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Day-to-Day Needs-based Activity-Travel Dynamics and Equilibria in Multi-State Supernetworks

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Abstract

This study proposes a new framework for studying activity-travel dynamics in multi-state supernetworks based on a needs-based theory. The framework postulates that temporal fluctuations in human needs are the source of dynamically generated activities, which in turn are manifested in the choice of day-to-day activity-travel patterns (ATPs) and dynamics in day-to-day traffic. Specifically, the utility of fulfilling a daily ATP is determined jointly by the degree of satisfying the disaggregate needs and the aggregate effects of traffic conditions when conducting the ATP. The needs are updated after completing the activities and so is the utility of fulfilling the ATP, which means that conducting the same ATP on two consecutive days is likely to derive different utilities. Thus, the choice of a daily ATP is determined not only by the needs based on past ATP choices but also on current traffic conditions and usage of facilities. We formulate these dynamics in terms of an activity-travel adjustment process, through which the proposed needs-based activity-travel dynamic system converges to a steady state, coinciding with the Wardropian user equilibrium and the equivalent mixed-strategy Nash equilibrium. This formalism integrates activity generation, activity-travel scheduling, and traffic flow evolution, and offers appealing explanations for day-to-day traffic dynamics and equilibria.

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Keywords: Activity-travel; need; day-to-day dynamics; equilibria; multi-state supernetwork

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1. Introduction

1.1. Day-to-day activity generation

It has been widely recognized that activity-based modeling (ABM) has become a dominant research paradigm in travel demand forecasting and analysis. A complete activity-based travel demand modeling framework normally encompasses three components, namely, activity generation, activity-travel scheduling, and activity-travel implementation (often referred to as trip assignment). These three components have been largely treated in isolation in previous studies. Several comprehensive activity-based travel demand systems have been developed by integrating the activity generation and activity-travel scheduling components (e.g., DAS, Bowman and Ben-Akiva, 2001; ALBATROSS, Arentze and Timmermans, 2004; MATSim, Balmer et al., 2006; SimAgent, Bhat et al., 2012). However, to consider the aggregate effects, these systems need external traffic assignment or simulation models in the implementation stage. Most applications rely on a single step integration. Others involve an iterative process in which the simulated travel times, based on traffic assignment models, serve as input to the next iteration of the activity-based model until an equilibrium is reached. Meanwhile, a few activity-travel assignment models have integrated the components of activity-travel scheduling and implementation (e.g., Lam and Yin, 2001; Lam and Huang, 2002; Li et al., 2010, 2014; Chow and Djavadian, 2015). Particularly, to incorporate multi-dimensional choice facets simultaneously, supernetwork representations have been adopted to achieve activity-travel pattern (ATP) based user equilibria. Based on multi-state supernetworks (Liao et al., 2010, 2011, 2013), Liu et al. (2015) represent a recent elaboration of dynamic user equilibrium that can capture multi-modal and multi-activity trip chaining at a high level of detail. To a large extent, these models considered activities within a one-day time frame.

However, as for activity generation, increasingly more scholars have recognized the necessity for extending the time-frame of ABM from one-day to multiple days, shifting to dynamic modeling of activity-travel behavior. Arentze and Timmermans (2009) made a concrete step in this direction and provided a theoretical development of the concept of “need”, stating that activities are conducted to satisfy particular needs, which fluctuate over time. After completing an activity, the corresponding need drops and then increases over time. The nature and speed of the changes depend on the activity. Owing to the heterogeneity of people’s preferences, activities may meet the needs at different levels with varying frequencies and may also increase or decrease the needs of other activities as side-effects. Pattabhiraman (2012) further developed a conceptual framework of the relationship between needs and activities based on inventory theory. This behavioral needs-based framework enhanced the conventional activity generation models. In that sense, the specific physical and psychological needs of a day have effects on the choice of activities, and thus daily activities are supposed to exhibit day-to-day dynamics. This seminal idea has been applied to address activity generation (e.g., Nijland et al., 2012, 2014) and activity scheduling (e.g., Märki et al., 2014; Chow and Nurumbetova, 2015) with a multi-day time frame or open planning horizon. Nevertheless, these models in principle need external modules that give feedback on the travel dynamics at the aggregate level.

1.2. Day-to-day traffic dynamics

Day-to-day traffic dynamics models predict the evolution of travel choices and traffic flows over time. These models can be classified along various dimensions: continuous time processes (e.g., Smith, 1984) versus discrete time processes (e.g., Cantarella and Cascetta, 1995; Watling, 1999); deterministic processes (e.g., Smith, 1983) versus stochastic processes (e.g., Watling and Hazelton, 2003; Parry and Hazelton, 2013). Under a rational adjustment process (RAP), as link flows evolve day-to-day, the aggregate travel costs of the traffic system based on the link travel costs fluctuate; and the flow patterns should be equivalent to those at a user equilibrium state if link flows become steady. To avoid path overlapping and flow non-uniqueness, day-to-day traffic assignment models that directly deal with link flow variables have been proposed and analyzed (He et al., 2010; Han and Du, 2012). In addition, many path-based dynamical systems approaches have been provided. Yang and Zhang (2009) classified the path-based RAP into five major categories, i.e., the simplex gravity flow dynamics (e.g., Smith, 1984), the network tatonnement process (e.g., Friesz et al., 1994), the route swapping adjustment process (e.g., Smith and Wisten, 1995), the projected dynamical system (e.g., Zhang and Nagurney, 1996), and evolutionary traffic dynamics (e.g., Wang et al., 2013). Furthermore, the route swapping approach has been classified into three major categories, i.e., the swapping towards...
the least costly route (e.g., Huang and Lam, 2002), pairwise swapping (e.g., Mounce, 2006), and swapping from above average cost routes to below average cost routes (e.g., Mounce and Carey, 2011).

Recently, some studies further extended the day-to-day model by considering departure time choice, stochastic context, bounded rationality (e.g., Cantarella and Watling, 2016; Guo et al., 2017; Ye et al., 2017) and even multi-modal traffic (e.g., Cantarella, et al. 2015; Li and Yang, 2016; Liu and Geroliminis, 2017). In addition, some studies incorporated the impacts of travel information in the day-to-day traffic dynamics (e.g., Bifulco et al., 2016; Liu et al., 2017; Li et al., 2018; Zhang et al., 2018). All these models satisfy the requirement of RAP, and focus on the day-to-day evolution process of traffic flows from non-equilibrium to equilibrium. However, these day-to-day traffic models only examine the traffic characteristics with fixed trip-based OD pairs over time. The adjustment processes are mainly channeled by the relation of travel cost and traffic flow, which after all can only be categorized as static processes from the view of dynamic activity-travel generation.

1.3. Day-to-day activity-travel dynamics

As discussed, existing traffic models tend to discard the varying activity-travel demands. Therefore, the aim of this study is to propose a new framework for dynamic activity-based travel demand analysis in multi-state supernetworks based on the needs-based theory. The framework postulates that temporal fluctuations in human needs are the source of dynamically generated activities, which in turn are manifested in the choice of day-to-day ATPs and dynamics in day-to-day traffic. To capture the evolution of activity-travel flows, we formulate a general activity-travel rational adjustment process (AT-RAP). Specifically, the utility of fulfilling an ATP of a day is determined jointly by the utility of satisfying the needs and the disutility of conducting the ATP. On the next day, the needs are updated, so are the utilities of ATPs, which means it is possible that conducting the same ATP at two consecutive days may derive different utilities. The transition of needs, activities, travel demand, and traffic flow from one day to the next day. To allow for the choice of ATPs of variational activities driven by needs, a multi-state supernetwork representation is introduced to accommodate the generation of ATPs. Given day-to-day dynamics is the focus of attention, within-day dynamics is not considered in the present study for convenience of conveying the major features. AT-RAP combines dynamic activity generation, activity-travel scheduling, and traffic flow evolution in a holistic framework. AT-RAP consistently considers ATP as a unit of analysis and seamlessly bridges the three phases of ABM. It offers appealing explanations of why traffic flows fluctuate day-to-day. Therefore, the behavior realism and modeling integrity over the existing travel demand models are substantially enhanced.

To that end, the remainder of this paper is organized as follows. Section 2 introduces the basic settings in multi-state supernetworks. Section 3 presents the mechanisms of dynamic needs and the flow adjustment process. Section 4 formulates activity-travel link (dis)utilities and ATP utilities. Section 5 analyzes the convergence and steady state of the needs-based activity-travel dynamical system. Section 6 illustrates the proposed modeling framework with a numerical example. Finally, a discussion and some concluding remarks are provided in Section 7.

2. The settings

2.1. The supernetwork

We consider an activity-based multi-state supernetwork SNK. As shown in Fig.1, \(h\) and \(h'\) denote an O-D pair, which refer to the same locations (i.e., home), but lie at the start and end state of conducting a daily activity program. The essence of multi-state supernetworks is that the activity-travel scheduling process is decomposed into path choice through different activity-travel states of conducting the daily activity programs. By splitting the integrated multi-modal transport network into private vehicle network (PVN) and public transport network (PTN), the links interconnecting PVN and PTN represent parking/picking-up private vehicles, and those interconnecting PTN and PTN represent conducting activities (see Liao et al., 2013, 2017 for detailed explanations). Thus, a consistent ATP from \(h\) to \(h'\) goes through a series of PTNs and PVNs of reachable states. For example, the ATP formed by the bold links in Fig.1 shows that the traveler leaves home by car to conduct an activity at \(a_1\) with parking at \(d_2\), then returns home and switches to bike to conduct another activity at \(a_2\) with parking at \(d_4\), and finally returns home. As seen, activity sequence is a part of the path choice in an ATP.
Let $l^s$ denote a link in $SNK$, where $r$ and $s$ represent the associated ATP and activity state, respectively. Specifically, activity state is a vector representing which activities are conducted or not, i.e., $s = \{s_a, a \in A\}$, where $A$ denotes the set of activities and $s_a$ is equal to 1 if activity $a$ has been conducted and 0 otherwise. Hereafter, let $l$ represent a link for short. In addition, there are four basic link types in $SNK$, that is, $l \in L_{PVN} \cup L_{PTN} \cup L_r \cup L_{PVN}$ is the set of physical links in $PVN$, which can only be accessed by private vehicles; $L_{PTN}$ is the set of physical links in $PTN$, where travelers can walk and take public transit; $L_r$ is the set of transfer links between transport modes; and $L_j$ is the set of activity links. This current study does not consider the direct flow interactions between links of $PTN$ and $PVN$. Note that there is a many-to-one mapping between activity links and an activity. For example, a shopping or entertainment activity may be conducted at one of multiple alternative locations, which correspond to different activity links in $SNK$. In view of this, let $aL$ be the set of links for conducting activity $a$.

Let $r$ denote a path, which is an ordered set of links, i.e., $\{l_1, \ldots, l_m\}$, and let $L_r$ be the set of links on path $r$. Logically, any path from $h$ to $h'$ defines an ATP, expressing the specific choice of mode/route, parking/activity location, and activity sequence. Therefore, we call ATP as path for short hereafter. The set of home locations is represented by $H$, i.e., $h \in H$. In order to formulate home-based paths, a home location is both the origin and the destination. Let $R_h$ denote the set of all feasible paths based on $h$. It should be noted that the ordered set of links could be an empty set, which is defined as a special path (i.e., stay-at-home without travel, for which the time is spent on in-home activities like eating, sleeping, working, and exercising, etc.). Note also that activity interdependency of household members is not considered in this study. Furthermore, let $F = (f_i^T, i \in I)^T$ denotes a vector of class-specific total numbers of travelers, where $f_i = (f_{ih}, h \in H)^T$ is its sub-vector for traveler class $i$. The vectors of class-specific path flow, link flow, and activity participation quantity are respectively denoted as $f = (f_i^T, i \in I)^T$, $x = (x_i^T, i \in I)^T$ and $q = (q_i^T, i \in I)^T$, where $f_i = (f_{ih}, h \in H, r \in R_h)^T$, $x_i = (x_i, l \in L)^T$ and $q_i = (q_{ih}, h \in H, a \in A)^T$ are their sub-vectors for traveler class $i$ in sequence. Overall, the geometric relationships among path flow, link flow, and activity participation can be expressed as

$$F_i = \Theta f_i, \quad x_i = \Delta f_i, \quad q_i = \Psi f_i, \quad \forall i \in I$$

where $\Theta = (\theta_{hr} | h \in H, r \in R_h)$ is the home-path incidence matrix and $\theta_{hr}$ is equal to 1 if path $r$ departs from $h$ and 0 otherwise. $\Delta = (\delta_{lr} | h \in H, r \in R_h, l \in L)$ is the link-path incidence matrix and $\delta_{lr}$ is equal to 1 if path $r$ from $h$ uses link $l$ and 0 otherwise; $\Psi = (\psi_{hr} | h \in H, r \in R_h, a \in A)$ is the activity-path incidence matrix and $\psi_{hr}$ is equal to

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**Fig. 1. Multi-state supernetwork representation.**
1 if path $r$ from $h$ includes activity $a$ and 0 otherwise. It is noted that Eq. (1) denotes the topology of a multi-state supernetwork and the aggregation of path flows on a link is activity state dependent. We denote a compact closed convex as $\Omega = \{ (f, x, q) | f_i = \Theta f_i, x_i = \Delta f_i, q_i = \Psi f_i, f_i \in R^m_{\geq 0}, f_i \}_\Omega$, where operator $\| \cdot \|$ denotes the mapping from a set to the number of its elements. The sets of feasible path flows, link flows, and activity participation quantities are $\Omega_x = \{ f | (f, x, q) \in \Omega \}$, $\Omega_x = \{ x | (f, x, q) \in \Omega \}$, and $\Omega_q = \{ q | (f, x, q) \in \Omega \}$, respectively. Furthermore, since travelers want to increase their utilities by conducting activities and meanwhile decrease their travel disutilities (generalized travel costs), we formulate the utility of fulfilling a path by the activity utilities deducting the travel disutilities on this path, which can be expressed as

$$V = U - C, \quad \forall (f, x, q) \in \Omega$$

In Eq. (2), $V = (V_{l}(i, h, r, h, r \in R_n)^T$, $U = (U_{l}(i, h, h, r \in R_n)^T$ and $C = (C_{l}(i, h, h, r \in R_n)^T$ respectively are the vectors of path utilities, activity link utilities and travel link disutilities, where $V_{l}(i, h, r \in R_n)$, $U_{l}(i, h, r \in R_n)$ and $C_{l}(i, h, r \in R_n)$ are the corresponding values obtained by traveler class $i$ traversing $r$ from $h$, respectively. Activity link utilities and travel link disutilities are considered additive, under network topology Eq. (1), which respectively are

$$U_i = \Delta^{-1} u_i, \quad C_i = \Delta^{-1} c_i.$$  

(3)

where $u_i$ and $c_i$ are the vector of link utilities and disutilities for traveler class $i$, respectively.

3. Needs-based activity-travel dynamical system

3.1. Activity need adjustment process

The notion that people’s daily activities are driven by the basic needs has long been recognized in the literature and put forward in several conceptual studies (e.g., Adler and Ben-Akiva, 1979; Axhausen, 2006; Timmermans and Arentze, 2006). Arentze and Timmermans (2009) were the first to develop an analytical model of dynamic activity generation based on the assumption that individuals’ activities are driven by a limited set of needs that tend to grow over time in autonomous processes. The central concept of the needs-based theory is that each activity of an individual’s activity repertoire satisfies one or more needs. It is postulated that activities have both decreasing and increasing effects on any given need and that the utility of conducting an activity increases with the needs it satisfies and decreases with the needs it induces. This original needs-based model has been broadly extended to incorporate more choice facets and to integrate activity-travel scheduling. For instance, Chow and Nurumbetova (2015) extended the HAPP model (Recker, 1995) to the multi-day horizon, in which every need is associated with a psychological inventory that reflects the level of satisfaction. When an activity is conducted, the corresponding need is (partly) satisfied and the psychological inventory is replenished by a quantity called the activity production. When the need is low, the psychological inventory is high and vice versa. Over time, the need builds up and the inventory gets depleted. The inventory is replenished when the individual performs an activity that satisfies the need. Although needs-based models take into account complex interactions between activities, needs, and persons (in a household context), we consider a simplified situation underlying the core concept where an individual is faced with decisions to conduct activities on the current day given the time elapses since the activities were conducted the last time (Nijland et al., 2012, 2014). It is also assumed that an activity is performed at most once within a day. If an activity is conducted multiple times a day, each has a different labeling. In that regard, the following assumption is made.

Assumption 1. There is a one-to-one mapping between a need and an activity, and the need to conduct an activity is positively correlated with the time elapse since it was done the last time.

The above assumption signifies that conducting an activity only satisfies one type of need, vice-versa. Once an activity is conducted regardless of the activity duration, the need decreases to a base level; otherwise, the need will cumulate over time. The iterative activity need is formulated as
\[ B_{ia}^{(n+1)} = \begin{cases} B_{ia}^{(n)} + K_{ia} \left( B_{ia}^{(n)} \right), & \text{if } a \text{ is not conducted by traveler class } i \text{ on day } n, \forall i \in I, a \in A, \\ B_{ia}^{(n)}, & \text{if } a \text{ is conducted by traveler class } i \text{ on day } n, \end{cases} \]  

where the above variables relate to travel class \( i \), of which \( B_{ia}^{(n)} \) is the need of activity \( a \) on day \( n \), \( B_{ia}^{(0)} \) is the reference need of activity \( a \) satisfying \( B_{ia}^{(0)} \geq 0 \), and \( K_{ia}(\cdot) \) is the need build-up rate of activity \( a \) as a function of the current need. Note that this build-up rate could decrease, remain unchanged, or even increase with regards to the current need. Eq. (4) only specifies the dynamic needs at the disaggregate level. However, at the aggregate level of activity-travel assignment, we cannot distinguish which person implements which specific activities. To update the aggregate choice of activities and needs, an alternative way is to suggest a mixed-strategy to depict the travelers’ activity choices. This mixed-strategy is expressed as \( (e_{iah}^{(n)}, 1 - e_{iah}^{(n)}) \), where \( e_{iah}^{(n)} \) is the probability of traveler class \( i \) conducting activity \( a \) from home \( h \) on day \( n \). Then, the adjusted need can be expressed in an expected form

\[ B_{ia}^{(n+1)} = e_{iah}^{(n)} B_{ia}^{(n)} + (1 - e_{iah}^{(n)}) \left( B_{ia}^{(n)} + K_{ia} \left( B_{ia}^{(n)} \right) \right), \quad \forall i \in I, h \in H, a \in A. \]  

where \( B_{ia}^{(n+1)} \) is the need of activity \( a \) for traveler class \( i \) from home \( h \) on day \( n + 1 \). To facilitate the presentation of the essential ideas, the following assumption is adopted.

**Assumption 2.** A traveler is inappreciable relative to the large number of travelers of each class at a residential area.

Under Assumption 2, according to the Weak Law of Large Numbers, it is known that for any arbitrarily small positive number \( \varepsilon \), with a sufficiently large \( F_{ih} \), there will be a very high probability that \( q_{inha}^{(n)} / F_{ih} \) will be close to \( e_{inha}^{(n)} \), i.e., \( \lim_{F_{ih} \to \infty} \Pr \{|q_{inha}^{(n)} / F_{ih} - e_{inha}^{(n)}| > \varepsilon\} = 0 \Rightarrow q_{inha}^{(n)} / F_{ih} \approx e_{inha}^{(n)} \). This simplification is made given the focus of modeling the effect chain from traffic flow via activity to need. According to Eq. (1), \( d_{inha} = \sum_{h \in H} \sum_{r \in R_h} \psi_{hora} f_{inha} \) is obtained. Finally, the need adjustment process is derived as

\[ B_{inha}^{(n+1)} = \left( 1 - \sum_{h \in H} \sum_{r \in R_h} \frac{\psi_{hora} f_{inha}^{(n)}}{F_{ih}} \right) \left( B_{ina}^{(n)} + K_{ina} \left( B_{ina}^{(n)} \right) \right) + \sum_{h \in H} \sum_{r \in R_h} \psi_{hora} f_{inha}^{(n)} B_{ina}^{(n)}, \quad \forall i \in I, h \in H, a \in A. \]  

### 3.2. Activity-travel flow adjustment process

In order to unify the macroscopic traffic flow and the microscopic individual travel choice, we introduce a mixed-strategy to depict the multi-traveler activity-travel choice. Specifically, we consider that any traveler faces with several alternative paths and chooses a mixed strategy, which can be expressed as a probability class-specific vector \( P_{ih} = (p_{ihr}, r \in R_h)^T, \ i \in I, h \in H \). \( P_{ih} \) is the probability of traveler class \( i \) choosing path \( r \) from home \( h \). Under Assumption 2, it is further known that \( f_{inha} \approx P_{ih} F_{ih} \) according to the Weak Law of Large Numbers. Then, we adopt an activity-travel rational adjustment process based on the route-swapping system proposed by Smith and Wisten (1995), Nagurney and Zhang (1997), and Mounce and Carey (2011), and so on. In this day-to-day dynamic system, it is assumed that all travelers have complete information of the previous days so that activity-travel flows are forced to evolve to a state with higher utilities. The swapping occurs only for paths of non-maximum utilities to those paths of maximum utility from the same \( h \) and \( i \). Overall, the detailed dynamical system is formulated as

\[ f^{(n+1)} = f^{(n)} + \lambda^{(n)} \Phi \left( f^{(n)} \right), \]  

where \( \lambda^{(n)} \) and \( \Phi \left( f^{(n)} \right) \) are a conversion coefficient and the flow transfer vector on day \( n \) respectively. The conversion coefficient reflects the travelers’ adjustment scale of path choice, which depends on how sensitive travelers
are to the difference between path utilities (Zhang and Nagurney, 1996). The sequence of conversion coefficients is considered as

$$\lambda^{(n)} = \frac{\lambda^0}{n/\omega},$$

(8)

where $\lambda^0$ represents the initial conversion coefficient and $\omega$ denotes the period of adjusting the conversion. $\left\lceil n/\omega \right\rceil$ means the period index the current day is in. It is noted that $\omega = 1$ implies that conversion coefficient changes every day and the larger $\omega$ the slower the conversion coefficient changes. Unless otherwise specified, throughout this paper, we assume $\lambda^{(n)}$ will not stay the same forever, mathematically that is, $\omega < \infty$.

Travelers try to employ an appropriate conversion coefficient based on the current benefits from day to day, so as to void the stagnant of their utility improvement. In addition, the flow transfer vector $\Phi(f)$ is given by

$$\Phi(f) = \sum_{i \in I, h \in H} \sum_{r \in R_h} \sum_{w \in \hat{R}_w} \sum_{n=1}^N f_{h, w}^{(n)} \left( \frac{V_{h, w}(f^{(n)}) - V_{h, w}(f)}{R_{h, w}^{(n)}} \right) \sigma_{r, w}^{(n)},$$

(9)

where $\hat{R}_w^{(n)} = \{ w \in R_h \mid V_{h, w}(f^{(n)}) \geq V_{h, w}(f), \forall r \in R_h \}$ is the set of best paths for class $i$ traveling from $h$ on day $n$ and $\sigma_{r, w}^{(n)}$ is the vector of swapping indicators from path $r$ to path $w$, which is set as 1 for the $r$-th element, $-1$ for the $w$-th element, and zeros for all other elements (Mounce and Carey, 2011). Behaviorally, the flow transfer vector ensures that it deducts flows from the paths with lower utilities and adds these flows onto the paths with the maximal utilities. The sum of the deductions is equal to the sum of the additions. Meanwhile, this swapping ensures that the new path flows are non-negative. Furthermore, substituting Eq. (2) into Eq. (9) yields

$$\Phi(f) = \sum_{i \in I, h \in H} \sum_{r \in R_h} \sum_{w \in \hat{R}_w} \sum_{n=1}^N f_{h, w}^{(n)} \left( \frac{U_{h, w}(f^{(n)}) - U_{h, w}(f)}{R_{h, w}^{(n)}} \right) \sigma_{r, w}^{(n)} + \sum_{i \in I, h \in H} \sum_{r \in R_h} \sum_{w \in \hat{R}_w} \sum_{n=1}^N f_{h, w}^{(n)} \left( \frac{C_{h, w}(f^{(n)}) - C_{h, w}(f)}{R_{h, w}^{(n)}} \right) \sigma_{r, w}^{(n)},$$

(10)

where the two items at the right-hand side can be seen as utility-dependent and disutility-dependent flow transfer vectors, respectively represented as $\Phi_U(f^{(n)})$ and $\Phi_C(f^{(n)})$. More specifically, $\Phi_U(f^{(n)})$ represents the force which swaps flows from the paths with lower activity participation utility to the paths with the higher activity participation utility; $\Phi_C(f^{(n)})$ represents the force which swaps flows from the paths with higher travel disutilities to the paths with the lower travel disutilities. Then, Eq. (10) can be simplified as

$$\Phi(f) = \Phi_U(f^{(n)}) + \Phi_C(f^{(n)}).$$

(11)

In addition, traffic congestion, as a negative network effect, makes travel disutility increase with increasing traffic flows. Therefore, based on disutility-dependent flow transfer, the traditional day-to-day studies mainly construct negative feedback of day-to-day traffic dynamic systems. However, travelers do not only wish to decrease their travel disutilities but also want to increase their activity participation utilities. Also, positive network effects, denoted as an additional user of a good/service has on the value of that product to others (Shapiro et al. 1998), is present in activity participation such as working together to create greater value, demand-side economies of scale, and choosing a restaurant by its popularity, and so on. Therefore, we try to capture the potential positive network effect and assume that the utility on an activity link increases with the increasing number of travelers using it. Note that the positive effects may not apply to all activities. Under the influence of $\Phi_U(f^{(n)})$, it generates a positive feedback of flow adjustment process. Taken together, the entire feedback system is shown in Fig. 2, where $y_{ij}(\cdot)$ and $z_{ij}(\cdot)$ are respectively related to the positive and negative network effects on link $l$ and will be discussed in detail in Section 4. It is noted that with the proposed system, the choice of route, mode, activity sequence, and activity location are all
unified into path choices. As a result, both positive and negative network effects are captured, and the positive and negative feedbacks are integrated into the needs-based activity-travel dynamical system. On each day, the path flows tend to change from the current pattern towards the desired pattern based on the path utility of the current day.

4. Activity-travel utility

4.1. Positive and negative network effects

To model the chain effects, we first formulate the activity-travel utility for traveler class $i$ traversing link $l$ from $h$ through path $r$, $u_{ihrl}(\cdot)$, as the following simplified form

$$u_{ihrl}(B_{ih}, x_i) = u_{ihrl}^0 y_i(x_i), \quad \forall i \in I, h \in H, r \in R_h, l \in L, a \in A.$$  \hspace{1cm} (12)

where $u_{ihrl}^0$ is the free flow utility for class $i$ traversing link $l$ from $h$ through path $r$, $y_i(x_i)$ is the positive network effect multiplier for link $l$. $u_{ihrl}^0$ is dependent of need-satisfaction over the past and realization of activities at the present day. Based on a classic utility formulation (Arentze and Timmermans, 2009), we adopt a specification for $u_{ihrl}^0$ as follows

$$u_{ihrl}^0 = \alpha_{ia} \rho_{ra} B_{ih}, \quad \forall i \in I, h \in H, r \in R_h, l \in L, a \in A.$$  \hspace{1cm} (13)

where $\alpha_{ia}$ is the parameter of marginal utility of activity $a$ on link $l$ and $\rho_{ra}$ is a scaling factor of activity $a$ due to the activity sequence in path $r$. Specifically, $\alpha_{ia}$ is positive if link $l$ corresponds to activity $a$ and 0 otherwise, which ensures that travel utility is zero on any non-activity link. In addition, $\rho_{ra}$ can be assigned with different values for different activity sequences. It is noteworthy that a path explicitly specifies the activity sequence, which can be discomposed into a progression of activity states. The combination of $r$ and $a$ implicitly disclose the activity state when conducting $a$. In that sense, the specification of $\rho_{ra}$ also includes the effects of activity states. For example, $\rho_{ra} < 1$ implies that conducting $a$ is disfavored at the activity state; $\rho_{ra} = 1$ implies that $a$ is independent of the activity state; and $\rho_{ra} > 1$ implies that $a$ is complementary with the foregoing activities in $r$. Regarding $y_i(x_i)$, without loss of generality, the following is assumed.

**Assumption 3.** The positive network effect multiplier for an activity link is equal to one when there is no traveler using it, while it is continuous and increasing with the flow on it, i.e., $y_i(0) = 1$ and $\frac{dy_i(x_i)}{dx_i} > 0$. That is to say, with other components unchanged, travelers obtain higher activity utilities with increasing activity participants.

On the other hand, although negative network effects are ubiquitous, most travel demand forecasting systems have only focused on congestion and increased travel times on road networks (e.g., Arnott et al. 1990). To increase the
travel behavior realism, a few studies have also considered negative network effects as queuing at parking locations (e.g., Fu and Lam, 2014) and discomfort (mainly attributed to the privacy loss, uncomfortable physical proximity, risk of sexual harassment, etc.) in public transit vehicles (e.g., Tian et al., 2007); while to the best of our knowledge, very few took crowdedness and decline of quality of services at activity locations (e.g., supermarkets and restaurants) into consideration (e.g., Liu et al., 2016; Fu and Lam, 2018). Above all, let $z_i(x_i)$ be the negative network effect on roads for private vehicles, in public transit vehicles, and at locations for activity and parking. It is associated with congestion delay, crowding discomfort, and decline of quality of service, and so on. Without of loss generality, we adopt the following assumption.

**Assumption 4.** The negative network effect multiplier for a link is equal to one when there is no traveler using it, while it is continuous and increasing with the flow on it, i.e., $z_i(0)=1$ and $dz_i(x_i)/dx_i > 0$.

We formulate the actual link travel disutility $c_i(x_i)$ as

$$c_i(x_i) = c_i^0 + g_i(x_i)t_i(x_i), \quad \forall l \in L,$$

(14)

where $c_i^0$ is fixed disutility independent of link flow, capturing factors such as transit ticket fare, parking fare and payment of using private vehicle in equal shares to link trip; $g_i(x_i)$ and $t_i(x_i)$ respectively are the disutility per unit time and link traverse time/activity duration (hereinafter referred to as link traverse time) on link $l$, which is composed of the variable part of link travel disutility. To be more specific, $g_i(x_i)$ can be calculated by

$$g_i(x_i) = \begin{cases} g_i^0, & \text{if } l \in L_{PN} \cup L_T, \\ g_i^0z_i(x_i), & \text{if } l \in L_{PN} \cup L_d, \end{cases}$$

(15)

where $g_i^0$ is the free flow traverse disutility per unit time on link $l$ and $z_i(x_i)$ is the negative network effect multiplier measuring discomfort due to crowdedness or decline of quality of services. As driving by private vehicle or transferring between modes is often in private space, we assume that its traverse cost per unit time is independent of link flow. However, in public transit vehicle or at an activity location, the traverse disutility per unit time is dependent on crowdedness or decline of quality of services. In addition, the link traverse time is formulated as

$$t_l(x_l) = \begin{cases} t_l^0z_l(x_l), & \text{if } l \in L_{PN} \cup L_T, \\ t_l^0, & \text{if } l \in L_{PN} \cup L_d, \end{cases}$$

(16)

where $t_l^0$ is the free flow traverse time and $z_l(x_l)$ measures delay time due to congestion. It is noted that the free flow traverse time on a public transit link already contains the waiting time due to the discrete departure intervals. We consider public transit in-vehicle times and activity durations are all fixed and thus link traverse times are independent of the flows. However, when driving on a road by private vehicles or transferring between modes, the traverse time is dependent on congestion. Substituting Eqs. (15) and (16) into Eq. (14), we have a unified function to model link disutility for all types of links as

$$c_i(x_i) = c_i^0 + g_i^0t_i^0z_i(x_i), \quad \forall l \in L.$$

(17)

Under Assumption 4, it is known that $z_i(0)=1$ and $c_i^0 + g_i^0t_i^0$ is the free flow travel disutility, which is independent of link flow while $z_i(x_i)$ is dependent on link flow.
4.2. Path utilities

As mentioned above, travelers want to increase their activity utilities and decrease their travel disutilities. Then, the link utility is equal to the activity utility (if any) deducting the travel disutility (if any), expressed as

\[ v_{ihr} = u_{ihr} - c_i, \quad \forall \, l \in L_a. \]  
(18)

On the path level, according to Eqs. (1)-(3), (12), (13) and (17), the path utility vector is formulated by

\[ V(B, f) = \Delta^{-1} u(B, \Delta f) - \Delta^{-1} c(\Delta f), \]  
(19)

where \( u(B, \Delta f) = (u_{ihr}, \forall i \in I, h \in H, r \in R_h, l \in L)^T \) is the vector of activity utilities with \( \|I\| \sum_{h \in H} \sum_{r \in R_h} \|L\| \) dimensions, \( B = (B_{ihr}, l \in I, h \in H, a \in A)^T \) is the vector of activity needs, and \( c(x) = (c_i(x), l \in L)^T \) is the vector of link travel disutilities. Based on the above setup, the following proposition is derived.

**Proposition 1.** The path utilities are continuous with the path flows.

**Proof.** \( V(B, f) \) can be regarded as an \( \|I\| \sum_{h \in H} \|R_h\| \) dimensional vector value function. Under Assumption 4, according to and Eqs. (3) and (16), it is obtained that path disutility \( C_{ihr} \) is continuous with link flows \( x \). According to Eq. (1), it is further obtained that link flows \( x \) is continuous with path flows \( f \). Therefore, \( C_{ihr} \) is continuous with \( f \). On the other hand, substituting Eqs. (6), (12) and (13) into Eq. (3), we have

\[ U_{ihr}^{(n)} = \sum_{ia} \delta_{ia} \alpha_{ia} \rho_{ia} \left( \left( 1 - \sum_{h \in H} \sum_{r \in R_h} \psi_{ihr} \xi_{ihr}^{(n-1)} \right) B_{ihr}^{(n)} + K_{ihr} \left( B_{ihr}^{(n)} \right) \right) y_i(x_i^{(n)}), \quad \forall i \in I, h \in H, r \in R_h. \]  
(20)

From Eq. (20), it is known that \( U_{ihr}^{(n)} \) is continuous with path flows \( f^{(n-1)} \) and link flows \( x^{(n)} \) under Assumption 3. Hence, for any \( i, h \) and \( r \), \( V_{ihr} \) is continuous with the current path flows \( f \). □

Proposition 1 also indicates that the vector of path utilities is determined by the series of the previous day-to-day path flows. During the adjustment process, a critical point of activity need is derived as follows. In addition, let \( \tilde{B}_{iha} \) make \( v_{ihr} (\tilde{B}_{iha}) = 0 \), and then according to Eqs. (12), (13), (17), (18) and Assumption 3, we have

\[ v_{ihr} = \alpha_{ia} \rho_{ia} \tilde{B}_{iha} y_i(x_i) - c_i - \gamma_i t_i z_i(x_i) = 0 \Leftrightarrow \tilde{B}_{iha} = \frac{c_i + \gamma_i t_i z_i(x_i)}{\alpha_{ia} \rho_{ia} y_i(x_i)}, \quad \forall i \in I, h \in H, a \in A. \]  
(21)

Substituting \( B_{iha}^{(n)} < \tilde{B}_{iha} \), we have \( v_{ihr}^{(n)} < 0 \) according to Eq. (21). If conducting any other activity in \( r \) is disfavored or independent of activity \( a \), there must exist a path \( r' \) which is identical to \( r \) except without activity link \( l \in L_a \). Let \( V_{ihr} \) be the utility of traveler class \( i \) using path \( r \) from \( h \), and thus the utility on path \( r' \) is \( V_{ihr} = V_{ihr} - \psi_{ihr} \). Due to \( v_{ihr} < 0 \), we have \( V_{ihr} > V_{ihr} \) and \( r \not\in \tilde{R}_{ih}^{(n)} \). Then, for any path \( r, r \not\in \tilde{R}_{ih}^{(n)} \) holds if there is a link \( l \in L_a \) satisfying \( B_{ihr}^{(n)} < \tilde{B}_{iha} \). It is noted that \( \tilde{B}_{iha} \) can be seen as a break-even need for traveler class \( i \) from \( h \). However, if there is an activity in \( r \) is complementary with activity \( a \), we cannot say \( r \not\in \tilde{R}_{ih}^{(n)} \) even \( B_{ihr}^{(n)} < \tilde{B}_{iha} \).

**Remark 1.** In case of conducting any other activity in \( r \) being disfavored or independent of activity \( a \), the path \( r \) is not in the set of best paths if a link corresponding to the activity is on path \( r \) when there is traveler class \( i \) who resides at \( h \) with the need for an activity lower than \( \tilde{B}_{iha} \). Then, the number of traveler class \( i \) using path \( r \) from \( h \) will be swapped out on the next day, i.e., \( f_{ihr}^{(n+1)} < f_{ihr}^{(n)} \), according to the definition of flow transfer vector \( \Phi(f) \). Furthermore, the relationship \( B_{ihr}^{(n)} \geq \tilde{B}_{iha} \) for any activity links on path \( r \) is a necessary condition of path \( r \) being in \( \tilde{R}_{ih}^{(n)} \) and the flow of traveler class \( i \) on path \( r \) will not reduce on the next day.
In addition, it is known that \( V_{thr} = \sum_{i=1}^{n} \delta_{thr}^i v_{thr}^i \) from Eq. (3). Then, from Eqs. (12) and (13), the path marginal utility for traveler class \( i \) with respect to activity link flow on path \( r \), i.e., \( \frac{\partial V_{thr}}{\partial x_{thr}} \), can be derived as

\[
\frac{\partial V_{thr}}{\partial x_{thr}} = \frac{\partial \sum_{i=1}^{n} \delta_{thr}^i v_{thr}^i}{\partial x_{thr}} = \alpha_{th} \rho_{th} B_{tha} \frac{dy}{dx} - g_{t} t_{i} \frac{dz}{dx}, \quad \forall i \in I, h \in H, l \in L,
\]

where \( \alpha_{th} \rho_{th} B_{tha} \frac{dy}{dx} \) and \( g_{t} t_{i} \frac{dz}{dx} \) respectively denote the marginal positive and negative network effects. As mentioned above, both items are non-negative under Assumptions 3 and 4. Furthermore, for any non-activity link \( l \), \( \alpha_{ta} \) is equal to zero and thus \( \frac{\partial V_{thr}}{\partial x_{thr}} < 0 \) always holds. For an activity link \( l \), we have

\[
\alpha_{ta} \rho_{ta} B_{tha} \frac{dy}{dx} > g_{t} t_{i} \frac{dz}{dx} \Leftrightarrow \frac{\partial V_{thr}}{\partial x_{thr}} > 0, \quad \forall i \in I, h \in H, l \in L, a \in A. \tag{23}
\]

\[
\alpha_{ta} \rho_{ta} B_{tha} \frac{dy}{dx} < g_{t} t_{i} \frac{dz}{dx} \Leftrightarrow \frac{\partial V_{thr}}{\partial x_{thr}} < 0, \quad \forall i \in I, h \in H, l \in L, a \in A. \tag{24}
\]

**Remark 2.** When the condition Eq. (23) holds, it means the marginal positive network effect for traveler class \( i \) is dominant on the activity link, which can cause positive feedback dominance in AT-RAP for traveler class \( i \). It may further lead to some activity links in SNK becoming more and more beneficial and then cause conformity behavior, namely *Bandwagon effect* (Leibenstein, 1950) or even polarization namely *Matthew effect* (Merton, 1968). Conversely, when the condition (24) holds, it means the marginal negative network effect for traveler class \( i \) is dominant on the activity link, which can cause negative feedback dominance in AT-RAP for traveler class \( i \). It may further lead to the utilities of used activity links being convergent and then flow non-polarization.

5. Properties of the dynamical system

5.1. Needs-based activity-travel equilibria

At the macroscopic level, the activity-travel flow assignment can be considered as a \( \sum_{i=1}^{n} \sum_{h \in H} F_{ih} \)-player non-cooperative game. Specifically, the travelers represent players, the path sets are action sets, the path utilities are pay-off depending on the strategies of the other travelers, and each traveler tries to maximize his/her path utility. As travelers make a mixed path choice \( P \), the activity-travel flow assignment is a mixed-strategy game. Under Assumption 3, the Wardropian user equilibrium, in this \( \sum_{i=1}^{n} \sum_{h \in H} F_{ih} \)-player non-cooperative mixed-strategy game, is defined as a state, in which no traveler wants to change his/her mixed path choice unilaterally. It can be expressed by finding a vector \( f \in \Omega_{B} \) such that the following condition holds:

\[
V_{thr}(B,f) = \max_{w \in R_{h}} V_{thr}(B,f), \quad \text{if} \quad f_{thr} > 0, \quad \forall i \in I, h \in H, r \in R_{h}, \tag{25}
\]

Eq. (25) is well known that, at an equilibrium state, for each traveler from \( h \) only those paths that have the maximal utility are used, and those paths that are not used should have utilities lower than or equal to the maximal utility. It is noted that Eq. (25) establishes an integrated model to formulate the flow assignment problem and obtain the user equilibria across multiple dimensions of activity-travel choices.

**Proposition 2.** On each day, there is at least one equilibrium path/link flow vector.

**Proof.** On each day, according to Eq. (6), it is known that the activity need vector \( B \) is determined by path flows on the previous day and independent of the path flows on the current day. From Eq. (1), we know the set of feasible path
flows is a compact closed convex set with \( \Omega_r \subset \mathbb{R}^{|\mathcal{I}|} \). In addition, the path utilities are continuous with the path flows (Proposition 1). Therefore, the user equilibrium satisfying Eq. (25) exists. Furthermore, due to the inclusion of 0-1 integer variables \( \delta_{irl} \) and \( \delta_{hra} \), \( \mathbf{V}(\mathbf{B}, \mathbf{f}) \) is non-convex on \( \Omega_r \). Consequently, Eq. (25) may have multiple local solutions. On the other hand, the set of feasible link flows is a compact closed convex set with \( \Omega_l \subset \mathbb{R}^{|\mathcal{I}|} \). From Proposition 1, the link utilities are continuous with the link flows and thus the equilibrium link flow vector exists. A necessary condition of the uniqueness of equilibrium link flows is the Jacobian matrix of \( \mathbf{v}(\mathbf{B}, \mathbf{x}) \) being positive definite. According to Eq. (24), for any \( i, h, r, l \), it is known that \( \partial \mathbf{v}(\mathbf{B}, \mathbf{x}) / \partial \mathbf{x}_{irl} \neq \partial \mathbf{v}(\mathbf{B}, \mathbf{x}) / \partial \mathbf{x}_{irl} \) if \( l = l' \) and 0 otherwise. By expanding the link utility Jacobian matrix, it is known that the link utility Jacobian matrix is not positive definite and thus \( \mathbf{v}(\mathbf{B}, \mathbf{x}) \) is not strictly monotonic on \( \Omega_l \). Therefore, there may be multiple equilibrium solutions of link flow vector. □

The equivalency between the Wardropian user equilibrium and a mixed-strategy Nash equilibrium in the traffic network has been proved by Haurie and Marcotte (1985) and Bell and Cassir (2002). Herein, we provide a proof for mixed activity-travel choice in the framework of multi-state supernetworks.

Proposition 3. On each day, a user equilibrium is equivalent to a \( \sum_{i\in\mathcal{I}} \sum_{h\in\mathcal{H}} F_{ih} \)-player non-cooperative mixed-strategy Nash equilibrium.

Proof. At the macroscopic level, under Assumption 3, \( f_{ih} \equiv p_{ih} F_{ih} \) holds for any \( i, h \) and \( r \) according to the Weak Law of Large Number, and thus \( f_{ih} = 0 \) holds if and only if \( p_{ih} = 0 \). According to Eq. (25), it obtains \( p_{ih} = 0 \) when \( r \notin \mathcal{R}_{ih} \). At the microscopic level, any traveler \( m \) class \( i \) from home \( h \) chooses a mixed strategy of path choice against all other traveler’s strategies \( \mathbf{P}^m \). This mixed strategy \( \mathbf{p}^m \) is considered as a probability vector of all feasible paths for \( i \) from home \( h \), i.e., \( \mathbf{p}^m = (p_{ihr}^m, r \in \mathcal{R}_h)^T \), where \( p_{ihr}^m \) is the probability that traveler \( m \) class \( i \) from home \( h \) choosing path \( r \). Furthermore, the path utility of traveler \( m \) from home \( h \) can be formulated in the following linear combination form

\[
V^m_{ih}(\mathbf{P}) = \sum_{r \in \mathcal{R}_h} p_{ihr} V^m_{ih}(\mathbf{P}^m, r), \quad \forall i \in \mathcal{I}, h \in \mathcal{H}, m \in F_{ih},
\]

where \( V^m_{ih}(\mathbf{P}^m, r) \) is the path utility of traveler \( i \) choosing pure path strategy \( r \) while all other travelers follow a mixed strategies \( \mathbf{P}^m \). For any home \( h \), since traveler class \( i \) are assumed to be homogeneous and have the same pure strategy set of path choice and thus they are replaceable. Therefore, for any home \( h \), both \( p_{ihr}^m = p_{ihr}^m = p_{ihr} \) and \( V^m_{ih}(\mathbf{P}^m, r) = V^m_{ih}(\mathbf{P}^j, r) \) hold for any class \( i \) traveler \( m \), \( j \) and path \( r \). Then, we have

\[
p_{ihr} > 0 \iff p_{ihr}^m > 0, \quad p_{ihr} = 0 \iff p_{ihr}^m = 0, \quad \forall i \in \mathcal{I}, h \in \mathcal{H}, m \in F_{ih}, r \in \mathcal{R}_h,
\]

where

\[
V^m_{ih}(\mathbf{P}^m, r) = \begin{cases} \max_{w \in \mathcal{R}_h} V^m_{ih}(\mathbf{P}^m, w) \Rightarrow p_{ihr}^m > 0, & \forall h \in \mathcal{H}, m \in F_{ih}, r \in \mathcal{R}_h, \\ \min_{w \in \mathcal{R}_h} V^m_{ih}(\mathbf{P}^m, w) \Rightarrow p_{ihr}^m = 0, & \forall i \in \mathcal{I}, h \in \mathcal{H}, m \in F_{ih}, r \in \mathcal{R}_h. \end{cases}
\]

Furthermore, due to \( f_{ih} \equiv p_{ih} F_{ih} \), we have

\[
V^m_{ih}(\mathbf{P}^m, r) \equiv V_{ih}(\mathbf{f}), \quad \forall i \in \mathcal{I}, h \in \mathcal{H}, m \in F_{ih}, r \in \mathcal{R}_h.
\]

Eq. (24) indicates that the utility of class \( i \) traveler \( m \) using pure path strategy \( r \) is approximately equal to the utility of path \( r \) corresponding to flow pattern \( \mathbf{f} \). Under Assumption 2, synthesizing Eqs. (27)-(29), we have

\[
V_{ih}(\mathbf{f}) = \max_{w \in \mathcal{R}_h} V^m_{ih}(\mathbf{f}) \Rightarrow p_{ihr} > 0 \iff p_{ihr}^m > 0 \iff V^m_{ih}(\mathbf{P}^m, r) = \max_{w \in \mathcal{R}_h} V^m_{ih}(\mathbf{P}^m, w), \quad \forall i \in \mathcal{I}, h \in \mathcal{H}, r \in \mathcal{R}_h,
\]
\[ V_{dr}(\mathbf{f}) < \max_{w \in R_h} V_{iwh}(\mathbf{f}) \Rightarrow p_{dr} = 0 \iff p_{ih}^m = 0 \iff V_{ih}^m(\mathbf{P}^{-m}, r) < \max_{w \in R_h} V_{iwh}(\mathbf{P}^{-m}, w), \quad \forall i \in I, h \in H, r \in R_h, \]  

(31)

Above all, it obtains that an activity-travel user equilibrium is equivalent to a mixed-strategy Nash equilibrium. □

It is noted that according to Nash (1951), there is at least one mixed-strategy Nash equilibrium to the above \( \sum_{i \in I} \sum_{h \in H} \sum_{r \in R_h} F_{ih} \) player non-cooperative game, which is consistent with Proposition 2. In addition, on each day, the activity need vector is the major determinant of the set of user equilibria. Furthermore, the user equilibrium set may change with \( B \) from day to day.

5.2. Convergence and stability analysis

In order to investigate the convergence of the proposed needs-based activity-travel dynamical system, it is necessary to construct two indicators to measure the convergences of activity needs and path flows, respectively. Particularly, each convergence indicator is considered as a gap function measuring how close the dynamical system is to a stationary or an equilibrium point. However, it is impractical to get the gap of path flows and activity needs between the current and equilibrium because the stationary is unknown in advance or there may not even be an equilibrium.

Therefore, we try to construct gap functions using the historical and current activity needs and path flows. Logically, the more travelers on those paths that have lower utilities than the maximal one, the larger gap should be obtained, which can be formulated as 

\[ \max_{i \in I} V_{iha}(\mathbf{P}^{(\alpha)}) - V_{iha}(\mathbf{P}^{(\alpha)}) \]  

Similarly, the gap function of activity needs can be formulated as 

\[ ||\mathbf{f}^{(\alpha)} - \mathbf{f}^{(\alpha)}|| \]  

Summing up the relative gaps across all paths of all travel classes, the need and flow convergence indicators can be respectively expressed as

\[ O_B(B^{(\alpha)}) = \sum_{i \in I} \sum_{h \in H} \sum_{r \in R_h} \left| \psi_{iha} f_{iha}^{(\alpha)} - B_{iha}^{(\alpha)} B_{iha}^{(\alpha-1)} \right|, \]  

(32)

\[ O_f(f^{(\alpha)}) = \sum_{i \in I} \sum_{h \in H} \sum_{r \in R_h} \frac{\max_{w \in R_h} V_{iwh}(\mathbf{f}^{(\alpha)}) - V_{iwh}(\mathbf{f}^{(\alpha)})}{\sum_{i \in I} \sum_{h \in H} \sum_{r \in R_h} f_{iwh}^{(\alpha)} V_{iwh}(\mathbf{f}^{(\alpha)})}. \]  

(33)

Proposition 4. AT-RAP converges to some stationary points coinciding with needs-based activity-travel equilibria.

Proof. Respectively substituting Eqs. (6) and (9) into Eqs. (32) and (33), it is known that both of the need convergence indicator \( O_B(B^{(\alpha)}) \) and the flow convergence indicator \( O_f(f^{(\alpha)}) \) are equal to zero if and only if the flow transfer vector \( \Phi(f) = 0 \). From Eq. (8), it is found that both \( \sum_{\alpha \in \mathcal{\alpha}} \lambda^{(\alpha)} = \infty \) and \( \sum_{\alpha \in \mathcal{\alpha}} (\lambda^{(\alpha)}) = 0 \) hold. Then, the AT-RAP dynamical system Eq. (7) can be seen as an Euler method in structure. As a result, the day-to-day \( f^{(\alpha)} \) converges to some stationary points even if the monotonicity of path flows is not guaranteed (Huang and Lam, 2002). Furthermore, since Eq. (7) is an Euler method, for any flow pattern \( f \in \Omega_f \) which satisfies the activity-travel user equilibrium condition Eq. (25) if and only if it is a stationary point for the dynamical system (see Theorem 2 in Zhang and Nagurney, 1996). □

Synthesizing Propositions 3, 4 and 5, it is known that through day-to-day AT-RAP, the needs-based activity-travel dynamical system converges to a steady state, which coincides with a mixed-strategy Nash equilibrium and the equivalent Wardropian user equilibrium. In addition, following the definition given by Nagurney and Zhang (1997), AT-RAP is stable if the Euclidean distance between the path flow pattern sequence \( f^{(\alpha)} \) initialized with any \( f^0 \) and any equilibrium flow pattern \( f \), namely, \( ||f^{(\alpha)} - f|| \) is monotone non-increasing from day to day. In this regard, let watershed need \( B_{iha}^* \) make \( \partial V_{iha}(\mathbf{B}_{iha}) = 0 \). From Eqs. (12), (13), (17), (18) and Assumption 3, we then have
\[ \frac{\partial v_{lhr}}{\partial x_{lhr}} = \alpha_a \rho_{a} h_{a} B_{lhr}^{\ast} \frac{\partial y_{l}}{\partial x_{l}} - c_{l}^{0} - g_{l}^{0} t_{l}^{0} \frac{\partial z_{l}}{\partial x_{l}} = 0 \Leftrightarrow B_{lhr}^{\ast} = \frac{g_{l}^{0} t_{l}^{0}}{\alpha_a \rho_{a}} \frac{\partial y_{l}}{\partial x_{l}}, \quad \forall i, h, a \in A. \]  

(34)

**Proposition 5.** If \( B_{lhr} \leq B_{lhr}^{\ast} \) holds for any \( i, h \) and \( a \) on each day, AT-RAP is stable.

**Proof.** As mentioned above, the AT-RAP dynamical system Eq. (7) is an Euler method, the criterion of the stability such that link utility \( \mathbf{v}(x) \) is monotone decreasing with link flows \( x \) (see Theorem 5 in Nagurney and Zhang, 1997). For non-activity links, under Assumption 4, the link marginal utility with respect to link flow can be derived as

\[ a \frac{\partial v_{lhr}}{\partial x_{lhr}} = -g_{l}^{0} t_{l}^{0} \frac{\partial z_{l}}{\partial x_{l}} < 0, \quad \forall i, h, r \in R_{h}, l \in L, \]  

(35)

For activity links, under Assumption 3, we then have

\[ \frac{\partial v_{lhr}}{\partial x_{lhr}} = \alpha_a \rho_{a} B_{lhr}^{\ast} \frac{\partial y_{l}}{\partial x_{l}} - g_{l}^{0} t_{l}^{0} \frac{\partial z_{l}}{\partial x_{l}} \begin{cases} > 0, & \text{if } B_{lhr} > B_{lhr}^{\ast}, \\ \leq 0, & \text{if } B_{lhr} \leq B_{lhr}^{\ast}, \end{cases} \quad \forall l \in L_{A}. \]  

(36)

From Eqs. (35) and (36), it is known that all link utilities are monotone decreasing with link flows when the condition \( B_{lhr} \leq B_{lhr}^{\ast} \) holds for any \( i, h \) and \( a \) on each day, and thus the AT-RAP is stable. □

**Remark 3.** On one day, when the positive network effect is very small on each activity link, that is, \( \frac{\partial z_{l}}{\partial x_{l}} \gg \frac{\partial y_{l}}{\partial x_{l}} \). Then, watershed need \( B_{lhr} \) is very large which easily guarantees \( B_{lhr} \leq B_{lhr}^{\ast} \) for any \( i, h \) and \( a \). And the flow on this activity tends to decrease according to Remark 2. As a result, the need may build-up from day to day, and it is massively likely to exist a combination of \( i, h \) and \( a \) making \( B_{lhr} > B_{lhr}^{\ast} \) on a certain day. Therefore, the sufficient conditions of stability of AT-RAP are often difficult to satisfy.

### 6. Numerical examples

#### 6.1. Basic setting

This section illustrates the proposed AT-RAP using a simple transportation network depicted in Fig. 3a. It shows that travelers live at location \( h \), and two activities \( a_{1} \) and \( a_{2} \) can be conducted in the center, where activity \( a_{1} \) can be conducted at location \( d_{1} \) and activity \( a_{2} \) can be conducted at location \( d_{2} \) or \( d_{3} \). Location \( h \) is connected with activity location \( d_{1} \) by private vehicle links 1, 2 and public transit links 3, 4. Activity location \( d_{1} \) is connected with \( d_{2} \) via walking links 5, 6; \( d_{1} \) is connected with \( d_{3} \) via private vehicle links 7, 8. In addition, there are three parking places located at \( h, d_{1} \) and \( d_{3} \), respectively.

We obtain the corresponding SNK as shown in Fig. 3(b). In this SNK, besides traffic links 1-8, other links numbered 9-17 are defined. Links 9, 10 and 11 represent conducting activity \( a_{1} \) at \( d_{1} \), conducting activity \( a_{2} \) at \( d_{2} \) and conducting activity \( a_{2} \) at \( d_{3} \), respectively. Links 12-17 represent picking up or parking at \( h, d_{1} \) and \( d_{3} \), respectively. Detailed parameter settings for all activity-travel links are shown in Table 1. The symbol “/” means “not relevant”. For simplicity, the need build-up rate for activities \( a_{1} \) and \( a_{2} \) are assumed to be constant, i.e., \( K_{i1}(\cdot) = k_{i1} \) and \( K_{i2}(\cdot) = k_{i2} \). It is further to suggest the functions for modeling positive and negative network effect multipliers as follows

\[ y_{i}(x_{l}) = 1 + \beta_{i} x_{l}^{\gamma_{i}}, \quad \forall l \in L, \]  

(37)
\[ z_l(x_i) = 1 + \eta \left( \frac{x_i}{\kappa_l} \right)^{\beta_l}, \quad \forall l \in L. \] (38)

where parameters can be assigned with different values for different types of links. For example, when \( \beta_l = 0 \) and \( \eta_l = 0 \), it implies there is respectively no positive and negative network effect on link \( l \) and then travelers incur the free flow travel utility or cost. When \( \gamma_l \) and \( \mu_l \) is equal to 1, it implies that the positive and negative network effect multipliers are in a linear form, respectively. Finally, according to Eqs. (17), (19), (20), (37) and (38), the path utility obtained by class \( i \) travelers traversing \( r \) from \( h \) on day \( n \) can be derived as

\[
V_{\text{tr}}^{(n)} = \sum_{i=1}^{m} \delta_{ai} \alpha_{ai} \rho_{ai} \left( 1 + \frac{\sum_{k=1}^{n} \psi_{hkr} f_{hkr}^{(n-1)}}{F_{hkr}} \right) \left( B_{ai}^{(n)} + k_{ai} \right) \left( 1 + \beta_l \left( \sum_{i=1}^{n} \sum_{r \in R_k} \delta_{hkr} f_{hkr}^{(n)} \right) \right) \right) \right), \quad \forall i \in I, h \in H, r \in R_h.
\] (39)

It is noted that there is a one-to-one correspondence between traveler and private vehicle. All travelers and private vehicles should departure from \( h \) and return to \( h' \) at the end of day trips. In this SNK, any route from \( h \) to \( h' \) is a feasible path. For example, a path denoted by the ordered link set \( \{12, 1, 15, 9, 5, 10, 6, 14, 2, 13\} \) shows that a traveler first picks up the car at home \( h \) and chooses link 1 to \( d_1 \) for parking and conducting \( a_1 \), after that, walks to \( d_2 \) for conducting \( a_2 \), and finally goes back to pick up the car at \( d_1 \) and drives home. In fact, any path is a one to one mapping with a combined choice of traffic route, mode, activity location, and sequence. We show the list of all feasible ATPs in Table 2. Suppose there are 10000 class 1 travelers and 5000 class 2 travelers in total at \( h \). Finally, set activity reference needs as \( B_{h}^{1} = 100, B_{h}^{2} = 20, B_{h}^{1} = 0, B_{h}^{2} = 30 \); set need build-up rates \( k_{h} = 20, k_{h} = 5, k_{h} = 5, k_{h} = 10 \); set parameters \( \lambda^0 = 0.05 \) and \( \omega = 30 \).

Fig. 3. (a) Traffic network and corresponding (b) multi-state supernetwork.
Table 1. Parameter settings of links

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<th>Link number</th>
<th>Link attribute</th>
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<th>$g_i^0$</th>
<th>$\kappa_i$</th>
<th>$t_i^0$</th>
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Table 2. Feasible paths

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6.2. Computational results

Initially, suppose all travelers of each class are evenly distributed on all the paths as they do not have any information on the current or past conditions of the transport system. For comparison, we first show the day-to-day needs-based activity-travel dynamics and equilibrium. First of all, from Figs. 4-6, it is observed that the adjustment processes and convergences of path flow, path utility, activity need, and activity location choice exhibit significantly different patterns. All these differences are derived from the different settings of activity needs. It implies that the chain effects from need via activity-travel to flow dynamics do exist. That is to say, ignorance of this chain may cause biases in the outcomes of travel demand forecasting. As shown in Fig. 4, activity needs and the numbers of users of activity locations change at first and then stabilize. Both the activity needs and location choices of class 1 travelers converge earlier than those of class 2 travelers.

To be more specific, as shown in Fig. 5a, for class 1 travelers, as the day progresses, the flows on paths 6, 7 and 10 fluctuate at first and then stabilize while other paths cannot attract traveler after the fifth day. After the flow adjustment convergence, there are 7885, 473 and 1642 travelers choosing paths 6, 7 and 10, respectively. As given above, reference needs of the two activities are relatively high and the utilities can cover the respective travel costs. In addition, conducting $a_1$ first and then $a_2$ is more favored by class 1 travelers. In particular, all used paths have activity sequence $\{a_1, a_2\}$. As shown in Fig. 5b, during the process, class 1 travelers realize that choosing paths 6, 7 and 10 delivers the higher utility and thus some continuously switch to them from other paths until a final equilibrium state is achieved, in which the used paths 6, 7 and 10 have the maximum and equal utility, that is, 140.9. These results are consistent with Propositions 3 and 5.
In Fig. 6a, for class 2 travelers, the flows on paths 5 and 10 fluctuate at first and then stabilize while other paths cannot attract traveler after ten days. After the adjustment convergence, there are 2840 and 2160 travelers choose paths 5 and 10, respectively. As given above, class 2 travelers’ reference need and need build-up rate of activity $a_i$ are 0 and 5, respectively. Therefore, the need of activity $a_i$ may be lower than the break-even need on some days. Then, the conducting utility of activity $a_i$ is very low on these days and it may not compensate for the corresponding travel costs. On the path level, in Fig. 6b, class 2 travelers realize that the utility of path 10 changes up and down
opposed to that of path 5. Thus, some class 2 travelers continuously adjust their activities and path choices until a final equilibrium state is achieved.

6.3. Influence of key parameters on the performance of the dynamic system

As mentioned in Eq. (39), $\beta_i$ and $\eta_i$ are the coefficients of positive and negative network effects, respectively. By respective setting $\beta_i = 10^{-3}$ and $\eta_i = 10^{-3}$ for all links and keeping other parameters the same as Section 6.1, the corresponding activity location choice is illustrated in Fig. 7. Specifically, as shown in Fig. 7a, when $\beta_i = 10^{-3}$, the positive network effect is small, which causes the dominance of negative feedback in AT-RAP. It further leads to both activity locations $d_2$ and $d_1$ being used by the two class travelers. Besides, it is also observed that the flow is fluctuant because there is at least one activity need higher than its watershed even under a tiny positive network effect. However, as shown in Fig. 7b, when $\eta_i = 10^{-3}$, the negative network effect is very small, which causes the dominance of positive feedback in AT-RAP. It further leads to a result for both classes that activity location $d_3$ attracts more and more flows while the flows at activity location $d_2$ keep going down to zero. This process can be seen as a Bandwagon effect or even a Matthew effect. These results confirm Remarks 2 and 3.

Furthermore, by setting the period of adjusting the conversion $\omega = \infty$ and keeping other parameters the same as Section 6.1, the day-to-day path flow adjustment processes of two class travelers are illustrated in Fig. 8. It is observed that the day-to-day flow adjustment process is not convergence. It is because that when $\omega = \infty$, we have $\lambda^{(\omega)} = \lambda^{0}$, means that travelers never change flow conversion coefficient, and then the proposed AT-RAP cannot be seen as an Euler method in structure. It is worth noting that the path flows of class 2 travelers are almost periodic due to periodic change in the need of activity 1.

![Fig. 7. The influences of tiny (a) $\beta_i$ and (b) $\eta_i$ on day-to-day activity location choices](image1)

![Fig. 8. Day-to-day path flow adjustment processes of (a) class 1 and (b) class 2 travelers under the case of $\omega = \infty$.](image2)
7. Conclusion

Understanding people’s activity-travel behavior is critical for effective policy-making in strategical and operational travel demand and traffic management. Over the past years, activity-based models have been gradually replacing trip-based models for travel demand analysis, due to inclusions of a valid representation of daily activity-travel behavior. However, three core components of activity-based modeling have been largely treated in isolation due to the lack of an integrative and cohesive modeling framework. In view of the advantages of the concept of needs, this study moves a step further by proposing a new framework of activity-based travel demand analysis in multi-state supernetworks based on the needs-based theory. It formulates the chain relationship among the dynamic needs, activities, and travel demand and traffic flows. We claim this formalism is the first of this kind combining dynamic activity generation, activity-travel scheduling, and traffic flow evolution in a strong sense. In addition, both positive and negative network effects are captured, while positive and negative feedback are integrated into the needs-based activity-travel dynamical system. As a result, it is found that through the day-to-day adjustment process, the proposed system converges to a steady state, which coincides with a mixed-strategy Nash equilibrium and the equivalent Wardropian user equilibrium. The proposed propositions are explored by simulated numerical examples. This study sheds light on how traffic flows and activity participation evolve dynamically and a full-fledged model development supports analyzing refined dynamic land-use transport demand management.

Nevertheless, several facets of this basic framework need further investigation. First, the mapping from multiple dimensions of physiological and psychological needs to multiple activities should be further developed. Second, the needs of individuals should be treated in the larger scope that incorporates interpersonal interactions, e.g. at a household level. Third, the mapping from dynamic needs to activity-travel utilities/disutilities needs sufficient empirical analyses. Fourth, it is also meaningful to integrate within-day dynamics and time-dependent (dis)utility functions in the model system. Depending on the time-dependent traffic flow propagation principles, travel times and disutilities other than formulated as BPR functions have significant impacts on the choice of ATPs and day-to-day traffic flows. For that matter, the dynamic activity-travel assignment model proposed in Liu et al. (2015) and the space-time supernetwork representation in Liao (2016) provide a solid knowledge base. Fifth, given the ubiquity of travel information and travel time uncertainty, incorporating them into travelers’ (dis)utility functions and choice mechanism possibly causes new network effects. Sixth, this study has the limitation of relying on an explicit enumeration of ATPs. Combing path generation methods (e.g., Liao and van Wee, 2017; Wang et al., 2019) is promising to enhance the model applicability. In line with these extensions, theoretical analyses on the properties of day-to-day dynamics and equilibria should also be established. We leave these modeling developments for our future research.

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References


Liu, W., Li, X., Zhang, F., Yang, H., 2017. Interactive travel choices and traffic forecast in a doubly dynamical system with user inertia and information provision. Transportation Research Part C 85, 711-731.


