# Data Driven Predictive Control Based on Orthonormal Basis Functions

Control Systems group

# Data Driven Predictive Control Based on Orthonormal Basis Functions

Ahmad Alrianes Bachnas, Roland Tóth, Siep Weiland

## Introduction

This work explores the concept of an adaptive *model predictive control* (MPC) scheme based on a flexible predictor model that utilizes *orthonormal basis functions* (OBFs). This model structure offers a trade-off between adaptation of model accuracy in terms of the expansion coefficients and the dynamical structure in terms of the basis functions. We show that this adaptation can maintain desirable control performance. Moreover, since OBF model structures can be seen as a generalization of *finite impulse response* (FIR) model structures, the incorporation of this scheme in FIR-based MPC is straightforward.

## LTI-OBF model parameterization

Given a complete orthonormal basis $\{\phi_i(z)\}_{i=1}^{\infty}$ of $\mathcal{RH}_2$, any stable LTI system with transfer function $F(z) \in \mathcal{RH}_2^{n_y \times n_u}$ can be written as[1]:

$$F(z) = \sum_{i=1}^{\infty} \mathrm{w}_i \phi_i(z), \tag{1}$$

where $\mathrm{w}_i \in \mathbb{R}^{n_y \times n_u}$ is a matrix of expansion coefficients.
The approximation of (1), in terms of a finite truncation, is defined as:

$$F_{\mathrm{OBF}}(z) = \sum_{i=1}^{n_\mathrm{b}} \mathrm{w}_i \phi_i(z). \tag{2}$$

Any such truncation results in an approximation error:

$$||F^{m,n}_{n_\mathrm{b}}||_{\mathcal{RH}_2} := \| F^{m,n}(z) - F^{m,n}_{\mathrm{OBF}}(z) \|_{\mathcal{RH}_2} = \sum_{i=n_\mathrm{b}+1}^{\infty} (\mathrm{w}_i^{m,n})^2, \tag{3}$$

where $m, n$ denotes a specific input-output channel. Arbitrarily low approximation error (3), can be achieved depending on the selection of the OBFs. By using the *Takenaka-Malmquist* functions as the OBFs:

$$\phi_i(z) = \frac{\sqrt{1-|\lambda_i|^2}}{z-\lambda_i} \prod_{j=1}^{i-1} \frac{1-\lambda_j^* z}{z-\lambda_j}, \tag{4}$$

the basis sequence can be generated by the inner-function:

$$G_b(z) = \pm \prod_{j=1}^{n_\mathrm{b}} \frac{1-\lambda_j^* z}{z-\lambda_j}, \tag{5}$$

where $\{\lambda_i\}_{i=1}^{n_\mathrm{b}} \subset \mathbb{D}$ are its poles. In this manner, the error (3) can be minimized by solving *inverse Kolmogov n-width* problem to select the poles of $G_b(z)$[1].

## Adaptability of the model structure

The adaptation goal is to minimize the approximation error of the model $||\tilde{F}^{m,n}_{n_\mathrm{b}}||_{\mathcal{RH}_2}$ in a closed-loop setting in case the underlying system, which is modeled during the commissioning stages, changes into a different system $\tilde{F}(z) \in \mathcal{RH}_2^{n_y \times n_u}$.
Different levels of adaptation are proposed:

1. If $||\tilde{F}^{m,n}_{n_\mathrm{b}}||_{\mathcal{RH}_2} \leq ||F^{m,n}_{n_\mathrm{b}}||_{\mathcal{RH}_2}$, then the adaptation is governed by re-estimating new coefficients $\tilde{\mathrm{w}}_i$.
2. If $||\tilde{F}^{m,n}_{n_\mathrm{b}}||_{\mathcal{RH}_2} \geq ||F^{m,n}_{n_\mathrm{b}}||_{\mathcal{RH}_2}$, but $||\tilde{F}^{m,n}_{n_\mathrm{b}+ln_\mathrm{b}}||_{\mathcal{RH}_2} \leq ||F^{m,n}_{n_\mathrm{b}+ln_\mathrm{b}}||_{\mathcal{RH}_2}$, then the adaptation is conducted by including more OBFs via repeating the filter bank, i.e. $\{\phi_i(z)\}_{i=n_{\mathrm{b}+1}}^{ln_\mathrm{b}} = \{\phi_i(z)\}_{i=1}^{n_\mathrm{b}} G_b^l(z)$.
3. If $l$ in the second level becomes relatively large, the poles of the OBFs are re-selected to maintain low model complexity.

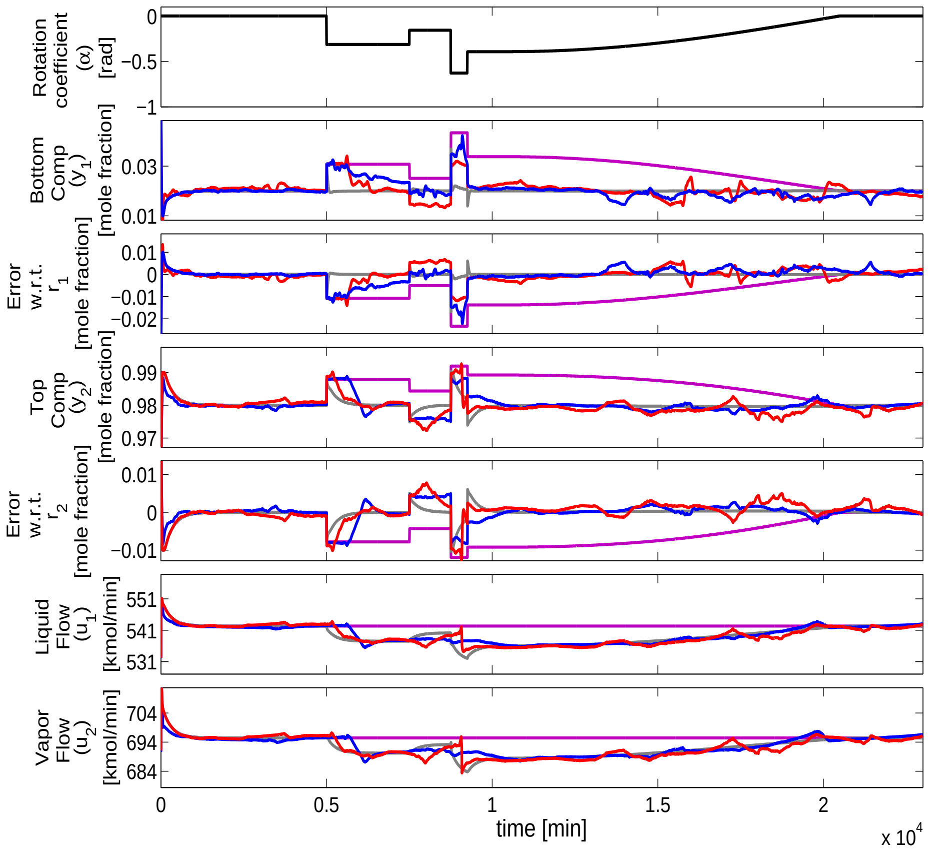


Figure 1: Performance of four different cases of predictive controller under rotation scenario. Fixed MPC (magenta line), Oracle MPC (grey line), MPC-OBF with LS estimation (red line), MPC-OBF with RLS estimation (blue line).

## Data-driven MPC

- Outputs of the predictor model (LTI-OBF):

$$\check{\mathrm{y}}_k^{H_p} = \Xi(k)\Psi x(k) + \Xi(k)\Upsilon u(k-1) + \Xi(k)\Theta\Delta\check{\mathrm{u}}_{k-1}^{H_u-1} \tag{6}$$

- Predictor matrix $\Xi(k)$ contain (re)estimation of $\hat{\mathrm{w}}_i$ per time instant $k$, which is accomplished in the PEM setting.
- OE noise model $\rightarrow$ LS estimator, MLE properties in terms of (2).
- State trajectory $x(k)$ corresponds to filtered input sequences.
- Standard quadratic cost function[2]:

$$V(k) = [\check{\mathrm{y}}_k^{H_p} - \check{\mathrm{r}}_k^{H_p}]^\top Q\, [\check{\mathrm{y}}_k^{H_p} - \check{\mathrm{r}}_k^{H_p}] + [\Delta\check{\mathrm{u}}_{k-1}^{H_u-1}]^\top R\, [\Delta\check{\mathrm{u}}_{k-1}^{H_u-1}], \tag{7}$$

  with reference $(\check{\mathrm{r}}_k^{H_p})$ and control increment sequence $\Delta\check{\mathrm{u}}_{k-1}^{H_u-1}$.
- Calculation of the control sequence under operational constraints $\rightarrow$ Constrained optimization problem$\rightarrow$ QP problem.

## Simulation study

- Tested on a binary distillation column benchmark model where the plant-model mismatch is induced by:

$$G_{\text{new}}(z) = \begin{bmatrix} \cos(\alpha(k)) & -\sin(\alpha(k)) \\ \sin(\alpha(k)) & \cos(\alpha(k)) \end{bmatrix} G(z). \tag{8}$$

- Comparison of four different MPC schemes (see Fig. 1).
- MPC with knowledge/estimation of the plant variations are able to calculate proper control actions to maintain the performance.

## References

[1] P.S.C. Heuberger, P.M.J. Van den Hof, B.Wahlberg, *Modelling and identification with rational orthonormal basis functions*, Springer, Heidelberg, 2005.

[2] J. Maciejowski, *Predictive control with constraints*, Prentice Hall, 2002.