may be poor as observed in, e.g., [7].

The control objective considered in this paper is to minimize the continuous-time error $e(t)$. Note that this includes both on-sample ($t = k\delta$) and intersample ($t \neq k\delta$) performance. Importantly, $u[k]$ should remain bounded, even in the presence of nonminimum-phase behavior. Trajectory $r(t)$ is assumed to be known a priori. In the next section, the concept of the proposed approach is presented.

### III. INVERSION FOR ON-SAMPLE AND INTERSAMPLE BEHAVIOR

In the proposed approach, the system is decomposed into two parts and inverted separately as illustrated in Fig. 2, where $H$ is decomposed as $H = H_1H_2$. The inversion of system $H_1$ aims at the intersample behavior. More specific, let $n_1$ be the state dimension of $H_1$, then $H_2$ is inverted such that there is exact state tracking of a desired state $x_1[k]$ every $n_1$ samples. The inversion of $H_2$ aims at the on-sample behavior through perfect output tracking for every sample.

Exact state tracking is experienced to yield good intersample behavior in multirate inversion [9], whereas exact output tracking yields good on-sample behavior in stable inversion [5]. Hence, the choice of the decomposition into $H_1$ and $H_2$ can be used to balance the on-sample behavior and the intersample behavior to the benefit of the continuous-time performance. The idea is conceptually illustrated in Fig. 3. An important observation is that a small on-sample error does not necessarily yield a small continuous-time error. The figure shows that the proposed approach provides a whole range of solutions that were non-existing before. The stable inversion and multirate inversion solution are recovered as the two extreme cases, see also Section VI-B.

The proposed approach requires the decomposition $H = H_1H_2$ in terms of state-space realizations and the desired state $\hat{x}_1[k]$ for $H_1$, see also Fig. 2. In Section IV, the desired state for the continuous-time system $H_c$ is presented. In Section V, the state-space decomposition $H = H_1H_2$ is presented. Based on these results, the complete approach is presented in Section VI.

### IV. DESIRED STATE FOR CONTINUOUS-TIME SYSTEM

In this section, the desired state for the continuous-time system is presented. Given a continuous-time reference trajectory $r(t)$ together with its $n - 1$ time derivatives and system $H_c$ in (1), the objective is to determine a bounded state $\hat{x}(t)$ such that $y(t) = C_c\hat{x}(t)$ yields $y^{(i)}(t) = r^{(i)}(t)$, $i = 0, 1, \ldots, n - 1$, where $(\cdot)^{(i)}$ denotes the $i$th time derivative of $(\cdot)$, i.e., such that $\hat{r}(t) = \hat{y}(t)$ where

$$\hat{r}(t) = \begin{bmatrix} r^{(0)}(t) \\ r^{(1)}(t) \\ \vdots \\ r^{(n-1)}(t) \end{bmatrix}, \quad \hat{y}(t) = \begin{bmatrix} y^{(0)}(t) \\ y^{(1)}(t) \\ \vdots \\ y^{(n-1)}(t) \end{bmatrix}.$$

A similar approach as in [9] is used based on the controllable canonical form given by Lemma 1, see also [14, Section 17.6]. The desired state is given by Theorem 2.

**Lemma 1** (Controllable canonical form). Let the transfer function of $H_c$ in (1) be given by

$$H_c(s) = C_c(sI - A_c)^{-1}B_c = \frac{B(s)}{A(s)}, \quad (4a)$$

$$A(s) = s^n + a_{n-1}s^{n-1} + \ldots + a_0,$$

$$B(s) = b_ms^m + b_{m-1}s^{m-1} + \ldots + b_0,$$

$$...$$

where $A(s)$ and $B(s)$ are polynomials.
Then the controllable canonical form $H_{ccf}(s) = T(H_e, T_{ccf})$ is given by

$$
\dot{x}_{ccf}(t) = A_{ccf}x_{ccf}(t) + B_{ccf}u(t), \quad y(t) = C_{ccf}x_{ccf}(t),
$$

where

$$
\begin{bmatrix}
  A_{ccf} & B_{ccf} \\
  C_{ccf}
\end{bmatrix} =
\begin{bmatrix}
  0 & 1 & 0 & \cdots & 0 & 0 \\
  0 & 0 & 1 & \cdots & 0 & 0 \\
  \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
  -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1} & b_0 \\
  1 & \frac{a_1}{s} & \frac{a_2}{s^2} & \cdots & \frac{a_{n-1}}{s^{n-1}} & 0
\end{bmatrix}
$$

(5c)

Then, the realizations

$$
H_{1f} = \begin{bmatrix} A & \Pi B D^{-1} \\ C & I - \Pi \end{bmatrix}, \quad H_{2f} = \begin{bmatrix} A \\ C(I - \Pi) B \end{bmatrix}
$$

are such that $H = H_{1f} H_{2f}$ in terms of transfer functions, i.e., $C(zI - A)^{-1} B + D = (C(zI - A)^{-1} \Pi B D^{-1} + I)(C(I - \Pi)(zI - A)^{-1} B + D)$.

Proof. Extension of [17, Corollary 11] to $D \neq I$.

If the $D$ matrix in Theorem 4 is singular, a bilinear transformation ([4, Section 3.4]; [15, Section 10.8.3]) can possibly be employed to obtain an equivalent system with nonsingular $D$ matrix. A multiplicative decomposition for the transformed system is obtained through Lemma 3. Applying the inverse transformation on the decomposed system yields the decomposition for the original system since $B(H_1 H_2) = B(H_1)B(H_2)$.

Importantly, Lemma 3 guarantees equivalence in terms of transfer functions, but not in terms of state-space realizations. Indeed, the decomposition of Lemma 3 yields nonminimal realizations of $H_{1f}, H_{2f}$ as both have state dimension $n$. By exploiting the modal form and using a suitable state transformation, the desired state-space decomposition for the proposed approach is obtained as given by Theorem 4.

Theorem 4 (State-space decompositions). Let $T_{mod} \in \mathbb{C}^{n \times n}$ be such that $H_{mod} = T(H, T_{mod}) = (A, B, C, D)$ is in modal form [18, Section 7.4] with nonsingular $D$. Let $H_{1f} H_{2f} = H_{mod}$ be the decomposition given by Lemma 3. Let $T_{per} \in \mathbb{R}^{n \times n}$ be such that

$$
T(H_{1f}, T_{per}) \equiv \begin{bmatrix} A_1 & 0 & B_1 \\ 0 & A_2 & 0 \\ C_1 & C_{1r} & I \end{bmatrix},
$$

$$
T(H_{2f}, T_{per}) \equiv \begin{bmatrix} A_1 & 0 & B_{2r} \\ 0 & A_2 & B_2 \\ 0 & C_2 & D \end{bmatrix},
$$

with $A_1 \in \mathbb{R}^{n_1 \times n_1}, A_2 \in \mathbb{R}^{n_2 \times n_2}, n_1 + n_2 = n$, and define

$$
H_1 \equiv \begin{bmatrix} A_1 \\ C_1 \end{bmatrix}, \quad H_2 \equiv \begin{bmatrix} A_2 \\ C_2 \end{bmatrix}.
$$

Furthermore, let $X \in \mathbb{R}^{n_1 \times n_2}$ satisfy

$$
A_1 X - X A_2 = B_1 C_2.
$$

Then, the state-space realization of $T(H, T_{per}T_{12})$ with

$$
T_{12} = \begin{bmatrix} I_{n_1} & X \\ 0_{n_1 \times n_2} & I_{n_2} \end{bmatrix},
$$
is identical to that of \( H_{mod} \).

**Proof.** The minimal realizations follow from the modal form and \( T_{per} \) in (12) and (13). Furthermore, \( \mathcal{T}(H_1H_2, T_{per}^{-1}T_{12}) = \mathcal{T}(H_1H_2, T_{12}) \mathcal{T}^{-1}(T_{per}) = H_{mod} \).

Theorem 4 yields a state-space decomposition \( H = H_1H_2 \) with identical state-space realizations. Note that such a decomposition always exists. Together with Theorem 2, Theorem 4 forms the basis for the proposed approach presented in the next section.

**Remark 2.** Note that \( V \) in Lemma 3 is related to the poles of \( H \), whereas \( V_\infty \) is related to the zeros of \( H \). Hence, the selection of \( V, V_\infty \) enables to assign the poles and zeros to either \( H_1 \) or \( H_2 \).

### VI. Approach

In the previous sections, preliminary results on the desired state and the state-space decomposition are presented. Based on these results, the proposed approach is presented in this section and special cases are recovered.

#### A. Exploiting stable inversion and multirate inversion

The proposed approach consists of two steps. First, stable inversion is applied to \( H_2 \) in (14) to obtain \( u \) such that \( y_2[k] = u_1[k] \), for all \( k \), see also Fig. 2. The solution is given by Theorem 5 and provides exact output tracking every sample. See [5, Section 4.2] for a proof.

**Theorem 5 (Inversion of \( H_2 \)).** Consider Fig. 2 and let \( H_2^{-1} \) be given by

\[
\begin{bmatrix} x_s[k + 1] \\ x_u[k + 1] \end{bmatrix} = \begin{bmatrix} A_s & 0 \\ 0 & A_u \end{bmatrix} \begin{bmatrix} x_s[k] \\ x_u[k] \end{bmatrix} + \begin{bmatrix} B_s \\ B_u \end{bmatrix} u_1[k],
\]

(17a)

\[
u[k] = \begin{bmatrix} C_s & C_u \end{bmatrix} \begin{bmatrix} x_s[k] \\ x_u[k] \end{bmatrix} + Du_1[k],
\]

(17b)

with \( |\lambda(A_s)| < 1 \) and \( |\lambda(A_u)| > 1 \). Then, \( y_2[k] = u_1[k] \), for all \( k \), if

\[
u[k] = C_s x_s[k] + C_u x_u[k] + Du_1[k],
\]

(18)

which is bounded for bounded \( u_1 \), where \( x_s \) follows from solving

\[
x_s[k + 1] = A_s x_s[k] + B_s u_1[k], \quad x_s[-\infty] = 0
\]

(19)

forward in time and \( x_u \) follows from solving

\[
x_u[k + 1] = A_u x_u[k] + B_u u_1[k], \quad x_u[\infty] = 0
\]

(20)

backward in time.

If \( u_1 \) is bounded, \( u \) in Theorem 5 is bounded by construction of \( x_s, x_u \), even if \( H_2 \) is nonminimum phase. The stable inversion solution in Theorem 5 achieves exact output tracking every sample and has infinite preactuation. Regular causal inversion is recovered as special case if the system is minimum phase, i.e., \( x_u \) is non-existing, see also [5].

Second, multirate inversion is applied to \( H_1 \) in (14) to obtain \( u_1 \). Note that by Theorem 5, \( y_2[k] = u_1[k] \), for all \( k \). The solution is based on lifting the state equation over \( n_1 \) samples. The solution is given by Theorem 6 and provides exact state tracking every \( n_1 \) samples.

**Theorem 6 (Inversion of \( H_1 \)).** Consider Fig. 2 with \( y_2[k] = u_1[k], \) for all \( k \), and let \( \hat{x}_1 \) be the desired state for system \( H_1 \) in (14). Consider the state equation lifted over \( \tau \) samples given by

\[
\hat{x}_1[q + 1] = A_1 \hat{x}_1[q] + B_1 u_1[q],
\]

(21)

with \( A_1 = A_1^n, \quad B_1 = [A_1^{n-1}B_1 \quad A_1^{n-2}B_1 \ldots B_1] \),

\[
u_1[q] = [u_1[kn_1] \quad u_1[kn_1+1] \ldots u_1[(k+1)n_1-1]],
\]

and \( x_1[q] = x_1[kn_1] \). Then, \( \hat{x}_1[q] = \hat{x}_1[q], \) for all \( q \), if

\[
u_1[q] = B_1^{-1} (\hat{x}_1[q + 1] - A_1 \hat{x}_1[q]),
\]

(22)

which is bounded for bounded \( \hat{x}_1 \).

**Proof.** See [9, Section IV.C].

Importantly, the inversion approach in Theorem 6 is based on the continuous-time system \( H_c \), rather than the discrete-time system \( H \). The approach yields exact state tracking, and hence output tracking, every \( n_1 \) samples and has \( n_1 \) samples preactuation. Note that \( u_1 \) is bounded if \( \hat{x}_1 \) is bounded, even if \( H_1 \) is nonminimum phase. More details can be found in, for example, [9], [10]. The desired state \( \hat{x}_1 \) in Theorem 6 is obtained by Procedure 7 which follows from Section IV and Section V.

**Procedure 7 (Desired state of \( H_1 \)).** Given \( H_c \) in (5), \( H \) in (2), and the decomposition \( H = H_1H_2 \) in Theorem 4, the following steps yields the desired state \( \hat{x}_1[k] \) in Theorem 6.

1. Obtain the controllable canonical form \( H_{cc,f} = \mathcal{T}(H_c, T_{cc}) \) according to Lemma 1.
2. Obtain the desired state \( \hat{x}(t) \) of \( H_c \) using Theorem 2.
3. Set the desired state of \( H \) to \( \hat{x}(k\delta) \).
4. Obtain the desired state of \( H_{mod} \), \( \hat{x}_{mod}[k] = T_{mod} \hat{x}[k], \) with \( H_{mod}, T_{mod} \) in Theorem 4.
5. Given \( H_1, H_2 \) in (14), let

\[
H_{12} = H_1H_2 = \begin{bmatrix} A_1 & B_1C_2 \\ C_1 & A_2 \end{bmatrix} \begin{bmatrix} B_1D_2 \\ C_1D_1C_2 \end{bmatrix}
\]

(23)

6. Obtain the desired state of \( H_{12} \), \( \hat{x}_{12}[k] = T^{-1}_{12} T_{per} \hat{x}_{mod}[k], \) with \( T_{12} \) in (16) and \( T_{per} \) satisfying (12) and (13).
7. Obtain the desired state for \( H_1 \), \( \hat{x}_1[k] = I_{n_1} 0_{n_1 \times n_2} \hat{x}_{12}[k]. \)

The combination of the inversion of \( H_2 \) in Theorem 5 and the inversion of \( H_1 \) in Theorem 6 constitutes the control input \( u \) in Fig. 2, which is bounded by design, also for nonminimum-phase systems. The design freedom is in the decomposition of \( H \) into \( H_1 \) and \( H_2 \) in Theorem 4. Equation (23) shows that the output is given by \( y[k] = C_1 x_1[k] + D_1 C_2 x_2[k] \) when either \( D_1 = 0 \) or \( D_2 = 0 \) as \( D = 0 \) in (2). If \( D_1 = 0 \), \( y[k] = C_1 x_1[k] \) and since inversion of \( H_1 \) in Theorem 5 ensures tracking of \( x_1 \) every \( n_1 \) samples, there is perfect output tracking every \( n_1 \) samples. If \( D_1 \neq 0 \), \( y[k] \) also depends on \( x_2[k] \) of \( H_2 \) and since inversion of \( H_2 \) in
Theorem 6 does not provide perfect state tracking, there is no perfect output tracking for \( y \) every \( n_1 \) samples. Hence, to guarantee exact on-sample tracking every \( n_1 \) samples, \( V, V_x \) in Theorem 4 are preferably chosen such that \( D_1 = 0 \).

In summary, input \( u[k] \) in Fig. 1 that minimizes \( e(t) \), in terms of both the intersample and on-sample behavior, is obtained by decomposing \( H \) as in Fig. 2 using Theorem 4, followed by inversion of \( H_2 \) using Theorem 5 and inversion of \( H_1 \) using Theorem 6. In the next section, special cases are recovered.

**Remark 3.** For strictly proper systems \( H_2 \), Theorem 5 can be applied to the bi-proper system \( H_2 \) obtained through time shifts \( H_2 = z^{d_2} H_2 \), where \( d_2 \) is the relative degree of \( H_2 \), see also [5, Remark 1]. If there are eigenvalues on the unit circle, i.e., there exist \( \lambda_i \) such that \( |\lambda_i(A)| = 1 \), then similar techniques as in [16] can be followed.

**Remark 4.** Note that \( B_1 \) in Theorem 6 is the controllability matrix of \( H_1 \) and hence \( B_1^{-1} \) exists if \( H_1 \) is controllable.

**B. Special cases**

The proposed approach provides a whole range of solutions as illustrated in Fig. 3. The stable inversion and multirate inversion solution are recovered as the two extreme cases and given by Corollary 8 and Corollary 9.

**Corollary 8 (Special case: stable inversion [5]).** The stable inversion solution for \( H \) is recovered from the proposed approach in Section VI-A as special case when \( H = H_2 \), i.e., \( H_1 = I \) and \( n_1 = 0 \).

**Corollary 9 (Special case: multirate inversion [9]).** The multirate inversion solution for \( H \) is recovered from the proposed approach in Section VI-A as special case when \( H = H_1 \), i.e., \( H_2 = I \) and \( n_1 = n \).

Importantly, although Theorem 5 yields exact output tracking of \( H_2 \) for every sample, the inversion of \( H_1 \) does not reduce to conventional multirate inversion of \( H_1 \) since the desired state \( \hat{x}_1 \) depends on the full system \( H_c \) and not only on \( H_1 \).

The proposed approach provides a whole range of solutions that were non-existing before, see also Fig. 3. In the next section, the advantages are demonstrated by application to a motion system.

**VII. APPLICATION TO A MOTION SYSTEM**

In this section, the approach proposed in Section VI is applied to a motion system. The results demonstrate the potential of the proposed approach.

**A. Setup**

The motion system is illustrated in Fig. 4(a) and based on the benchmark system in [5]. The continuous-time transfer function from input \( u \) to output \( y \) is given by

$$H_c = \frac{0.3125}{s^2} \frac{s^2 + 15s + 1500}{s^2 + 37.5s + 3750}$$  \hspace{1cm} (24)

and is minimum phase. The discretized system (2) with sampling time \( \delta = 0.02 \) s has the transfer function

$$H = \frac{5.13 \times 10^{-5}(z + 0.842)}{(z - 1)^2} \frac{z^2 - 1.249z + 0.742}{z^2 - 0.5415z + 0.4724}$$  \hspace{1cm} (25)

and is also minimum phase. The Bode diagrams of \( H_c \) and \( H \) are shown in Fig. 4(b). Reference trajectory \( r \) is shown in Fig. 5. The system is controlled in open loop.

**B. Results**

Three different solutions are compared in simulation: the proposed approach with \( n_1 = 2 \), the special case \( n_1 = 0 \), i.e., stable inversion in Corollary 9, and the special case \( n_1 = 4 \), i.e., multirate inversion in Corollary 8. The error signals for the three approaches are shown in Fig. 6.

The results in Fig. 6(a) show that the special case of stable inversion achieves exact tracking every sample, but poor intersample behavior. The results for the special case of multirate inversion in Fig. 6(b) show good intersample behavior, but moderate on-sample behavior since the solution only yields exact tracking every \( n = 4 \) samples.

The results of the proposed approach are shown in Fig. 6(c). The results show good intersample behavior with exact on-sample tracking every \( n_1 = 2 \) samples. Hence, it outperforms the special case of stable inversion in terms of on-sample performance. At the same time, it outperforms the special case of multirate inversion in terms of intersample performance.

The results demonstrate the potential of the proposed approach on a motion system as it outperforms the existing solutions.
A discrete-time system inversion approach is developed that balances the on-sample and intersample behavior for the purpose of continuous-time performance. The multirate inversion and stable inversion approaches are recovered as special cases. Application to a motion system demonstrates the advantages of the proposed approach.

In this paper, a multiplicative decomposition is considered. Ongoing research focuses on an additive decomposition of which preliminary results can be found in [19]. Ongoing research also focuses on extension to non-equidistantly sampled systems [20]. For these systems, which are essentially linear periodically time-varying (LPTV), poor intersample behavior is a well-known problem.

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This work is part of the research programmes Robust Cyber-Physical Systems (RCPS) (No. 12694) and VIDI (No. 15698) which are both (partly) financed by the Netherlands Organisation for Scientific Research (NWO). The work is supported by JSPS KAKENHI Grant Number 18H05902.