Infrared thermography of sorptive heating of thin porous media – Experiments and continuum simulations

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ABSTRACT

We have studied the imbibition of water from a stationary nozzle into thin, moving porous media that are suspended in air, as well as the accompanying evaporation and condensation processes. Due to sorptive heating and the latent heat associated with the phase change processes, the temperature of the porous medium becomes non-uniform. We have measured the temperature distributions using infrared thermography as a function of substrate speed. Moreover, we developed a numerical model coupling Darcy flow and heat transfer in the thin porous medium with gas flow, heat and water vapor transport in the surrounding gas phase. The numerical simulations reproduce the measurements very well and point at an intricate buoyancy-induced gas-phase convection pattern.

Keywords:
Infrared thermography
Moisture distributions
Porous media
Sorption
Heat of wetting

1. Introduction

Imbibition and drying processes of thin porous media are of paramount importance in technological applications such as paper making [1–8], printing [9–28] and coating [29–37]. Owing to the strong absorption bands of water in the near infrared (NIR) spectral region, NIR spectroscopy has been used for moisture content analysis during paper making and drying processes [38–48]. Similarly, infrared (IR) thermography has been used as an indirect method for the determination of moisture distributions in paper [49–58].

Aslannejad et al. have used IR thermography to characterize the temperature rise at the wetting front accompanying the vertical imbibition of water into a sheet of dry paper driven by capillary rise [58]. Evaporation and condensation processes were eliminated by means of impermeable plastic sheets that were contacting the paper on both sides. They measured temperature amplitudes for a single sheet of paper up to approximately 1 °C. The authors concluded from their results that evaporation/condensation processes do not significantly contribute to temperature changes in bulk porous media.

Using IR thermography, we have studied temperature changes accompanying aqueous imbibition into thin porous media that are in direct contact with the ambient atmosphere. Our motivation is to gain insight into the thermal aspects of solvent imbibition and subsequent drying of inkjet printed sheets. For paper initially equilibrated to an ambient relative humidity of about 40%, we measured temperature increases up to 3 °C. As we show, the recondensation of evaporated water by far exceeds the sorptive heating at low substrate speeds and cannot be disregarded.

The geometry of our setup resembles that of an automatic scanning absorptometer [59,60]. The important differences are that the paper is suspended in air and that the constant substrate speed is slow. Thus, imbibition in the thickness direction of the paper sheets can be considered instantaneous compared to the time required for attaining a quasi-steady moisture distribution in the horizontal direction. An IR camera captures the temperature field of the moving porous substrate. Besides conducting systematic experiments we also developed a numerical model for one-dimensional flow and heat transfer in the substrate coupled to a two-dimensional model for flow as well as heat and vapor transport in the adjacent gas phase.

The following section contains a detailed description of the experimental setup and the material properties of the paper samples. In Section 3 we present the theoretical model based on 1D unsaturated single-phase flow and heat transfer in a thin porous medium, the 2D equations for gas flow, water vapor and heat convection and diffusion in the adjacent gas phases as well as the relevant boundary and interfacial conditions. In Section 4 we describe the experimental results, which are compared with and serve as a validation for the numerical results that are discussed in Section 5.

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2. Experimental

Fig. 1(a, b) shows a schematic and a photograph of the experimental set-up. It consists of a sheet of paper mounted 6 mm above an aluminum base plate fastened onto a motorized translation stage (Newport, model UTS 100CC). A stationary brass injector with 3 supply holes of diameter 1 mm is connected to a motorized syringe pump (KD Scientific, model Gemini 88 plus) and supplies water onto the suspended sheet of paper. The gap distance between the bottom surface of the injector and the paper surface was maintained at approximately 0.5 mm. The constant speed of motion $U_{sub}$ of the substrate is varied in the range 0.2–3 mm/s. The dimensions of the substrates are on the order of 256 pixels, spectral sensitivity range 7–13 μm.

Fig. 1(c) shows a typical temperature map acquired approximately 10 s after injection of water. The ambient temperature and the relative humidity (RH) are measured during the experiments (General Electric, model RH2350 plus). Experiments have been performed at ambient conditions with room temperature varying in the range 23–27 °C and RH varying in the range 40–60%.

The paper types studied comprise typical coated and calendered (Mondi, DNS HSI CF, porosity 41.3%) and uncoated, uncalendered (Mondi, DNS HSI NF, porosity 35.8%) printing papers. We used demineralized water for our experiments (BWT Ministil P-12, electrical conductivity 1.0 μS/cm).

3. Theoretical model

We have developed a continuum scale model for one-dimensional (1D) water imbibition into unsaturated thin porous media including evaporation/condensation and heat transfer effects that is coupled to a two-dimensional (2D) model for gas-phase convection, vapor transport and heat transfer in the ambient atmosphere above. These fully-coupled set of equations are solved using the finite-element software Comsol Multiphysics 5.3a.

Table 1 provides a list of the variables and parameters used in the model along with their units and numerical values.

3.1. Transport in the thin porous medium domain

Our starting point is the single-phase transport equation for liquids in unsaturated porous media [61]

$$\phi \rho_w \frac{\partial S_w}{\partial t} + \nabla \cdot (\rho_w \mathbf{u}) = 0, \quad (1)$$

where $\phi$ is the porosity, $S_w = V_{liq}/(\phi V)$ is the degree of saturation, i.e. the ratio of the liquid volume $V_{liq}$ contained in a volume element $V$ of the porous medium and the volume available to fluid $\phi V$. Moreover, $\rho_w$ is the mass density of liquid water and $\mathbf{u}$ the Darcy velocity

$$\mathbf{u} = -\frac{K_{rw}(S_w)}{\mu_w} \frac{\partial p_c}{\partial S_w} \nabla S_w, \quad (2)$$

where $K_{rw}(S_w)$ is the relative permeability of water, $K_1$ is the intrinsic permeability of the porous medium, $\mu_w$ is the dynamic viscosity of water and $p_c$ is the capillary pressure. We assume that $p_c$ varies with $S_w$ according to the Van Genuchten relation [63,64]

$$p_c(S_w) = p_{gz}(S_w^{1/n_g} - 1)^{(1/n_g)}, \quad (3)$$

where $p_{gz}$, $n_g$ and $n_r$ are adjustable parameters that depend on the average pore size and the pore size distribution. Since we only consider imbibition with a monotonic water distribution, we need not take hysteretic effects into account. As is commonly assumed, we set $m_r = 1 - 1/n_r$. Aslannejad et al. [65,66] determined typical values of $p_{gz}$, $m_g$ and $n_r$ for coated and uncoated printing paper. We assumed that the relative permeability is given by the Van Genuchten relation [63,64]

$$K_{rw}(S_w) = S_w^m \left[ 1 - \left( 1 - S_w^{1/n_g} \right)^{m_r} \right]^2, \quad (4)$$

where $k_r = -0.4$ is a connectivity factor, which we treated as a fitting parameter.

In using Eq. (1) as the transport equation for the imbibition of water into the porous medium, the presence of air is neglected.
Table 1
List of symbols and parameters.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Value</th>
<th>Units</th>
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</thead>
<tbody>
<tr>
<td>(a_w)</td>
<td>Water activity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c_g)</td>
<td>GAB Isotherm constant</td>
<td>18.647</td>
<td></td>
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<tr>
<td>(c_{p,air})</td>
<td>Specific heat capacity of air</td>
<td>1000</td>
<td>J/kg K</td>
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<td>(c_{p,w})</td>
<td>Volume averaged specific heat capacity</td>
<td>1400</td>
<td>J/kg K</td>
</tr>
<tr>
<td>(c_{p,s})</td>
<td>Specific heat capacity of solid</td>
<td>4180</td>
<td>J/kg K</td>
</tr>
<tr>
<td>(C_1)</td>
<td>Prefactor of heat of wetting</td>
<td>(1.2 \times 10^6)</td>
<td>J/kg K</td>
</tr>
<tr>
<td>(D_{h,0})</td>
<td>Exponential prefactor of heat of wetting</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>(h_n)</td>
<td>Water vapor diffusivity in air</td>
<td>(0.28 \times 10^{-4})</td>
<td>m²/s</td>
</tr>
<tr>
<td>(\Delta H_h)</td>
<td>Heat of wetting</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(j)</td>
<td>Mass flux of water vapor</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(k)</td>
<td>GAB Isotherm constant</td>
<td>0.85</td>
<td></td>
</tr>
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<td>(k_{aw})</td>
<td>Thermal conductivity of air</td>
<td>0.025</td>
<td>W/mK</td>
</tr>
<tr>
<td>(k_{aw}-)</td>
<td>Volume averaged thermal conductivity</td>
<td>0.025</td>
<td>W/mK</td>
</tr>
<tr>
<td>(k_{c})</td>
<td>Relative permeability parameter</td>
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<td></td>
</tr>
<tr>
<td>(k_{rw})</td>
<td>Relative permeability parameter</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>(k_s)</td>
<td>Absolute permeability</td>
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<td>m²</td>
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<td>Thermal conductivity of water</td>
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<td>W/mK</td>
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<td>(M_0)</td>
<td>GAB Isotherm constant</td>
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<td>(m_g)</td>
<td>Van Genuchten exponent</td>
<td>1-(1/n_{gh})</td>
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</tr>
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<td>(M_w)</td>
<td>Molar mass of water</td>
<td>18 \times 10^{-3}</td>
<td>kg/mol</td>
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<td>(n_{gh})</td>
<td>Capillary pressure exponent</td>
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<td>(p)</td>
<td>Pressure of the gas</td>
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<td>Pa</td>
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<td>Ambient pressure</td>
<td>1 \times 10^5</td>
<td>Pa</td>
</tr>
<tr>
<td>(p_i)</td>
<td>Capillary pressure</td>
<td></td>
<td>Pa</td>
</tr>
<tr>
<td>(p_{ps})</td>
<td>Pressure scaling parameter</td>
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<td>Pa</td>
</tr>
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<td>(p_{sat})</td>
<td>Saturation vapor pressure</td>
<td></td>
<td>Pa</td>
</tr>
<tr>
<td>(q)</td>
<td>Heat source</td>
<td></td>
<td>W/m³</td>
</tr>
<tr>
<td>(r_a)</td>
<td>Relative humidity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R_{dry,air})</td>
<td>Specific gas constant of dry air</td>
<td>287</td>
<td>J/kg K</td>
</tr>
<tr>
<td>(R_{w,air})</td>
<td>Specific gas constant of water vapor</td>
<td>461.5</td>
<td>J/kg K</td>
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<td>(S_{as})</td>
<td>Residual air saturation</td>
<td>0.1</td>
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</tr>
<tr>
<td>(S_{rw})</td>
<td>Residual water saturation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(S_{av})</td>
<td>Degree of saturation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(T)</td>
<td>Temperature of the porous medium</td>
<td>-</td>
<td>K</td>
</tr>
<tr>
<td>(T_{amb})</td>
<td>Ambient temperature</td>
<td>298.15</td>
<td>K</td>
</tr>
<tr>
<td>(T_{gas})</td>
<td>Temperature of the gas</td>
<td></td>
<td>K</td>
</tr>
<tr>
<td>(t_{p})</td>
<td>Paper thickness</td>
<td>0.1 mm</td>
<td>mm</td>
</tr>
<tr>
<td>(u)</td>
<td>Darcy velocity</td>
<td></td>
<td>m/s</td>
</tr>
<tr>
<td>(\bar{\mathbf{u}})</td>
<td>Velocity of the gas phase</td>
<td></td>
<td>m/s</td>
</tr>
<tr>
<td>(U_{sub})</td>
<td>Speed of the substrate</td>
<td></td>
<td>m/s</td>
</tr>
<tr>
<td>(x)</td>
<td>Horizontal coordinate</td>
<td></td>
<td>m</td>
</tr>
<tr>
<td>(y)</td>
<td>Vertical coordinate</td>
<td></td>
<td>m</td>
</tr>
<tr>
<td>(x_{k})</td>
<td>Dynamic viscosity of air</td>
<td>1.81 \times 10^{-5}</td>
<td>Pa s</td>
</tr>
<tr>
<td>(\mu_w)</td>
<td>Dynamic viscosity of water</td>
<td>8.9 \times 10^{-4}</td>
<td>Pa s</td>
</tr>
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<td>(\rho_{av})</td>
<td>Volume averaged density</td>
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<td>kg/m³</td>
</tr>
<tr>
<td>(\rho_{gas})</td>
<td>Mass density of air-vapor mixture</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>(\rho_{h,0})</td>
<td>Concentration of water vapor</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>(\rho_{s})</td>
<td>Mass density of solid</td>
<td>1500</td>
<td>kg/m³</td>
</tr>
<tr>
<td>(\rho_{sat})</td>
<td>Saturation conc. of water vapor in air</td>
<td>0.02287</td>
<td>kg/m³</td>
</tr>
<tr>
<td>(\rho_{m})</td>
<td>Mass density of water</td>
<td>1000</td>
<td>kg/m³</td>
</tr>
<tr>
<td>(\phi)</td>
<td>Porosity</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This is justified by the large viscosity ratio \(\mu_w/\mu_{aw} \approx 50\) and the small thickness of the porous medium. Eq. (1) applies to systems without mass loss. In our case, evaporation and condensation effects will induce changes in the local saturation, which depend on the local temperature. This requires addition of a source and sink term \(j\) in Eq. (1) and to simultaneously solve for heat transfer [62]

\[
\dot{Q} = \frac{\partial S_{w}}{\partial t} + \nabla \cdot (\rho_{w} \bar{\mathbf{u}}) = J. \tag{5}
\]

\[
(\rho c_p)_{air} \frac{\partial T}{\partial t} + (\rho c_p)_{air} \bar{\mathbf{u}} \cdot \nabla T = \nabla \cdot (k_{w} \nabla T) + \dot{Q}. \tag{6}
\]

where \(\dot{Q}\) accounts for heat generation processes in the bulk (unit \(W/m^3\)). The average of the product of mass density and specific heat capacity at constant pressure is defined as [62]

\[
(\rho c_p)_{as} = \phi S_{w} \rho_{w} c_{p,w} + (1 - S_{w}) \rho_{m} c_{p,m} + (1 - \phi) \rho_{s} c_{p,s}. \tag{7}
\]

Similarly, the average thermal conductivity is assumed to be

\[
\kappa_{av} = \phi S_{w} \kappa_{w} + (1 - S_{w}) \kappa_{air} + (1 - \phi) \kappa_{s}. \tag{8}
\]

Here the subscripts ‘s’ and ‘air’ refer to the intrinsic properties of the solid [69,67,68] (not including the pore space) and air, respectively. In practice, the terms containing properties of air can be neglected, owing to the low numerical values of \(\kappa_{av}\) and \(\rho_{air}\).

We now transform Eqs. (5) and (6) into a coordinate system moving with speed \(U_{sub}\) into the negative \(x\)-direction and perform an average in the \(y\)-direction across the thickness \(t_{p}\) of the porous medium, which yields the one-dimensional evolution equations

\[
\frac{\partial S_{w}}{\partial t} + \frac{\partial}{\partial x} (\rho_{w} \bar{\mathbf{u}}) = J. \tag{9}
\]

\[
(\rho c_p)_{air} \frac{\partial T}{\partial t} + (\rho c_p)_{air} \bar{\mathbf{u}} \cdot \nabla T = \frac{\partial}{\partial x} \left( \kappa_{w} \frac{\partial T}{\partial x} + \dot{Q} \right). \tag{10}
\]

Information on the source and sink terms \(j\) and \(\dot{Q}\) will be provided in Section 3.3.

3.2. Gas-phase domain

In the gas-phase domain, the incompressible Navier-Stokes equation

\[
\rho_{gas} \left( \frac{\partial \bar{\mathbf{u}}}{\partial t} + \bar{\mathbf{u}} \cdot \nabla \bar{\mathbf{u}} \right) = -\nabla p + \rho_{gas} \bar{\mathbf{g}} + \mu_{gas} \nabla^2 \bar{\mathbf{u}} \tag{11}
\]

is solved, as the relevant speeds are much smaller than the speed of sound. Here, \(\rho_{gas}\) is the mass density of water vapor–air mixtures, \(\bar{\mathbf{u}}\) the velocity, \(p\) the pressure and \(\bar{\mathbf{g}}\) is the gravitational acceleration. We assumed that the viscosity of the gas phase \(\mu_{gas}\) is unaffected by the water vapor concentration and substituted \(\mu_{gas} = 1.84 \times 10^{-5} \text{ Pa s}\). Gas flow is driven by the motion of the translation stage and by buoyancy effects, both as water vapor is lighter than air and the temperature is non-uniform. Consequently, we need to solve a coupled system of equations for gas flow, heat transfer and mass transfer.

For the time evolution of the water vapor density \(\rho_{H_2O}\) in the gas phase, the convection-diffusion equation

\[
\frac{\partial \rho_{H_2O}}{\partial t} + \bar{\mathbf{u}} \cdot \nabla \rho_{H_2O} = \nabla \cdot (D_{H_2O} \nabla \rho_{H_2O}) \tag{12}
\]

is solved. Here, \(D_{H_2O} = 2.8 \times 10^{-5} \text{ m}^2/\text{s}\) is the diffusivity of water vapor in air at room temperature \((T = 25 \text{ °C})\). For the dynamics of the gas temperature distribution, we solve the heat transfer equation

\[
(\rho c_p)_{air} \frac{\partial T_{gas}}{\partial t} + (\rho c_p)_{air} \bar{\mathbf{u}} \cdot \nabla T_{gas} = \nabla \cdot (k_{air} \nabla T_{gas}) \tag{13}
\]

where we have substituted thermal properties of dry air \((\rho c_p)_{air} = 1.18 \text{ kg/m}^3, c_{p,air} = 1.005 \text{ kJ/(kg K)}\), \(k_{air} = 0.0235 \text{ W/(m K)}\), as the ones for humid air do not differ appreciably.

We treat the mixtures of water vapor and air as an ideal gas. The dependence of the saturated vapor pressure on temperature is determined from the following empirical relation [70]

\[
\rho_{sat}(Pa) = 610.78 \exp \left[ \frac{17.2694}{T^3/238.3} \right]. \tag{14}
\]

Using the ideal gas law, the mass density of water vapor is determined as
\[ \rho_{H_2O} = \rho_{H_2O}^0 M_W / (RT), \]  
\[ \rho_{gas} = \frac{p_{gas,amb}}{R_{dry,amb} \cdot T_{dry,amb}} = \frac{p_{gas,amb}}{R_{dry,amb}} \left[ 1 - \frac{\rho_{H_2O}}{\rho_{H_2O}^0} \left( 1 - \frac{R_{dry,amb}}{R_{dry,H_2O}} \right) \right], \]

where \( p_{amb} = 101300 \text{ Pa} \) is the ambient pressure, \( R_{dry,amb} = R_{dry} = 287.04 \text{ J/(kg K)} \) and \( R_{dry,H_2O} = 461.5 \text{ J/(kg K)} \) are the specific gas constants of dry air and water vapor, respectively.

### 3.3. Evaporation rate, sorptive heating and evaporative cooling

We assume that the evaporation process is diffusion-limited, which is appropriate for near-ambient conditions. Consequently, the rate of mass loss \( j \) from the paper is given by

\[ j = \frac{D_{H_2O}}{t_p} \frac{\partial \rho_{H_2O}}{\partial y}, \]

where \( t_p \approx 100 \mu \text{m} \) is the paper thickness. The heat source/sink term \( q \) is given by [71–75]

\[ q = j \frac{L_{H_2O}}{t_p} + h_w \frac{\partial T}{\partial y} (T - T_{amb}) \]

\[ q \approx 0.6 \text{ cm} + \rho_w \phi \left( \frac{\Delta H}{t_p} - U_{sub} \frac{\partial s}{\partial y} \right), \]

where \( L_{H_2O} = 44.203 \text{ kJ/mol} \) is the enthalpy of evaporation of water and \( h_w = k_w / d = 4.33 \text{ W/m}^2/\text{K} \) is a heat transfer coefficient, where \( d = 6 \text{ mm} \) is the height of the suspended paper sheet above the Al base plate.

The first term in Eq. (18) proportional to \( j \) describes heat removal or addition due to water evaporation or condensation. The second and third terms represent Newtonian cooling due to the Al baseplate and the conductive thermal coupling with the 2D domain. The last term refers to sorptive heating effects due to dynamic changes in \( S_w \) as well as the motion of dry paper into a wet zone. We represent the differential heat of wetting [76–82]

\[ \Delta H = C_1 \exp(-C_2[MC]) \]

with fit parameters \( C_1 = 1500 \text{ kJ/kg} \) and \( C_2 = 25 \). Here,

\[ [MC] = \frac{\phi \rho_w}{(1 - \phi) \rho_0} S_w \]

is the moisture content (units: kg water per kg dry paper).

The coupling of the paper domain to the gas domain is done by assuming continuity of temperature, i.e.

\[ T_{gas}(x, y = 0, t) = T(x, t) \]

and assuming a vapor density \( \rho_{H_2O}(x, y = 0, t) \) determined by the relevant vapor sorption isotherm (VSI) of paper. We assume that the VSI is given by a Guggenheim-Anderson-de Boer (GAB) isotherm, which can reproduce the sorption behavior of cellulose-based materials well [83–87]

\[ [MC] = \frac{M_0 C_g k_{aw}}{(1 - k_{aw} \phi_w) (1 + (C_g k_{aw} - 1) k_{aw})}. \]

Here \( a_w \) is the water activity and \( M_0 = 0.061 \), \( C_g = 18.647 \) and \( k = 0.85 \) are typical fit-parameters for paper [83–87]. In local thermodynamic equilibrium, the water activity is equal to the local relative humidity \( r_s \)

\[ a_w = r_s \equiv \frac{\rho_{H_2O}(x, y = 0, t)}{\rho_{sat}(T(x, y = 0, t))}. \]

where \( \rho_{sat} \) is defined by Eqs. (14) and (15). Eqs. (20) and (21) couple \( \rho_{H_2O} \) with \( S_w \), which closes our set of equations.

### 3.4. Initial and boundary conditions

In this section, the relevant initial and boundary conditions for the coupled domains are described. Fig. 2 shows a schematic of the computational domains and the relevant boundary conditions (BCs).

#### 3.4.1. Paper domain

We assume that at \( t = 0 \) the moisture content of the paper is in equilibrium with ambient conditions, i.e. the initial value of saturation \( S_w(t = 0) \) is determined by Eq. (20) and the sorption isotherm Eq. (21)

\[ S_w(t = 0) = \frac{(1 - \phi) \rho_0}{\phi \rho_w} [MC](a_w = r_s). \]

The initial temperature is \( T_{amb} \). The BCs are assumed to be \( S_w(x = 0) = S_l \), which is close to 1, and \( S_w(x = L_1) = S_r \), which is set equal to \( S_w(t = 0) \). The thermal BCs are \( T(x = 0) = T_{amb} \) and \( T(x = L_1) = T_{amb} \). Here, \( x = 0 \) and \( L_1 \) are the position of the left and right domain boundaries in Fig. 2. The latter is sufficiently remote from the nozzle such that it does not affect the temperature distribution for small \( x \).

#### 3.4.2. Gas-phase domain

The initial gas velocity is set to zero. The initial vapor concentration and temperature correspond to ambient conditions

\[ U(x, y, t = 0) = 0, \]

\[ p(x, y, t = 0) = p_{amb}, \]

\[ \rho_{gas}(x, y, t = 0) = r_h \rho_{sat}(T_{amb}), \]

\[ T_{gas}(x, y, t = 0) = T_{amb}. \]

The BCs for the top boundary (\( y = L_2 \)) represent ambient conditions

\[ U(x, y = L_2, T = T_{amb}) = 0, \]

\[ T(x, y = L_2, T = T_{amb}) = T_{amb}. \]

\[ \rho_{gas}(x, y = L_2, T = T_{amb}) = \rho_{H_2O} = r_h \rho_{sat}(T_{amb}). \]

![Fig. 2. Schematic of the computational domains and the boundary conditions for the 2D gas-phase and the 1D porous medium domains.](image-url)
\[ p(x, y = L_2, t) = p_{\text{amb}}, \]
\[ p_{\text{gas}}(x, y = L_2, t) = r_s p_{\text{sat}}(T_{\text{amb}}), \]
\[ T_{\text{gas}}(x, y = L_2, t) = T_{\text{amb}}. \]

The BCs for the lateral boundaries are
\[ u_x(x, y, t) = 0 = \partial u_y / \partial x = 0 \] (left),
\[ u_x(x, y, t) = 0 = u_y \] (right),
\[ \mathbf{n} \cdot \nabla p_{\text{gas}} = 0, \]
\[ T_{\text{gas}}(x, y, t) = T_{\text{amb}}. \]

i.e. no-slip and no-penetration BCs for the velocity and no-adsoption BCs for the water vapor.

4. Experimental results

Fig. 3(a, b) shows measured temperature distributions \( T(x, t) \) for the uncoated paper type and with the substrate moving at 0.2 and 1 mm/s, respectively, along the negative \( x \)-axis. Far away from the nozzle the temperature remains at ambient level. Close to the nozzle, \( T \) falls below ambient temperature due to evaporative cooling from a region where a liquid water meniscus resides on top of and is in contact with the porous substrate. Its precise extension depends on the wetting properties of the paper, the volume of water underneath the nozzle prior to commencing the substrate motion, the vertical gap separation of the nozzle from the paper and the settings of the syringe pump. Generally it ranges between about 0.5–3 mm. At a certain distance from the nozzle, a temperature maximum occurs that exceeds ambient level by 2–3 K, which is caused by recondensation of water vapor on dry regions of the substrate as well as sorptive heating due to moving a dry substrate into a stationary wet zone.

It can be seen from both figures that the position of the maximum of the temperature curve does not change strongly in time implying that the system is in a quasi-steady state. The differences in amplitude and width of the observed \( T(x, t) \) profile are caused by the different substrate speeds. For a higher value of \( U_{\text{sub}} \), the \( T(x, t) \) profile becomes narrower. We quantify the shape of the curves by the two parameters \( \Delta T_{\text{max}} \) and \( \Delta x_{\text{max}} \), which correspond to the maximum temperature rise above ambient and its full width at half maximum relative to ambient conditions.

Fig. 4(a) shows the variation of \( \Delta T_{\text{max}} \) with substrate speed \( U_{\text{sub}} \) for two different types of paper along with the result of the model introduced in Section 3. A maximum is observed for a speed of \( U_{\text{sub}} \approx 1 \text{ mm/s} \). At much lower or higher substrate speeds, \( \Delta T_{\text{max}} \) seems to approach a constant. There is good agreement between the experiments and the model within experimental scatter.

In Fig. 4(b) the width \( \Delta x_{\text{max}} \) of the temperature peaks is plotted as a function of substrate speed \( U_{\text{sub}} \). The symbols represent experimental data, the solid and dashed lines correspond to powerlaw relations \( \Delta x_{\text{max}} \sim |U_{\text{sub}}|^{-0.7} \) and \( \sim U_{\text{sub}}^{-1} \), respectively. The simulations reproduce the trend of the measurements very well.
5. Discussion

5.1. Moisture transport in the paper substrate

Fig. 5(a) shows numerical simulations of the degree of saturation \( S_w(x) \) for three values of \( U_{sub} \) and the same conditions as in Fig. 4. The position \( x = 0 \) corresponds to the apex of the nozzle (or, more precisely, the extension of the water meniscus ahead of the nozzle). For higher substrate speeds, the penetration distance \( x_d \) of the wetting front decreases and the spatial saturation gradients \( \Delta S_w(x) \) increase. The parameter \( x_d \) is defined as the \( x \)-coordinate where \( S_w = 0.4 \). The inset shows \( x_d \) as a function of \( U_{sub} \), which is well-represented by a powerlaw \( x_d \sim |U_{sub}|^{-1} \) (solid line) for \( |U_{sub}| > 0.1 \) mm/s. In other words, \( U_{sub} \) controls how far ahead of the nozzle the wetting front can propagate for high substrate speeds. There is a second regime for very slow speeds \( |U_{sub}| < 0.1 \) mm/s, where evaporative mass loss rather than convection controls the position of the wetting front and where \( x_d \) becomes independent of \( U_{sub} \).

Fig. 5(b) shows the temperature change \( \Delta T_w(x) = T(x, t) - T_{amb} \) at steady state for the same conditions as in Figs. 5(a) and 4. The curves look qualitatively similar to the experimental temperature profiles in Fig. 3. Close to the nozzle, there is a colder zone \( \Delta T < 0 \) due to evaporative cooling. Further ahead there is a local temperature maximum \( \Delta T_{\text{max}} > 0 \), after which \( \Delta T \) diminishes to zero. The position of the temperature maximum occurs at relatively low values of \( S_w \), i.e. close to the wetting front. Its presence is caused by two separate effects:

- Recondensation of water vapor that evaporated from the more saturated region closer to the nozzle. The condensation is accompanied by the liberation of latent heat. It is affected by the convective and diffusive transport of water vapor in the gas-phase domain.
- The heat of wetting \( \Delta H_w \) due to the steady motion of dry substrate material into the stationary wet zone, which becomes more pronounced at lower values of \( S_w \) [see Eq. (19)].

According to Eq. (18), the position of the maximum of \( \Delta T_w \) is determined by the product of \( \partial S_w/\partial x \) and \( \Delta H_w \), as far as sorption is concerned. Since \( \Delta H_w \) decays with increasing \( S_w \), the maximum is located close to the wetting front rather than at the inflection point of \( S(x) \). The contribution from vapor condensation is contained in the term proportional to \( j(x) \), which is controlled by the vapor convection and diffusion. Interestingly, the non-uniformity of the saturation curve \( S_w \) in Fig. 5(a) induces a built-in horizontal gradient in \( \rho_{w,0} \) via the interfacial condition Eq. (22). One expects also the condensation-induced heat flux to reach a maximum close to the wetting front, because it is the position closest to the source where substantial condensation can occur. However, the spatial distribution will be broader due to the diffusive and convective vapor transport mechanisms. The superposition of the two contributions results in a peak close to the wetting front with a relatively slow decay in the positive \( x \)-direction, consistent with Figs. 3 and 5(b).

We note that the peak positions, which are approximately the same in both Figs. 3(a, b), change significantly in Fig. 5(b). The reason is that the quantity of water pumped out of the nozzle was higher in the experiments than assumed in the simulations. In the simulations we assume that the contact line of the water meniscus on the paper is always located at the nozzle apex at \( x = 0 \). In the experiments this proved challenging to control, leading to some experiments where the contact line was ahead of the nozzle apex and others where it was slightly behind it.

Fig. 6 illustrates the influence of the substrate permeability parameter \( K_s \) on the temperature profiles. A larger value of \( K_s \) implies that the wetting front extends further, i.e. leads to a larger value of \( x_d \). As both \( \partial S_w/\partial x \) decreases and the gas-phase transport becomes impaired because it has to cover a larger distance, the temperature profile becomes broader and decreases in amplitude for large \( K_s \). The constant prefactor \( p_2 \) of the capillary pressure in Eq. (3) effectively is a multiplicative factor in Eq. (2). Consequently, varying \( p_2 \) has the same impact on \( \Delta T_w \) as a variation of \( K_s \).

Fig. 7 shows the effect of the ambient humidity on the maximum temperature change \( \Delta T_{\text{max}} \). Since \( \Delta H_w \) diminishes for higher values of \([MC]\) and since less water can condense on more saturated paper, \( \Delta T_{\text{max}} \) goes to zero for \( r_h \) approaching 1. For the same reason, \( \Delta T_{\text{max}} \) increases significantly for dry paper equilibrated with a low humidity atmosphere in the limit \( r_h \rightarrow 0 \).
5.2. Convection-diffusion in the gas phase

In the gas-phase, convection is induced by the motion of the substrate and by buoyancy, as water vapor is lighter than air and the temperature is non-uniform. The convection influences the water vapor distribution and thus the diffusion-limited evaporative mass transfer into and out of the 2D domain. Consequently, it affects both the amplitude and the width of the temperature profile of the porous domain. Fig. 8 shows contour plots of the concentration of water vapor in air for four different values of $U_{\text{sub}}$. The lateral extent of the region with enhanced vapor concentration is determined primarily by the width of the wet zone $x_d$ and decreases for increasing $|U_{\text{sub}}|$.

Fig. 9 presents contour plots of the $x$-coordinate of the gas-phase velocity $u(x,y)$. Close to the substrate, the convection velocity is determined by the BC $u_x = U_{\text{sub}} < 0$. Above the substrate, a convection roll is observed. Surprisingly, the speed of convection in the gas phase by far exceeds the substrate speed. Moreover, while the gas velocity is aligned with the direction of substrate motion in the region immediately above the substrate for $|U_{\text{sub}}| \geq 0.4$ mm/s, a change of direction is visible in Fig. 9(a, b) close to the nozzle exit for $|U_{\text{sub}}| \leq 0.4$ mm/s. This flip is due to the antagonistic effects of the thermally-induced and water-vapor concentration-induced buoyancy terms. A high water vapor concentration occurs primarily near the nozzle exit, which favors a rising velocity. Together with the leftwards substrate motion, a convection roll rotating clock-wise is induced. The same holds for the thermal buoyancy induced by the temperature maximum. For low speeds, however, both the extension of the wet zone $x_d$ and the degree of evaporative cooling significantly increase. The corresponding thermally-induced buoyancy now favors a falling velocity and thus a locally counter-clockwise rotation.

5.3. Scaling analysis

The scaling analysis of this problem is challenging, as five coupled partial differential equations belonging to two different domains are involved. Nevertheless it is possible to rationalize the behavior of the temperature distribution in the limits of very high and very low substrate speeds.

For high $|U_{\text{sub}}|$, the convection induced by the substrate motion in the negative $x$-direction balances the Darcy flow in the positive $x$-direction. Evaporative mass loss is negligible, because the spatial extent of the wet zone quantified by $x_d$ is very short. In steady state the balance of Darcy flow and substrate motion yields

$$u(i) \sim \phi_S U_{\text{sub}}.$$  

Using Eq. (2) and $x_d$ as the relevant lateral lengthscale, this leads to the scaling relation $x_d \sim U_{\text{sub}}$, in perfect agreement with the numerical results shown in the inset in Fig. 5(a).

In the limit of small $|U_{\text{sub}}|$, the extension of the wet zone is not controlled by the substrate motion, but rather by the evaporative mass loss. This explains why $x_d$ approaches a constant for decreasing $|U_{\text{sub}}| \to 0$ in the inset in Fig. 5(a). The extension of the wet zone becomes considerably large, which implies that a wide zone is subjected to evaporative cooling and thus a reduced temperature $T < T_{\text{amb}}$. The evaporatively cooled substrate temperature saturates for small $|U_{\text{sub}}|$, because the sorption term proportional to $U_{\text{sub}}$ becomes negligible and the thermal coupling with the base plate becomes dominant. This cold zone induces a thermal-buoyancy-induced flow in the gas phase that is driven by the temperature difference between the nozzle and the substrate. This flow is locally in the positive $x$-direction consistent with the finding e.g. in Fig. 9(a). Consequently, in the limit of small $|U_{\text{sub}}|$, the horizontal gas flow speed and the gas temperature distribution in the vicinity of the nozzle go approximately constant and so does the evaporation rate $j$. Thus, all terms in Eq. (10) are small for steady state and small $|U_{\text{sub}}|$ and thus large $x_d$, except $q$. The wetting term in $q$ is negligible and the remaining terms indicate a scaling of $\Delta T_{\text{max}}$ that is independent of $U_{\text{sub}}$ in perfect agreement with the trend observed in Fig. 4(a).
For high \( |U_{\text{sub}}| \), the heat released by wetting becomes significantly higher than the heat released by vapor recondensation. In this regime, the flow speed in the gas phase is determined by \( U_{\text{sub}} \) rather than buoyancy effects. All horizontal gradient terms scale as \( x^2 \sim U_{\text{sub}} \). As the typical values of the Peclet number \( Pe = U_{\text{sub}} x_d / D_{\text{H2O}} \) are 0.1 are both small and approximately constant, the typical length scales in the \( x \) - and \( y \)-directions in the gas phase are equal. Therefore, the mass flux \( j \) is expected to scale approximately linearly with \( U_{\text{sub}} \), which implies that the term \( jL_e / M_w \) in Eq. (18) is negligible compared to the wetting term proportional to \( U_{\text{sub}}^2 \) in the recondensation region. The same holds for the heat conduction term and the term proportional to \( h_g \) in Eq. (18). Consequently, all non-negligible terms in Eq. (10) scale as \( U_{\text{sub}}^2 \), which means that again \( \Delta T_{\text{max}} \) should become constant. This is visible in Fig. 4(a) only approximately, as \( \Delta T_{\text{max}} \) decreases by only about 30\% upon an increase in speed from 1 mm/s to 10 mm/s. At speeds higher than 10 mm/s the trend of \( \Delta T_{\text{max}} \) approaching a constant becomes more apparent (data not shown), however, the validity of our 1D model becomes somewhat questionable in this regime.

At \( U_{\text{sub}} = 1 \) mm/s, the heat released due to wetting and recondensation are of similar magnitude and the locations of the maximum heat release for both effects essentially coincide, leading to a global maximum of \( \Delta T_{\text{max}} \).

5.4. Limitations of our model

1. We consider a one-dimensional model in the paper domain, i.e. the \( T \) and \( S_h \) distributions are assumed to be independent of \( z \). This is permissible for thin substrates but places an upper limit \( U_t \) on the substrate speed \( U_{\text{sub}} \). A suitable criterion may be that \( x_d \) should remain larger than the substrate thickness \( t_s \), which indicates that \( U_t > 10 \) mm/s according to the inset of Fig. 5(a).

2. Equivalently, setting up a 1D-model for the paper domain comprised by Eqs. (9) and (10) implies that the timescale for imbibition in the thickness direction (and for releasing the heat of wetting, see Section 3.3) are short compared to the lateral timescale \( x_d / U_{\text{sub}} \).

3. The air phase is explicitly disregarded in the paper domain, only the liquid-phase saturation is considered. This is permissible as there is no net airflow in the thickness direction of the paper that could enhance evaporation or condensation. Moreover, pushing away of air by the liquid-filled zone does not induce any pressure drop, as the air escapes essentially without resistance in the vertical direction owing to the thinness of the paper. Moreover, the heat capacity of the displaced air is negligible compared to that of water or paper.

4. We neglected the hysteretic behavior of the capillary pressure. This does not restrict the validity of our model, as we only consider (quasi-) steady state wetting of a substrate, i.e. hysteretic behavior cannot manifest itself.

5. We assume laminar flow in the gas phase, which is appropriate for the relatively low Reynolds numbers \( \leq 50 \) resulting from the simulations.

6. We assume the temperature of the nozzle to be ambient, whereas evaporative cooling can reduce it slightly.

7. We do not explicitly account for the exchange of liquid between the pore space and the interior of the paper fibers, neither for the water transport inside the fibers.

8. We neglected evaporation from the underside of the paper. This is permissible as the space underneath is narrow, closed and cannot take up much humidity. Moreover, for the same reason there is no significant gas convection in this region.

6. Summary and conclusions

We have studied the temperature increase of paper substrates when they come in contact with liquid water by means of infrared thermography. The heating is due to the release of latent heat upon water vapor condensation on the dry regions and the release of the heat of wetting. We have conducted systematic experiments using both coated and uncoated paper types. Moreover, we developed a coupled numerical model taking into account vapor convection and diffusion in the gas phase as well as liquid flow in the thin porous medium coupled with heat transfer. The numerical results reproduce the experimental results well.

The results are relevant to inkjet printing, where infrared thermography can be used to monitor the imbibition of water into paper substrates and its subsequent evaporation. The simulations point at a complex buoyancy-induced gas-phase convection pattern, that changes qualitatively as the substrate speed is altered. This suggests that an external flow should be applied to facilitate a robust process control.

Declaration of Competing Interest

None.

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References


