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Integrated design for a CVT: dynamical optimization of actuation and control

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Abstract:
With increasing demands on more energy and fuel efficient vehicles, one can achieve the goal by improving the vehicle powertrain system. A continuously variable transmission (CVT) allows the engine or electric machine to operate on its optimal operation points. The optimal operation points are high efficiency points that lead to reduced energy consumption of the vehicle. However, this type of transmission may still have relatively high actuation losses (depending on the actuation type), which hinders the energy saving benefits. Classically, the plant (e.g., actuation system, variator) of the CVT was separately designed from the control design. In this paper, an integrated optimal CVT variator and actuation control design is presented. The aim of the new design is to minimize the CVT mass (pulley sheaves, belt), tracking error and control effort. To achieve this goal, a nested optimization framework is implemented to obtain an optimal transmission system design over a selected drive cycle. The results show that the optimized CVT design yields non-compromising tracking performance, however, with much smaller variator mass (-46%) and control effort (-62%).

Keywords: Co-design, optimization, continuous variable transmission, vehicle powertrains

1. INTRODUCTION

With systems becoming more complex, achieving a true optimal system design can be a challenge due to the existing mathematical coupling (e.g. via shared variables in constraints and objectives) between the corresponding subsystems (e.g., between the controller and the plant). Combined optimal plant and control design has been shown to result in an improved system performance [1], [2]. However, a combined system-level optimization might result in a nonconvex optimization problem, even when the separate plant and control optimization subproblems are convex [3]. Therefore, a suitable design framework must be developed in order to overcome this challenge. The method for combined plant and control optimization is referred as co-design.

The combined plant and control design has been a subject of discussion for the past few years. Generally, the co-design formulation can be expressed as the weighted sum of the plant and control objectives [3]:

$$\min_{\mathbf{x_p} \subseteq \mathbb{R},\, \mathbf{x_c} \subseteq \mathbb{R}} w_p\, J_p(\mathbf{x_p}) + w_c\, J_c(\mathbf{x_p}, \mathbf{x_c})\,, \tag{1}$$

subject to:

$$\mathbf{g_p}(\mathbf{x_p}) \leq \mathbf{0}\,,\ \mathbf{h_p}(\mathbf{x_p}) = \mathbf{0}\,,$$
$$\mathbf{g_c}(\mathbf{x_c}, \mathbf{x_p}) \leq 0\,,\ \mathbf{h_c}(\mathbf{x_c}, \mathbf{x_p}) = 0\,,$$

where $\mathbf{x_i}$ are the design parameters, $\mathbf{g_i}$ and $\mathbf{h_i}$ are the inequality and equality constraints, and J_i are the objective functions, w_i are the weighting parameters, for the plant and control system, $i \in \{p, c\}$ with $w_i \in \mathbb{R}$, respectively.

A vehicle powertrain is a complex dynamical system, which consists of multiple components at the subsystem level, e.g., engine, alternator, and a transmission. On the present day, it is desirable to design a vehicle with high energy efficiency. A way to improve the energy efficiency is by utilizing a continuously variable transmission (CVT) in the drivetrain. A CVT allows smooth shifting performance. Moreover, a CVT can operate the engine (or electric machine) at the optimal operation points, which results in a more efficient energy consumption. However, the efficiency of this type of transmission strongly depends on its actuation design, which hinders the energy saving capability that it potentially offers.

For automotive applications, a higher power density (kW/kg) and reduced size of transmissions (packaging) are desired. Smaller transmission dimensions are also beneficial in terms of production cost. Furthermore, as investigated in [4], the power capacity of a CVT is dependent on the ratio coverage of the transmission, which is determined by the physical parameters of the CVT, such as the pulley center distance and wedge angle. Other than increasing the power capacity, reduced wedge angle gives added benefits, such as reduced size and increased efficiency.

In this work, an integrated design method of a CVT variator (pulleys and v-belt) and its control system is proposed. The CVT design will be optimized dynamically over a complete drive cycle profile. In this work, we limit ourselves to optimizing the variator pulley sheaves and

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belt. The optimization of the actuation system (motors) and mechanism is seen as future work.

This paper is arranged as follows. Section 2 discusses the plant design modeling. The co-design formulation for the CVT is explained in Section 3. The results of the proposed co-design for CVT on a vehicle powertrain over a drive cycle is elaborated in Section 4. Lastly, the conclusion and future work are presented in Section 5.

2. PLANT MODEL: ELECTROMECHANICALLY ACTUATED CONTINUOUS VARIABLE TRANSMISSION

In this work, the CVT that will be studied is the electromechanically actuated continuously variable transmission (EMPACT). This type of CVT, as shown in Fig. 1, has electric servomotors as the actuation system, which prompt the pulley sheaves on the primary and secondary side. In this section, the behaviour and mathematical formulations of EMPACT will be discussed.

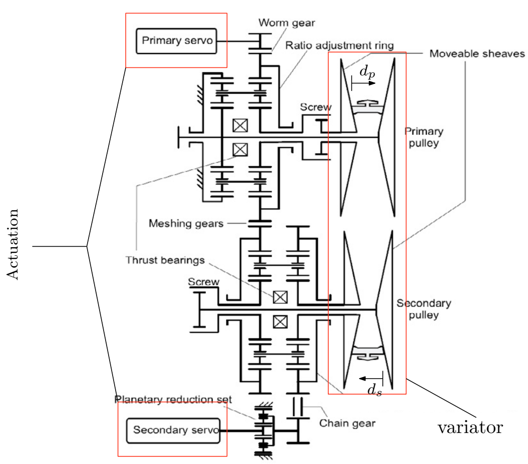

Fig. 1. Schematic diagram of EMPACT CVT [5]

2.1 CVT geometric model

The schematic diagram of the CVT variator is depicted in Fig. 2. The CVT geometric ratio is defined as the ratio between the running radii on the primary and the secondary side,

$$r_g = \frac{R_p}{R_s}\,. \tag{2}$$

Additionally, the speed ratio is defined as,

$$r_s = \frac{\omega_s}{\omega_p}\,, \tag{3}$$

where ω_p and ω_s are the primary and secondary rotational speeds, respectively. The movable pulley positions d_p and d_s determine the corresponding running radii of the CVT at the primary and secondary side,

$$R_p = \frac{d_p}{2\tan\beta} + R_o\,;\;\; R_s = \frac{-d_s}{2\tan\beta} + R_o\,, \tag{4}$$

where β is the pulley wedge angle and R_o is the running radius at $r_g = 1$. The position of the movable pulley sheave results in different wrap angles φ_i, which are the span of the belt on the pulley sheaves, and are formulated as:

$$\varphi_p = \pi + 2\varphi\,;\;\; \varphi_s = \pi - 2\varphi\,, \tag{5}$$

with

$$\varphi = \sin^{-1}\left(\frac{R_p - R_s}{a}\right). \tag{6}$$

where a is the center distance between the primary and secondary pulley, φ_p and φ_s are the belt wrap angles, respectively.

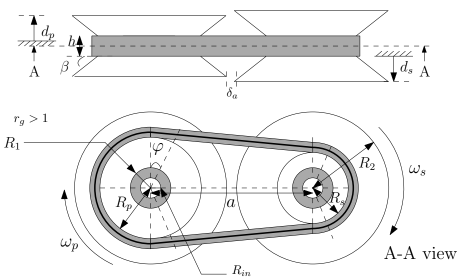

Fig. 2. CVT variator diagram

Due to friction, there exists a slip v in the variator, which is the difference between the tangential speed on the primary and secondary side, $v = \frac{\omega_p\,R_p - \omega_s\,R_s}{\omega_p\,R_p}$. Here, the slip is considered to be small, such that $\omega_p\,R_p \approx \omega_s\,R_s$, and therefore, $r_g \approx r_s$.

2.2 Variator dynamics and clamping forces

Several models have been derived to describe the transient behaviour of the CVT variator. Here, the Carbone, Mangialardi, and Mantriota (CMM) model will be utilized to describe the behaviour of the variator ratio dynamics [6]. This mathematical model describes the behaviour of the ratio change as a function of the clamping force and also the pulley deformation. The rate of change of the speed ratio is then given by,

$$\dot{r}_g = 2\,\omega_p\,\Delta \frac{1+\cos^2\beta}{\sin(2\beta)}\,c(r_g)\left[\ln\left(\frac{F_p}{F_s}\right) - \ln\left(\frac{F_{p,ss}}{F_{s,ss}}\right)\right], \tag{7}$$

where ω_p is the rotational speed of the driving (primary) pulley, Δ is the pulley deformation, β is the pulley wedge angle. F_p and F_s are the primary and secondary clamping forces, and the subscript ss indicates the steady-state value when $\dot{r}_g = 0$. The term $c(r_g)$ is a function of r_g that relates the dimensionless speed ratio with the difference of the logarithmic steady state and applied clamping forces. The pulley deformation Δ is affected by the clamping forces, and can be expressed as [7]:

$$\Delta = (1 + 0.02(F_s - 20)) \cdot 10^{-3}\,, \tag{8}$$

where F_s is the secondary clamping force in kN. The clamping forces F_p and F_s are given by,

$$F_i = \int_{\varphi_i} \frac{S_i(\theta) - C_i(\theta) - F_b - F_c}{2\sin\beta}\,\mathrm{d}\theta\,, \tag{9}$$

where $i \in \{p, s\}$, S_i and C_i are the tension and compression forces over the pulleys, F_b and F_c are the centrifugal forces of the band and belt blocks, $F_b = \rho_b v_b^2$ and $F_c = \rho_c v_b^2$, respectively [8]. The belt velocity v_b is assumed to be $v_b = \omega_p R_p = \omega_s R_s$. The calculation of the clamping forces are also dependent on ρ_b and ρ_c, which are the mass per unit length of the bands and the blocks, respectively.

3. CO-DESIGN FORMULATION

A co-design strategy is proposed to obtain an improved EMPACT plant and control design for a specific drive cycle. The key performance indicators (KPIs) are packaging (dimensions), efficiency, performance, and cost. Here, we focus on minimizing the variator mass, which is expressed as a function of the variator dimensions (plant objective) and the tracking performance and actuation effort (control objective) of the CVT system. The KPIs can be interpreted as the system wide objectives. Further, we use optimal control techniques in order to not compromise the tracking performance of the control system.

The CVT actuation controller should provide the corresponding signal that minimizes the tracking error and control effort for the actuation system. However, due to the presence of the coupling between the geometrical properties and the reference for control, the response of the CVT is also dependent on the choice of the plant design parameters, namely β and a, which becomes a challenge in finding the combined plant and control optimal design.

3.1 Plant Design Problem

To derive the plant design objective J_p, the CVT pulley sheaves are considered to be identical truncated cones and the metal belt is assumed as a v-belt. The mathematical formulation of the variator mass is defined as,

$$M_v = \rho_{pu}\, V_{pu} + \rho_b\, V_b\,. \tag{10}$$

V_{pu} and V_b are the pulleys and belt volume of the variator, given by

$$V_{pu} = 4\pi(R_2 - R_1)\tan\beta\Big[\frac{1}{3}(R_1^2 + R_1R_2 + R_2^2) - R_{in}^2\Big]\,, \tag{11}$$

$$V_{be} = \frac{(b_1 + b_2)b_3}{2} L_b\,, \tag{12}$$

where R_1 and R_2 are the pulley top and bottom radii, R_{in} is the radius of the shaft, L_b is the length of the belt, and b_1, b_2, and b_3 are the belt parameters, respectively, as depicted in Figs. 2 and 3.

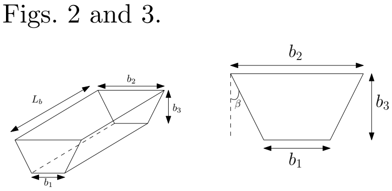

Fig. 3. CVT belt geometries

As seen in Fig. 2, the sheave radius R_2 determines the value of the center distance a. Hence, the plant design objective is mathematically expressed as,

$$\min_{\mathbf{x_p} \subseteq \mathcal{X}_p \subseteq \mathbb{R}} J_p = M_v(\mathbf{x_p})\,, \tag{13}$$

subject to:

$$\beta - \beta_{max} \leq 0 \tag{14}$$

$$-\beta + \beta_{min} \leq 0 \tag{15}$$

$$h_{min} \leq \tan\beta(R_2 - R_1) \leq h_{max} \tag{16}$$

$$R_{i,min} \leq R_i \leq R_{i,max} \tag{17}$$

$$2R_2 - a + \delta_a \leq 0 \tag{18}$$

where h_{min} and h_{max} and β_{min} and β_{max} are the minimum and maximum pulley heights and wedge angles; $R_{i,min}$ and $R_{i,max}$ are the minimum and maximum pulley radii for $i = \{1, 2\}$; a is the pulley center distance; δ_a is the minimum pulley distance; and $\mathbf{x_p}$ are the plant design parameters,

$$\mathbf{x_p} = \{R_1, R_2, \beta\}\,,$$

where the set $\mathbf{x_p}$ is a subset of set $\mathcal{X}_p$, given by,

$$\mathcal{X}_p = \{\mathbf{x_p} \in \mathbb{R} \,|\, \mathbf{g_p}(\mathbf{x_p}) \leq \mathbf{0}\,,\, \mathbf{h_p}(\mathbf{x_p}) = \mathbf{0}\}\,.$$

3.2 Control Design Problem

This work focuses only on the speed ratio control of CVT. The reference for the primary servomotor to realize the desired ratio values is influenced by the plant design parameters (i.e., β, R_1, R_2), which indicates the coupling that exists between the plant parameters and the control. Here, firstly, the primary servomotor reference design model for the primary rotational position is discussed. This serves as input to the servomotor control problem accordingly. The ratio r_g is not directly measurable, and must, therefore, be estimated from the measureable variables. For the EMPACT, the measured variables are the servomotor rotations θ_{mp} and θ_{ms} and the primary ω_p and secondary ω_s shaft speeds. The electric servomotors actuate the movable pulley sheaves to realize the required clamping forces necessary to perform ratio shifting. Hence, the ratio control of the EMPACT CVT can be approached as a servomotor control problem [5].

3.2.1 Servomotor reference design

Mathematically, the pulley sheave positions can be expressed as a function of the servomotor positions,

$$d_p = \frac{\theta_{mp}}{r_w}\, s\, z \tag{19a}$$

$$d_s = d_p + \frac{\theta_{ms}}{r_r\, r_c}\, s\, z \tag{19b}$$

where θ_{mp} and θ_{ms} are the motor angular positions of the primary and secondary actuations; r_r, r_c, and r_w are the reduction gear constants; s is the screw pitch, and z is the ratio between the sun and annulus of the planetary gear set connecting the servomotor and the screw.

The desired reference ratio trajectory $r_{g,r}$ can be translated as the reference for the primary servomotor $\theta_{mp,r}$. As $r_{g,r}$ is known *a priori*, $\dot{r}_{g,r}$ can be obtained using the trajectory. Then, using the CMM model that is described in (7), the required clamping force ratio $\frac{F_p}{F_s}\Big|_r$ that yields the desired ratios can be calculated by the relation:

$$\frac{F_p}{F_s}\bigg|_r = \exp\!\left(\dot{r}_g\, \frac{\sin(2\beta)}{2\,\omega_p\,\Delta\,(1+\cos^2\beta)\, c(r_g)}\right) \cdot \frac{F_{p,ss}}{F_{s,ss}}\,. \tag{20}$$

The steady state clamping force ratios $\frac{F_{p,ss}}{F_{s,ss}}$ as a function of gear ratio values can be computed *a priori* using the geometrical and physical variator model as described by [6]. To reduce the calculation time for the design optimization, some models are reduced as parametric representations of the relevant design parameters. The steady-state clamping force ratio can be approximated as,

$$\frac{F_{p,ss}}{F_{s,ss}} = \begin{cases} a_1\beta r_g + a_2\beta + a_3\,, & \text{for } r_g \leq 1 \\ b_1 + b_2\beta^2\ln(r_g)\,, & \text{for } r_g > 1 \end{cases} \tag{21}$$

with fitted constants $a_1 = 1.2204$, $a_2 = -1.65$, and $a_3 = 1.0738$, and $b_1 = 0.9968$ and $b_2 = 6.2373$.

Similarly, the resulting clamping force ratio $\frac{F_p}{F_s}$ can be approximated as a function of the primary wrap angle φ_p and β,

$$\frac{F_p}{F_s} = \begin{cases} 0.6793\varphi_p - 2.1303\beta - 0.47\,, & \text{for } r_g \leq 1\,, \\ 0.8409\varphi_p - 3.4886\beta - 0.456\,, & \text{for } r_g > 1 \end{cases} \tag{22}$$

Using (4), (5), (6), and (22), the corresponding $d_{p,r}$ that yields the desired clamping force ratio $\frac{F_p}{F_s}\big|_r$ can be obtained. Furthermore, via the relation in (19a), the desired reference trajectory for the primary servomotor actuation $\theta_{mp,r}$ can be determined.

3.2.2 Servomotor control design

The primary servomotor type is selected to be a DC motor. The dynamics is given by:

$$\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\begin{bmatrix}\theta_{mp}\\ \omega_{mp}\\ i\end{bmatrix}}_{\xi} = \underbrace{\begin{bmatrix}0 & 1 & 0\\ 0 & -b/I & K_t/I\\ 0 & -K_e/L & -R/L\end{bmatrix}}_{\mathbf{A}}\begin{bmatrix}\theta_{mp}\\ \omega_{mp}\\ i\end{bmatrix} + \underbrace{\begin{bmatrix}0\\ 0\\ 1/L\end{bmatrix}}_{\mathbf{B}} u\,, \tag{23}$$

$$\mathbf{y} = \underbrace{\begin{bmatrix}1 & 0 & 0\end{bmatrix}}_{\mathbf{C}}\begin{bmatrix}\theta_{mp}\\ \omega_{mp}\\ i\end{bmatrix} \tag{24}$$

where θ_{mp} and ω_{mp} are the primary servomotor angular position and rotational speed, i is the motor current, K_t and K_e are the torque and voltage constants, b is the friction constant, L is the inductance, and R is the armature resistance. I is the rotor inertia, u is the control input which is in voltage, and finally the state variables are given by $\xi = [\theta_{mp}\,, \omega_{mp}\,, i]^\top$.

In order to design a controller such that the error between the desired and actual trajectory is minimized, the state space model of the system is augmented such that the error $\varepsilon = \int_0^T (\theta_{mp,r}(t) - \theta_{mp}(t))\,\mathrm{d}t$ is included as an additional state. The feedback gains are found by utilizing an LQR formulation. A feedforward term k_{ff} is included to minimize the steady state error. The augmented state is expressed as:

$$\frac{d}{dt}\underbrace{\begin{bmatrix}\xi\\ \varepsilon\end{bmatrix}}_{\xi_a} = \underbrace{\begin{bmatrix}\mathbf{A} & 0\\ \mathbf{C} & 0\end{bmatrix}}_{A_a}\begin{bmatrix}\xi\\ \varepsilon\end{bmatrix} + \underbrace{\begin{bmatrix}\mathbf{B}\\ 0\end{bmatrix}}_{B_a} u + \begin{bmatrix}0\\ -1\end{bmatrix}\theta_{mp,r}\,, \tag{25}$$

where the control input is given by

$$u = -\mathbf{k_{fb}} \cdot \xi + k_{ff} \cdot \varepsilon\,, \tag{26}$$

where $\mathbf{k_{fb}}$ and k_{ff} are the control gains. The proposed control structure is depicted in Fig. 4.

The primary actuation controller calculates the required voltage u to the servomotor such that the desired ratio is tracked, while still minimizing the energy consumption. The cost function for the controller is given by

$$J_c = \int_0^{t_f} \xi_\mathbf{a}^\top \mathbf{Q}\xi_\mathbf{a} + u^\top \mathbf{R} u\,\mathrm{d}t\,, \tag{27}$$

where the minimum of the cost function can be found by solving the algebraic Riccati equation. The weight matrices $\mathbf{Q}$ and $\mathbf{R}$ are commonly chosen by the user to have acceptable balance between minimizing error and control effort that yields satisfactory performance. However, as in the co-design framework the plant design parameters (i.e., β, R_1, R_2) will be varied, the corresponding reference trajectory for the servomotor reference $\theta_{mp,r}$ are also changed.

Hence, the weight matrices $\mathbf{Q}$ and $\mathbf{R}$ are selected such that the error between the desired and actual ratio trajectory is minimized for a plant design. The optimal solution of (27) is found by solving the Riccati equation,

$$\mathbf{A_a^\top P} + \mathbf{PA_a} - \mathbf{PB_a R^{-1} B_a^\top P} + \mathbf{Q} = \mathbf{0}\,, \tag{28}$$

as long as the conditions $\mathbf{P} > \mathbf{0}\,, \mathbf{Q} \geq \mathbf{0}\,, \mathbf{R} > \mathbf{0}$ are satisfied. $\mathbf{P}$ is a positive definite matrix that satisfies the algebraic Riccati equation. The weight matrices $\mathbf{Q}$ is a $m \times m$ matrix and $\mathbf{R}$ is a $n \times n$ matrix, where m is the rank of $\mathbf{A_a}$ and n is the rank of the input $\mathbf{u}$. The gains $\mathbf{k_{fb}}$ and k_{ff} are determined by solving the Riccati equation $[\mathbf{k_{fb}} \quad k_{ff}] = \mathbf{R^{-1}\,B_a^\top P}$.

In the EMPACT CVT, the primary side is often used to perform ratio tracking, while the secondary side is used to regulate the slip in the variator. The minimum required secondary clamping force to ensure torque transfer is given by

$$F_{s,r} = \frac{\alpha\,T_{si}\,\cos\beta}{2\,\mu\,R_s}\,, \tag{29}$$

where α is a safety factor, commonly chosen to be 1.3, T_{si} is the belt internal torque exerted on the secondary pulley, and μ is the effective traction coefficient. The design of secondary actuation controller is outside of the research scope.

3.3 Co-design problem formulation

The combined plant and control design problem formulation of the CVT is written as:

$$\min_{\mathbf{x_p} \subseteq \mathcal{X}_\mathbf{p}, \mathbf{x_c} \subseteq \mathcal{X}_\mathbf{c}} w_p\,M_v(\mathbf{x_p}) + w_c \int_0^{t_f} \xi_\mathbf{a}^\top \mathbf{Q}\xi_\mathbf{a} + u^\top \mathbf{R} u\,\mathrm{d}t\,, \tag{30}$$

subject to: (15)-(18) and $\mathbf{P} > \mathbf{0}\,, \mathbf{Q} \geq \mathbf{0}\,, \mathbf{R} > \mathbf{0}$.

The optimized design parameters are defined as $\mathbf{x_p} = \{R_1, R_2, \beta\}$ and $\mathbf{x_c} = \{\mathbf{Q}, \mathbf{R}\}$, and $\mathbf{x_p} \subseteq \mathcal{X}_p$, $\mathbf{x_c} \subseteq \mathcal{X}_c$. The selection of the weights w_p and w_c are important in finding the Pareto front of the optimization problem. In this work, the weights w_i for $i = p, c$ are selected to be 1. Furthermore, in this specific problem, the parameters β and a serve as couplings between the plant design (minimizing variator mass, a more compact CVT) and the control design (minimizing speed ratio tracking error) problem.

4. NESTED OPTIMIZATION APPROACH

A nested optimization approach will be implemented to dynamically obtain an optimal CVT design for the selected drive cycle. This is because the plant design parameters (i.e., β, R_1, R_2) have influence on the control design (minimizing ratio tracking error). The nested approach has an outer loop that optimizes the combined design objective (30) by varying the plant design parameters, and an inner loop that optimizes the control performance. For this problem, the ratio control problem is solved using an

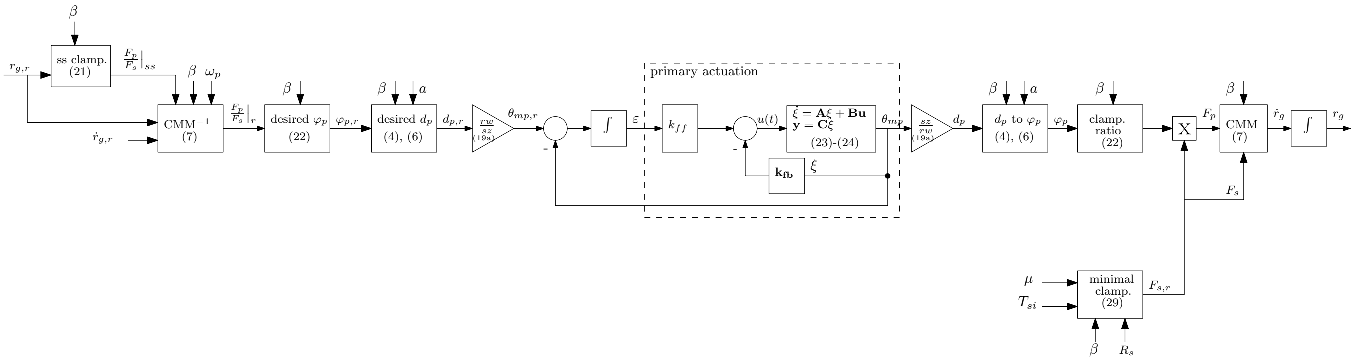

Fig. 4. Control scheme for EMPACT ratio control

LQR formulation with augmented state. Additionally, the weight matrices $\{\mathbf{Q}, \mathbf{R}\}$ are obtained by optimization. The proposed framework is depicted in Fig. 5.

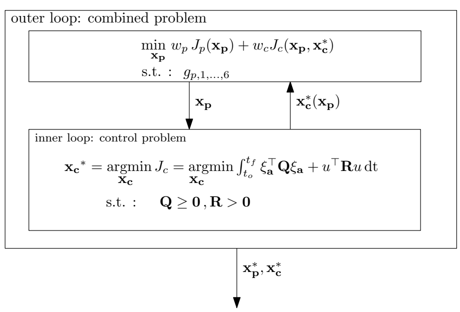

Fig. 5. Nested optimization framework for the EMPACT CVT

4.1 Reference speed trajectory

The performance of the optimized CVT throughout a given driving profile will be analyzed. The reference CVT ratio $r_{g,r}$ is determined such that the engine is operated near, if not at, the optimal operation line of the engine. The NEDC drive cycle is selected to investigate the benefits of utilizing CVT in a traditional vehicle powertrain with a 2.0 L gasoline engine. The NEDC drive cycle is used to determine the required CVT ratio values such that the vehicle is operated at the engine optimal operation line.The wheel speed ω_{wh} is defined as

$$\omega_{wh} = \frac{v_{wh}}{R_{wh}}\,, \tag{31}$$

where v_{wh} is the vehicle speed from the drive cycle and R_{wh} is the wheel radius. The torque demand at the wheel is calculated using

$$T_{wh} = (F_r + F_a + F_i)\, R_{wh}\,, \tag{32}$$

where F_r is the rolling resistance, F_a is the aerodynamic drag, and F_i is the force due to inertia of the vehicle. Furthermore, the ratio is selected such that the required engine power to drive the vehicle is generated with minimum fuel consumption. To determine the ratio, several assumptions are considered. At $\omega_{wh} = 0$, the engine is at idle, $\omega_e = \omega_{id}$. Throughout the CVT operation over the NEDC drive cycle, the efficiency of the transmission is treated to be constant $\eta_t(t) = \eta_t$. The power demand at the wheels is used to determine the power required from the engine,

$$P_{e,r} = \frac{P_{wh}}{\eta_t} = \frac{T_{wh}\,\omega_{wh}}{\eta_t}\,. \tag{33}$$

The engine optimal line is found to be the combination of engine speed and torque ω_e^* and T_e^* that yield the minimum fuel consumption. Using the optimal operation points, the desired ratio reference that yields optimal fuel consumption can be calculated. The desired speed ratio $r_{g,r}$ is defined as,

$$r_{g,r} = \min\left(\max\left(r_{g,min}, \frac{\omega_{wh}\, i_{FDR}}{\omega_e^*}\right), r_{g,max}\right)\,, \tag{34}$$

where ω_e^* is the engine speed at the optimal operation line, and i_{FDR} is the final drive ratio.

The obtained ratio reference is used to perform co-design on the CVT variator and ratio controller dynamically for the complete NEDC profile. The vehicle parameters used throughout the simulation study are summarized in Table 1.

Table 1. Simulation Parameters

	Symbol	Value	Units
Vehicle mass	m_s	1180	kg
Roll. resistance	c_r	0.0174	–
Drag. resistance	c_d	0.29	–
Vehicle frontal area	A_f	2.38	m^2
Air density	ρ_a	1.225	kg/m^3
Rotational mass	m_{ro}	2	%
Wheel radius	R_{wh}	0.35	m
Final drive ratio	i_{FDR}	3	–
Engine idle speed	ω_{id}	78.53	rad/s
CVT efficiency	η_t	0.9	–

4.2 Design results

The CVT variator design optimization and the speed ratio controller over a complete drive cycle are demonstrated. The simulation parameters used in this study is summarized in Table 2. This subsection discusses the results obtained from the study of optimizing the CVT design compared to a baseline model [1].

It can be seen from the results that the optimized CVT is able to follow the desired ratio trajectory despite having a lower variator dimensions throughout the drive cycle, as summarized in Table 3. Using the proposed nested optimization framework, the mass of the pulley sheaves

[1] Variator geometries from Jatco CK2 with EMPACT actuation system for reasons of comparison

Table 2. Simulation Parameters

	Symbol	Value	Units
Motor inertia	I	$31.4 \cdot 10^{-6}$	$kg \cdot m^2$
Motor friction constant	b	$2.1 \cdot 10^{-3}$	$N \cdot m/krpm$
Motor inductance	L	2.75	mH
Motor torque constant	K_t	$52.96 \cdot 10^{-3}$	Nm/A
Motor voltage constant	K_e	5.5	V/krpm
Motor armature resistance	R_a	0.47	Ω
Range of β	$[\beta_{min}, \beta_{max}]$	[6,15]	°
Range of R_1	$[R_{1,min}, R_{1,max}]$	[15,30]	mm
Range of R_2	$[R_{2,min}, R_{2,max}]$	[60,90]	mm
Inner radius	R_{in}	7.5	mm
Range of ratio	$[r_{g,min}, r_{g,max}]$	[0.5,2.5]	–
Weight	w_p	10	–
Weights for $i \in \{c, 1, 2\}$	w_i	1	–
Belt width	b_1	28	mm
Belt thickness	b_3	24	mm
Screw pitch	s	$4/(2\pi \cdot 1000)$	rad/m
Reduction gear	r_w	16	–
Initial ratio	$r_{g,o}$	0.5	–
Planetary gear set ratio	z	2	–
Mass density	ρ_b, ρ_{pu}	7850	kg/m^3

can potentially be reduced from 6.86 to 3.69 kg (-46%). The optimized parameter β for the new variator is found to be 6°, R_1 to be 22.8 mm, R_2 to be 66.7 mm, and the pulley center distance a to be 136.4 mm. Despite the reduced dimensions, the optimized CVT variator yields comparable performance to the baseline. The resulting ratio tracking performance is depicted in Fig. 6.

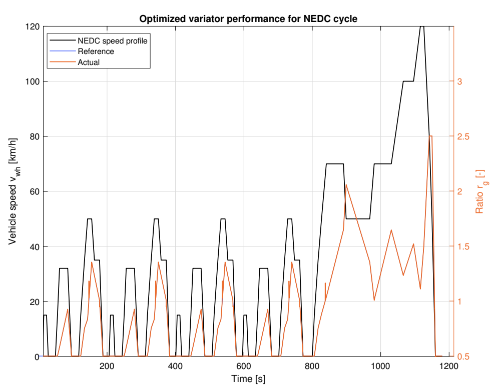

Fig. 6. Results for the optimized model for NEDC drive cycle

Table 3. Nested co-design results for NEDC drive cycle

		Symbol	Baseline	Optimized	Unit
J	Plant objective	J_p	6.86	3.69	kg
	Control objective	J_c	7.6	2.89	10^{10}
	Variator volume	V_v	8.74	4.70	$10^{-4}m^3$
$\mathbf{x_p}$	Wedge angle	β	11.0	6	°
	Top outer radius	R_1	26.3	22.8	mm
	Bottom outer radius	R_2	82.5	66.7	mm
$\mathbf{x_c}$	LQR weight	$Q(1)$	2.065	0.768	–
	LQR weight	$Q(2)$	0.001	0.001	–
	LQR weight	$Q(3)$	0.001	0.001	–
	LQR weight	$Q(4)$	10^4	10^4	–
	LQR weight	R	0.01	0.01	–
$\mathbf{K}, r_{cov}, a$	Center distance	a	168	136.4	mm
	Ratio coverage	r_{cov}	5.33	4.23	–
	Feedback gain	$k_{fb}(1)$	2.4439	2.355	–
	Feedback gain	$k_{fb}(2)$	0.035	0.035	–
	Feedback gain	$k_{fb}(3)$	3.54	3.497	10^{-4}
	Feedforward gain	k_{ff}	62.38	63.88	–

5. CONCLUSIONS AND FUTURE WORK

In this study, a nested optimization framework for the EMPACT CVT to obtain a combined plant and control design for the NEDC drive cycle has been succesfully implemented. The proposed approach calculates the minimized CVT variator's mass, as well as the corresponding actuation control parameters that yields the best possible ratio tracking performance.

Based on the results, it is seen that the variator design could be reduced in terms of size and mass without compromising the ratio tracking performance. At the same time, the corresponding optimal controller for the new optimized design is found using the proposed framework. Using the co-design framework explained in this paper, the mass of the pulley sheaves can potentially be reduced up to 46.2%, from 6.86 to 3.69 kg. It was also obtained that the wedge angle β for the CVT variator can be reduced from 11° to 6°, and center distance a from 169 to 136.4 mm. Furthermore, with the new optimized parameters, the ratio coverage of the CVT is reduced from 5.33 to 4.23. It is seen that the optimized CVT design yields comparable performance to the baseline CVT, despite the smaller size.

In this work, the CVT efficiency is not modeled. In the future, the actual CVT efficiency as a function of the relevant physical design parameters should be included in the co-design framework. Currently, only ratio controller is considered in this work as part of the co-design framework for CVT. The framework could also be extended to include slip controller on the secondary side of the variator. Lastly, in the future, the potential of using CVT in a full electric vehicle powertrain will be investigated.

REFERENCES

[1] H. K. Fathy, P. Y. Papalambros, A. G. Ulsoy, and D. Hrovat, Nested plant/controller optimization with application to combined passive/active automotive suspensions, in Proc. American Control Conference, Denver, USA, 2003.

[2] J. A. Reyer and P. Y. Papalambros, Combined optimal design and control with application to an electric DC motor, in Journal of Mechanical Design, vol. 124, pp. 183–191, 2002.

[3] H. K. Fathy, J. Reyer, P. Papalambros, G. Ulsoy, On the coupling between the plant and controller optimization problems, in Proc. American Control Conference, Virginia, USA, 2001.

[4] A. Brandsma, J. van Lith, E. Hendriks, Push belt CVT developments for high power applications, in International Congress on Continuously Variable Power Transmission CVT, Eindhoven, The Netherlands, 1999.

[5] T. W. G. L. Klaassen, The EMPACT CVT : dynamics and control of an electromechanically actuated CVT, PhD Thesis, Eindhoven University of Technology, 2007.

[6] G. Carbone, L. Mangialardi, G. Mantriota, The Influence of Pulley Deformations on the Shifting Mechanism of Metal Belt CVT, Journal of Mechanical Design, vol. 127, pp. 103–113, 2005.

[7] G. Carbone, L. Mangialardi, B. Bonsen, C. Tursi, P. A. Veenhuizen, CVT dynamics: Theory and experiments, Mechanism and Machine Theory, vol. 42, pp. 409–428, 2007.

[8] B. G. Vroemen, Component control for the Zero Inertia powertrain, PhD thesis, Eindhoven University of Technology, 2001.