Performance evaluation of optical cross-connects by saddlepoint approximation
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Performance Evaluation of Optical Cross-Connects by Saddlepoint Approximation

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Abstract—The impact of in-band crosstalk on the transmission performance of optical cross-connects, incorporating (de)multiplexers and space switches, is studied. A statistical description of the receiver decision variable that yields a performance analysis in good agreement with experiment is given. Bit error rate and power penalties are calculated using the so-called saddlepoint approximation which is numerically simple and gives accurate results.

Index Terms—Error analysis, optical communication, optical cross-connects, optical crosstalk.

I. INTRODUCTION

OPTICAL cross-connects are regarded as a promising solution to the increasing demand of routing flexibility and transport capacity of broadband communication systems. An example of the structure of an optical multiwavelength cross-connect is presented in Fig. 1.

Linear crosstalk in cross-connects can be classified as in-band or interband crosstalk, according to whether it has the same nominal wavelength as the desired signal or not. The effect of interband crosstalk can be reduced by concatenating narrow-bandwidth optical filters. In-band crosstalk, however, cannot be removed as the signal and the crosstalk operates at the same wavelength. The deteriorating effect of in-band crosstalk is further intensified in cascaded optical nodes due to its accumulative behavior. This paper studies the effect of in-band crosstalk on the error performance of optical cross-connects. It has been observed that the crosstalk induced noise shows a highly non-Gaussian (bounded) statistics [1]. The use of an approximate Gaussian (nonbounded) distribution results in performance analyzes predicting greater penalties than those using a bounded distribution [2]; see Fig. 6.

In this paper, a statistical description of the receiver decision variable is given through the moment generating function (mgf). The performance evaluation is carried out with the help of the so-called saddlepoint approximation, using the mgf for the decision variable, that is numerically simple and gives accurate results. The analysis takes into consideration the effects of linear random polarization, nonideal extinction ratio, and receiver thermal noise together with transmitted data statistics. Power penalties due to inband crosstalk have been measured in an experimental setup that uses a directly modulated light source. Experimental results are in good agreement with the theory.

The paper is structured as follows: In Section II, the model of the system under analysis is presented. Section III presents the derivation of the mgf of the decision variable while Section IV introduces the saddlepoint approximation for calculating error probabilities. Section V describes the experiments. Comparison of experimental results and theory is also presented. Finally, in Section VI, summarizing conclusions are drawn.

II. SYSTEM MODEL

We consider an optical signal which has traversed an optical cross-connect consisting of (de)multiplexers and space switches (Fig. 1). The equivalent baseband form of the total
The optical field is given by
\[ S(t) = \text{Re}\left\{ \mathcal{S}(t) \exp(j\omega_0 t) \right\} \]
where, in general, \( \mathcal{S}(t) \) is the envelope (modulation) of the input optical signal (\( \mathcal{S}(t) \)), expressed as the real part of a complex field function
\[ \mathcal{S}(t) = A(t) \eta \exp(j\phi(t)) \]
where \( \omega_0 = 2\pi f \) is the optical frequency, \( f \) is the phase, and \( A(t) > 0 \) is the optical pulse shape. The vector \( \eta \) indicates the state of linear polarization. \( S_{\text{sh}}(t) \) and \( S_{\text{ch}}(t) \) represent the optical field, equivalent baseband form, of the desired signal and crosstalk interferer, respectively.

The output of the photodetector \( I_{\text{ph}}(t) \) is a shot noise process characterized by a photoelectron intensity \( \lambda(t) \). The time varying intensity of the photoelectron process is proportional to the instantaneous optical signal power. The instantaneous optical power is proportional to the squared magnitude of the electromagnetic field quantity. Hence, the photoelectron intensity can be written as
\[ \lambda(t) = \frac{1}{2} \frac{n}{\hbar} |S_{\text{sh}}(t)|^2 \text{ photoelectrons/s} \]
where \( n \) is the photodetector quantum efficiency and \( \hbar \) is Planck’s constant. This relation provides a connection between the electro-magnetic field model and the photon model of light, constituting the so called semiclassical approach of optical detection [3].

To continue the analysis, we return to the description of the optical field of the desired signal and the crosstalk, \( S_{\text{sh}}(t) \) and \( S_{\text{ch}}(t) \), respectively.

\[ S_{\text{sh}}(t) = \sqrt{b_k} A_s(t) \eta_s e^{j\phi_s(t)} \]
\[ S_{\text{ch}}(t) = \sqrt{c_k} A_c(t) \eta_c e^{j\phi_c(t)} \]
where \( \epsilon \) is the component power crosstalk parameter: the ratio of leakage crosstalk to signal power. The quantity \( b_k \) \((k = 0, \pm 1, \pm 2 \pm \cdots) \) is introduced to represent the binary symbols: \( b_k \in \{\rho, 1\} \) \((0 \leq \rho < 1) \). For the case of perfect extinction ratio we have \( \rho = 0 \). \( \phi_s \) and \( \phi_c \) are real (we consider only linear polarization states) unit vectors representing the signal and crosstalk polarization state, respectively.

It is convenient to normalize the optical field (to avoid carrying the factor \( \hbar \pi \) along in further calculations) so that the photoelectron intensity can be written as
\[ \lambda(t) = \frac{1}{2} |S_{\text{sh}}(t)|^2 = \frac{1}{2} |S_{\text{sh}}(t)|^2 + |S_{\text{ch}}(t)|^2. \]

III. THE MOMENT GENERATING FUNCTION

The postdetector filter is assumed to be an integrator over the time interval \([0, T]\). With no loss of generality we consider the time interval \([0, T]\) \((k = 0)\) and denote the decision variable by \( Z = \mathcal{Z}(T) \), where
\[ Z = \int_0^T [I_{\text{sh}}(t) + I_{\text{ch}}(t)] dt 
= X_{\text{sh}} + X_{\text{ch}} \]

\( X_{\text{ch}} \) is a zero mean, Gaussian distributed random variable (r.v.) with variance \( \sigma_{\text{ch}}^2 \) given by
\[ \sigma_{\text{ch}}^2 = \frac{\sqrt{2} K_B T_k T}{q e R_L} \]
where \( K_B \) is the Boltzmann’s constant, \( T_k \) the temperature in Kelvin, \( q e \) the electron charge, and \( R_L \) the receiver resistance load. The mgf of the decision variable is
\[ M_Z(s) = E\{e^{sZ}\} = M_{\text{ch}}(s) M_{\text{sh}}(s) \]
where \( M_{\text{ch}} \) is the mgf for a zero-mean Gaussian variable with variance \( \sigma_{\text{ch}}^2 \)
\[ M_{\text{ch}}(s) = e^{\sigma_{\text{ch}}^2 s^2/2} \]
\( M_{\text{sh}}(s) \) is the mgf of \( X_{\text{sh}} \), the filtered shot noise contribution to the decision variable \( Z \). The product of mgf in (11) is a consequence of the stochastic independence of the shot and thermal noise.

The filtered shot noise is well modeled by a doubly stochastic Poisson process with intensity \( \lambda(t) \). Hence, for the case of an integrator postdetection filter, \( M_{\text{sh}} \) is given by [7]
\[ M_{\text{sh}}(s) = M_{\Lambda}(e^s - 1) \]
where \( M_{\Lambda}(s) = E\{e^{s\Lambda}\} \) and \( \Lambda = \int_0^T \lambda(t) dt \) is the Poisso parameter.
A. Single Crosstalk Source

For the case of a single crosstalk source the parameter $\Lambda$ has the form

$$\Lambda = \frac{1}{2} \int_0^T |S_{sc}(t)|^2 dt$$

$$= m(\xi_0^2 + \alpha \xi_0^2) + 2m \sqrt{\xi_0^2 \xi_0^2} \cdot \xi_0 \cos (\phi_x - \phi_x). \quad (14)$$

The bit alignment between the signal and crosstalk interferer is assumed to be perfect. Expression (14) is derived under the assumption that the relative phase difference is constant at least within one bit duration. The phase difference $\phi_x - \phi_x$ is assumed to be a uniformly distributed random variable in the interval $[0, 2\pi]$. The probability distribution function (pdf) of the variable $\xi = \cos (\phi_x - \phi_x)$ is the so-called arcsine distribution. The pdf of $\xi$ is given by [8]

$$f(\xi) = \begin{cases} \frac{1}{\sqrt{1-\xi^2}}, & -1 < \xi < 1 \\ 0, & \text{elsewhere.} \end{cases} \quad (15)$$

Experimental measurements have shown that the statistics of in-band crosstalk induced noise approaches the form described by (15) [1].

The signal and crosstalk are assumed to exhibit linear polarizations with random, independent orientation angles $\theta_s$ and $\theta_x$, respectively. The parameter $\Lambda$ takes the form

$$\Lambda = m(\xi_0^2 + \alpha \xi_0^2) + 2m \sqrt{\xi_0^2 \xi_0^2} \xi(\theta_s, \theta_x)$$

$$= m(\xi_0^2 + \alpha \xi_0^2) + 2m \sqrt{\xi_0^2 \xi_0^2} \xi(\theta_s, \theta_x) \quad (16)$$

where the function $\xi(\theta_s, \theta_x)$ is given by [4]

$$\xi(\theta_s, \theta_x) = \cos (\theta_s - \theta_x) \quad (17)$$

$\theta_s - \theta_x$ is taken to be uniformly distributed in $[0, 2\pi]$. The pdf of $\zeta$ is given by the doubled, nonnegative part of an arcsine distribution [8]

$$f(\zeta) = \begin{cases} \frac{2}{\pi \sqrt{1-\zeta^2}}, & 0 < \zeta < 1 \\ 0, & \text{elsewhere.} \end{cases} \quad (18)$$

The mgf for $\Lambda$ is derived from the pdf of the random variables involved in it. The result is (see Appendix for a derivation)

$$M_{\Lambda}(s) = \exp \left[ sm(\xi_0^2 + \alpha \xi_0^2) \right] I_0(s \sqrt{\xi_0^2 \xi_0^2}) \quad (19)$$

where $I_0(x)$ is the modified Bessel’s function of zero order. The final expression of the mgf for $Z$ is then

$$M_{Z}(s) = M_{\Lambda}(e^s - 1)M_{\lambda}(s). \quad (20)$$

B. Multiple Crosstalk Sources

This section treats the case of in-band crosstalk when $N$ interfering fields are present. We assume that each interferer has relative crosstalk power $\epsilon$. The expression for the decision variable takes then the following form:

$$Z = mb_0^2 + 2\sqrt{\epsilon}m \sum_{n=1}^{N} \sqrt{b_0^2 \xi_0^2} \xi(s) \cdot X_{n} \pm \cos (\phi_x - \phi_x)$$

$$+ 2 \epsilon m \sum_{j=n+1}^{N} \sum_{n=1}^{N} \sqrt{b_0^2 \xi_0^2 \xi_0^2} \cos (\phi_x - \phi_x)$$

$$+ \epsilon m \sum_{n=1}^{N} b_0^2 + X_{n}. \quad (21)$$

The decision variable (21) consists of the signal term, the signal-crosstalk beat terms, the crosstalk-crosstalk beat terms, self crosstalk beat term, and the receiver thermal noise. The third terms (crosstalk-crosstalk beat terms) have a variance smaller by $O(\sqrt{\epsilon})$ than the signal-crosstalk beating terms. However, in this paper the crosstalk-crosstalk beat terms are not neglected, but considered statistically independent and will be included in the performance analysis.

The error probability analysis is conducted by a weighted statistically average of the error probability for each value $\mu$ of the $N$ crosstalk term being simultaneously “one.” This probability is given by the binomial distribution

$$p(\mu) = \frac{N!}{(N-\mu)!\mu!} p^\mu (1-p)^{N-\mu}. \quad (22)$$

Hence, the average error probability $P_e$, for a given threshold $\alpha$, is given by

$$P_e = \sum_{\mu=0}^{N} P_e(\alpha, \mu)p(\mu). \quad (23)$$

The $N$ crosstalk sources are considered statistically independent. Hence, the mgf for $Z$ is easily derived using (19) due to the fact that the mgf of a sum of independent r.v is the product of the mgf for each r.v in the sum. Note that the effect of nonperfect extinction ratio is also easily incorporated in the analysis by considering the total crosstalk field as the sum of $\mu$ field terms of amplitude $A(t)$ and $\nu = N - \mu$ field terms with amplitude $\sqrt{\nu}A(t)$. The bit-error rate for a given $\mu$ is calculated by the saddlepoint approximation; see Section IV-A.

IV. PERFORMANCE ANALYSIS

The question is to evaluate the average error rate $P_e$ of the system under discussion. We are going to treat the case of amplitude shift keying (ASK) modulation format. The error probability, given that a binary “one” is transmitted is

$$q_-(\alpha) = P_{T1}(Z < \alpha) \quad (24)$$

where $\alpha$ denotes the decision threshold. Similarly, the error probability, given a binary “zero” is transmitted is

$$q_+(\alpha) = P_{T0}(Z > \alpha). \quad (25)$$

Assuming that the symbols are a priori equally probable, the average error probability is

$$P_e = \frac{1}{2} (q_-(\alpha) + q_+(\alpha)). \quad (26)$$

A. Analysis by Saddlepoint Approximation

The saddlepoint approximation (spa) has been proposed by Helstrom [9], as an efficient and numerically simple tool for analyzing communication systems. The spa has shown a reasonably high degree of accuracy in the analysis of optical communication systems, e.g., [10].
As shown in [9], the tail probability \( q_+(\alpha) \) is approximately equal to
\[
q_+(\alpha) \approx \frac{\exp[\Phi(s_0)]}{\sqrt{2 \pi \Phi''(s_0)}}
\] (27)
the so-called saddlepoint approximation. The function \( \Phi(s) \) is related to the mgf for \( Z, M_Z(s) \) by
\[
\Phi(s) = \ln[M_Z(s)] - s\alpha - \ln |s|.
\] (28)
The parameter \( s_0 \) is the positive root of the equation
\[
\Phi'(s) = 0
\] (29)
and \( \Phi''(s_0) \) stands for the second derivative of (28) at \( s = s_0 \).
The lower probability tail
\[
q_-(\alpha) = \int_{-\infty}^{\alpha} p(z) \, dz
\] (30)
is approximated by
\[
q_-(\alpha) \approx \frac{\exp[\Phi(s_1)]}{\sqrt{2 \pi \Phi''(s_1)}}
\] (31)
with \( s_1 \) equal to the negative root of (29). See [9] or [3] for further details. The error probability is minimized by adjusting the detection threshold \( \alpha \). The optimum value of \( \alpha \) and the parameters \( s_0, s_1 \) may be found numerically by solving an appropriate set of equations [3].

V. EXPERIMENTAL RESULTS AND DISCUSSION

The experimental arrangement depicted in Fig. 2 has been used to model the crosstalk interference in a cross-connect system. As transmitter, a DFB laser which has an unmodulated linewidth of 50 MHz at center wavelength of 1550 nm is directly driven by a pulse pattern generator. The generator produces repetitive \( 2^7 - 1 \) PRBS of 2.5 Gb/s electrical signals. The extinction ratio is measured to be 8 dB. The laser light is divided into two paths. One path is regarded as the desired signal and the other the crosstalk. The crosstalk path is further divided into \( N \) channels by an \( 1 \times N \) photonic splitter. An optical attenuator, and polarization controller are located and adjusted to give each crosstalk channel an equal interference to the desired signal and obtain matched polarizations at the receiver. Fiber delays with different lengths are used to decorrelate all crosstalk channels.

In the experiment only three fiber delays are used with a different length of 500 m which far exceeds the laser coherence length. At the end the crosstalk channels are combined by a photonic coupler. The desired signal after being interfered by the crosstalk is detected and examined using the bit error rate tester. As receiver, an InGaAs PIN photodetector with a responsivity of 0.9 A/W followed by a transimpedance amplifier has been used. The detector’s sensitivity is about -26dBm for a bit error rate of \( 10^{-9} \). The electrical amplifier (EA) can give a maximum gain of 32 dB and has a noise figure of 5 dB. The electrical filter for suppressing the receiver thermal noise has a bandwidth of 1.75 GHz. The performance is measured using a fixed decision-threshold at midway between “one” and “zero.”

Fig. 3 shows the output of the receiver when there is no crosstalk source added in the system. We can see the presence of receiver thermal noise in bit “one” and “zero” as well. Next, the crosstalk channels are added to the signal channel. As an example, Fig. 4 gives a plot of the signal channel contaminated by three crosstalk channels of \( -20 \) dB each, relative to the signal channel power. The envelope of the interference is not constant. At the edges of the pulses where the frequency variation due to chirp are maximum, small distortion can be observed. The shapes of the envelopes are further varied by bit delays as the results of different fiber delay used in the experiment setup.

Measured and theoretical bit-error rate curves for a single crosstalk source and different values of \( \epsilon \) are presented in Fig. 5. In Fig. 6 measured results are presented for power penalties together with the theoretical curves, calculated by the spa using the derived statistics for the receiver decision variable (solid lines). The result are in good agreement with the theory considering that discrepancies may arise due to additional penalties introduced by the signal processing and measurement errors. Analysis with linear randomly polarized signals resulted in power penalties non substantially different from those obtained for the worst case: precisely matched signal and crosstalk polarizations. This observation is in good agreement with an earlier published result stating that systems with randomly polarized fields show a statistical preference for near-worst-case operation [4]. In the experiment the polarization of signal and crosstalk are matched to simulate the worst case situation.

Measurements of crosstalk-induced power penalties in an optical cross-connect switch have been reported in [5] and [6]. The experimental setup reported in [5] and [6] uses an external modulated light source in contrast to a directly modulated source used in our experiment making a direct comparison of results difficult. Power penalties measured using a directly modulated source are reported in [2] for a single crosstalk source and at lower bit rate than that employed in the present work. Our result (see Fig. 6) shows the same general
Fig. 3. The detected laser pulse pattern (a) and eye diagram (b) used in the crosstalk measurement. The laser is directly modulated with 2.5 Gb/s $2^7 - 1$ pseudo-random binary sequences.

appearance as that in [2]: good agreement with experiment, assuming a bounded statistics for crosstalk. The results for power penalties yielded by the Gaussian approximation are also shown in Fig. 6 (dash-dot lines). It can be observed that the analysis using a Gaussian distribution yields considerably greater power penalties than the bounded statistics approach, and than the measurement results.

VI. CONCLUSION

Performance analysis of in-band crosstalk in an optical cross-connect has been studied using a comprehensive statistical approach. Supporting measurements, using a directly modulated light source, appear to confirm the theoretical analysis with reasonably accuracy. The saddlepoint approximation yields results in good agreement with the experimental data while the Gaussian approximation predicts greater penalties. Furthermore, the spa is numerically simple. It is shown that at a bit-error rate of $10^{-9}$ component crosstalk levels less than $-24$ dB yield power penalties lower than 1 dB for a single crosstalk source; while for three interferers crosstalk levels less than $-30$ dB result in power penalties of below 1 dB.

APPENDIX

This appendix gives a short derivation of the mgf for the signal-crosstalk term of the receiver decision variable in (16).
As we know the pdf of $\zeta$, cf. (18), the unconditioned mgf $M_\zeta(s)$ can be written as

$$M_\zeta(s) = \int_{-\infty}^{\infty} \frac{e^{s\zeta}}{\sqrt{1 - \zeta^2}} d\zeta.$$  

An analytical expression for (35) can be found by using [12, eq. (6.567)]. Finally, the result, which is used in the derivation of (19), is

$$M_\phi(s) = I_0^2(s/2).$$  

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REFERENCES


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