Changing students’ beliefs about the relevance of mathematics in an advanced secondary mathematics class

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ABSTRACT

This study shows that using authentic contexts for learning differential equations in a differentiation-by-interest setting can enhance students’ beliefs about the relevance of mathematics. The students in this study were studying advanced mathematics (wiskunde D) at upper secondary school in the Netherlands. These students are often not aware of the relevance of the mathematics they have to learn in school. More insights into the application of mathematics in other sciences can be beneficial for these students in terms of preparation for their future study and career. A course differentiating by student interest with new context-rich curriculum materials was developed in order to enhance students’ beliefs about the relevance of mathematics. The intervention aimed at teaching differential equations through guided small-group tasks in scientific, medical or economical contexts. The results show that students’ beliefs about the relevance of mathematics improved, and they appreciated experiencing how the mathematics was applied in real-life situations.

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Education; task design; relevance of mathematics; differentiation by interest; differential equations

1. Introduction

Many students in secondary education appear not to be aware of the relevance and importance of mathematics for their future education and working career (Onion, 2004), including those students with a considerable talent in mathematics (Musto, 2008). Few students seem to develop insights into how the mathematical concepts they learn relate to authentic situations they may encounter in everyday life, even when they enjoy studying mathematics (Borko et al., 1992).

Students in the final years of secondary education in the Netherlands are offered an additional mathematics course aimed at preparing students for university education with a strong foundation in mathematics. Although the course was originally intended to emphasize the connection of mathematics to other sciences, research shows that students taking the course when being taught using the regular course materials fail to see the relevance of mathematics (Van Elst, 2013).

In this paper, we report on the design and evaluation of new, dual-purpose curriculum materials: one aim was to teach the theory of ordinary differential equations to students; another to improve students’ beliefs about the relevance of mathematics through working
with these materials. We used the concepts of design research to (re-) design, implement and evaluate these new curriculum materials for the advanced mathematics course in upper secondary education (year 12).

One of our pilot studies had shown that students taking the advanced mathematics course were interested in a wide range of future studies. For this reason, the new curriculum materials differentiated by student interest, offering them mathematical tasks based on a variety of biological, medical and social/economic problems. We aimed at enhancing our students’ understandings of the ways mathematics might be relevant in their future university education and subsequent career, by showing these students how the theory of ordinary differential equations was applied in some of their individual fields of interest.

The results of the subsequent interventions with these new materials lead to insights into the engagement of the students with the new materials, and how student beliefs about the relevance of mathematics could be changed/enhanced using purposefully designed curriculum materials.

Our main research question is:

How can a learning strategy based on differentiation by interest for teaching ordinary differential equations enhance students’ views on the relevance of mathematics?

Our data collection strategy was based on answering the following three sub-questions that were formulated to contribute to answering our main research question:

- How did students’ beliefs about the relevance of mathematics change during the intervention?
- How did the students experience the content of the new course materials?
- How did the students perform in the course examinations after the intervention?

2. Theoretical background

In the 1970s research concerned with the attitude of students towards learning mathematics recognized ‘the importance and relevance of mathematics’ as one of the affective factors influencing students’ attitude (Aiken, 1974). In those studies, several questionnaires measuring the attitude of students towards (learning) mathematics used a scale for measuring students' beliefs about the usefulness of mathematics (e.g. the Attitude towards Mathematics questionnaire (Martinot, Kuhlemeier, & Feenstra, 1988), the Fennema-Sherman Mathematics Attitude test (Fennema & Sherman, 1976), the Attitudes toward Learning Mathematics questionnaire (Wong & Chen, 2012)).

Previous studies on students’ perceptions about the relevance of mathematics in general and for their own future study and career have been conducted predominantly with the aim to develop understandings of the perceived relationship with the mathematical performance of students in general (e.g. Farooq & Shah, 2008; Schoenfeld, 1989), or to explain gender differences (Meece, Wigfield, & Eccles, 1990). From these studies, it became clear that failing to see the relevance of mathematics was not only related to a negative attitude of students towards mathematics. Moreover, also motivated students questioned the relevance of mathematics. Hence, these studies showed that many students, and not just the disaffected ones, failed to see the relevance of mathematics and how it would be used in their individual lives (Musto, 2008).
Teaching students about the relevance and the purpose of the mathematics they learn at school has become an important factor when designing new courses. A common notion associated with the relevance of mathematics is that the mathematics should be contextualized in appropriate situations (Julie, 2002).

In the literature, the meaning of the word context is ambiguous. For example, it can refer to the classroom setting, the framework of an intervention, or the ‘cover story’ of a task. In this paper, we use the notion of context to describe what Clarke and Helme (1998) call figurative context: the real-world scenario and setting a learning task or assignment is embedded in.

Sealey and Noyes (2010) concluded that mathematics should be presented in some context, be purposeful and enable all students to acquire awareness of the important role of mathematics in society. Tomlinson (2000) described which contexts were real and purposeful in students’ minds, and inspired them to learn mathematics:

Students will learn best when they can make a connection between the curriculum and their interests and life experiences. (p. 6).

In order to prepare new curriculum materials, to be used to enhance students’ beliefs about the relevance of mathematics, we needed to set the mathematics in contexts that appeal to students with all kinds of interests. It was not likely that one context would appeal to all students. Students differed in many ways: e.g. their readiness to learn, styles of learning, their experiences but also their interests (Tomlinson, 2000). In the literature two types of interest have been identified as crucial: Personal interest, activated internally; and situational interest, environmentally activated and important in catching students’ attention (Schraw, Flowerday, & Lehman, 2001). One of the strategies described by Schraw et al. (2001) to increase situational interest was offering students meaningful choices. The positive connection between choice and interest is widely recognized (Flowerday & Schraw, 2000), and can also be derived from the self-determination theory (Deci, Vallerand, Pelletier, & Ryan, 1991).

The concept of offering choices did not guarantee a connection with students’ interests. The offered options students could choose from had to be meaningful to them if they were to be relevant to students’ personal interests and goals, not too numerous or complex, yet not too easy, and they should be congruent with the values of the students’ social background (Katz & Assor, 2007).

Only appealing to the students was not enough for an offered context to be used for educational purposes. They were also seen as the vehicles to introduce and apply the mathematical concepts at stake. Ideally, the mathematical concepts learned by the students should be indifferent to the choices students made when offered dissimilar problems and contexts. This asked for a topic area where the mathematical concepts and principles could be applied to a wide variety of real-life problems. Amongst others, Gallegos (2009) identified the theory of differential equations as an ideal tool to model phenomena of various natures.

The theory of ordinary differential equations is one of the fundamental mathematical subjects for many contemporary science and engineering studies. It is invariably part of the first or second-year curriculum at any university of science and technology. According to Boyce (1994), the topic of differential equations has traditionally been taught as a pure analytical course. Several types of differential equations are presented along with the analytical method of obtaining their solution. However, in the last decades the focus of
teaching differential equations has gradually shifted to a contextualized, problem-based approach. More emphasis is placed on the modelling process, on the applications of differential equations in real life, on graphical methods to research differential equations and on numerical methods to solve differential equations by computer (Boyce, 1994).

Several studies have described their positive experiences with the use of Realistic Mathematics Education (RME) in the teaching and learning of ordinary differential equations (e.g. Kwon, 2002; Rasmussen & King, 2000). Students were challenged in an inquiry-oriented approach to reinvent key mathematical ideas from the theory of differential equations. Using problems situated in realistic settings students were challenged to create their own analytical, graphical, and numerical methods for analysing and solving differential equations. Huber (2010) has reported on the effective use of Interdisciplinary Lively Applications Projects (ILAPs) in an ordinary differential equations course to apply solution methods to situations that students might encounter in other disciplines. In the projects, students had to transform a problem from a real-world setting into a mathematical model, solve that model, and then interpret the results in the real-world scenario. The described projects had ties to mechanical engineering, environmental engineering, physical education, and even mythology.

The connection between the application of differential equations and the relevance of mathematics we were trying to establish in our study has been recognized in more places. Gallegos (2015) justified the implementation of a new differential equation course through modelling and technology at a Mexican university as a means to allow students to recognize the importance of mathematics in everyday life situations. Flegg, Mallet, and Lupton (2012) described promising results in changing students’ perceptions of the relevance of mathematics during a first-year engineering mathematics course in an explorative study in engineering education. They also used a context-based approach in which mathematics was applied to recognizable engineering problems.

To summarize, in order to enhance students’ beliefs about the relevance of mathematics we intended to design new curriculum materials for teaching the theory of differential equations which offered students a choice between contexts in order to differentiate by students’ individual interests.

3. Materials and methods

In this section, we describe the setting of our study, the design of the curriculum materials, the student selection, and data collection strategy for our intervention, and the validation of the questionnaire on the relevance of mathematics.

3.1. The setting: upper secondary education in the Netherlands

The Dutch education system consists of six years of primary education, and 4–6 years of secondary education depending on the level of education (De Putter-Smits & Van Driel, 2016). The highest level of secondary education is the six years (K-7 to K-12) pre-university education called VWO, which provides students access to university. For every student at VWO level mathematics is a mandatory course.

In 2007 an advanced mathematics course called ‘wiskunde D’ (mathematics D) was introduced to better prepare students for technical and engineering studies at university
level. The course was intended for students with considerable mathematical talent and was added to the curriculum to further improve these students’ algebraic skills, to offer challenging and engaging mathematics, and to emphasize its connection to other sciences (cTWO, 2007). The course is therefore aimed at students with an interest in science and engineering. It includes mathematical topic areas which are part of every first-year university curriculum: e.g. complex numbers, analytic geometry, and differential equations. Students are offered the opportunity to take this advanced course in addition to their regular mathematics course.

It might be expected that students who take this advanced course in mathematics are convinced that mathematics has relevance. Studies on the implementation of ‘wiskunde D’ have shown that students praised the course as being challenging and fun, but many students also mentioned that it was not clear to them why the mathematics in the course was useful for their future study and career (Van Elst, 2013). Cheung (2012) found that although teachers of ‘wiskunde D’ stated that the content of the course was well suited as a preparation for a future education in a technical or engineering environment, the curriculum did not sufficiently emphasize the applications of mathematics and the connections to other sciences.

3.2. Design of the curriculum materials

Using a design-research approach (Van den Akker, Gravemeijer, McKenney, & Nieveen, 2006) an intervention consisting of small-group guided tasks on ordinary first-order differential equations was developed. The design process consisted of several cycles, where the various tasks were developed, implemented, tested and evaluated to improve the design of the curriculum materials. In the first part of the design process, two small pilot studies were performed to gain more insight into the way the students engage and interact with the new materials.

In a first pilot study (in 2014) we looked at student interest and motivation to work on dissimilar real-life problems. As we suspected that the interest of our students was much broader than engineering, we asked students \( n = 26 \) to sort eight real-life problems from ‘most interesting problems to work on’ to ‘most boring problems to work on’. We also compared these rankings with their plans for their future study.

The results of this study clearly showed that the students in the advanced mathematics course had diverse views on which real-life problems were ‘most interesting problems to work on’. Hence, differentiation by student interest had been chosen as one of the criteria for the development of the new curriculum materials. The new materials should draw on student interest by offering them a choice of contexts or problem situations, which attracted them the most. Therefore, bio-medical and social-economic problems were used besides engineering and scientific problems in our new materials.

Later that year, in a second pilot study the same students worked in small groups (two to four students) on a first prototype of the new materials. Students could choose between two problem-solving tasks: one based on the most popular engineering problem from the first pilot study and one set in the most popular bio-medical context. Both tasks had the same structure based on the model of guided reinvention (Stephan, Underwood-Gregg, & Yackel, 2014). The ways the students engaged with the new materials were observed and
Table 1. The structure of the fifteen guided tasks.

<table>
<thead>
<tr>
<th>Phase</th>
<th>Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Introduction</td>
<td>In 2–3 pages the context is introduced, the problem is posed and a justification is provided why a solution to the problem is of use.</td>
</tr>
<tr>
<td>2. Formulating a hypothesis</td>
<td>Calling on prior knowledge of the students they are asked to formulate a hypothesis for the solution of the posed problem.</td>
</tr>
<tr>
<td>3. Guided reinvention</td>
<td>The type of differential equation (DE) used as the mathematical model to solve the problem is introduced by a guided reinvention in only a few steps.</td>
</tr>
<tr>
<td>4. Graphical and algebraic exploration of the DE</td>
<td>Using slope fields and algebraic exercises the students research the shape and nature of the solutions of the differential equation.</td>
</tr>
<tr>
<td>5. Introduction of the new mathematical concept</td>
<td>A new mathematical concept is introduced to solve the new type of differential equation. In the first tasks this is an algebraic method. In later tasks the focus gradually shifts to a numerical approach.</td>
</tr>
<tr>
<td>6. Solving the DE</td>
<td>The students are guided through the process of solving the differential equation and finding a solution to the problem using the newly introduced mathematical concept.</td>
</tr>
<tr>
<td>7. Interpreting the solution and evaluation of the hypothesis</td>
<td>The students are asked to interpret their solution within the context and to reflect on their own hypothesis they formulated in phase 2.</td>
</tr>
<tr>
<td>8. Reproduction of the learned mathematical concept</td>
<td>Each task ends with a second, related problem within the same context. Students are asked, without further guidance, to use steps 3, 6 and 7 again and solve the second problem as a reproduction exercise to recap the most important mathematical steps of the task.</td>
</tr>
</tbody>
</table>

recorded. In a class discussion after finishing the tasks, the students were asked for their feedback on the structure, the content and the cognitive demand of the tasks.

Using the evaluation of the second pilot study the general structure of the tasks was re-designed to match the feedback from the students. From this design process emerged a general structure for guided tasks intended for small groups of two to three students. The framework consisted of eight phases as displayed in Table 1.

A total of 15 guided tasks were designed using this framework. Each task took approximately 2–3 lessons to complete. Students studied five types of differential equations with these tasks. The materials were intended to differentiate by student interest, as for each type of differential equation students could choose from three problems set in three contexts: a scientific, a biomedical, or a social-economic context (Table 2). All students worked on the same type of differential equation in one round of tasks, regardless of their choice of problem.

Students were not restricted to one area of context. They were free to choose the problems they wanted to work on. Only 3 of the 103 students picked five problems with the same type of context. Most students varied with their choice. During the intervention, a popular path of choice (8 out of 103 students) was 1. The Chernobyl disaster, 2. The Axe effect, 3. Second hand smoke, 4. A new oil crisis? and 5. Alcohol consumption consisting of three scientific, one social-economic and one biomedical context.

The role of the teacher was to guide the process as a coach and time keeper, and not to act as the teacher explaining the theory of differential equations in a traditional front-teaching style. It was expected that the students would develop understandings of the mathematical
Table 2. Overview of the curriculum materials: 15 tasks for teaching 5 types of differential equations.

<table>
<thead>
<tr>
<th>Differential equation</th>
<th>Scientific contexts</th>
<th>Biomedical contexts</th>
<th>Social-economic contexts</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y' = ay )</td>
<td>Chernobyl disaster</td>
<td>Salmonella</td>
<td>Painted by Vermeer</td>
</tr>
<tr>
<td></td>
<td>Radioactive decay</td>
<td>Food poisoning</td>
<td>Art forgery</td>
</tr>
<tr>
<td>( y' = ay + \beta )</td>
<td>Lake IJssel</td>
<td>Vancomycin</td>
<td>The Lynx effect</td>
</tr>
<tr>
<td></td>
<td>Salt to fresh water conversion</td>
<td>Intravenous infusion</td>
<td>Advertisement</td>
</tr>
<tr>
<td>( y' = ay + f(t) )</td>
<td>Second hand smoke</td>
<td>Crime Scene Investigation</td>
<td>Market of seasonal products</td>
</tr>
<tr>
<td></td>
<td>Carbon monoxide levels</td>
<td>Estimated time of death</td>
<td>Price fluctuations</td>
</tr>
<tr>
<td>( y' = ay^2 + \beta y )</td>
<td>A new oil crisis</td>
<td>Ebola in Sierra Leone 2014</td>
<td>USA census 1790–2050</td>
</tr>
<tr>
<td></td>
<td>World oil reserve decay</td>
<td>Epidemic growth</td>
<td>Population growth</td>
</tr>
<tr>
<td>( y' = f(y, t) )</td>
<td>Felix Baumgartner skydive</td>
<td>Alcohol consumption</td>
<td>Effects of global warming</td>
</tr>
<tr>
<td></td>
<td>Free fall modelling</td>
<td>Blood alcohol levels</td>
<td>Sea level rising</td>
</tr>
</tbody>
</table>

Table 3. Overview of the number of students in the intervention.

<table>
<thead>
<tr>
<th>School year</th>
<th>Group/School</th>
<th>Number of participants</th>
<th>Completed both questionnaires</th>
</tr>
</thead>
<tbody>
<tr>
<td>2015–2016</td>
<td>Group 1 (school A)</td>
<td>19</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>Group 2 (school A)</td>
<td>17</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>Group 3 (school B)</td>
<td>13</td>
<td>8</td>
</tr>
<tr>
<td>2016–2017</td>
<td>Group 4 (school A)</td>
<td>20</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>Group 5 (school A)</td>
<td>16</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>Group 6 (school B)</td>
<td>18</td>
<td>13</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>103</td>
<td>79</td>
</tr>
</tbody>
</table>

concepts whilst working on the tasks. After each round of tasks, the teacher showed the class that all three parallel problems were solved using the same type of ordinary first-order differential equation, to underline the power of the applied mathematical model.

3.3. Participants

To gain insights into students’ interactions with the new curriculum materials, and to measure the effect of the new materials on students’ beliefs about the relevance of mathematics, an intervention was set up and carried out in two consecutive school years.

The participants were selected from two schools in the Netherlands. School A was located in a rural area in the south of the country, which only offered education at pre-university (VWO) level. School B was located in a city in the south of the country and offered education at two levels, one of which VWO.

Six advanced mathematics classes participated in the intervention. In three classes the intervention started in January of 2016. As the response rate of these classes was low (67%) we decided to do a second intervention to increase the total number of students. Another three classes started the intervention in January of 2017. A total of 103 students participated in the interventions. Of those, 79 (77%) completed the pre- and post-intervention questionnaires (see Table 3).
Table 4. Relevance of mathematics scale from the Attitude towards Mathematics questionnaire (translated from Dutch).

<table>
<thead>
<tr>
<th>#</th>
<th>Item</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mathematics is of importance to get a job in the future.</td>
</tr>
<tr>
<td>2</td>
<td>I can see the benefit of mathematics for other subjects in school.</td>
</tr>
<tr>
<td>3</td>
<td>I think that mathematics will not be of any use in most professions.</td>
</tr>
<tr>
<td>4</td>
<td>You do not necessarily need any mathematics in future life.</td>
</tr>
<tr>
<td>5</td>
<td>I believe mathematics is of no real value.</td>
</tr>
<tr>
<td>6</td>
<td>The mathematics you learn in school is of little use in real life.</td>
</tr>
<tr>
<td>7</td>
<td>Most of the mathematics we learn in school, comes in handy later in life.</td>
</tr>
<tr>
<td>8</td>
<td>Mathematics is very useful in many everyday situations.</td>
</tr>
</tbody>
</table>

3.4. Data collection strategies

We used the 8-item Likert scale relevance of mathematics from the Attitude towards Mathematics questionnaire (Martinot et al., 1988) to quantify students’ beliefs about the relevance of mathematics (see Table 4). Pre- and post-tests were administered in order to measure change in students’ beliefs about the relevance of mathematics for this particular topic.

Prior to the intervention we conducted a pre-test to measure students’ beliefs about the relevance of mathematics before they worked with the new curriculum materials. We used the results of the pilot studies and the pre-tests to replicate the results by Van Elst (2013) who reported that students taking an advanced course in mathematics fail to see the relevance of mathematics.

Second, we conducted a post-test using the same questionnaire at the end of the intervention. We analysed the data and compared the paired results of the pre- and post-test of each student. Our hypothesis was a positive change in students’ opinion on the relevance of mathematics. We used a paired samples t-test on the results of the pre- and post-test to see whether their opinion on the relevance of mathematics had changed in the period the intervention took place and used Cohen’s $d$ to calculate the effect size of the change (Cohen, 2013).

At the end of the intervention students were asked to complete a survey on the redesigned course and the new curriculum materials. Students were asked to rate the tasks they worked on, to name positive and negative experiences during the course, and if and how the course and the used materials contributed to their beliefs about the relevance of mathematics.

Finally, examination results from school A were collected and compared using a t-test to examination results from the examination year before the intervention. This was done to see if the intervention influenced the learning of the mathematical concepts of ordinary first-order differential equations in any way compared to the traditional teaching method of differential equations.

The various means of data collection and their purpose during the intervention are shown in Table 5.

3.5. The validation of the questionnaire on the relevance of mathematics

The relevance of mathematics-scale was validated for use in our setting in a pilot study in December 2014. A total of 85 students from three schools in their final year of their
Table 5. Data collection strategies and their purpose.

<table>
<thead>
<tr>
<th>Data collection strategies</th>
<th>Purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td>Questionnaire on relevance of mathematics</td>
<td>Measure change in students’ beliefs about the relevance of math</td>
</tr>
<tr>
<td>Survey on the new course and its materials</td>
<td>Explore the way the students experienced the course and new materials</td>
</tr>
<tr>
<td>Exam results</td>
<td>Measure effect of new curriculum materials on students’ grades</td>
</tr>
</tbody>
</table>

Table 6. Validation of the relevance of mathematics scale; pilot study versus data from Martinot et al.

<table>
<thead>
<tr>
<th></th>
<th>Pilot study</th>
<th>Martinot et al. (1988)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>α = 0.89</td>
<td>α = 0.89</td>
</tr>
<tr>
<td>Year 12</td>
<td>N</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td>85</td>
<td>17.99</td>
</tr>
<tr>
<td>Year 12 advanced</td>
<td>32</td>
<td>20.81</td>
</tr>
</tbody>
</table>

secondary education completed the questionnaire. We found Cronbach’s alpha for the scale to be $\alpha = 0.89$. The mean score over the 8-items of the relevance of mathematics-scale was 17.99 with 0 being the lowest possible score (8 times score 0 = “totally disagree”) and 32 being the highest possible score (8 times score 4 = “totally agree”). Out of these 85 students 32 took the advanced mathematics course ‘wiskunde D’. This group had a mean score of 20.81 over the 8-item scale (see Table 6).

Comparing our results with Martinot et al. (1988) the Cronbach’s alpha we found was the same $\alpha = 0.89$, leading us to deem the scale valid for use in our experiment. Martinot et al. (1988) used the questionnaire in their study on students in the first three years of secondary education in the Netherlands. They found mean scores of 21.95 for first-year students ($n = 96$) to 21.09 for students in the third year of secondary education ($n = 143$) (Table 6).

The students in the advanced mathematics course in our pilot study had a lower mean score than the students Martinot et al. (1988) reported on. What stands out is the standard deviation scores we found for both the complete group and the subgroup of students taking the advanced course indicating a wide spread in the results.

4. Results

In this section, we answer the three research sub-questions, by discussing the results on students’ beliefs about the relevance of mathematics, on students experience with the new curriculum materials, and on their exam grades after the intervention.

4.1. How did students’ beliefs about the relevance of mathematics change during the intervention?

From the 103 students who participated in the intervention, 79 completed the questionnaire on both the pre- and post-test. For these students, we found a mean score of 20.78 with a standard deviation of 4.88 on the relevance of mathematics-scale at the pre-test. If we take a closer look at the distribution of the scores of the 79 students it appears to have a bimodal form with two peaks, one around 18–20 and one around 24–26 (Figure 1). The
The mean score of the 79 students on the pre-test before the intervention and post-test after the intervention is depicted in Table 7. A paired samples t-test on the repeated measures of their individual pre- and post- scores gave a $p = 0.048$, barely making the threshold of significance, and Cohen’s $d = 0.19$ indicating that the size of the measured effect is small.

To get a better notion of the effects of the intervention we also investigated the separate data of the sub-group of 39 of the 79 students who scored below the median of 21 on the pre-test.

The mean for this sub-group is 16.79 on the pre-test and 19.72 on the post-test. In Figure 2 the distribution of the scores on the post-test for this subgroup of 39 students is shown.

Comparing these scores of the pre- and the post-test using a paired samples $t$-test on their individual scores gave $p < 0.001$ indicating a significant change on the relevance of mathematics scale for this subgroup of students with a pre-test score below the total group median (see Table 7). For this sub-group, Cohen’s $d$ equals 0.61 which indicates a much larger effect on this sub-group of students compared to the whole group.

### 4.2. How did the students experience the content of the new course materials?

Of the 103 participants in the intervention 91 completed the survey on the re-designed course and the use of the new curriculum materials. Overall students rated the new course with a 6.9 on a scale from 1 to 10. Subdivided by the six participating classes the 5.4 of group
2 in 2015–2016 stands out with all other mean scores between 6.9 and 7.5 (see Table 8). In the optional remarks students from this group complained mainly about many computer problems they encountered and occasionally about the lack of guidance by their teacher.

Out of the 91 students 65 agreed with the statement ‘The tasks gave me a good impression how mathematics can be applied in other sciences’ of which 17 strongly (see Figure 3).

This result was confirmed by the answers students gave to the open question to name positive points about the course. Twenty-seven of the 91 students mentioned ‘experience the application of mathematics’ as a positive point of the course. Autonomy in solving the tasks and learning the theory and working in groups were the other positive points mentioned several times (see Table 9).

When asked for negative points about the course the lack of traditional, teacher-led instruction and guidance by the teacher was mentioned most, by 22 out of 91 students. Remarkable was that 10 out of 14 students from group 2 who rated the module with a 5.4 average mentioned this as a downside of the course. Other students complained that they did not fully understand the tasks or complained about computer problems. Three students thought working in groups was a negative point of this course (see Table 10).

### 4.3. How did the students perform on the examination of the course after the intervention?

The 72 students of school A sat an examination after the interventions of 2016 and 2017. The theory of differential equations was one of the four mathematical subjects tested in this...
exam. The students from the advanced mathematics class in 2015 from the same school acted as the control group. This class took the traditional, teacher-led course in differential equations and sat the same examination. To be able to properly compare the results, the same exam (with the same questions on all four mathematical topics up to changing some numeric values) was used in each of the examinations of 2015, 2016 and 2017 in school A.

The students from the control group performed better on all four subjects of the examination with a mean score of 66.8% for the full exam, compared to a mean score of 61.7% for the students who participated in the intervention. The same difference was found when comparing the scores for the questions on differential equations. Like for the full exam, the students in the control group scored better with about the same relative margin (see Table 11). The students in the intervention performed worse than the control group in
Table 11. School A: Examination grades after the intervention compared to the grades the year before.

<table>
<thead>
<tr>
<th>Examination year</th>
<th>N</th>
<th>Full Exam</th>
<th>Partial Exam on differential equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>2015 traditional course</td>
<td>26</td>
<td>66.8%</td>
<td>70.8%</td>
</tr>
<tr>
<td>2016 &amp; 2017 after intervention</td>
<td>72</td>
<td>61.7%</td>
<td>64.8%</td>
</tr>
</tbody>
</table>

Every part of the exam but they did not perform significantly better or worse on questions about differential equations compared to their performance on all the questions in the exam. Due to the anonymity of the examination scores it was not possible to isolate the scores of the sub-group that scored low on the pre-test of the questionnaire.

5. Conclusions

In this section, we answer the main research question by providing conclusive remarks on students’ beliefs about the relevance of mathematics after working with and experiencing the new curriculum materials. Moreover, we explain the resulting examination grades, and suggest implications for future research on teachers’ use of the materials.

Concerning students’ beliefs about the relevance of mathematics, over three consecutive years a total of 112 students taking the advanced mathematics course ‘wiskunde D’ in higher secondary education in the Netherlands were questioned on their beliefs about the relevance of mathematics using the validated relevance of mathematics-scale. A mean score of 20.79 over the 8 items confirmed the findings by Van Elst (2013) that many of students were not convinced about the relevance and use of mathematics. However, the large standard deviations implied that there was a large spread among the results, and thus among students’ beliefs about the relevance of mathematics.

The described bimodal distribution of the students’ scores on the relevance of mathematics-scale at the pre-test could imply that among the group of 79 students we had a group of students who had strong beliefs about the relevance of mathematics with their scores centred around the peak at 24–26 and a group of students with relative weak beliefs about the relevance of mathematics with their scores centred around 18–20 (Figure 1). It is not likely that the group of students already convinced about the relevance of mathematics was affected much by the intervention. This might have affected the results of the total group on the relevance of mathematics-scale, resulting in a $p = 0.048$ value with Cohen’s $d = 0.19$ on the paired samples t-test of the scores of the pre- and post-test. It is also likely that the effects of the intervention would be more profoundly visible if we looked at the separate data of the sub-group of 39 of the 79 students who scored below the median of 21 on the pre-test.

The significant change in students’ beliefs about the relevance of mathematics, $p < 0.001$ with a Cohen’s $d = 0.61$, among the sub group with a score below the median in the pre-test indicates a much stronger effect of the intervention on students with low beliefs about the relevance of mathematics.
These results suggest that students not only learnt the basic concepts of the theory of differential equations in this course on differential equations, but that the course also enabled students to enhance their views on the relevance of mathematics. Hence, it can be concluded that working with the guided tasks contributes not only to the conceptual understanding of the mathematics involved but also to students’ productive disposition about the relevance of mathematics, hence developing a more complete understanding and improving mathematical proficiency (Kilpatrick, Swafford, & Findell, 2001).

Concerning students’ experience with the new curriculum materials, from the survey we conclude that the students appreciated the new course materials and recognized their purpose. They rated the course with 6.8 out of 10, with only one group scoring below 6.9, with 5.4. This one group complained about ICT problems and lack of guidance by the teacher.

Most students recognized and appreciated the ‘deeper’ goal of the tasks, namely to show the relevance of mathematics. 65 of the 91 students agreed with the statement ‘The tasks gave me a good impression how mathematics can be applied in other sciences’, and 27 students named the experience how mathematics was applied in real-life situations as one of the positive points of the new course.

Other positive points mentioned were the autonomy students felt whilst working on the tasks, and working in small groups. What stood out was the call for a more teacher-led, traditional type of instruction. The lack of teacher-led instruction was mentioned 22 times as a negative point about the course. It is likely that students were not used to this form of education and the lack of teacher-led instruction made them unsure about their learning. Perhaps adding some teacher-led instruction at the end of each task, summarizing the newly introduced mathematical concepts in a plenary, could help students feel more confident about their learning the mathematics in this new course. These kinds of knowledge consolidation moments have also been introduced to other similar context-based projects in the Netherlands (Janssen, 2009).

Regarding students’ examination grades, from our results it can be concluded that it was possible to incorporate improving students’ beliefs about the relevance of mathematics as a second learning goal alongside the learning of the mathematical concepts in a differential equations course for higher secondary education, without negatively influencing the mathematical learning goal. Relative to their overall performance the students in the intervention performed about the same on their examination as the students in the year prior to the intervention.

Although in the course teachers did not use teacher-led front teaching, the teachers were still crucially important. As a coach they had to guide the students while they were working on the tasks. Knowledge about the 15 contexts and the mathematical concepts was therefore essential in order to be able to support students. Hence, teachers would have to acquaint themselves with (the concepts of) the course materials, and take on the role as a coach in the classroom. In terms of further research, we think that how to suitably guide (and perhaps teach) students while working on these tasks would be an appropriate research and development field.

To conclude, we suggest that learning differential equations in relevant context helps to improve students’ beliefs about the relevance of mathematics. Further research could identify various relevant contexts to incorporate in the learning of other mathematical subjects to further improve these beliefs, preferably at a younger age.
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