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An Improved Dropping Algorithm for Line-Of-Sight Massive MIMO with Max-Min Power Control

A. Farsaei, A. Alvarado, F. M. J. Willems, U. Gustavsson

Abstract—In line-of-sight massive MIMO, there is a nonnegligible probability that two users become highly correlated, which leads to a reduction in the achievable sum-rates of linear precoders. In this paper, threshold values of a previously proposed dropping algorithm are found analytically to avoid repeating a large number of simulations to find the optimal threshold. These thresholds allow us to improve conjugate beamforming (CB) and zero-forcing (ZF) sum-rates with max-min power control. By using the proposed threshold values, the CB and ZF sum-rates are maximized, when there are only two correlated users. In addition, by using the derived thresholds, a modified dropping algorithm is proposed for channels with any number of correlated users. The results of the simulation scenarios show that at signal to noise ratio of 20 dB and 120 antennas at the base station, the modified algorithm improves the average CB and ZF sum-rates up to 36% and 5%, respectively.

Index Terms—Conjugate beamforming, line-of-sight, massive MIMO, max-min power control, zero-forcing.

I. Introduction

Massive MIMO is a promising technology for 5G wireless networks [1]. One of the key properties massive MIMO relies on is favorable propagation (FP) [2], which implies that the channel vectors to the users become mutually orthogonal when the number of antennas at the base station (BS) tends to infinity. FP provides a large number of spatial degrees of freedom, which results in high data throughput and radiated-energy efficiency [2], [3]. Moreover, in FP environments, linear precoders, e.g., conjugate beamforming (CB) or zero-forcing (ZF), are optimal in terms of downlink capacity [2].

Studying line-of-sight (LOS) environments is of great importance since the real propagation environment is likely to fall between LOS and independent and identically distributed Rayleigh [4]. Under LOS propagation environments, it is of great interest to provide uniformly good service to all the users [5]. This is achieved by using max-min power control, which maximizes the fairness among the users.

In LOS environments, there is a nonnegligible probability that two users become highly correlated [2]. This makes the propagation environment unfavorable, which leads to a reduction in the downlink capacity and the sum-rate of linear precoders [6], [7]. To improve CB and ZF sum-rates with max-min power control, a dropping algorithm was proposed in [5]. In the algorithm in [5], the BS drops some users to make the

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correlation between the channel vectors of the remaining users be less than a threshold. This threshold is found via numerical simulations for a large number of realizations with the same scenario, i.e., the number of users, the path loss, and total transmit power. The drawback of this approach is that new simulations are required for every new scenario. A similar approach is used in [8] for parallelogram planar arrays, and in [9] for a scheduling algorithm in massive MIMO.

In this paper, the dropping problem is analyzed for CB and ZF with max-min power control in LOS environments. We are interested in maximizing the minimum signal to interference plus noise ratio (SINR) among the served users. The contributions of this paper are as follows. First, we derive the threshold values used in the dropping algorithm of [5]. The derived thresholds avoid repeating numerical simulations as done in [5], [8]. Second, we prove that by using the derived thresholds, the CB and ZF sum-rates for a channel of K users with only two correlated users, are maximized. Lastly, the derived thresholds are used to modify the dropping algorithm in [5]. Numerical simulations show that the modified algorithm improves the CB and ZF sum-rates considerably.

This paper is organized as follows[1]. In Sec. II, the system model is given. The dropping problem is analyzed in Sec. III. The simulation results are presented in Sec. IV. Finally, Sec. V concludes the paper.

II. System Model

The model for the downlink channel from an M-antenna BS to K single-antenna users with linear precoding is shown in Fig. 1. The intended zero-mean, uncorrelated and unit variance symbols $\boldsymbol{s} = (s_1,...,s_K)^T \in \mathbb{C}^{K \times 1}$ are precoded by a diagonal matrix $\mathrm{diag}(\boldsymbol{p})$ and a linear precoding matrix $\boldsymbol{U} \in \mathbb{C}^{M \times K}$ with unit-norm column vectors $\boldsymbol{u}_i$. The power control vector is $\boldsymbol{p} = (\eta_1,...,\eta_K)^T$, where $\eta_i \in \mathbb{R}^+$ with $i = 1,...,K$ are the max-min power control coefficients. The power constraint at the BS is $\sum_{i=1}^{K} \eta_i^2 = P_{\mathrm{tot}}$. The precoded vector $\boldsymbol{x} \in \mathbb{C}^{M \times 1}$ is transmitted through a general channel $\boldsymbol{H} = (\boldsymbol{h}_1,...,\boldsymbol{h}_K)^T \in \mathbb{C}^{K \times M}$, where $\boldsymbol{h}_i$ is the channel vector from the BS to user i. The received signal for user i is:

$$y_i = \boldsymbol{h}_i^T \boldsymbol{u}_i \eta_i s_i + \sum_{\substack{j=1 \\ j \neq i}}^{K} \boldsymbol{h}_i^T \boldsymbol{u}_j \eta_j s_j + n_i, \qquad (1)$$
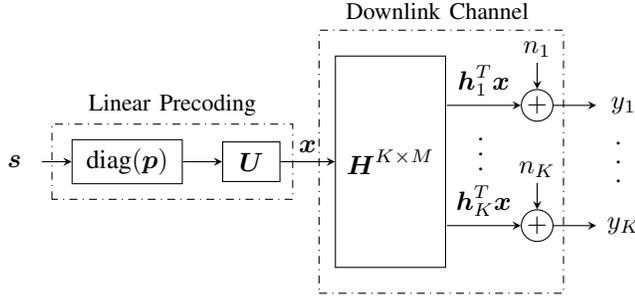
Fig. 1. The model of the downlink channel with linear precoding.

where n_i is complex AWGN noise with variance N_0. For a given channel matrix, SINR for each user can be expressed as

$$\text{SINR}_i = \frac{|\boldsymbol{h}_i^T \boldsymbol{u}_i|^2 \eta_i^2}{\sum_{j=1, j \neq i}^{K} |\boldsymbol{h}_i^T \boldsymbol{u}_j|^2 \eta_j^2 + N_0}. \tag{2}$$

To elaborate on the derived thresholds, we will focus on a specific channel in Sec. III, which is defined as follows.

Definition 1. A correlated channel of size K is a channel with K single-antenna users in which user $K-1$ and user K are the only correlated users ($\|\boldsymbol{h}_{K-1}\| \geq \|\boldsymbol{h}_K\|$), and the other users are mutually orthogonal. The spatial correlation of user $K-1$ and user K is given by:

$$\rho = \frac{\boldsymbol{h}_K^H \boldsymbol{h}_{K-1}}{\|\boldsymbol{h}_K\| \|\boldsymbol{h}_{K-1}\|}. \tag{3}$$

Consider a correlated channel of size K. If the BS with the power constraint P_{tot} drops user (DU) K, user 1 to user $K-1$ will experience FP. Therefore, the following sum-rate R^{DU} is achieved with CB or ZF with max-min power control using the Lagrangian multiplier:

$$R^{\text{DU}} = (K-1) \log_2 \left(1 + \frac{P_{\text{tot}}/N_0}{\sum_{i=1}^{K-1} \frac{1}{\|\boldsymbol{h}_i\|^2}} \right). \tag{4}$$

In what follows, the CB and ZF sum-rates for a correlated channel of size K are derived.

A. The CB Sum-Rate

To find the CB sum-rate, we need to find the SINR of each user with CB. The unit-norm column vectors of CB are found by:

$$\boldsymbol{u}_i^{\text{CB}} = \frac{\boldsymbol{h}_i^*}{\|\boldsymbol{h}_i\|}, \quad i = 1, ..., K. \tag{5}$$

Replacing (5) in (2), the following SINRs are found for the users:

$$\text{SINR}_i^{\text{CB}} = \frac{\|\boldsymbol{h}_i\|^2 \eta_i^2}{N_0}, \quad i = 1, ..., K-2 \tag{6}$$

$$\text{SINR}_{K-1}^{\text{CB}} = \frac{\|\boldsymbol{h}_{K-1}\|^2 \eta_{K-1}^2}{\|\boldsymbol{h}_{K-1}\|^2 |\rho|^2 \eta_K^2 + N_0}, \tag{7}$$

$$\text{SINR}_K^{\text{CB}} = \frac{\|\boldsymbol{h}_K\|^2 \eta_K^2}{\|\boldsymbol{h}_K\|^2 |\rho|^2 \eta_{K-1}^2 + N_0}. \tag{8}$$

The max-min power control coefficients, $\eta_i, i = 1, ..., K$ are found by maximizing the minimum SINR among the users given by (6)–(8) using bisection method as explained in [5]. However, due to FP of user 1 to user $K-2$, the CB SINR corresponding to the max-min power control is derived analytically by solving a quadratic equation. The largest root always results in a negative power allocation for the correlated users, while the smallest root always results in a positive power allocation for all the users. The smallest root is found by:

$$z = \frac{P_{\text{tot}}}{2N_0\gamma_1} + \frac{1 + \frac{\gamma_2}{\gamma_1}}{2|\rho|^2} - \sqrt{\left(\frac{P_{\text{tot}}}{2N_0\gamma_1} + \frac{1 + \frac{\gamma_2}{\gamma_1}}{2|\rho|^2} \right)^2 - \frac{P_{\text{tot}}}{N_0\gamma_1|\rho|^2}}, \tag{9}$$

where

$$\gamma_1 = \sum_{i=1}^{K-2} \frac{1}{\|\boldsymbol{h}_i\|^2} \quad \text{and} \quad \gamma_2 = \frac{1}{\|\boldsymbol{h}_{K-1}\|^2} + \frac{1}{\|\boldsymbol{h}_K\|^2}. \tag{10}$$

Using (9) for the CB SINR, R^{CB} is found as:

$$R^{\text{CB}} = K \log_2 \left(1 + z \right). \tag{11}$$

B. The ZF Sum-Rate

To find the ZF sum-rate, we need to find the ZF SINR values. The pseudo-inverse of the channel $\boldsymbol{A} = \boldsymbol{H}^H (\boldsymbol{H}\boldsymbol{H}^H)^{-1}$ is required to find $\boldsymbol{u}_i^{\text{ZF}}$. Due to the definition of the correlated channel of size K, there are only two off-diagonal elements for $(\boldsymbol{H}\boldsymbol{H}^H)^{-1}$, which makes it simple to find $\boldsymbol{A}$. Then, each column of $\boldsymbol{A}$ is normalized to have unit-norm, which results in the following $\boldsymbol{u}_i^{\text{ZF}}$:

$$\boldsymbol{u}_i^{\text{ZF}} = \frac{\boldsymbol{h}_i^*}{\|\boldsymbol{h}_i\|}, \quad i = 1, ..., K-2 \tag{12}$$

$$\boldsymbol{u}_{K-1}^{\text{ZF}} = \frac{1}{\sqrt{1 - |\rho|^2}} \left(\frac{\boldsymbol{h}_{K-1}^*}{\|\boldsymbol{h}_{K-1}\|} - \rho^* \frac{\boldsymbol{h}_K^*}{\|\boldsymbol{h}_K\|} \right), \tag{13}$$

$$\boldsymbol{u}_K^{\text{ZF}} = \frac{1}{\sqrt{1 - |\rho|^2}} \left(\frac{\boldsymbol{h}_K^*}{\|\boldsymbol{h}_K\|} - \rho \frac{\boldsymbol{h}_{K-1}^*}{\|\boldsymbol{h}_{K-1}\|} \right). \tag{14}$$

Replacing (12)–(14) in (2), the SINR for user i is found by:

$$\text{SINR}_i^{\text{ZF}} = \frac{\|\boldsymbol{h}_i\|^2 \eta_i^2}{N_0}, \quad i = 1, ..., K-2 \tag{15}$$

$$\text{SINR}_{K-1}^{\text{ZF}} = \frac{\|\boldsymbol{h}_{K-1}\|^2 (1 - |\rho|^2) \eta_{K-1}^2}{N_0}, \tag{16}$$

$$\text{SINR}_K^{\text{ZF}} = \frac{\|\boldsymbol{h}_K\|^2 (1 - |\rho|^2) \eta_K^2}{N_0}. \tag{17}$$

By maximizing the minimum SINR among the users same as (4), the following ZF sum-rate R^{ZF} is found:

$$R^{\text{ZF}} = K \log_2 \left(1 + \frac{P_{\text{tot}}/N_0}{\frac{\gamma_2}{1 - |\rho|^2} + \gamma_1} \right). \tag{18}$$

III. Improved Dropping Algorithm

For a correlated channel of size K, the following Theorem gives the optimal dropping strategy in terms of the sum-rate.
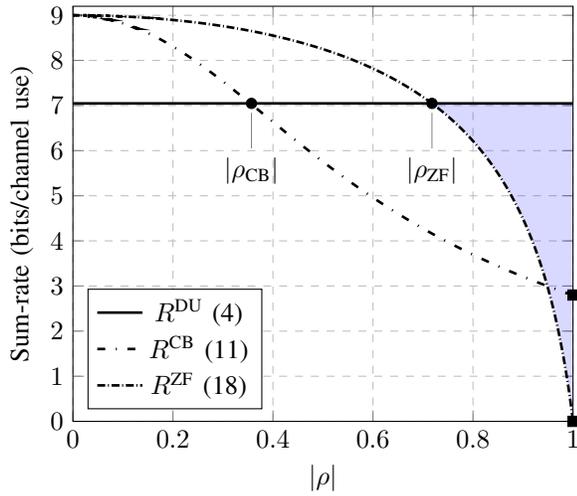
Fig. 2. The comparison of R^{DU}, R^{CB}, and R^{ZF} as a function of $|\rho|$ for a correlated channel of size 3 at SNR $= 7$ when $M = 64$. The shaded area shows the extra sum-rate that the BS can achieve by using the proposed threshold for the dropping with ZF. The thresholds for CB ($|\rho^{\mathrm{CB}}|$) and for ZF ($|\rho^{\mathrm{ZF}}|$) are shown with black circles.

Theorem 2. *Consider a correlated channel of size K with ρ. If a BS with P_{tot} drops one of the correlated users based on*

$$\begin{cases} drop\ user\ K & when\ |\rho| > |\rho^{\mathrm{ZF}}| \\ no\ dropping & when\ |\rho| \le |\rho^{\mathrm{ZF}}|, \end{cases} \quad (19)$$

where $|\rho^{\mathrm{ZF}}|$ is given by:

$$|\rho^{\mathrm{ZF}}| = \sqrt{1 - \frac{\gamma_2}{\frac{P_{tot}/N_0}{(2^\alpha - 1)} - \gamma_1}}, \quad (20)$$

and $\alpha = R^{\mathrm{DU}}/K$, then, the ZF sum-rate with max-min power control is maximized.

Proof. By solving $R^{\mathrm{DU}} \ge R^{\mathrm{ZF}}$ for $|\rho^{\mathrm{ZF}}|$ using (4) and (18), one can verify (20), which shows that the ZF sum-rate is maximized by using the dropping algorithm. $\square$

The results of Theorem 2 are true for any channel matrix. However, our emphasis is on LOS environments in which there is a nonnegligible probability that two users become highly correlated [2]. The same analysis can be done for CB, while $|\rho^{\mathrm{CB}}|$ is found by solving $R^{\mathrm{CB}} = R^{\mathrm{DU}}$ using the bisection method [10, sec. 7.1.1.]. This is possible since z (see (9)), and consequently R^{CB} (see (11)) are strictly decreasing functions of $|\rho|$. In Fig. 2, R^{DU}, R^{CB}, and R^{ZF} are shown as a function of $|\rho|$ for a correlated channel of size 3. The threshold value for CB (and ZF) is found using the bisection method (and using (20)), which is exactly the intersection of R^{DU} and R^{CB} (and R^{ZF}) in Fig. 2 shown by a black circle. By using the derived threshold, the BS with CB (and ZF) can achieve R^{DU} when $|\rho| > |\rho^{\mathrm{CB}}|$ (and $|\rho| > |\rho^{\mathrm{ZF}}|$). Thus, compared to a BS, which does not drop any user, the dropping with the proposed threshold values avoids a large losses in the sum-rate (for instance, the shaded area in Fig. 2 is the loss in the ZF sum-rate). More importantly, by using the derived thresholds, the maximum CB and ZF sum-rates are achieved.

Although R^{CB} is lower than R^{ZF} for most values of $|\rho|$ (see Fig. 2), R^{CB} becomes close to 1 bit/channel use for each user when $|\rho| \to 1$ while R^{ZF} tends to zero. By analyzing (11) and (18), the behaviors of R^{CB} and R^{ZF} when $|\rho| \to 1$ are found as:

$$\lim_{|\rho| \to 1} R^{\mathrm{CB}} = K \log_2(1 + \frac{\|\boldsymbol{h}_K\|^2 \eta^2_{K_{(|\rho|=1)}}}{\|\boldsymbol{h}_K\|^2 \eta^2_{K_{(|\rho|=1)}} + N_0}), \quad (21)$$

$$\lim_{|\rho| \to 1} R^{\mathrm{ZF}} = 0, \quad (22)$$

which explains the behavior in Fig. 2 (see the black squares).

We conclude this section by modifying the dropping algorithm in [5] using the proposed thresholds. This is explained for ZF in Algorithm 1. By having the channel matrix $\boldsymbol{H}$, the maximum spatial correlation among the users $|\rho^{\mathrm{max}}|$ is found:

$$|\rho^{\mathrm{max}}| = \max_{i,j \neq i} |\rho_{ij}|. \quad (23)$$

If $|\rho^{\mathrm{max}}|$ is larger than $|\rho^{\mathrm{ZF}}|$, one of the correlated users, who has the highest sum correlation (considering the distance difference) to the remaining users (see lines 4-5 of Algorithm 1), is dropped. Then, the channel matrix, $|\rho^{\mathrm{max}}|$, and $|\rho^{\mathrm{ZF}}|$ are updated to check whether dropping is still required. This procedure is repeated until there is no need for dropping any user. The same algorithm is found for CB using $|\rho^{\mathrm{CB}}|$. The simulation results in the next section show the effectiveness of the modified dropping algorithm.

Algorithm 1 Modified Dropping algorithm for a channel of K users with ZF

Input: $\boldsymbol{H}, K, P_{\mathrm{tot}}/N_0$
1: find $|\rho^{\mathrm{max}}|$ using $\boldsymbol{H}$
2: find $|\rho^{\mathrm{ZF}}|$ based on (20)
3: **while** $|\rho^{\mathrm{max}}| > |\rho^{\mathrm{ZF}}|$ **do**
4:     find m and n the indexes of users associated with $|\rho^{\mathrm{max}}|$
5:     **if** $\sum_{l \neq m,n} \frac{|\rho_{ml}|}{\|\boldsymbol{h}_m\|} > \sum_{l \neq n,m} \frac{|\rho_{nl}|}{\|\boldsymbol{h}_n\|}$ **then**
6:       drop user m
7:     **else**
8:       drop user n
9:     **end if**
10:    $K = K - 1$
11:    update $\boldsymbol{H}$ by removing the row of the dropped user
12:    update $|\rho^{\mathrm{max}}|$ using $\boldsymbol{H}$
13:    find $|\rho^{\mathrm{ZF}}|$ based on (20)
14: **end while**

IV. SIMULATIONS

Consider a channel of $K = 8$ users, where the users are uniformly distributed in a field-of-view (FoV) of $[30°, 150°]$. The carrier frequency is set to 30 GHz. A single-cell is analyzed, where the users are assumed to be at the cell-edge (200 m) to study the worst case scenarios. The minimum distance between the two users is assumed to be a wavelength. In LOS environments, the element ij of $\boldsymbol{H}$, denoted by h_{ij} is modeled by [11, Sec. 7.2.2.]

$$h_{ij} = \frac{\sqrt{\beta_i}}{\sqrt{M}} e^{-jkd_i} e^{+jk(j-1)\Delta \cos(\phi_i)}, \quad j = 1, ..., M, \quad (24)$$
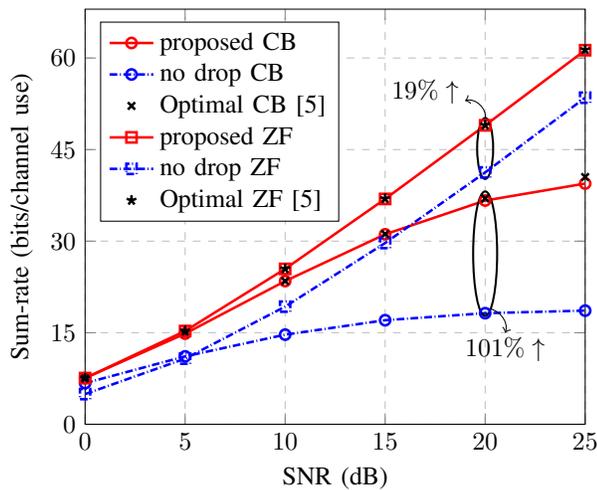
Fig. 3. The average R^{CB} and R^{ZF} using the modified dropping algorithm (solid lines) compared to no dropping (dashed lines) for $K = 8$ and $M = 40$. At SNR = 20 dB, R^{CB} and R^{ZF} are improved by 101% and 19%, respectively.
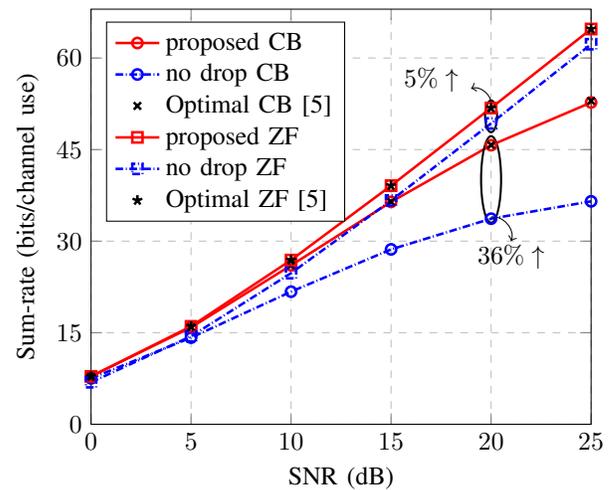

Fig. 4. Same as Fig. 3 for $M = 120$.

where β_i is the large-scale fading, k is the wave number, d_i is the distance from user i to the array, Δ is the spacing between the antenna elements, and ϕ_i specifies the direction of user i with respect to the array (see [11, fig. 7.3. (b)]). The antenna spacing is assumed to be $\Delta = \lambda/2$. Two scenarios where the BS has $M = 40$ and $M = 120$ antennas are considered to evaluate the advantages of using the modified dropping algorithm for CB and ZF. The threshold values $|\rho^{\text{CB}}|$ and $|\rho^{\text{ZF}}|$ are found for each SNR in the modified dropping algorithm. The simulation is run for 10000 random realizations.

The average R^{CB} and R^{ZF} over SNR for the modified dropping algorithm (solid lines) compared to when the BS does not drop any user (dashed lines) are shown in Fig. 3 and Fig. 4 for $M = 40$ and $M = 120$, respectively. The modified dropping algorithm improves the sum-rate in all SNRs for both CB and ZF. For instance, at SNR = 20 dB, R^{CB} and R^{ZF} for $M = 40$ are improved by 101% and 19%, respectively (see Fig. 3). By increasing the antenna at the BS to $M = 120$, the improvements become 36% and 5% for CB and ZF, respectively. For $M = 120$ the transmit power at the BS is reduced 3 times to have a fair comparison with $M = 40$. By increasing M, the expected spatial correlation among the users decreases [5, Theorem 13.]. This shows for $M = 120$ fewer users are dropped, which limits the improvement for $M = 120$ compared to $M = 40$. The results show that the modified dropping algorithm is beneficial for the BS, especially for small M. The average sum-rates with the dropping algorithm of [5] using the optimal threshold are shown in Fig. 3 and Fig. 4. By using the modified dropping algorithm, one can avoid searching for the optimal threshold as in the dropping algorithm of [5] with a negligible loss in the average sum-rate.

V. CONCLUSIONS

In this paper, the dropping problem is analyzed for the CB and ZF with max-min power control in LOS environments. Threshold values are analytically derived to be used in the previously proposed dropping algorithm to avoid repeating a large number of simulations to find the optimal threshold. By using the derived threshold for the dropping, the sum-rate is maximized when there are only two correlated users. Furthermore, a modified dropping algorithm is proposed for channels with any number of correlated users. The results of the simulation scenarios show that at SNR of 20 dB and 120 antennas at the BS, the modified dropping algorithm improves the average CB and ZF sum-rates up to 36% and 5%, respectively.

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