The shape factor of conduction in a multiple channel slab and the effect of non-uniform temperatures

Citation for published version (APA):

DOI:
10.1016/S0017-9310(96)00293-1

Document status and date:
Published: 01/01/1997

Document Version:
Publisher’s PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:
• A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher’s website.
• The final author version and the galley proof are versions of the publication after peer review.
• The final published version features the final layout of the paper including the volume, issue and page numbers.

Link to publication

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

• Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
• You may not further distribute the material or use it for any profit-making activity or commercial gain
• You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the “Taverne” license above, please follow below link for the End User Agreement:
www.tue.nl/taverne

Take down policy
If you believe that this document breaches copyright please contact us at:
openaccess@tue.nl
providing details and we will investigate your claim.
The shape factor of conduction in a multiple channel slab and the effect of non-uniform temperatures

F. L. A. GANZEVELS and C. W. M. VAN DER GELD†
Eindhoven University of Technology, Faculty of Mechanical Engineering, P.O. Box 513, 5600 MB
Eindhoven, the Netherlands

(Received 26 March 1996 and in final form 8 July 1996)

INTRODUCTION

In a parallel plate compact heat exchanger made of a polymer the heat resistance of the plate is important of the performance for the heat exchanger [1, 2]. This is because the heat conduction coefficient is relatively low, a typical value of 0.19 W m⁻¹ K⁻¹ for PolyVinylDiFluoride. The overall heat transfer generally depends on the shape of the polymer slab. Many studies were devoted to the quantification of the effect of the shape. Langmuir [3], Kutateladze [4], Sunderland and Johnson [5], Lewis [6] and Hahne and Grigull [7] introduced shape factors to take this dependency in account. In this paper conduction of heat in the flow direction of the coolant is assumed to be negligible. Hahne and Grigull [7] summarize expressions for the shape factors of many two-dimensional (2D) geometries. However, only few shape factors seem to exist for a slab cooled from two sides containing multiple channels in a row. A typical cross-section of the geometry with multiple coolant channels as studied in this paper is schematized in Fig. 1. Here \( d_1 \) denotes the length of the coolant channel in the direction of the gas flow and \( d_2 \) the length perpendicular to the gas flow direction. The coolant channels are centred at a distance \( s \) from each other and the distance between each centre and the gas surface is \( h \). The edges of the inner channels are not acute but slightly curved. Typical values of \( d_1, d_2 \) and \( h \) are 1.37, 1.47 and 1.00 mm, respectively [1].

With dropwise condensation, the plate is partly wetted and the temperature at the gas side wall is not uniform since the condensation enthalpy is mainly released at the feet of the condensing drops. A typical wetted area fraction in dropwise condensation for air-stream mixtures is 36% [1]. This non-uniformity of the gas-sided temperature might affect the shape factor and hence the heat resistance of the condenser plate. This effect is therefore investigated in this paper.

THEORY

In the two-dimensional geometry of Fig. 1 the heat flux \( q \), in W m⁻², is given by [8]

\[
q = Q/A_1 = \frac{\int \frac{\partial T}{\partial n_1} dA_1}{A_1}
\]

if the heat flux is steady and the heat conduction coefficient \( \lambda \) is independent of the temperature. Here \( n_1 \) is the outward normal of surface \( A_1 \). The shape factors, \( S \) and \( F \), are defined as:

\[
S = \frac{\int n_1 dA_1}{(T_1 - T_0)}
\]

and

\[
F = \frac{h}{\int n_1 dA_1}/(T_1 - T_0)
\]

and yields \( Q = \lambda \cdot S \cdot (T_1 - T_0) \) and \( q = \lambda \cdot F \cdot (T_1 - T_0)/h \) with \( T_1 \) the average temperature of the PVDF plate at the gas side and \( T_0 \) that of the coolant side (see Fig. 2). \( S \) represents the shape factor of Langmuir [3] as used by many others, e.g. Hahne and Grigull [7]. However, the factor \( F \), apart from being dimensionless, has the following advantages:

(1) For a piece of slab that is not shaped in a special manner, i.e. which is flat, \( F \) has the value 1.

(2) It has the straightforward interpretation of the ratio \( h/\theta \) with \( h \) defined in Fig. 1 and \( \theta \) a 'conductive' mean thickness.

\( S \) is used throughout this paper for ease of reference. Values of \( F \) will be indicated at some places.

The Laplace equation for conduction with the present assumptions reads

\[
\frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial y^2} = 0
\]

\( \Theta \) is the heat resistance of the slab which is now given by

\[ R^* = h/(\lambda \cdot A_1 \cdot F) = 1/(\lambda \cdot S) \] with \( A_1 = L \cdot W, L \) and \( W \) the lengths of the plate in the direction of the coolant and gas, respectively. The total heat resistance \( R_{tot} \) is \( 1/(R_1 + R_2 + R_3) \) with \( R_1 \) and \( R_2 \) the heat resistances by convection towards surfaces \( A_1 \) and \( A_2 \), respectively, i.e. \( R_1 = 1/(A_1 \cdot h_{conv}) \) with \( h_{conv} \) denoting the convective heat transfer coefficient, \( Q = (T_1 - T_0)/R_{conv} \).

† Author to whom correspondence should be addressed.‡ The heat resistance of the slab is now given by.
## NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>surface [m$^2$]</td>
</tr>
<tr>
<td>$a_1,a_2,a_3$</td>
<td>coefficients</td>
</tr>
<tr>
<td>$d_1$</td>
<td>length coolant channel in direction of the gas flow [m]</td>
</tr>
<tr>
<td>$d_2$</td>
<td>length coolant channel perpendicular to the gas flow direction [m]</td>
</tr>
<tr>
<td>$F$</td>
<td>shape factor</td>
</tr>
<tr>
<td>$h$</td>
<td>distance between gas side surface and centre coolant channel [m]</td>
</tr>
<tr>
<td>$h_{conv}$</td>
<td>heat transfer coefficient [W m$^{-2}$ K$^{-1}$]</td>
</tr>
<tr>
<td>$L$</td>
<td>length of heat exchanger plate in coolant direction [m]</td>
</tr>
<tr>
<td>$\bar{Q}$</td>
<td>heat flow rate [W]</td>
</tr>
<tr>
<td>$q$</td>
<td>heat flux [W m$^{-2}$]</td>
</tr>
<tr>
<td>$R$</td>
<td>radius [m]</td>
</tr>
<tr>
<td>$R^*$</td>
<td>heat resistance [K W$^{-1}$]</td>
</tr>
<tr>
<td>$r$</td>
<td>effective radius, $d_1^{(1-r)} \cdot d_2^2$ [m]</td>
</tr>
<tr>
<td>$S$</td>
<td>shape factor [m]</td>
</tr>
<tr>
<td>$s$</td>
<td>distance between two successive channels [m]</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature [K]</td>
</tr>
<tr>
<td>$W$</td>
<td>length of heat exchanger plate in gas flow direction [m]</td>
</tr>
<tr>
<td>$x$</td>
<td>coordinate in gas flow direction [m]</td>
</tr>
<tr>
<td>$x^*$</td>
<td>$x$-coordinate where the step occurs [m]</td>
</tr>
<tr>
<td>$y$</td>
<td>coordinate perpendicular to the gas flow direction [m].</td>
</tr>
</tbody>
</table>

### Greek symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>thermal conductivity [W m$^{-1}$ K$^{-1}$]</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>dimensionless temperature $(T - T_0)/(T_1 - T_0)$</td>
</tr>
</tbody>
</table>

---

**Fig. 1.** Schematic of cross-section of finite slab with coolant channels.

**Fig. 2.** Boundary conditions of a single cell of Fig. 1 with isothermals schematized.
with dimensionless temperature \( \Theta = (T - T_0) / (T_1 - T_0) \). The heat flux and the shape factor have been found by numerically solving the differential equation (3) for constant temperatures \( T_1 \) and \( T_0 \) at the gas and coolant side, respectively.‡ Discretisation using the Crank–Nicholson method yields

\[
\frac{\partial^2 \Theta}{\partial x^2}_{m,n} = \frac{\Theta_{m+1,n} + \Theta_{m-1,n} - 2\Theta_{m,n}}{(\Delta x)^2}
\]

and

\[
\frac{\partial^2 \Theta}{\partial y^2}_{m,n} = \frac{\Theta_{m+1,n} + \Theta_{m-1,n} - 2\Theta_{m,n}}{(\Delta y)^2}.
\]

The central node \( \Theta_{m,n} \) of each Crank–Nicholson cell \([8, 9]\) is estimated according to

\[
\Theta_{m,n} = \frac{1}{2} (\Delta y)^2 \left( \frac{\Theta_{m+1,n} + \Theta_{m-1,n}}{(\Delta x)^2} + \frac{\Theta_{m,n+1} + \Theta_{m,n-1}}{(\Delta y)^2} \right)
\]

for step sizes \( \Delta x \) and \( \Delta y \) in x and y direction, respectively.

Because of symmetry only one cell of Fig. 1 has to be computed. The temperatures at \( A_1 \) and \( A_2 \) are known (see Figs. 1 and 2) and the gradients \( \partial \Theta / \partial n = n \cdot \nabla \Theta \) at the other boundaries are zero. All temperatures except that of the boundaries are set to zero as a first guess. New temperatures at the nodes are now calculated with equation (4). The process is reiterated until a steady state has been achieved. Convergence is said to be reached if the relative difference between successive calculated values, \[ (T_{m,n}^{(m)} - T_{m,n}^{(m-1)}) / T_{m,n}^{(m)} \] is less than \( 10^{-5} \). For small \( d_1 (d_1 < 3.6 \times 10^{-4} \text{m}) \) the number of iterations necessary is about 200 and for larger \( d_1 \) this is about 750. The distance between the nodes of adjacent cells is \( 4 \times 10^{-5} \text{m} \) in both x and y directions. Increasing the number of nodes did not affect the value of \( F \).

Alternatively, temperatures are computed for the same boundary conditions with the aid of the finite element package FIDAP\textsuperscript{TM}, version 7.52.

### CYLINDRICAL CHANNELS

For a row of cylindrical channels in a semi-infinite slab, as shown in Fig. 1, Kutateladze \([4]\) gives the shape factor

\[
S_{\text{cyl}} = \frac{A_1}{s} \cdot \pi / \sin \left( \frac{s}{a_1 \pi R} \right)
\]

with \( R \) the radius of the cylinder, \( s \) the distance between two successive cylinders, \( h \) the distance between the centre of the cylinder and the surface and \( A_1 = 2 \cdot \pi \cdot L \cdot L \) being the length of the plate in the direction of the coolant. By selecting \( A_1 \) in this way, \( Q_{\text{net}} = 1 \cdot S \cdot (T_1 - T_0) \) obviously is the heat flux rate to both sides of a piece of slab with length \( s \): thus, \( Q = Q_{\text{net}} \).

In Fig. 3 the dashed line represents relation (5). For six radii the shape factor has been computed with the finite element package FIDAP\textsuperscript{TM} for \( s = 2 \text{mm} \) as indicated by FEM in Fig. 3. For small radii there is fair agreement but for larger radii Kutateladze’s relation yields too high values for \( S_{\text{cyl}} \).

The fitted coefficients are \( a_0 = 0.76 \pm 0.08 \) and \( a_1 = 1.26 \pm 0.05 \). These values are given with a 95% confidence interval \([11]\).

### RECTANGULAR CHANNELS

To obtain an expression for the shape factor for the bundle of rounded rectangular channels of Fig. 1 the Kutateladze’s relation is adapted once more. As opposed to equation (6), the radius \( R \) is replaced by an effective radius \( r = d_1^{0.5} - d_2^{0.5} \). This yields

\[
S = \frac{(A_1)}{s} \cdot a_0 \pi / \ln \left( \frac{s}{a_1 \pi R} \sin \left( \frac{\pi h}{s} \right) \right)
\]

and

\[
F = \left( \frac{h}{a_1} \right) \cdot S.
\]

Note that \( A_1/s = 2 \cdot L \), as before.

The shape factor has been computed both with the difference scheme (see Fig. 4) described above and with the finite element package FIDAP\textsuperscript{TM}, using the grids depicted in Fig. 4. The results of the Crank–Nicholson method and the FIDAP\textsuperscript{TM} runs agree within 0.5%. The coefficients of equation (7) have been fitted to the computational results given in Fig. 5, yielding \( a_0 = 0.67 \pm 0.02 \), \( a_1 = 0.728 \pm 0.006 \) and \( a = 0.86 \pm 0.02 \) for uniform temperatures at the boundary.

The value 0.86 for \( a \) can be compared with the corresponding value 0.75 for a single rectangular cell in an infinite slab. In this case the shape factor is given by [4]

\[
S = 5.7 + \frac{d_1}{2d_2} \cdot \ln \left( \frac{3.5h}{d_1^{0.25} + d_2^{0.75}} \right)
\]

in which \( h = d_1 + d_2 \) have the same meaning as in equation (7) and Fig. 1. The effective radius, \( r \), is dominated by \( d_2 \) since when the wall thickness, \( (h - d_2) \), goes to zero the heat resistance goes to that of a thin plate, \( R_p = (h - d_2)/\lambda \).

The shape factor for channels with the geometry of Fig. 1 is about 5% smaller than that of exactly rectangular channels. In this case computations with finite difference scheme and with FIDAP\textsuperscript{TM} have yielded \( a_0 = 0.70 \pm 0.02 \), \( a_1 = 0.730 \pm 0.005 \) and \( a = 0.88 \pm 0.02 \), to be used in equation (7).

In practical applications the slab is cooled by water flowing through the channels and heated by gas condensing at the gas-sided surface \( A_1 \), see the introduction and [2]. Underneath condensing drops the plate temperature is higher than elsewhere [2]. To simulate this, the temperature is given a Heaviside step function of distance \( x \), see Fig. 2. The step occurs at \( x = x^* \), i.e. in \([0, x^*]\), the temperature is taken to be \( 1.05 \cdot T_1 \) and in \([x^*; 1] \) it is \( T_1 \). The average wall

\[
\text{shape factor for cylindrical channels}
\]

\[
F_{\text{cyl}} / R 
\]

\[
0.76 \pm 0.04 \text{[computed with FIDAP\textsuperscript{TM}]} \]

\[
0.86 \pm 0.04 \text{[computed with FIDAP\textsuperscript{TM}]} \]

Fig. 3. Computations of the shape factor for a row of cylindrical channels in an infinite slab.

† Conformal mapping \([4, 10]\) is another method to determine the shape factor. However, the geometry of Fig. 1 is too complex to estimate the shape factor of this geometry by conformal mapping.

‡ Technical Notes 2495
temperature is \( T_i = (1 + 0.05 \cdot x^*/s) \cdot T_i \) and replaces \( T_i \) in equation (2) and related equations. The resulting shape factor is named \( S_{x,v} \). With uniform temperature, it is given by \( S_{o} = S_{ov} \). Computations are performed with FIDAP\textsuperscript{TM} for 10 different \( x^*/s \) values. The number of nodes in the grid is 2038 and each element has four nodes (quadrilateral elements). Further refining the grid changed the outcome by 0.1%, typically. Figure 6 summarizes the results for \( \text{DEV} = 100\% \cdot (S_{x,v} - S_{o})/S_{o} \). At places where \( T_i < T_1 \), the dimensionless temperature \( \Theta \) exceeds 1. The non-uniformity of the temperature does not significantly affect the shape factor and the heat resistance of the condenser plate. For wetted area fractions of about 36% that are typical in drop-wise condensation of air-steam mixtures [1] the deviation of the shape factor is about 0.3%. In Fig. 7 the computed isothersms are plotted for the case \( x^*/s = 0.35 \).

**CONCLUSIONS**

For the geometry of Fig. 1 the shape factor \( S_r \) defined by equation (2), is given by equation (7) with \( a_0 = 0.67 \pm 0.02 \), \( a_1 = 0.728 \pm 0.006 \) and \( a = 0.86 \pm 0.02 \). It holds for \( 0.60 \leq r/s \leq 0.83 \), with \( r \) the effective radius given by \( d^0/4 \) or \( d^0/8 \). For exactly rectangular channels equation (7) can also be used, with coefficients \( a_1 = 0.70 \pm 0.02, a_1 = 0.730 \pm 0.005 \) and \( a = 0.88 \pm 0.02 \). It holds for \( r/s < 0.85 \). The shape factor of Kutateladze (5) for cylindrical channels is only valid for radii smaller than 30% of \( s \). For larger radii the modified relation (6) can be used with \( a_0 = 0.76 \pm 0.08 \) and \( a_1 = 1.26 \pm 0.05 \).

If the gas-sided wall temperature is not uniform but follows a prescribed step function, the shape factor differs only by 0.3% if the average wall temperature is used.
Throughout this paper the classical definition of the shape factor, $S$, has been used. However, the authors believe that the shape factor $F$ as defined in equation (2) is more useful for reasons given below equation (2).

**REFERENCES**


6. G. K. Lewis, Shape factors in conduction heat flow for...


