Identifying Thermal Dynamics for Precision Motion Control *

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Abstract: Thermal-induced deformations are becoming increasingly important for the control performance of precision motion systems. The aim of this paper is to identify the underlying thermal dynamics in view of precision motion control. Identifying thermal systems is challenging due to large transients, large time scales, excitation signal limitations, large environmental disturbances, and non-linear behavior. An approach for non-parametric identification is developed that is particularly suitable for thermal and mechanical aspects in mechatronic systems. In particular, prior knowledge of several domains can be directly specified. Additionally, the non-parametric model is used as a basis for parameter estimation of a grey-box model. The presented methods form a complete framework that facilitates the implementation of advanced control techniques and error compensation strategies by providing high-fidelity models, enabling increased accuracy and throughput in high precision motion control.

Keywords: Frequency Response Function, Thermo-mechanical systems, Transient reduction, Parameter estimation, Local Parametric Modeling, Grey-box modeling

1. INTRODUCTION

Impressive progress in advanced motion control of precision mechatronics has led to a situation where thermal-induced deformations are a major error source (Oomen, 2018). Precision motion systems are capable of positioning up to the nano-meter scale. These precise movements are essential in several industrial applications, e.g., the manipulation of the sample in an electron microscope and the manufacturing of integrated circuits. As a result of these advancements made in motion control, the position errors are very small and the thermally induced deformations have relatively become more significant on the overall system performance of precise motion systems, and are no longer negligible. Heat sources causing the deformations within precision mechatronics are typically heat dissipation from actuators and encoders or environmental temperature fluctuations.

To meet these ever increasing demands to enhance the throughput and positioning accuracy, thermal deformations must be analyzed and compensated for through an appropriate thermo-mechanical model. Accurate modeling of precise thermo-mechanical systems is complex, e.g., due to uncertain parameters and contact resistances. Earlier solutions to compensate for the deformations in electron microscopes, for instance, can not cope with large deformations and strongly depend on model quality (Salmons et al., 2010; Tărău et al., 2011). Therefore, an accurate parametric model is desired for a model-based approach. Ideally, using a limited amount of temperature measurements combined with an accurate thermo-mechanical model enables the use of error-compensation techniques (Koevoets et al., 2007; Evers et al., 2019).

Accurate modeling of thermo-mechanical behavior is achieved using Frequency Response Function (FRF) identification, key requirements are accuracy and experimentation costs. Currently, experimental modeling of thermal systems is often done by sequential excitation of system inputs by individually applying a sinusoidal or step-like excitation until steady state is achieved (Ljung, 1999). Due to the large time constants in thermal systems, this method quickly becomes tedious for an increasing number of inputs and frequency range.

In sharp contrast, modeling for advanced motion control, see, e.g., Oomen (2018); Voorhoeve et al. (2018); Evers et al. (2018a) is fast, accurate and inexpensive. These recent developments, the local parametric methods, see Pintelon and Schoukens (2012) for initial results in this direction. The Local Polynomial Method (LPM) (Pintelon et al., 2010) exploits the local smoothness of the transient term that otherwise would cause a bias. Both the transient contribution and system dynamics are modelled with a polynomial in a small frequency window. The local rational method (LRM) (McKelvey and Guérin, 2012), is an extension of the LPM that can lead to improvements over the LPM (Geerardyn et al., 2014). However, the LRM is non-convex due to the rational parameterization. In addition, the variance is only accurately computed for a high SNR, since measured output signals are included in the regression matrix. A newly developed rational parametrization with prescribed poles (LRMP), introduced in Evers et al. (2018a) and applied in Evers et al. (2018b), yields supe-
ior estimation accuracy over the LPM while maintaining linearity in the parameters. Although thermo-mechanical interactions are increasingly relevant, the modeling of this behavior in view of control is a key challenge. And although estimating FRFs using a local modeling approach shows promising results by suppressing the transients, these techniques are not yet applied to thermal dynamics in precision motion systems. The methods are mainly developed and applied on (lightly damped) mechanical systems, see, e.g., Geerardyn et al. (2014); Van Keulen et al. (2017); Voorhoeve et al. (2018). Excitation signal design, transient and disturbance reduction are, for example, quantities that need to be re-evaluated. The aim of this paper is to develop a framework for advanced identification suitable for thermal dynamics and experimentally validate this on an experimental setup.

The main contributions of this paper are:

1. A concise overview of the challenges in thermo-mechanical system identification
2. Fast and accurate multi-variable FRF estimations of thermal systems
3. Improving the low-frequency estimation error by incorporating additional sensor inputs
4. Parameter estimation of thermal systems using a grey-box approach

This paper is organized as follows. In Section 2, the challenges of identifying thermal systems are presented. In Section 3, an approach for improved non-parametric FRF estimation is presented, followed by a grey-box approach that utilizes the improved FRFs to estimate model parameters. Finally, in Section 4, the full framework is applied onto an experimental setup.

2. THERMAL SYSTEM IDENTIFICATION: CHALLENGES AND PROBLEM FORMULATION

In this section, several challenges in identification of thermal dynamics for precision motion systems are presented.

2.1 Industrial challenges

Deformations induced by thermal gradients are increasingly relevant in several industrial applications, see, e.g., Mikroniek (2014) for a selection. Examples include, warping and wafer edge deformation in lithography applications, thermal-induced drift in Transmission Electron Microscopy (TEM) (Tarău et al., 2011) and frame deformations in machine tools (Koevoets et al., 2007; Weck et al., 1995). While the full temperature field is relevant for the prediction of thermal induced deformations, the expansion is often most relevant in a single Degree of Freedom (DOF), e.g., perpendicular to the electron beam in TEM applications.

2.2 Experimental setup

In this section the experimental setup is presented. The experimental setup is considered in 1D, this simplification is valid for many industrial applications where the thermal behavior is often analyzed in 1D, e.g., in the tool-path direction in machine tools (Weck et al., 1995). The setup consist of a round uniform cylinder with a length of 250 mm and a diameter of 25 mm. The system has two heat inputs and five temperature outputs, in the form of power resistors and negative thermal coefficient (NTC) thermistors, respectively. A picture of the experimental setup including its inputs and outputs is shown in Fig. 1. The experimental setup consist of two versions; version 1 is a full aluminium cylinder, while version 2 consist of two aluminium cylinders with a small piece polyoxymethylene (POM) in between that acts as a high thermal resistance, as displayed in Fig. 1. These two variants of the setup represent different industrial use cases, a full aluminum design or a common mixed-material system design. Typically, thermal properties of aluminum are accurately known, however the thermal conductivity of POM is uncertain. The conductivity of POM varies between 0.22 to 0.39 W/mK at 20°C depending on the manufacturing process of the material. This provides an excellent benchmark for a grey-box parameter estimation problem.

\[ y(t) = \frac{\partial T}{\partial t}(t) = C e^{At} x_0 + \int_{t_0}^{t} C e^{A(t-\tau)} B u(\tau)d\tau + D u(t) \quad (1) \]

**Transient response**

Thermal actuators are often limited to positive input signals or to binary sequences (Monteyne et al., 2013). As a result of the heat input, the temperature of the system increases, and therefore components in the system expand. This could exceed the measurable temperature or deformation range. Especially with systems with multiple inputs, where the applied heat input is cumulative. The design of thermal excitation signals should have an low mean input while the input spectra remains rich. Designing optimal excitation signals remains a challenge for accurate FRF estimation.

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Environmental disturbances The response of thermal systems is highly influenced by environmental disturbances, and therefore the identification accuracy often deteriorates. Typically, these stochastic environmental disturbances are dominated by day/night cycles or varying temperature fluctuations. The temperature of the environment is considered as an additional input to calibrate model parameters based on experimental data gathered under transient conditions. This facilitates the use of high-fidelity models for advanced model-based control, enabling further advances in precision motion control.

2.3 Problem formulation

In precision motion control thermo-mechanical interactions are increasingly relevant, yet the modeling of this behavior in view of control is a key challenge. In this paper, estimating the FRF is taken as a first step towards high-fidelity modeling. The estimated FRF can be used directly, e.g., for controller tuning Karimi and Zhu (2014). In this work, the FRF is used as a basis for a grey-box approach to calibrate model parameters based on experimental data gathered under transient conditions. This facilitates the use of high-fidelity models for advanced model-based control, enabling further advances in precision motion control.

3. ADVANCED THERMAL SYSTEM IDENTIFICATION

In this section, a framework for advanced system identification is presented.

3.1 Non-parametric frequency response function estimation

Consider a causal linear time invariant (LTI) system in an open-loop identification setting as shown in Fig. 2. The response \( y(n) \) to input \( u(n) \) of a discrete LTI system is as follows

\[
y(n) = \sum_{m=-\infty}^{\infty} g(n-m)u(m) + \nu(n), \tag{2}
\]

with \( g(n) \) the impulse response of the system and \( \nu(n) \) the additive noise contribution. The discrete Fourier Transform (DFT) of

\[
X(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x(n)e^{-j2\pi nk/N}, \tag{3}
\]

Application of the DFT to (2) results into

\[
Y(k) = G(\Omega_k)U(k) + T(\Omega_k) + V(k). \tag{4}
\]

Where, \( \Omega_k = e^{-j2\pi nk/N}, Y(k), U(k), V(k) \) are the DFT of \( y(n), u(n), \nu(n) \), \( G(\Omega_k) \) is the frequency response function of the dynamic system, and \( k \) denotes the \( k^{th} \) frequency bin. Here, \( T(\Omega_k) \) represents the combination of the system and noise transient terms. These transients are the result of the truncation of an infinite response to a finite measurement interval.

Traditionally, the empirical transfer function estimation (ETFE) is used to derive the FRF (Ljung, 1999; Pintelon and Schoukens, 2012). The ETFE is defined as

\[
\hat{G}_{\text{ETFE}}(\Omega_k) = \frac{Y(k)U(k)^{-1}}{Y(k)U(k)^{-1} + V(k)U(k)^{-1}}. \tag{5}
\]

For thermal systems, the transient contribution is increasingly significant. While the ETFE can often yield acceptable results on mechanical systems, since the transient is significantly shorter than the measurement period, for thermal systems the estimation accuracy is severely biased due to the transient component. To reduce the estimation error, the transient should be explicitly addressed during the FRF estimation.

3.2 Local Rational method with Prescribed poles (LRMP)

To cope with data gathered under transient conditions, a local modeling method is adopted. The method is developed in Evers et al. (2018a) and it uses a local rational parameterization of \( G(\Omega_k) \) and \( T(\Omega_k) \) in (4). The system dynamics and transient are parametrized for a small local frequency window \( k + r \) as

\[
G(\Omega_{k+r}) = \sum_{b=1}^{N_k} \theta_{Gb} B_b(\Omega_{k+r}, \zeta) \tag{6}
\]

\[
T(\Omega_{k+r}) = \sum_{b=1}^{N_k} \theta_{Tb} B_b(\Omega_{k+r}, \zeta), \tag{7}
\]

where \( B_b \) is an orthonormal basis with \( \zeta \) the pre-scribed poles. This parametrization is linear in the parameters, i.e., the optimization is convex, while having the advantages of using a rational function. Moreover, the orthonormal basis straightforwardly allows for the inclusion of prior knowledge on the system dynamics through the pre-scribed poles \( \zeta \). Generally, thermal systems are of first order with inherently stable poles. Commonly, a first estimation of the time constants of the system can be made and included as prior knowledge through \( \zeta \) to improve the estimation error of the FRF, see, e.g., Evers et al. (2018b).

3.3 Incorporating additional inputs

To reduce the effect of environmental disturbances on the system identification problem measurements of the ambient temperature are incorporated as an additional input (Ljung, 1999). Measuring the environment and treating it like an input is commonly done for identifying vibration isolation systems, see, e.g., Beljen et al. (2018). One of the main environmental disturbances is ambient temperature fluctuations. The temperature of the environment...
is spatially dependent, and therefore ambient temperature measurements are only valid under the assumption that the ambient temperature is uniform for the thermal system. Otherwise, multiple ambient temperature measurements are needed, which leads into correlating input signals making the identification procedure more complex. The measured environmental disturbances can easily be incorporated as an additional input, yielding
\[ Y(k+r) = \hat{G}(\Omega_{k+r}) \left[ \frac{U(k+r)}{D(k+r)} \right] + T(\Omega_{k+r}) + V(k+r) \]  
where \( D(k+r) \) is the DFT of the measured environmental disturbance. Here, \( \hat{G} \) is now a 1 × 2 multi-variable system model due to the additional system input.

3.4 Parametric models for control and error compensation

In this section, the parametric model of the thermal system and the grey-box parameter estimation approach are presented.

**Thermal modeling** Consider a thermal system, e.g., the setup in Sec. 2.2, where the thermal dynamics are described by the heat equation
\[ c_p \rho \frac{\partial T(x,t)}{\partial t} = \kappa \frac{\partial^2 T(x,t)}{\partial x^2} + h(T(x,t) - T_\infty(t)) + Q(x,t). \]  
With \( T(x,t) \) the temperature at position \( x \), \( T_\infty(t) \) the ambient temperature, \( Q(x,t) \) the heat flux, \( h \) the convection coefficient, \( \kappa \) the thermal conductivity, \( \rho \) the material density, and \( c_p \) the specific heat. The heat transfer due to radiation is linearized and combined in the convection coefficient \( h \). By spatial discretization of (9), a parameterized model can be generated in state space representation by
\[ \hat{G}(\Omega_k, \varphi) := \begin{cases} \hat{T}(t) = A(\varphi)T(t) + B(\varphi)u(t) \\ \hat{y}(t) = C(\varphi)T(t) \end{cases} \]  
where \( \varphi \in \mathbb{R}^{N_p \times 1} \) the parameter set with \( N_p \) the number of parameters such as material constants and contact resistances.

**Grey box identification** The parameterized model (10) contains uncertain parameters \( \varphi \) that limit the prediction accuracy and suitability for advanced control. The aim of grey box identification is to calibrate the parameter set \( \varphi \) such that the model (10) is suitable for control. The grey-box approach is based on minimizing the discrepancy between the measured non-parametric FRF and the FRF of the parametric model with the following cost function
\[ J = \min_{\varphi} \left\{ \| W(\Omega_k) \left( \hat{G}(\Omega_k) - \hat{G}(\Omega_k, \varphi) \right) \|^2 \right\}. \]  
Here, \( \hat{G}(\Omega_k) \) is the measured non-parametric FRF, obtained by applying the approach presented in Sec. 3.1, \( \hat{G}(\Omega_k, \varphi) = C(\varphi)\hat{\Omega}_k I - A(\varphi)^{-1}B(\varphi) \) is the FRF of the parametric model (10), \( W(\Omega_k) \in \mathbb{C}^{N_y \times N_y} \) is a dynamic weighting filter depending on the variance of the FRF at each frequency, and \( N_x \) and \( N_y \) are the number of inputs and outputs, respectively. By minimizing (11) the parameter set \( \varphi \) is calibrated such that the model (10) best describes the experimental system. This facilitates the use of the calibrated model for advanced control and error compensation techniques. Alternatively, a direct approach, not going through a non-parametric model, can be used. This can potentially yield similar accuracy. However, this would require the estimation of the initial state \( T_0 \) as well, here a non-parametric model allows for an intermediate check.

4. CASE STUDY: 1D THERMAL SYSTEM

In this section, the proposed identification methodology is applied to the experimental setup presented in Sec. 2.2.

4.1 Frequency response function estimation

In this section, the FRF of the experimental setup is estimated by using the local parametric method as presented in Sec. 3.1.

**Transient suppression** Since the input to the system is constrained to be positive, the excitation input \( u_1(t) \) is selected as a Random Phase Multisine (RPMS) with offset
\[ u_1(t) = \sum_{k=1}^{N} A_k \sin(2\pi k t / N + \phi_k) + \Delta. \]  
Here, \( N \) is the number of samples in a period, \( A_k \) is the amplitude of the sinusoidal signal at frequency \( k \) and \( \phi_k \) is a uniformly distributed phase on \( [0, 2\pi) \) such that \( \mathbb{E}\{e^{i\phi_k}\} = 0 \) and \( \Delta \) is an offset to enforce \( u_1(t) \geq 0 \). The RPMS is a common excitation signal in system identification, it is a summation of sinusoidal signals that has specific characteristics (Pintelon and Schoukens, 2012) in the frequency domain, e.g., the amplitude spectrum is pre-determined and the phase distribution is random.

The temperature response \( T_1 \) to the input \( u_1 \) is shown in Fig. 3. Initially, the temperature response consist of a first order step response including higher order dynamics due to the offset \( \Delta \) in the excitation signal. After the initial transient has settled, the output consist of the response of the excitation signal and environmental disturbances.

Two sub-records of the same dataset are considered, one includes the first two periods, which consist of a significant
Incorporating ambient temperature measurements In this section a measurement of the ambient temperature $T_\infty$ is used as an additional excitation input, see Eq. (8), to improve the estimation accuracy at low-frequencies. The procedure yields an estimate for a $1 \times 2$ plant model

$$
\hat{G}(\Omega_k) = [G(\Omega_k)_{u_1 \rightarrow T_1}, G(\Omega_k)_{T_\infty \rightarrow T_1}]
$$

(13)

containing the transfer function between the input $u_1$ and $T_1$, i.e., the desired FRF, and the transfer function between $T_\infty$ and $T_1$, where the latter can be used for a disturbance sensitivity analysis.

In Fig. 5, the amplitude estimation of the FRF $G(\Omega_k)_{u_1 \rightarrow T_1}$ and the $3\sigma$ uncertainty bounds are shown. The results show the estimation using only the input $u_1$, shown in red (a), and using $u_1$ and the ambient temperature measurements as an additional input, shown in blue (b). The amplitude estimation and variance at medium to high frequencies are similar for both estimations. However, using only the input $u_1$ a large variance is obtained in the low frequency region. By then incorporating the ambient measurement as an additional input, the variance of the estimation of the FRF is reduced significantly.

4.2 Parameter estimation

In this section, the parameters of a Multi-Input Multi-Output (MIMO) lumped mass model, i.e., a model in the form (10), are calibrated by minimizing the discrepancy between the parametric model and the non-parametric FRF estimation using (11). In Fig. 6 the estimated non-parametric FRF and the calibrated parametric model, of the version 2 setup, is shown. Clearly, the estimated parametric model is within the $3\sigma$ uncertainty of the FRF estimation. The conductivity of the slice of POM material is estimated at 0.32 W/mK, which is well within the range supplied by the manufacturer. The procedure yields a MIMO high fidelity parametric model of the experimental system that is suitable for advanced control techniques and error compensation strategies that enable increased performance in precision motion control.

5. CONCLUSION

The identification framework presented in this paper enables the fast and accurate identification of thermal dynamics in view of precision motion control. By applying the local parametric method an improved non-parametric FRF estimate is obtained. Furthermore, by explicitly taking into account the transient contributions a significant reduction in measurement time is achieved when compared to classical methods. Moreover, the estimation error for low-frequencies is significantly reduced by incorporating additional sensor data that makes use of environmental disturbances to provide additional excitation input. Building on the improved FRF estimation, a grey-box parameter calibration approach is presented that yields high fidelity parametric models of the thermo-dynamical system. The proposed methodology is applied to a multi-variable experimental setup. The method achieves significant improvements in estimation accuracy, and a reduced experimentation time by suppressing the transient and disturbance contributions. The presented methods form a framework that facilitates the implementation of advanced control techniques and error compensation strategies, enabling increased accuracy and throughput in high precision motion control.
The FRF is used for a grey-box parameter estimation, yielding a high fidelity parametric model (black solid).

**REFERENCES**


