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Deployable structures using non-singular rigid foldable patterns

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Abstract

The opportunities of fully foldable structures are well utilized in some industries, but are only rarely applied in the built environment. In the first part of this study, a number of folding typologies are investigated and their (un)favourable properties are related to the built environment. In the second part, the results of the first part are exploited in order to design an adaptable pavilion.

The conclusion of the first part of the study is that there are multiple folding typologies which can generate a wide variety of forms. However, there is especially one typology with a high potential to translate the folding patterns into real structures. This folding typology is called non-singular, rigid foldable. The advantage of this typology is that the individual surfaces do not bend during the folding motion, while the degrees of freedom (DOF) are only dependent on geometric characteristics of the folding patterns.

The second part of the study uses the results from the first part in order to design an adaptable pavilion. To design this pavilion a variant study is performed with folding patterns which belong to the stable adaptive typology. The variants are compared to each other with respect to their: structural performance, innovative appearance, effective floor space range and ease of transportation as well as deployability.

From this study the final variant is studied in more detail and a structural analysis is performed based on the Eurocode for temporary structures. In this non-linear structural analysis, the structure is modelled in various configurations. The resulting design leads to an innovative pavilion which is able to transform in multiple configurations by only moving the support points, while it is stable for every possible state.

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Keywords: Deployable structures, Textile Skins, Adaptive Systems, Foldable patterns, Temporary structures

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1. Introduction

1.1. Foldable structures in the Built Environment

The opportunities of fully foldable structures are well utilized in some industries, but are only rarely applied in the built environment. In the first part of this study, a number of folding typologies are investigated and their (un)favourable properties are related to the built environment. In the second part, the results of the first part are exploited in order to design an adaptable pavilion. .

1.2. Folding typologies

The authors explored extensively various folding patterns, which show a potential for structural applications [1] and as a consequence these patterns were categorized based on their structural properties and potential for the built environment. This section lists the characteristics of these typologies and gives the individual conclusion [2], [3].

and [5]

Non-rigid foldable patterns

- Huge form freedom due to the deformation of the individual surfaces
- Stresses are introduced by deformation of the surface during the folding process and it is (almost)
- impossible to stabilize due to the flexibility of the surfaces

Rigid and non-rigid foldable patterns

- Rigid foldable for in-plane movement
- Only non-rigid foldable for out-of-plane movement and thus limited adaptability

Singular, rigid foldable patterns

- Can be easy realized with rigid panels and hinges
- Can be stabilized by “freezing” all hinges in the desired configuration

Non-singular, rigid foldable patterns

- Can be easy realized with rigid panels and hinges
- The structure can be stabilized by fixing the supports

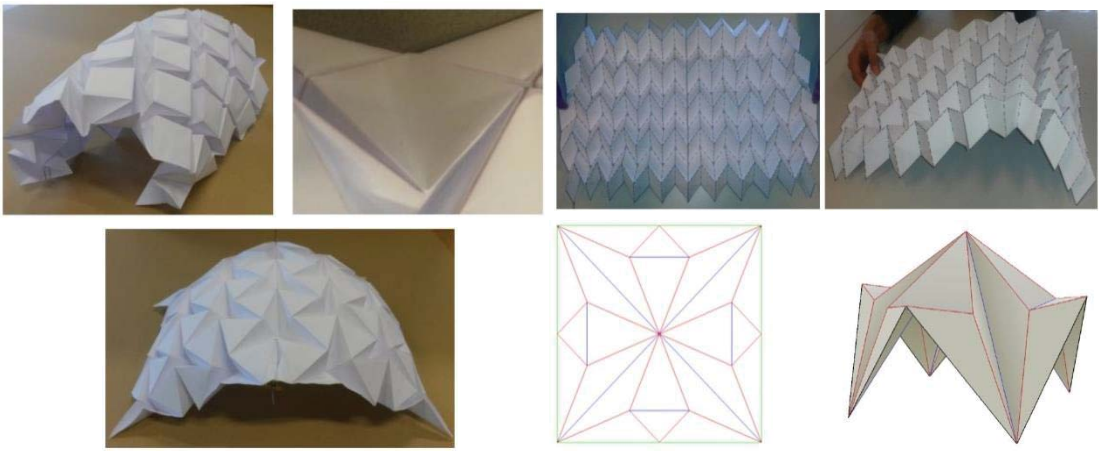

Figure 1: Top left (2x): Non-rigid foldable pattern; Top right (2x): Rigid and non-rigid foldable pattern; Bottom left (1x): Singular, rigid foldable pattern; Bottom right (2x): Non-singular, rigid foldable pattern

2. Folding typologies

2.1. Non-singular, rigid foldable patterns

The difference with singular folding patterns is that non-singular patterns have a clear mid-point. Although non-singular patterns only consist of one base pattern, it is possible to further subdivide the pattern into sub-patterns which are rotated around the mid-point. An example of a non-singular, rigid foldable pattern is shown in Figure 2, which is called the *quod* pattern. For the modelling of the geometries the “Freeform origami” software” by Tachi has been used [4]

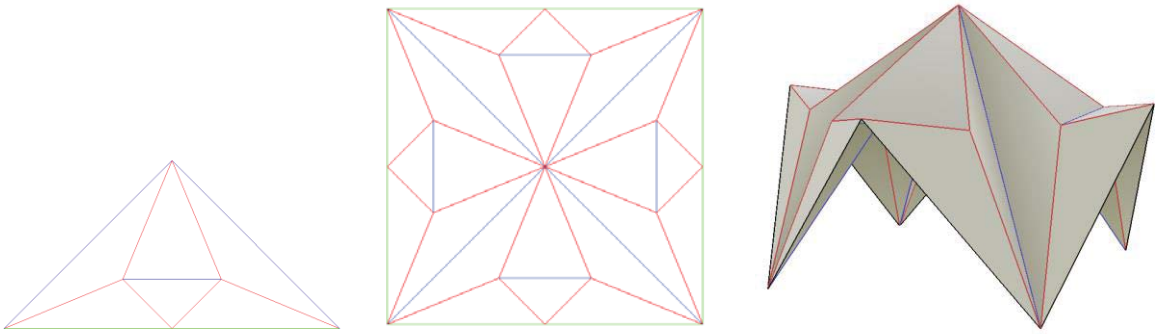

Figure 2. (a) sub-pattern (b) quod patterns (c) folded geometry

From a structural design point of view, rigid foldable patterns have the advantage that it is possible to schematize the geometry as a structure with hinged connected bars. This is done by schematizing the folds as bars and the vertices as ball hinges. The resulting structure consists of bars which are hinged connected to each other in the vertices. A physical model of the schematization of the *quod* pattern is shown in Fig. 3.

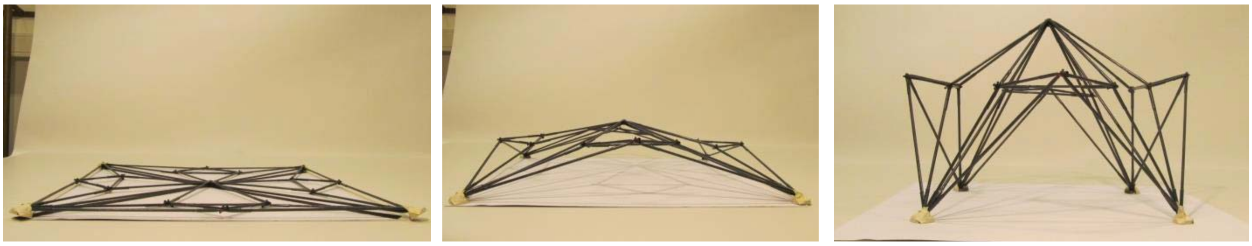

Figure 3. *Hinged connected bar model*, in different folding states

For structures which fully consist of hinged connected bars, it is possible to determine the number of degrees of freedom based on the number of bars and vertices.

Tachi [5], [6] showed that it is possible to calculate the degrees of freedom of structures within this typology by a simplified formula, based on the number of boundary edges of the patterns. This is possible because all folding patterns within this typology are based on triangles whereby a relation is present between the number of folds, vertices and boundary edges. The simplified formula presented by Tachi is shown in equation 1.

$$N_{Eo} - 3 = \text{DOF}$$

$$8 - 3 = 5 \text{ DOF} \qquad (1)$$

N_{Eo} = number of boundary edges

Equation 1: Formula to calculate the DOF of non-singular, rigid foldable structures (Tachi, 2010)

The number of boundary edges is defined as the number of lines along the edges of the patterns which run from intersection point to intersection point. This means that the *quod* pattern consists of 8 boundary edges (two for each side). Substituting this number in formula 1, it follows that the DOF of the adaptable geometry is 5.

When the geometry is rigid, it has to be connected to the surrounding environment, whereby rigid body motion is prevented by adding 6 support components (three for translation and three for rotation). The total number of independent support components required to transform the adaptable *quod* pattern into a stable structure is 11 (5+6).

The advantage of the non-singular, rigid foldable folding typology is that the DOF are only dependent on the number of boundary edges. This means that if the number of boundary edges is not modified, the DOF remain the same, regardless of the internal (triangular) pattern. It is therefore possible to generate an infinite number of patterns with the same number of DOF. Three examples of non-singular rigid folding patterns with 11 DOF are shown in Fig. 4.

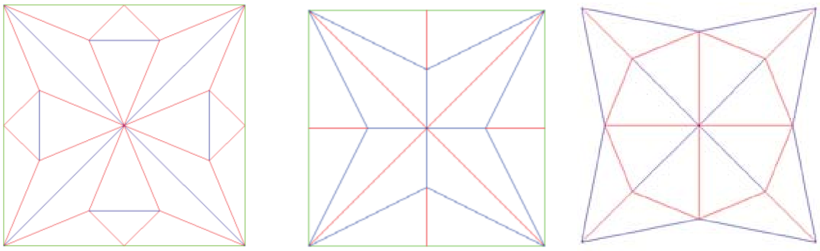

Figure. 4. Example of folding patterns with 8 boundary edges and corresponding 11 DOF

When studying at the folded geometry of the *quod* shown in Fig. 4 it follows that the geometry has four columns where the supports are located in the same XY plane. By pinning these supports, 12 (three for each point) support components are added whereby the adaptable geometry is transformed into a stable (singular statically undetermined) structure because $12 \geq 11$. This means that the global geometry is dependent on the location of the supports while the structure is stabilized by pinning the supports. Structures which possess these properties are referred as *Stable adaptive structures.*

As was described in the previous chapter, the DOF of non-singular rigid folding patterns are dependent on the number of boundary edges. When the DOF of the patterns is known, it is also possible to determine the number of support components needed in order to convert the folding pattern into a stable structure. The minimum number of support components needed is equal to the number of DOF plus 6 (due to rigid body motion).

When the required number of support components is known, they are distributed over the points located on the edge of the pattern. For each pinned point there are three support components added (translation in x-, y- and z direction). This means that also the number of points that have to be pinned can be determined. The derivation of the formula to calculate the number of boundary points that have to be pinned is shown below.

$$N_{Eo} - 3 = \text{DOF}$$

$$\text{DOF} + 6 \leq \text{number of support components}$$

$$\frac{N_{Eo} + 3}{3} \leq \text{number of pinned points}$$

Equation 2: Derivation of formula to calculate the number of points that have to be pinned.

There is a geometrical rule added to equation 2 in order to ensure that all pinned points are located on the same XY plane (the ground) in the folded situation. This geometrical rule holds that half of the vertices at the edge of the pattern consist of support points, and may only be connected to each other with a maximum of two boundary edges.

In other words, the vertices which lie on de boundary edge of the pattern are alternately support points (on the ground level) and non-supported points (located in a higher XY plane). Due to this rule, there are always enough support points to pin, and local instability is prevented because the support components are uniform distributed over the pattern.

2.2. Concept design adaptable pavilion

The final goal of the study is to design an adaptable pavilion for the built environment. The conclusion of this chapter is that non-singular, rigid foldable patterns have a high potential to achieve this goal due to the following reasons:

1. It is possible to construct non-singular, rigid foldable structures with rigid panels and hinges because the individual surfaces do not deform during the folding process. This means that it is possible to use conventional (stiff) building materials to make the panels and hinges.
2. Due to the non-singularity of the folding patterns, the DOF and corresponding support component necessary to stabilize the structure are known.
3. It is possible to relate the number of support components required to stabilize the structure with the number of support points which are still standing on the ground in the folded state. This results in adaptable structures where the global geometry is dependent on the location of the supports, while the structure is stabilized by pinning the supports.

Due to the above mentioned reasons the non-singular, rigid foldable typology has a high potential for application in the built environment as adaptable structure.

Folding patterns create stable adaptive structures if they meet the following geometrical rules:

- The folding pattern fully consists of triangles
- The supports are connected to each other with a maximum of two boundary edges.
- In the unfolded state, there is a maximum of two support points present which lie on the same line. This excludes singular folding patterns.
- The pattern is foldable
- The number of support components is uniform distributed over the support points, with a maximum of three (pinned).
- The total number of independent support components must be equal to or more than E_0 +3,
- E_o= the number of boundary edges

Every folding pattern which complies with the above mentioned rules belongs has the following properties:

- Adaptable by moving the supports
- Stabilized by pinning the supports
- Constructed of rigid panels (or triangular frame) and hinges

Add a photo of the quod in the large state (see my email on thursday) and replace the photo of onle leg up by the more 'extreme ' photo present in the same mail

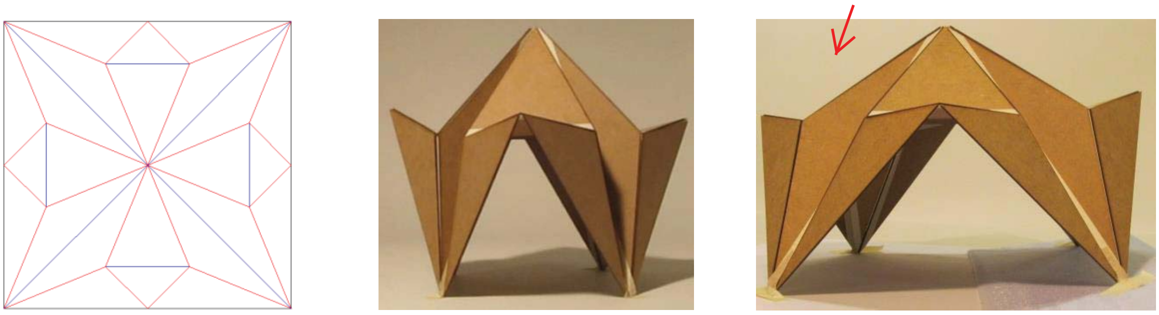

Figure 5 Adaptable geometry (left) changes in a stable adaptive structure (middle and right)

Based on the geometrical rules, it is possible to generate an infinite number of structures which possess these properties. Examples of folding patterns which create *stable adaptive structures* are shown in figure 6. More examples of stable adaptive structures (with their folded geometry) are presented in [1].

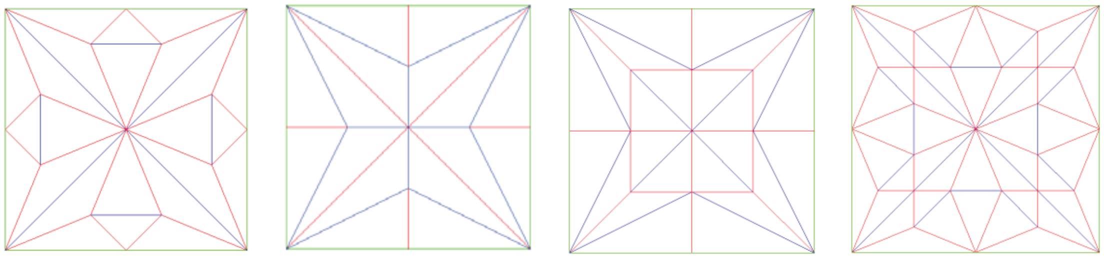

Figure 6 Example of folding patterns which generate stable adaptive structures in the folded state

Besides folding patterns which are based on a totally different pattern, it is also possible to modify already existing folding patterns in the *stable adaptive structures* typology by modifying the underlying base pattern. For instance, by modifying the angles between the folds the length of the folds is changed while the number of boundary edges remains the same, meaning that the structure is still stabilized by pinning the supports.

It is even possible to modify the number of supports by modifying the angle between the boundary edges of the base pattern. For example, the base pattern used in the *quod* pattern can also be used for designing an triangular or pentagon folding pattern as is shown in figure 7.

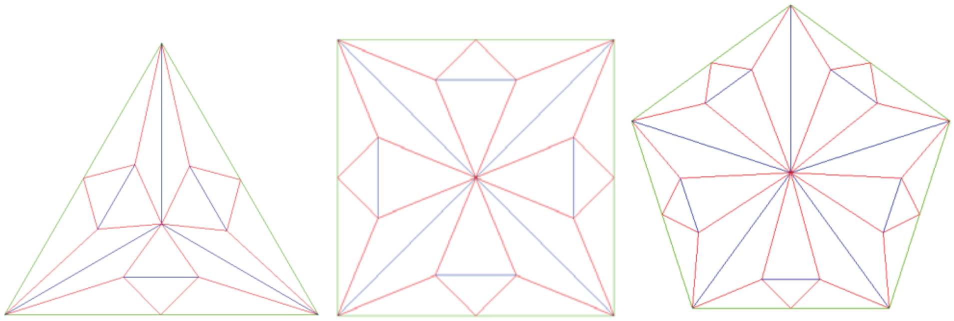

Figure 7 Triangular, quadrilateral and pentagon folding pattern based on ‘the breaking surface’ base pattern

The folding patterns shown in figure 7 are all stable when the supports are pinned. The only difference between the patterns is whether the structures are statically determined or undetermined as is shown in table 1.

Polygon	**Number of boundary edges (polygon x2)**	**Needed support components $(N_{EO} +3)$**	**Number of supports**	**All supports pinned**
Triangle	6	9	3	Statically determined (3x3=9)
Quadrangular	8	11	4	1 statically undetermined (4x3=12)
Pentagon	10	13	5	2 statically undetermined (5x3=15)

Table 1: Properties of triangular, quadrilateral and pentagon folding pattern based on ‘the breaking surface’ base pattern

This example shows that it is possible generate an infinite number of stable adaptive structures based on only one base pattern. The possibilities to modify the base patterns are explained in more detail in [1].

3. Adaptable pavilion

The results from the previous chapters are utilized to design an adaptable pavilion. The pavilion will be used for promotional purposes at festivals, sport events and product launches. This function is chosen due to the special combination of appearance and properties of the structures, which are part of the stable adaptive typology. It is the intention that the unique appearance will draw the audience attention, while the properties are favourable for the owners to transport, deploy and modify for changing demands. The design criteria for the adaptable pavilion are listed below:

Innovative appearance
The appearance of the adaptable pavilion is an important criterion for the adaptable pavilion for two reasons. Firstly, the pavilion has a promotional function, meaning that the primary function of the pavilion is to draw the audience attention. Secondly, an innovative appearance corresponds to the innovative principles on which the structure is based.

Adaptable floor space
The main advantage of structures within the stable adaptable typology is that they can vary the dimensions, including floor space and dependent on location and time.

Transportable & Deployable
An additional advantage of this type of structures is that it is possible to transport and assembled them in a 2d plane (because they originate from a folding pattern). For this reason the packaging volume is limited and the assembly process is simplified to a 2-dimensional structure.

3.1. Variants

Based on the geometrical rules described in the previous sections, it is possible to generate a wide variety of folding patterns which belong to the stable adaptive typology. Even though the patterns are based on the same geometrical rules, the appearances can significantly differ. Four examples are shown in Fig. 8.

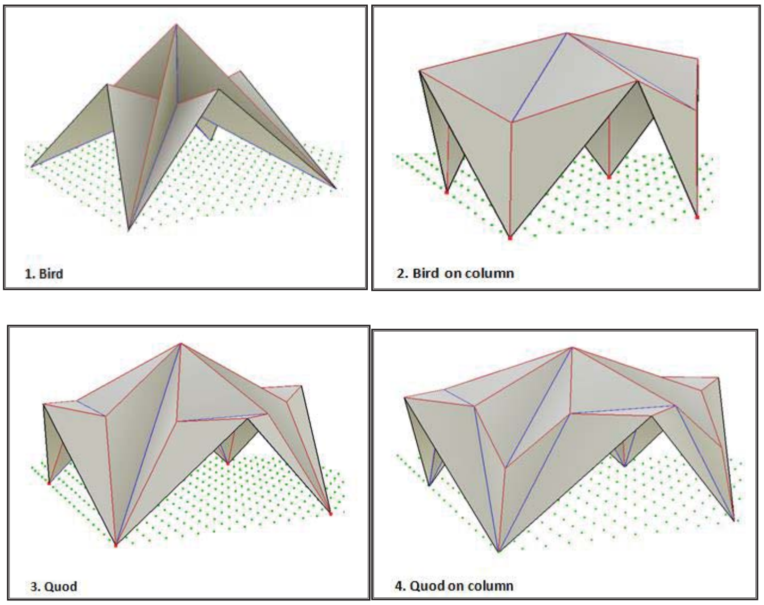

Figure.8:.Variants within the stable adaptable structures typology

These four variants are compared to each other based on their geometrical properties. The relevant geometrical properties for the comparison study are described below.

The minimum dimensions
The minimum dimension of a folding pattern, is the dimension of the boundary edges in the unfolded (flat) state, which is necessary to create a geometry with an usable height of approximately 2.2 meters in the folded state. The minimum dimension of the folding patterns is dependent on the effectiveness of the pattern.

Effective floor space range
The effective floor space range is the adaptable floor space range which the structure can generate, while the effective height is still acceptable. The range is dependent on the minimum dimensions and the effectiveness of the pattern.

Minimum folding angle
The minimum folding angle is the smallest angle between two adjacent surfaces during the folding motion in the effective floor space range. The minimum folding angle is important because it determines the minimum rotation angle which the hinges have to fulfil without limiting the adaptability.

Variety of triangles
All variants are fully based on triangles. However, each pattern consists of a different number of triangles with different dimensions. The number triangles is dependent on the number of folds and influences the complexity of the structure. This complexity affects the way in which the structure is transported and deployed.

Number of hinges
The minimum number of hinges necessary to translate the folding pattern into a real structure is dependent on the number of fold lines of the patterns. When assumed that for every (internal) fold line two hinges are necessary, it becomes possible to calculate the total number of hinges.

Estimation of the weight
When the total length of the frame is known and the profile of the bars is estimated, it is possible to estimate and compare the global weight of the frames. It is mentioned that, in this preliminary comparison study, the profiles are assumed to be the same for every variant and are not yet validated in a finite element program. The calculated weight is therefore just for giving a global estimation.

3.2. Comparison and conclusion

Table 2 shows an overview of the geometrical properties of the variants.

	Bird	Quod	Quod on column	Bird on column
Minimum dimensions	10m	8 m	8m	8m
Floor space range	25-35m^2	10-36m^2	16-36m^2	9-36m^2
Minimum folding angle	0^0	25^0	90^0	110^0
Number of triangles	16	24	32	16
Variety of triangles	2	4	4 -5	2
Total length of frame	207 m	197 m	208,2 m	124,8
Number of hinges	40	64	88	40
Weight frame (40-3)	196 kg	186 kg	196 kg	118 kg
Estimated total weight	276 kg	275 kg	310 kg	183 kg

Table 2. Comparison geometrical properties

It can be seen from table 2, that each variant has some different strengths and weaknesses and iIt follows from the schematization that each variant has some different design criteria which they approach. The final choice of the variant is therefore dependent on the starting point(s) which is considered as most important. This choice is mainly dependent on the function of the pavilion and the preference of the designer.

It was mentioned above, that the appearance of the adaptable pavilion is an important characteristic due to two reasons. Firstly, because of its promotional function and secondly, because it emphasizes the innovative principles on which the structure is based.

For these reasons, the emphasis in this study is on the appearance of the pavilion and the *Quod on column* variant is chosen to be studied in more detail. Although it has to be mentioned, that with a different function (or focus), a different choice could be made.

Figure 9. Visualization of the *quod on column variant*

4. Conclusion

The potential of fully foldable structures in the built environment is investigated in study by investigating the properties of several folding typologies. The conclusion of these investigations is that especially the non-singular, rigid foldable typology has a high potential to be applied in the built environment. This is the reason why, this typology is used, in the context of this study, to generate a new typology of structures which is called stable adaptive structures. This typology comprises of structures which are able to modify their global geometry by moving their supports, while the structures are stabilized by only pinning the supports, regardless of their position.

With a set of geometrical rules it is possible to generate a wide variety of structures which all belong to this typology. The properties of the stable adaptive structures are used to design an adaptable pavilion where the actual form (configuration) is dependent on the requirements such as floor space, appearance and transportability. In a variant study, four variants were compared to each other, after which the so called quod on column was chosen as the final design. The main reason for choosing this variant is due to its effective floor space range in combination with its innovative appearance, which emphasizes the innovative principles behind the pavilion.

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