The observation of Bose-Einstein condensation (BEC) in a dilute ultracold gas of alkali atoms [1–3] in 1995 is generally considered to be one of the most fascinating applications of atomic cooling techniques. Since these pioneering experiments, the field of BEC continues to lead to surprising results. Recently, Myatt et al. observed an unexpected slow decay of overlapping $^{87}$Rb condensates in two different spin states [4], due to a near coincidence in two scattering lengths [5,6]. In contrast, a recent experiment by Söding et al. [7] shows such a surprisingly large decay rate of trapped ultracold Cs in the $f, m_f = 4, 4$ state that the realization of BEC appears to be impossible. The other potentially viable hyperfine state, $f, m_f = 3, -3$, does not offer a solution since recent observations show it also decays anomalously quickly [8]. The fast decay occurs even at very weak magnetic fields where it is normally highly suppressed. In this Letter, we theoretically explain these observations and point to ways to circumvent the fast decays. Interatomic collisions, responsible for the decays, are therefore crucial to understand.

The interactions between cold cesium atoms were previously studied in 1993 [9], using the limited experimental information available [10]. New data and progress in theory have led us to reinvestigate this problem. Here, we use a larger set of cold-collision data that consists of atomic fountain frequency shifts and elastic scattering rates. The phase parameters from this analysis completely characterize cold collisions between two ground-state cesium atoms. We show that the total set of experimental data can be described in a consistent picture, a fascinating aspect of which is that a Feshbach resonance happens to dominate $V_{\text{ind}}$ for cesium. It is an interaction of the spins via modes of the electronic orbital degrees of freedom, instead of via modes of the electromagnetic field as in the case of $V_{\text{dip}}$. Its strength and radial form factor, comparable to the factor $1/r^3$ in $V_{\text{dip}}$, have been predicted by an ab initio calculation [12].

Table I summarizes the experimental data for our analysis. This analysis includes the atomic hyperfine interaction of each atom, the relative motion of the two nuclei, the above $V_{\text{dip}}$ and $V_{\text{ind}}$ terms, and the $S = 0$ and $I$ interaction potentials at long range $r > 20a_0$. The less well-known $r < 20a_0$ parts are treated by means of boundary conditions at $r_0 = 20a_0$ for the rapidly oscillating $S = 0$ and $I$ radial wave functions. The boundary conditions consist of the local phases $\phi_S^0, \phi_I^0$ of these oscillating wave functions at $r_0$ and their first derivatives $\dot{\phi}_S^0, \dot{\phi}_I^0$, with respect to $E$ and $l(l + 1)$, which summarize the “history” of the collision in the part of space $r < r_0$. The dependence on the derivative parameters is weak, owing to the small $E$ and $l$ ranges involved in cold collisions. We calculate these using the best available singlet [13] and triplet [14] potentials. The generally fractional $s$-wave vibrational quantum numbers at dissociation, $v_{\text{DS}}$ and $v_{\text{DT}}$ (modulo 1), for the singlet and triplet potentials are essentially equivalent to the first two phase parameters. They, however, provide for more direct physical insight being a measure of how far the last bound or the first unbound Cs + Cs state is from the dissociation threshold.

We begin by considering the first three experimental observations in Table I. These impose very restrictive conditions in the $v_{\text{DS}}, v_{\text{DT}}$ parameter plane. The observation of a near-zero energy potential resonance [15] in the totally polarized, pure triplet $(4, 4) + (4, 4)$ elastic collision selects a narrow strip $-0.08 < v_{\text{DT}} < +0.17$ (bold solid lines in Fig. 1). In contrast, the extremely small elastic cross section in the $(3, -3) + (3, -3)$ collision observed at about $30 \mu K$ [16] imposes a restriction in both
$v_{DS}$ and $v_{DT}$, due to the mixed singlet-triplet character of the initial spin state. Interestingly, it corresponds to a Ramsauer-Townsend minimum associated with a Feshbach resonance, which is nearby. Together, these conditions already localize the parameters rather strictly to about 1.5% of the area of the parameter plane. Another highly constraining quantity is the large $\delta \nu_{30} + \delta \nu_{40}$ total fountain collisional frequency shift [10]. Beyond the dotted lines the calculated shift goes rapidly to 0 and changes sign, in strong disagreement with experiment. Next we apply a least-squares analysis to all entries (1)–(9) in Table I. We find a very narrow $\chi^2$ minimum of 4.2 for essentially 7 degrees of freedom, indicated by the shaded area:

$$v_{DS} = -0.096 \pm 0.005, \quad v_{DT} = -0.065 \pm 0.005.$$  

(1)

Our calculations show that the regions selected in the $v_{DS}, v_{DT}$ plane depend only weakly on the strength of the indirect spin-spin interaction.

The result (1) allows us to calculate the triplet and singlet scattering lengths:

$$a_T = a_{4,4} = -350_{-35}^{+30} a_0, \quad a_S = -208 \pm 17 a_0.$$  

(2)

The excellent localization in the $v_{DS}, v_{DT}$ plane defined by the preceding experimental observations is also a good starting point to predict the decay rates. It is expected that the inelastic decay rates are strongly enhanced by the proximity of potential resonances or Feshbach resonances, since both facilitate the penetration of the colliding atoms to short distances, where the inelastic transitions take place through $V^{\text{dp}}$ and $V^{\text{ind}}$. For the $|4,4\rangle$ state, we find the decay to be dominated by $V^{\text{ind}}$. It is enhanced by the proximity of the same pure triplet potential resonance that enhances the above-mentioned elastic scattering cross section $\sigma_{|4,4\rangle}$. We indeed find the largest $G_{|4,4\rangle}$ values concentrated in the same horizontal strip in Fig. 1 [17]. To a good approximation, they are independent of $v_{DS}$. Item 11, the decay rate constant $G_{(3,-3)+(3,-3)}$, is even more restrictive. It has a pronounced Feshbach resonance dependence on $v_{DS}, v_{DT}$, leading to an excessively strong decay. The largest rates are concentrated in the region bounded by the thin solid lines in Fig. 1, which supports the previous conclusion.

An important aspect of future work aspiring to achieve BEC in Cs is avoiding the fast decay rates. With that in mind, we present in Fig. 2 the calculated $B$ dependence of the decay rate constant $G_{(3,-3)+(3,-3)}$ at 1 $\mu$K for the strength of $V^{\text{ind}}$ consistent with the measured $G_{|4,4\rangle}$.
In addition, we calculate the scattering length \( a_{3,-3} \) for atoms in the \([3, -3]\) state. In contrast to the \([4, 4]\) decay, we find a number of Feshbach resonances for fields up to 200 G, with a clear correspondence between the resonance features in \( G_{(3, -3) + (3, -3)} \) and \( a_{3,-3} \). The resonance closest to \( B = 0 \) is responsible for the enhancements of the (in)elastic rates and frequency shifts occurring prominently in the data of Fig. 1. For fields between 110 and 200 G the rate constant has dropped 2 orders of magnitude relative to the value \( 1 \times 10^{-12} \text{ cm}^3 \text{s}^{-1} \) observed in the Paris experiment, in full agreement with the measured large stability of atoms in this state at fields \( B = 170 \text{ G} \) [16,19]. The realization of BEC for large gas samples, however, would require such a reduced decay in combination with a positive scattering length, an effectively repulsive interatomic interaction, while the calculated \( a_{3,-3} \) is equal to about \(-240a_0\). BEC does appear to be possible, however, for a small enough condensate: the relevant ratio \( a_{3,-3} / a_{10} \) of the scattering length to the length scale of the harmonic oscillator ground state in the Paris experiment is not very different from that of the Rice trap [2], for which both experiment and theory have shown that a condensate is stable for fewer than 1000 atoms.

Another possibility, previously considered for laser-cooled clocks [20], is to use a different isotope of cesium. The most attractive possibility for BEC would be \(^{135}\text{Cs}\). For different isotopes the radial phases \( \phi_s^0, \phi_T^0 \) scale very accurately proportional to the square root of the atom mass [9], as expected in the WKB approximation. Applying the \( \sqrt{m} \) scaling rule to the \(^{133}\text{Cs} \) singlet and 

\[
\begin{align*}
\nu_{DS}^{(135\text{Cs})} &= 0.074 \pm 0.015, \\
\nu_{DT}^{(135\text{Cs})} &= 0.371 \pm 0.015.
\end{align*}
\]

Here, we use the above-mentioned potentials to determine the number of triplet radial nodes for \(^{133}\text{Cs} \) within \( r_0 \) to be \( 40 \pm 1 \) and the number of singlet nodes \( 138 \pm 1 \). Using (3), we can calculate all desirable quantities for \(^{135}\text{Cs} \) ground-state collisions, in particular,

\[
\begin{align*}
a_T &= a_{4,4} = +138 \pm 5a_0, \\
a_S &= +500^{+100}_{-70}a_0.
\end{align*}
\]

We conclude from the positive value of \( a_T \) that a stable totally polarized Bose condensate is possible in a \(^{135}\text{Cs} \) vapor. Also, the ratio of the rates of elastic collisions to inelastic, depolarizing collisions is favorable: using the calculated values [21] for these quantities, we find the "practical" limit [22] attainable by forced evaporative cooling is 120 nK, a factor of 8 lower than for the \([4, 4] \) state of \(^{133}\text{Cs} \).

For the \([3, -3]\) state the conditions are even more favorable because there is no adjacent Feshbach resonance. Over the field range from 0 to 1000 gauss, we find the scattering length to be nearly constant and positive:

\[
a_{3,-3}(B) = +163^{+15}_{-8}a_0.
\]

In the same range the rate constant is always less than \( 10^{-14} \text{ cm}^3 \text{s}^{-1} \) and, for field values below 5 gauss, less than \( 10^{-15} \text{ cm}^3 \text{s}^{-1} \). A \(^{135}\text{Cs} \) Bose condensate in the \([3, -3]\) state is thus expected to be stable. Also, this isotope allows for evaporative cooling to very low temperatures: the practical cooling limit is 0.1 nK and, for field values below 5 gauss, even lower by a factor of 100.

The precision of the present analysis also leads to much more accurate \( \delta \nu_{3,0} \) and \( \delta \nu_{4,0} \) frequency shifts for \(^{135}\text{Cs} \):

\[
\begin{align*}
\delta \nu_{3,0} &= -15 \pm 4 \text{ mHz}, \\
\delta \nu_{4,0} &= 1.3^{+0.02}_{-0.04} \text{ mHz},
\end{align*}
\]

for an energy of 1 \( \mu \text{K} \), where they have nearly reached their zero energy limit. The opposite sign of these shifts will allow the cold collision frequency shift to be canceled as proposed in Ref. [20].

Summarizing, we have developed a consistent picture for collisions of ultracold cesium atoms by analyzing a set of data, consisting of atomic fountain frequency shifts and elastic scattering rates. We have shown that the fast decays observed recently are due to the influence of potential and Feshbach resonances, in combination with an indirect spin-spin interaction. We have discussed the prospects for the realization of Bose-Einstein condensation in ultracold gas samples of both \(^{133}\text{Cs} \) and \(^{135}\text{Cs} \). The prospects are particularly favorable for \(^{135}\text{Cs} \) in the \([3, -3]\) state.
[17] The absolute magnitude of the measured rate constant $G_{4,4}(4,4)$ points to a strength of $V_{4}$ times larger than predicted by Mies *et al.* [12]. An indication for a similar discrepancy was found for Rb [18].
[21] We find $\sigma(4,4) = 13.4 \times 10^{-12}$ cm$^2$ and $G_{4}(4,4) = 0.263 \times 10^{-12}$ cm$^3$ s$^{-1}$.