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Inter-Frame Polar Coding with Dynamic Frozen Bits

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***Abstract*—A new inter-frame correlated polar coding scheme is proposed, where two consecutive frames are correlated-encoded and assist each other during decoding. The correlation is achieved by dynamic configuration of the frozen bits. The frozen bits of the second frame partially depend on the unfrozen bits of the first frame in encoding and the number of bits that are viewed as frozen by decoder is alterable in different decoding modes. Using this new encoding/decoding scheme, a failed decoded frame can be decoded again with extra information which corrects the errors in its highly unreliable unfrozen bits. Thus the probability of successful decoding is improved. Simulation results show that the performance of the proposed polar codes outperforms that of the classical counterpart significantly with negligible memory and complexity increment.**

***Index Terms*—Polar codes, Inter-frame correlated, Dynamic frozen bits.**

I. Introduction

POLAR codes were proposed by Arıkan in 2008 and have proven to be capacity-achieving for binary-input discrete memoryless channels under successive cancellation (SC) decoding [1]. However, the finite-length performance under SC decoding is not satisfactory. Recent research work has been focused on improving the error-correction performance of polar codes at moderate and short code length.

SC list (SCL) decoding expands the search range of decoding, improving the error-correction performance significantly [2]. In cyclic redundancy check (CRC) aided SCL (CA-SCL) decoding, the source information contains a built-in CRC, which reduces the number of codewords with the minimum Hamming weight, further providing error-correction performance gain [3]. The optimal strategy of CRC design is studied in [4]. SC stack decoding [5] and SC flip decoding [6], [7] bring substantial complexity reduction with respect to SCL decoding but the error-correction performance does not enhance compared with that of SCL decoding. Belief propagation (BP) decoding is an iterative decoding scheme with lower decoding latency [8]. However, there is a gap between its error-correction performance and that of CA-SCL decoding. Polar subcodes with dynamic frozen symbols are proposed in [9], where frozen bits values are determined by some information bits before them in the same frame. This kind of polar coding scheme has large minimum distance, but it only shows advantage over CA-SCL decoding when the list size is sufficiently large (above 32). Star polar subcodes further introduce the concept of star trellis, which enables a parallel decoding and obtains a slight error-correction performance gain with respect to polar subcodes in specific cases [10]. Research on SC flip decoding shows that in most failed decoding cases, the first error occurs in unfrozen bits with low reliabilities, i.e., the ones with small average log-likelihood ratio (LLR) magnitudes. Once it is corrected, the probability of successful decoding increases significantly.

Inspired by this observation, we propose an inter-frame polar coding scheme with dynamic frozen bits. The values of frozen bits are set dynamically in order to make two consecutive frames correlated. The correlation is leveraged to correct the errors in the highly unreliable unfrozen bits of a failed decoded frame. Our results for a polar code of length 1024 and rate $1/2$ show a coding gain of 0.28 dB over classical polar coding scheme at the block error rate (BLER) of 10^{-4}.

II. Polar Coding and CA-SCL Decoding

A polar code of length N is constructed with a vector of relative reliabilities of bit indices $\mathbf{v} = \{v_0, \ldots, v_{N-1}\}$, where bit index v_i is less reliable than bit index v_j if $i < j$. The classical polar coding scheme divides all N source bits into two sets according to $\mathbf{v}$ and the number of unfrozen bits K. Unfrozen bits set $\mathcal{A} = \{v_{N-K}, \ldots, v_{N-1}\}$ contains the indices of K bits with higher reliabilities, and those of the remaining $N-K$ bits form frozen bits set $\mathcal{A}_c = \{v_0, \ldots, v_{N-K-1}\}$. In order to achieve a reasonable error-correction performance for polar codes with finite length, a CRC of length r is concatenated with polar codes and CA-SCL decoding is used. The r CRC bits and the $K-r$ information bits are assigned to the bits with indices in $\mathcal{A}$. The frozen bits with indices in $\mathcal{A}_c$ are fixed to predefined values known to the decoder.

CA-SCL decoding is a bit-by-bit sequential decoding algorithm that follows the SC decoding schedule. Unlike SC decoding that estimates each unfrozen bit based on its calculated LLR value, CA-SCL decoding estimates each unfrozen bit as either 0 or 1. In order to limit the exponential growth in the complexity of the decoder, at each bit estimation step, L candidate paths are allowed to survive using a path metric. In this paper, we use the LLR-based formulation of the path metric in [11]. After the estimation of the last bit in CA-SCL decoding, CRC is performed on the final L candidate paths. For the paths that pass the CRC, the one with the best path metric is output as the decoding result. If there is no path in the final L candidate paths that pass the CRC, the path with the best path metric is output as the decoding result.

III. Inter-Frame Polar Coding

A. Inter-Frame Correlated Encoding Scheme

Fig. 1 illustrates the classical polar encoding scheme and the proposed one. The proposed polar coding scheme uses the same set $\mathcal{A}$ for information and CRC bits. However, the difference between the proposed and the classical polar coding scheme is that some of the bits in $\mathcal{A}_c$ are not fixed to predefined values that are known to the decoder. In this paper,

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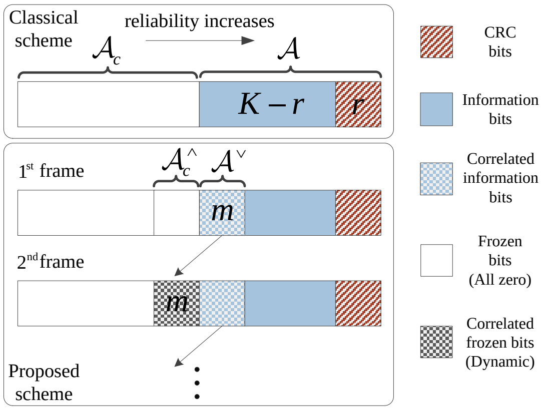

Fig. 1. The classical and the proposed polar encoding schemes.

Algorithm 1: Frozen bits assignment scheme

for *the i-th $(i \geq 1)$ transmitted frame* **do**
  for $k \leftarrow 0$ **to** $N-K-1$ **do**
    if $i = 1$ **then**
      $C_i^{v_k} = 0$;
    else
      if $k < N-K-m$ **then**
        $C_i^{v_k} = 0$;
      else
        $C_i^{v_k} = C_{i-1}^{v_{k+m}}$;

we set the predefined values that are known to the decoder to 0.

Let $\mathcal{A}_c^{\wedge} = \{v_{N-K-m}, \ldots, v_{N-K-1}\}$ denote the set of m most reliable frozen bits (MRFBs) and $\mathcal{A}^{\vee} = \{v_{N-K}, \ldots, v_{N-K+m-1}\}$ denote the set of m most unreliable unfrozen bits (MUUBs). Note that $\mathcal{A}_c^{\wedge} \in \mathcal{A}_c$, $\mathcal{A}^{\vee} \in \mathcal{A}$, and $|\mathcal{A}_c^{\wedge}| = |\mathcal{A}^{\vee}| = m$. We assign the MRFBs of a frame with the MUUBs of its preceding frame and we keep the remaining frozen bits as zeros. More formally, let $C_i^{v_k}$ denote the value of the v_k-th bit of the i-th transmitted frame. The frozen bits assignment scheme is summarized in Algorithm 1.

B. Inter-Frame Assisted Decoding Scheme

Let α_i represent the vector of channel LLR values for the i-th received frame and $\widehat{C}_i^{v_k}$ represent the decoding estimation of bit v_k for the i-th received frame. We use CA-SCL decoding algorithm to decode each frame. The classical CA-SCL decoder sets $\widehat{C}_i^{v_k} = 0$ for $v_k \in \mathcal{A}_c$, and estimates the values of K unfrozen bits according to the output of CA-SCL decoding algorithm. As a result, the information from other frames is not required and whether the decoding succeeds or not does not affect the decoding of other frames. However, since consecutive frames are correlated-encoded in the proposed scheme, the decoding of a frame may require information from the two adjacent frames and the decoding result impacts the next decoding decision directly.

The proposed inter-frame assisted (IFA) SCL (IFA-SCL) decoding scheme is summarized in Algorithm 2. It uses an indicator f to indicate whether the decoding succeeded

Algorithm 2: IFA-SCL decoding scheme

Initialize $g = 0$;
for *the i-th $(i \geq 1)$ received frame* **do**
  if $i = 1$ **then**
    M0;
  else
    if $f = 0$ **then**
      M1;
      if $f = 0$ **then**
        Store $\widehat{C}_i^{v_k}$ for $v_k \in \mathcal{A}^{\vee}$;
      else
        Store α_i and $\widehat{C}_i^{v_k}$ for $v_k \in \mathcal{A}_c^{\wedge}$;
    else
      M2;
      if $f = 0$ **then**
        Store $\widehat{C}_i^{v_k}$ for $v_k \in \mathcal{A}_c^{\wedge}$ and $v_k \in \mathcal{A}^{\vee}$;
        if $g = 0$ **then**
          M3;
        else
          $g = 0$;
      else
        $g = 1$;

$(f = 0)$ or failed $(f = 1)$, an indicator g that shows whether consecutive decoding failures occurred $(g = 1)$ or not $(g = 0)$ in the decoding of the two latest frames, and is composed of four decoding modes as follows:

M0 Perform CA-SCL decoding on frame i with the frozen bits defined as

$$\widehat{C}_i^{v_k} = 0, \quad 0 \leq k < N-K. \tag{1}$$

Set f in accordance with the decoding result.

M1 Perform CA-SCL decoding on frame i with the frozen bits defined as

$$\begin{cases} \widehat{C}_i^{v_k} = 0, & 0 \leq k < N-K-m, \\ \widehat{C}_i^{v_k} = \widehat{C}_{i-1}^{v_{k+m}}, & N-K-m \leq k < N-K. \end{cases} \tag{2}$$

Set f in accordance with the decoding result.

M2 Perform CA-SCL decoding on frame i with the frozen bits defined as

$$\widehat{C}_i^{v_k} = 0, \quad 0 \leq k < N-K-m. \tag{3}$$

Set f in accordance with the decoding result.

M3 Perform CA-SCL decoding on frame $i-1$ with α_{i-1} and partial frozen bits $\widehat{C}_{i-1}^{v_k}$, $N-K-m \leq k < N-K$, fetched from memory, and define other frozen bits as

$$\begin{cases} \widehat{C}_{i-1}^{v_k} = 0, & 0 \leq k < N-K-m, \\ \widehat{C}_{i-1}^{v_k} = \widehat{C}_i^{v_{k-m}}, & N-K \leq k < N-K+m. \end{cases} \tag{4}$$

In order to determine whether the decoding is successful or not, we utilize the small undetected error probability of CRC [12] and define a successful decoding as when the decoding output passes the CRC. Algorithm 2 shows that if the decoding of frame $i-1$ succeeds, the decoding of frame i is executed in mode **M1**. The decoder uses the values of MUUBs of frame $i-1$ as those of MRFBs of frame i. On the other hand, if the decoding of frame $i-1$ fails, the decoding of frame i is executed in mode **M2**. The decoder regards the MRFBs as unfrozen bits since correct values of these bits cannot be obtained from frame $i-1$. The indicator g ensures that frame $i-1$ undergoes a secondary decoding operation in mode **M3**, only if both frames $i-2$ and i are decoded correctly. In this case, the MRFB values of frame $i-1$ which are taken from frame $i-2$, and the MUUB values of frame $i-1$ which are taken from frame i, are correct. This guarantees that the frames help each other in order to improve error-correction performance.

In the proposed inter-frame polar coding scheme, the frozen bits are dynamic in two ways: first, their values are not all fixed to predefined values that are known to the decoder, and second, the number of them changes in different decoding modes. It should be noted that the proposed scheme is different from the information re-transmission scenario in which all or part of a frame is re-transmitted until a successful data delivery is acknowledged. In the proposed scheme, the MUUB values of frame $i-1$ are assigned to the MRFBs of frame i. Therefore, the code rate remains unchanged. However, the information re-transmission scheme inevitably reduces the code rate in order to re-transmit the same frame.

C. Complexity Analysis

The proposed IFA-SCL decoding scheme does not modify the underlying CA-SCL decoding processes. Therefore, different decoding modes use the same CA-SCL decoder. Considering M quantization bits for channel and internal LLR values, the memory requirement of the CA-SCL decoder with list size L can be approximated[1] as [13]

$$M\left(N+L\left(N-1\right)\right)+L\left(2N-1\right)\ [\text{bits}]\,. \tag{5}$$

The IFA-SCL decoder requires additional $MN+3m$ bits to store N channel LLR values of a failed decoded frame, m bit estimations of a failed decoded frame and at most $2m$ bit estimations of a successfully decoded frame according to Algorithm 2. Thus, the memory required by the proposed IFA-SCL decoding scheme is

$$M\left(2N+L\left(N-1\right)\right)+L\left(2N-1\right)+3m\ [\text{bits}]\,. \tag{6}$$

It should be noted that only a secondary decoding attempt in mode **M3** leads to extra computational complexity in the proposed IFA-SCL decoding scheme which makes the total computational complexity of decoding a frame almost twice as that of the classical scheme. However, as will be shown in Section IV, the decoding in mode **M3** occurs with a small probability for good channel conditions or large list sizes. This results in a small increment in the average computational complexity for the proposed IFA-SCL decoding scheme, in comparison with that of the classical CA-SCL decoding.

IV. Simulation Results

To verify the effectiveness of the proposed strategy, the performance of the inter-frame polar coding scheme is evaluated. Additive white Gaussian noise (AWGN) channel model is assumed and binary phase-shift keying (BPSK) modulation is adopted. For the simulation results in this section, the relative reliability sequence in the 5G standard [14] is used for the polar code of length $N=1024$, and for the polar code of length $N=2048$, the relative reliability vector is generated by the method in [15], using a design signal-to-noise ratio of 2 dB. A CRC of length $r=16$ in the 5G standard with generator polynomial $D^{16}+D^{12}+D^{5}+1$ is used for CA-SCL decoding and the rate of the code is defined as $R=\left(K-r\right)/N$.

Fig. 2 shows the BLER and bit error rate (BER) of polar codes of different code parameters with respect to energy per bit to noise power spectral density ratio ($\mathrm{E_b/N_0}$). It can be seen that the proposed inter-frame polar coding scheme with IFA-SCL decoding algorithm brings a performance gain of 0.28 dB at a BLER of 10^{-4} in comparison with the classical polar coding scheme with CA-SCL decoding when $N=1024$, $R=1/2$, $L=16$, and $m=40$. It is worth mentioning that the BLER performance of the proposed inter-frame polar coding scheme with $N=1024$, $R=1/2$, $L=16$, and $m=40$ is 0.15 dB better than that of the classical polar coding scheme of the same code length and rate with $L=32$, and it is almost similar to that of the classical polar coding scheme of the same code rate and list size with $N=2048$, at a target BLER of 10^{-4}. This result is particularly important since considering $M=6$, the memory requirement of the proposed scheme with $N=1024$, $R=1/2$, $L=16$, and $m=40$ is 143368 bits, while it is 268064 bits for the classical polar coding scheme with $N=1024$ and $L=32$, and 274320 bits for the classical polar coding scheme with $N=2048$ and $L=16$. The BLER gain of the proposed scheme in comparison with the classical scheme is also seen when $R=1/3$. Fig. 2 also provides the BLER and BER of star polar subcodes in [10] and it can be seen that the proposed IFA-SCL decoding scheme provides 0.1 dB performance gain with respect to star polar subcodes when $N=2048$, $R=1/2$, $L=32$, and $m=48$, at a target BLER of 10^{-4}. A similar trend can also be observed when comparing BER of different schemes in Fig. 2.

Fig. 3 displays the effect of the value of m on error-correction performance of the proposed inter-frame polar coding scheme at $\mathrm{E_b/N_0}=2.0$ dB and $R=1/2$, where two code lengths of 1024 and 2048, and two list sizes of 8 and 16 are considered. Empirically, it can be seen that in all cases, there is a specific value of m with which the best BLER performance can be achieved. This is due to the fact that for small values of m, there are not enough additional frozen bits to help the re-decoding process of a failed decoded frame in mode **M3**, while for large values of m, too many frozen bits are considered as unfrozen to decode a frame in mode **M2**.

Table I reports the average and the maximum computational complexity of the proposed IFA-SCL decoding scheme in comparison with that of the classical CA-SCL decoding scheme in terms of the number of arithmetic operations performed by each scheme[2]. The number of arithmetic operations is calculated by decoding 10^{6} frames with $N=1024$, $R=1/2$, $L=16$, $r=16$, and $m=40$ at different $\mathrm{E_b/N_0}$ values. It can be seen that while the proposed scheme

[1] A small memory is required to store path metrics which is neglected here.

[2] For the classical scheme, the average and the maximum number of arithmetic operations are equivalent.

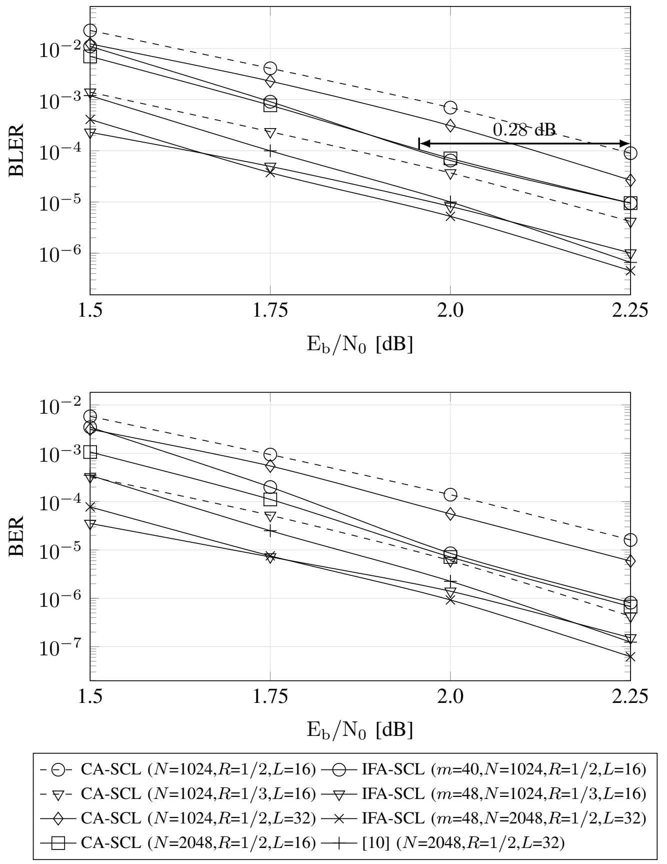

Fig. 2. BLER (top) and BER (bottom) comparisons between the proposed polar coding scheme, the classical polar coding scheme, and star polar subcodes of [10], with different decoding parameters using a CRC of length 16.

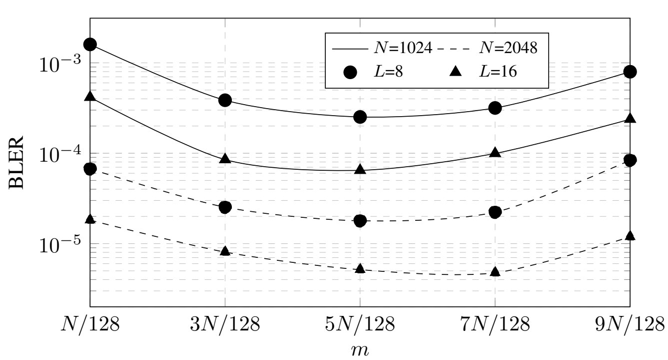

Fig. 3. Error-correction performance of the proposed polar coding scheme with different values of m when $R = 1/2$, CRC length is 16, and $E_b/N_0 = 2.0$ dB.

has a maximum computational complexity $\mathcal{C}^{\max}_{\text{proposed}}$ which is about twice as that of the classical CA-SCL decoding scheme $\mathcal{C}_{\text{classical}}$, it has an average computational complexity $\mathcal{C}^{\text{average}}_{\text{proposed}}$ which is only up to 2% higher than that of the classical CA-SCL decoding scheme. It is worth mentioning that the average computational complexity gap between the proposed and the classical decoding schemes reduces as the E_b/N_0 value increases.

TABLE I
AVERAGE COMPUTATIONAL COMPLEXITY COMPARISON IN TERMS OF THE NUMBER OF ARITHMETIC OPERATIONS PERFORMED

E_b/N_0	1.5 dB	1.75 dB	2.0 dB	2.25 dB
$\mathcal{C}_{\text{classical}}$	126882	126882	126882	126882
$\mathcal{C}^{\text{average}}_{\text{proposed}}$	128920	127340	126986	126890
$\mathcal{C}^{\max}_{\text{proposed}}$	251818	251818	251818	251818

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V. CONCLUSION

An inter-frame polar coding scheme with dynamic frozen bits is proposed to improve the error-correction performance of polar codes. In the proposed decoding scheme, consecutive frames share information to help each other when a decoding operation fails. We showed that for a polar code of length 1024 and rate $1/2$, the proposed inter-frame polar coding scheme can provide an error-correction performance gain of 0.28 dB over the classical scheme at the block error rate of 10^{-4}. Moreover, we showed that this error-correction performance advantage is achieved with negligible increment in memory requirements and average computational complexity.

REFERENCES

[1] E. Arıkan, "Channel polarization: A method for constructing capacity-achieving codes for symmetric binary-input memoryless channels," *IEEE Trans. Inf. Theory*, vol. 55, no. 7, pp. 3051–3073, Jul. 2009.

[2] I. Tal and A. Vardy, "List decoding of polar codes," *IEEE Trans. Inf. Theory*, vol. 61, no. 5, pp. 2213–2226, May. 2015.

[3] K. Niu and K. Chen, "CRC-aided decoding of polar codes," *IEEE Commun. Lett.*, vol. 16, no. 10, pp. 1668–1671, Oct. 2012.

[4] Q. Zhang, A. Liu, X. Pan, and K. Pan, "CRC code design for list decoding of polar codes," *IEEE Commun. Lett.*, vol. 21, no. 6, pp. 1229–1232, Jun. 2017.

[5] K. Niu and K. Chen, "Stack decoding of polar codes," *Electron. lett.*, vol. 48, no. 12, pp. 695–697, Jun. 2012.

[6] O. Afisiadis, A. Balatsoukas-Stimming, and A. Burg, "A low-complexity improved successive cancellation decoder for polar codes," in *Asilomar Conf. on Signals, Syst. and Comput.*, Nov. 2014, pp. 2116–2120.

[7] C. Condo, F. Ercan, and W. J. Gross, "Improved successive cancellation flip decoding of polar codes based on error distribution," in *IEEE Wireless Commun. and Netw. Conf. Workshops*, Apr. 2018, pp. 19–24.

[8] A. Elkelesh, M. Ebada, S. Cammerer, and S. ten Brink, "Belief propagation list decoding of polar codes," *IEEE Commun. Lett.*, vol. 22, no. 8, pp. 1536–1539, Aug. 2018.

[9] P. Trifonov and V. Miloslavskaya, "Polar subcodes," *IEEE J. on Sel. Areas in Commun.*, vol. 34, no. 2, pp. 254–266, Feb. 2016.

[10] P. Trifonov, "Star polar subcodes," in *IEEE Wireless Commun. and Netw. Conf. Workshops*, Mar. 2017, pp. 1–6.

[11] A. Balatsoukas-Stimming, M. B. Parizi, and A. Burg, "LLR-based successive cancellation list decoding of polar codes," *IEEE Trans. Signal Process.*, vol. 63, no. 19, pp. 5165–5179, Oct. 2015.

[12] M. El-Khamy, J. Lee, and I. Kang, "Detection analysis of CRC-assisted decoding," *IEEE Commun. Lett.*, vol. 19, no. 3, pp. 483–486, Mar. 2015.

[13] S. A. Hashemi, C. Condo, F. Ercan, and W. J. Gross, "Memory-efficient polar decoders," *IEEE J. on Emerging and Sel. Topics in Circuits and Syst.*, vol. 7, no. 4, pp. 604–615, Dec. 2017.

[14] "3GPP TS RAN 38.212 v1.2.1," Dec. 2017. [Online]. Available: http://www.3gpp.org/ftp/Specs/archive/38_series/38.212/38212-f30.zip.

[15] I. Tal and A. Vardy, "How to construct polar codes," *IEEE Trans. Inf. Theory*, vol. 59, no. 10, pp. 6562–6582, Oct. 2013.