Golay code

References

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GOLAY CODE – From a purely mathematical point of view, the Golay codes are the most interesting codes constructed as yet (1996). The *binary Golay code* $\mathcal{G}_{23}$ is a 12-dimensional subspace of $\mathbf{F}_2^{23}$ with the property that any two vectors (i.e., words) differ in at least 7 positions (they have *distance* $d = 7$). In coding terminology, $\mathcal{G}_{23}$ is a $[23, 12, 7]$ *binary code*, i.e., a 3-error-correcting code (cf. **Error-correcting code**). Similarly, the *ternary Golay code* $\mathcal{G}_{11}$ is a $[11, 6, 5]$ ternary code. It was shown by A. Tietäväinen and J.H. van Lint (see [4]) that the Golay codes are the only non-trivial e-error-correcting *perfect codes* with $e > 1$ over any alphabet Q for which $|Q|$ is a prime power. A *perfect e-error-correcting code* is a subset of Q^n such that every vector in the space has distance at most e to a unique codeword.

An *extension of a code* C of length n is the set of words of length $n+1$ obtained by adjoining an extra coordinate to all the words of C in such a way that the sum of the $n+1$ coordinates is 0. The extended codes $\mathcal{G}_{24}$ and $\mathcal{G}_{12}$ are of interest in group theory because their automorphism groups are the 5-transitive Mathieu groups M_{24} and M_{12} (cf. also **Mathieu group**).

For design theory (cf. also **Design with mutually orthogonal resolutions**; **Block design**), the Golay codes are important for the following reason. The words of weight (i.e., number of non-zero coordinates) 8 in $\mathcal{G}_{24}$ are the blocks of the (unique) **Steiner system** $S(5, 8, 24)$. Similarly, the words of weight 6 in $\mathcal{G}_{12}$ support the blocks of the (unique) Steiner system $S(5, 6, 12)$.

For each of the codes, several constructions are known. E.g.,

1) Make a list of the numbers $0, 1, \ldots, 2^{24}-1$ written binary as vectors in $\mathbf{F}_2^{24}$. Delete each vector that has distance less than 8 to a previous vector that has not been deleted. At the end of this procedure, 4096 vectors will remain. They form a *linear code*, in fact $\mathcal{G}_{24}$.

2) In the spaces $\mathbf{F}_2^{23}$ and $\mathbf{F}_3^{11}$, consider the codes of length $n = 23$, respectively $n = 11$, generated by the vectors $\vec{c}_i$ $(1 \le i \le n)$ for which $c_{i,j} = 1$ if $j - i$ is a non-zero square and 0 otherwise. One thus obtains the binary and the ternary Golay code.

3) Consider the (11×11)-circulant matrix with top row (01000111011). This is the incidence matrix of the unique 2- $(11, 6, 3)$-design. Form P by bordering this matrix with a column of 1's in front and a row of 1's on top, with a 0 in the upper left-hand corner (cf. **Bordering method**). Then adjoin I_{12} in front of P. One obtains a (12×24)-matrix G in which every row has eight 1's (except the top row, which has 12). The rows of G generate $\mathcal{G}_{24}$.

4) As in 3), form a (5×5)-circulant with top row $(0, 1, -1, -1, 1)$ and border it on top with a row of 1's. To this, adjoin I_6 in front to form a (6×11)-matrix G. The rows of G generate the $[11, 6, 5]$ ternary Golay code.

For other constructions and more theory of these codes, see the references.

M.J.E. Golay (1902–1989) was a Swiss physicist who worked in many different fields. He is known for his work on infrared spectroscopy and the invention of the capillary column, but to mathematicians mainly for his discovery of the two Golay codes.

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GORYACHEV–CHAPLYGIN TOP – A rigid body rotating about a fixed point, for which:

a) the principal moments of inertia $\lambda = (\lambda_1, \lambda_2, \lambda_3)$, with regard to the fixed point, satisfy the relation $\lambda_1 = \lambda_2 = 4\lambda_3$;

b) the centre of mass belongs to the equatorial plane through the fixed point;

c) the principal angular momentum is perpendicular to the direction of gravity, i.e., $\langle m, \gamma \rangle = 0$.

First introduced by D. Goryachev [4] in 1900, the system was later integrated by S.A. Chaplygin [3] in terms of hyper-elliptic integrals (cf. also **Hyper-elliptic integral**). The system merely satisfying a) and b) is not algebraically integrable, but on the locus, defined by c), it is; namely, it has an extra invariant of homogeneous degree 3:

$$Q_4 = (m_1^2 + m_2^2)m_3 + 2m_1\gamma_3.$$