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Manifold Based Control for a 1-DOF Beam with One-Sided Spring Using Feedback Linearization

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Abstract

Vibration reduction in a harmonically excited 1-DOF beam with one-sided spring is realized by forcing the system from a long-term stable $\frac{1}{2}$ subharmonic response towards a coexisting unstable harmonic response of lower vibration amplitude using feedback linearization. The control effort can be kept small once the unstable harmonic response is stabilized, because this response is a natural solution of the uncontrolled system. To reduce control effort, the approximated stable manifold is used to determine the desired trajectory for control, because at the stable manifold, the system state evolves towards the unstable harmonic response without control effort. The stable manifold is approximated using the stable eigenvectors of the monodromy matrix. Due to the local validity of the approximation, a two-stage control strategy is implemented. In the first stage, the system state is controlled directly towards the unstable harmonic response, to reach the region where the stable manifold can be approximated accurately by the stable eigenvectors. In the second stage, the system state is controlled towards the stable eigenvectors, and evolves towards the unstable harmonic response with hardly any control effort.

1 Introduction

Active vibration control can be effective in reducing the amount of wear, damage, and noise, occurring in harmonically excited nonlinear mechanical systems such as suspension bridges [1], ships colliding at fenders, or rattling gears, due to coexisting long-term responses at certain frequency ranges. Among these responses there always exists a harmonic response that is the most favorable with respect to vibration amplitude. However, if the harmonic response is unstable, the system will have a long-term response with higher vibration amplitude and non-harmonic frequency. Our objective is to stabilize the unstable harmonic response to achieve vibration amplitude reduction, while keeping the control effort small due to the fact that two characteristics of the uncontrolled system are exploited. First, the unstable harmonic response is a natural solution, so no control effort is needed once it is stabilized. Second, there exists an invariant manifold, the so-called stable manifold, that leads to the unstable harmonic response without control effort.

Much research has been done for the control of long-term chaotic response [2]. However, long-term chaotic response is not always encountered in real-life applications due to high levels of damping, or due to the small frequency ranges in which it occurs. Therefore, this research is focused on long-term periodic response, especially the stable $\frac{1}{2}$ subharmonic response, because long-term periodic response often occurs in practical situations for reasonable levels of damping, and within large frequency ranges. Furthermore, the control of long-term chaotic response, based on the method of Ott et al. [3], can only be applied when the system state is sufficiently close to the unstable harmonic response. In case of chaotic behavior, this condition will be satisfied within a system-dependent time span. However, in case of long-term periodic response, this condition will not be satisfied, so active control based on feedback linearization [4] will be used to drive the system state close to the unstable harmonic response, within a time span that is related to the available control effort.

A two-stage control strategy is applied to a 1-DOF model of a beam with one-sided spring, that can be related to suspension bridges. In the first stage, the system state is controlled from a long-term stable $\frac{1}{2}$ subharmonic response towards the unstable harmonic response, as close as necessary for the approximation of the stable manifold to be valid. In the second stage, the system state is controlled towards the approximation of the stable manifold, that is represented by the stable eigenvectors of the monodromy matrix. The overall control effort can be kept small, because at the stable manifold the system state evolves towards the unstable harmonic response without control effort.

2 1-DOF Beam with One-Sided Spring

The controlled 1-DOF beam with one-sided spring, see Fig. 1a, is modeled as:

$$M\ddot{q} + B\dot{q} + Kq + F_{nl}(q) = F_e + u, \qquad (1)$$

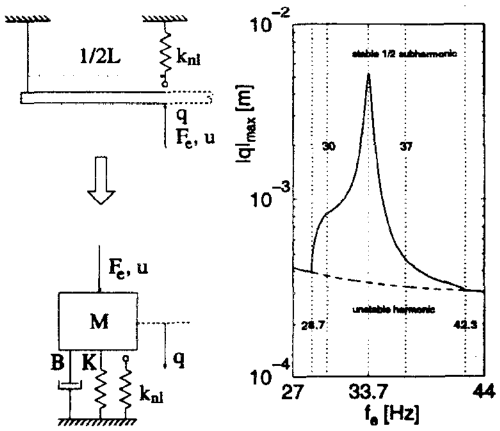
with q the displacement of the middle of the beam, $M = 5.15$ [kg] the mass, $B = 16.84$ $[\frac{\text{Ns}}{\text{m}}]$ the damping, and $K = 34410.56$ $[\frac{\text{N}}{\text{m}}]$ the stiffness of the 1-DOF beam. The force of the one-sided spring $F_{nl}(q)$ equals:

$$F_{nl}(q) = \begin{cases} k_{nl}q & \text{if } q \geq 0, \\ 0 & \text{if } q < 0, \end{cases} \qquad (2)$$

with $k_{nl} = 82500$ $[\frac{\text{N}}{\text{m}}]$ the stiffness of the one-sided spring. The one-sided spring is a nonlinear spring that adds stiffness to the beam when the beam is subjected to positive displacements q. The harmonic excitation force F_e equals:

$$F_e = c_1\omega^2 \cos(\omega t), \ \omega = 2\pi f_e, \qquad (3)$$

with $c_1 = 0.98\ 10^{-3}$ [kgm]. The control force u will be the subject of section (4).



(a) 1-DOF model (b) periodic behavior

Figure 1: 1-DOF beam with one-sided spring

Stabilizing the unstable harmonic response is very advantageous from a control point of view. This can be seen in equation (1), where the unstable harmonic response q_d represents a natural solution of the uncontrolled system, or:

$$M\ddot{q}_d + B\dot{q}_d + Kq_d + F_{nl}(q_d) = F_e. \qquad (4)$$

So no control effort is needed once the unstable harmonic response is stabilized.

A frequency domain representation of the long-term periodic behavior of the uncontrolled 1-DOF model is displayed in Fig. 1b. It shows the different kinds of response in maximum absolute value of the displacement q as a function of the excitation frequency f_e. The advantage of stabilizing the unstable harmonic response, from a vibration point of view, can be seen when looking at the frequency range between 28.7 and 42.3 Hz, where a stable $\frac{1}{2}$ subharmonic response coexists with an unstable harmonic response of much lower vibration amplitude; up to 10 times smaller around 33.7 Hz.

3 Invariant Manifold

Exploiting the invariant manifold, to reduce control effort in stabilizing the unstable harmonic response, can be very advantageous. A manifold is called invariant when every initial condition in the set remains in the set. The invariant manifold of the unstable harmonic response is composed of a stable and an unstable manifold. The stable manifold is the infinite set of initial conditions that approach the unstable harmonic response with increasing time; the same holds for the unstable manifold, but now the unstable harmonic response is approached with decreasing time [5]. The stable manifold can be used to reduce control effort, because once the system state is controlled at the stable manifold, the system drives the state towards the unstable harmonic response, without control effort.

The invariant manifold can be calculated using the procedure given by Parker and Chua [5], implemented in the finite element package DIANA. However, the calculation of the invariant manifold is time consuming, while the usage within a control design is complicated, because huge data sets are necessary to describe the invariant manifold accurately. Therefore, the eigenvectors of the monodromy matrix are used to approximate the invariant manifold around the unstable harmonic response. Such an eigenvector is called stable when it has a Floquet multiplier with absolute value smaller than one, and is called unstable when it has a Floquet multiplier with absolute value larger than one; the Floquet multipliers are the eigenvalues of the monodromy matrix. The stable eigenvector is an approximation of the stable manifold, and represents the direction in which the unstable harmonic response attracts, whereas the unstable eigenvector is an approximation of the unstable manifold, and represents the direction in which the unstable harmonic response repels.

To visualize the invariant manifold and its approximation, within the context of stabilizing the unstable harmonic response, Poincaré maps are used as displayed in Fig. 2 for $t_0 = 0$; five elements can be distinguished: the unstable harmonic response (one dot marked with **1**),

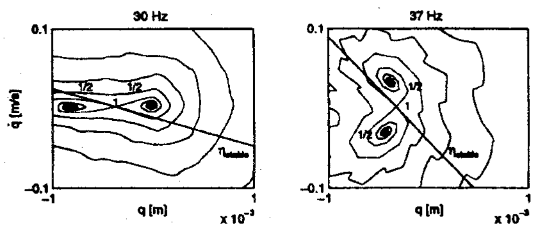


Figure 2: Poincaré maps of the 1-DOF model

the stable $\frac{1}{2}$ subharmonic response (two dots marked with **1/2**), the unstable manifold (the curve starting at **1**, while ending at **1/2**), the stable manifold (the curve ending at **1**, while filling the entire state space), and the stable eigenvector η_{stable}.

The accuracy in which the stable manifold is approximated by the stable eigenvectors, is both frequency and time dependent. The frequency dependency is shown in Fig. 2; especially the angled shape of the stable manifold at 37 Hz can limit the applicability of the stable eigenvectors as an approximation for the stable manifold. The usage of the stable eigenvectors at the resonance frequency of 33.7 Hz, is restricted to such a small region around the unstable harmonic response, that this frequency is omitted in the example. The time dependency results in a varying region around the unstable harmonic response within the excitation period, in which the stable manifold can be approximated accurately by the stable eigenvectors. This region remains, on average, above the level γ; see section (4).

4 Control Based on Feedback Linearization

The control used for stabilizing the unstable harmonic response, is based on feedback linearization [4], and is composed of a part that compensates the existing behavior, and a part that achieves the desired behavior. Based on the 1-DOF model of equation (1), the compensation of the existing behavior is effectuated by:

$$u = B\dot{q} + Kq + F_{nl}(q) - F_e + Mv. \qquad (5)$$

The desired behavior q_d is achieved with:

$$v = \ddot{q}_d - K_D(\dot{q} - \dot{q}_d) - K_P(q - q_d), \qquad (6)$$

where the values for $K_D \geq 0$ and $K_P \geq 0$ can be chosen using linear design techniques such as pole placement, or linear quadratic optimization. Feedback linearization results in the following error equation:

$$\ddot{e} + K_D\dot{e} + K_P e = 0, \; e = q - q_d. \qquad (7)$$

Using Lyapunov's method, this equation can proved to be asymptotically stable in a global sense, resulting in an error e that approaches zero as time increases.

The desired behavior (q_d, $\dot{q}_d$, and $\ddot{q}_d$) can be chosen to be the unstable harmonic response [6]. Once this response has been stabilized, no control effort u will be needed, because the unstable harmonic response is a natural solution of the uncontrolled system; see equation (4). The desired behavior is calculated beforehand, by calculating the unstable harmonic response, and is approximated by a truncated Fourier series, that enables easy derivation of the desired displacement, velocity, and acceleration needed for the control.

The desired behavior (q_d, $\dot{q}_d$, and $\ddot{q}_d$) can also be chosen to be on the approximated stable manifold, because on the stable manifold the system itself drives the state towards the unstable harmonic response, without control effort. The points on the approximated stable manifold, that define the desired behavior, are chosen as the points closest to the momentary state x, or:

$$x_r = x_d + \alpha\varepsilon(\beta)\eta_{stable}, \; \varepsilon(\beta) = \begin{cases} 1 & \text{if } \beta \geq 0, \\ 0 & \text{if } \beta < 0, \end{cases} \qquad (8)$$

and:

$$\alpha = \frac{\eta^T_{stable}\tilde{e}}{\eta^T_{stable}\eta_{stable}}, \; \tilde{e} = \begin{vmatrix} q - q_d \\ \dot{q} - \dot{q}_d \end{vmatrix}. \qquad (9)$$

The usage of the approximated stable manifold as the desired behavior for control, is only possible in the neighborhood of the unstable harmonic response, where the stable eigenvectors of the monodromy matrix approximate the stable manifold sufficiently accurate. This restriction requires a two-stage control. In the first stage, the system state is controlled directly towards the unstable harmonic response, until it approaches the unstable harmonic response sufficiently close. In the second stage, the system state is controlled towards the stable eigenvectors to obtain control effort reduction, because the system drives the state towards the unstable harmonic response. The switching between stages is effectuated by choosing β from equation (8) as:

$$\beta = \left(1 - \frac{2||e^*||}{\gamma + ||e^*||}\right), \; e^* = \begin{vmatrix} \frac{1}{|q|_{max}} & 0 \\ 0 & \frac{1}{|\dot{q}|_{max}} \end{vmatrix} \tilde{e}. \qquad (10)$$

The implementation of the two-stage control results in the following control force u:

$$\begin{aligned} u &= B\dot{q} + Kq + F_{nl}(q) - F_e \\ &+ M(\ddot{q}_r - K_D(\dot{q} - \dot{q}_r) - K_P(q - q_r)). \end{aligned} \qquad (11)$$

5 Example

To illustrate the ability of the two-stage controller to stabilize the unstable harmonic response, and to use the approximated stable manifold to reduce control effort, a comparison is made between two cases. In case 1, the system state is controlled, in the second stage, towards the stable eigenvectors, thus, stabilizing the unstable harmonic response indirectly. In case 2, the system state is controlled, in the second stage, directly towards the unstable harmonic response. In the first stage, the system state is controlled, for both cases, directly towards the unstable harmonic response. This can be seen in Fig. 3, where the initial state is chosen on the stable $\frac{1}{2}$ subharmonic response. Switching be-

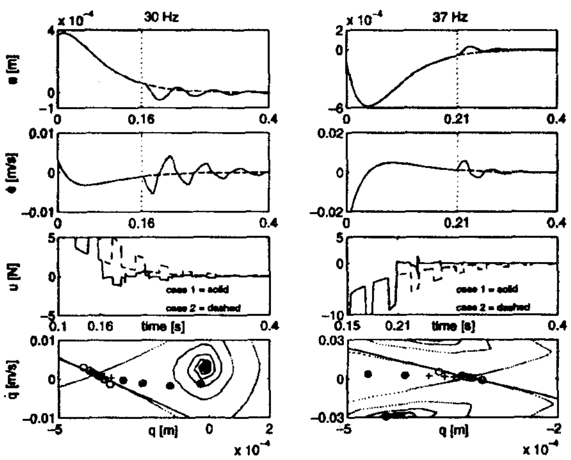


Figure 3: Controlled behavior of the 1-DOF model

tween the first and the second stage, takes place when the relative error e^* in equation (10) becomes smaller than 20%, or $\gamma = 0.2$. In the Poincaré maps of Fig. 3, it can be seen that after switching, the system state belonging to case 1, represented by the o-symbols, is on the stable eigenvectors, while the system state belonging to case 2, represented by the +-symbols, continues its orbit ignoring the presence of the stable manifold completely. The differences between the two cases, after switching to the second stage, are the control effort, and the reduction of the error between the actual state and the state represented by the unstable harmonic response. The reduction of the error, however, is in both cases comparable due to the choice of the control parameters for case 2, that are related to the Lyapunov exponents. In the time series of Fig. 3, it can be seen that the control effort after switching, on $t \approx 0.16$ [s] at 30 Hz, and on $t \approx 0.21$ [s] at 37 Hz, becomes much smaller for case 1 compared to case 2. The control effort reduction at 30 Hz is smaller compared to the control effort reduction at 37 Hz, because the region where the stable eigenvectors approximate the stable manifold accurately, is larger at 37 Hz than at 30 Hz. It can also be seen in the time series of Fig. 3, that the control effort after switching reduces to a low value, because motions almost perpendicular to the direction of the flow of the system state need little control effort.

6 Conclusions

The two-stage control can be successful in reducing overall control effort when stabilizing the unstable harmonic response of a harmonically excited 1-DOF beam with one-sided spring. The overall control effort reduction depends on the region where the stable manifold can be approximated accurately by the stable eigenvectors of the monodromy matrix. The control effort needed to force the system state within this region, in the first stage, can be large in general, and reduces the relative control effort reduction obtained in the second stage; this occurs at the resonance frequency of 33.7 Hz, due to the discontinuity of the stable manifold. This region could be enlarged using higher order approximations of the stable manifold.

At the stable manifold, the time needed for the system state to evolve towards the unstable harmonic response, solely depends on the system dynamics. However, controlling the system state directly towards the unstable harmonic response, makes it possible to choose this time via the choice of the control parameters.

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