An open source implementation of the IDR(S)Stab(L) solver

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1. Introduction

-Linear systems are the **backbone of simulations.**

-For small systems the **robust LU-decomposition** is often used.

-Linear systems in 2D and 3D simulations are too large to tackle with this method.

-An **alternative to LU** is to use iterative methods, for example **BiCGStab** in the Eigen C++ library [1].

-**IDR(S)Stab(L) is a generalization of BiCGStab** [2] and is applicable to a wider range of systems.

-Here the Eigen-implementation of BiCGStab is compared with our implementation of IDR(S)Stab(L). For matrices from the Matrix Market [3] and a 2D model problem.

2. Setup

Computational setup:

Intel(R) Core(TM) i7-5820K CPU @ 3.30GHz 6C/12T

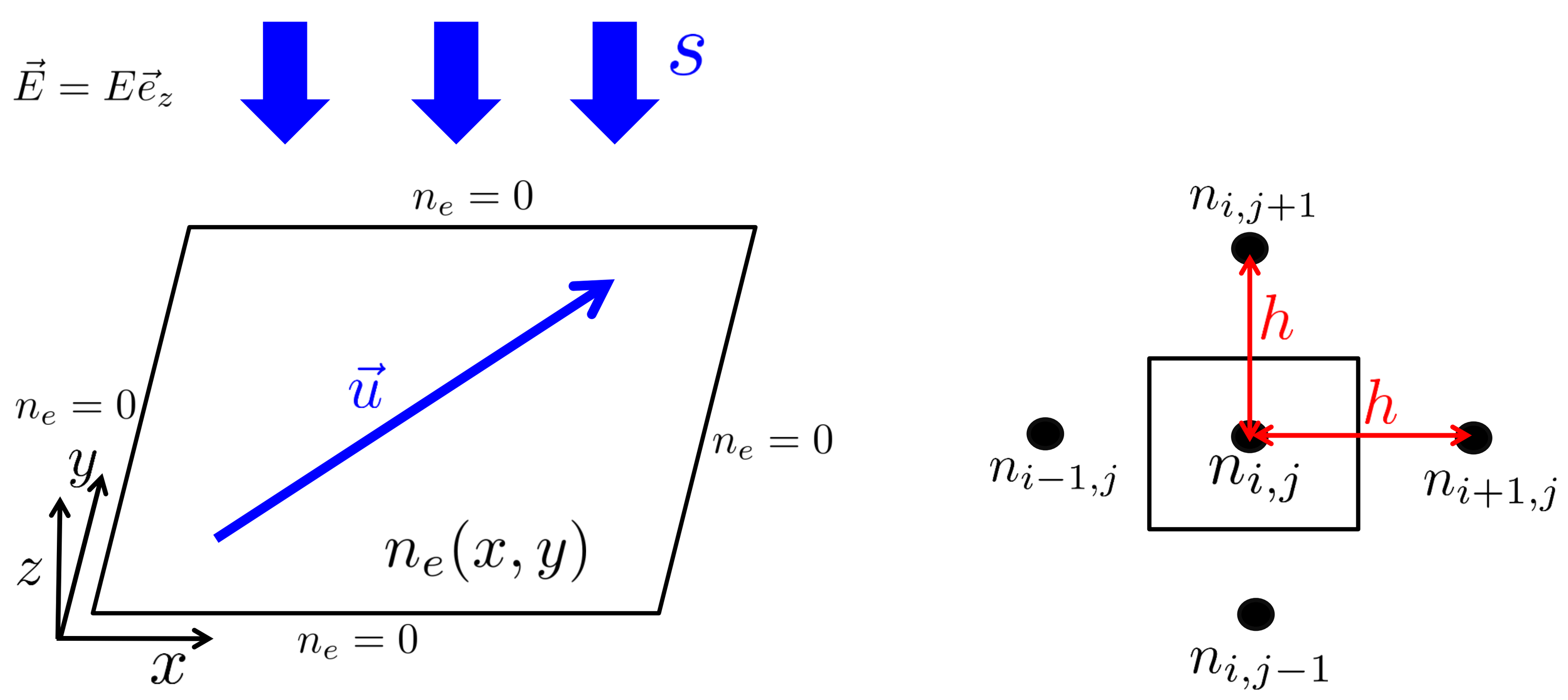
GCC 7.4.1, Eigen 3.3.7, optimization flags: -fopenmp, -O3 –march=native

2D model problem:

Electrons accelerated toward a plate with a layer of neutral gas, compute e- density n_e.

Parameters: Advection u diffusion $\mathcal{E}$, influx s, reaction β.

Length=1 → non-dimensional $\beta L^2/\mathcal{E}$ (Damköhler) and $uL/\mathcal{E}$ (Peclet) numbers.



$$-\mathcal{E}\nabla^2 n_e + \vec{u}\cdot\nabla n_e - \beta n_e = s, \qquad \vec{u} = u\vec{e}_x + u\vec{e}_y$$

Discretization:

-Finite Volume Method (FVM)

-Exponential scheme / Homogeneous flux scheme [4]

-161x161 grid with gridspacing h

-Resulting scheme:

$$\left[\tfrac{1}{2}u - \tfrac{1}{2h}\mathcal{E}Q\right] n_{i+1,j} + \left[\tfrac{1}{2}u - \tfrac{1}{2h}\mathcal{E}Q\right] n_{i,j+1} +$$
$$\left[\tfrac{2}{h}\mathcal{E}Q - \beta h\right] n_{i,j} +$$
$$\left[-\tfrac{1}{2}u - \tfrac{1}{2h}\mathcal{E}Q\right] n_{i-1,j} + \left[-\tfrac{1}{2}u - \tfrac{1}{2h}\mathcal{E}Q\right] n_{i,j-1} +$$
$$= hs$$

$$Q := \frac{P}{e^P - 1} - \frac{P}{e^{-P} - 1}, \qquad P := \frac{uh}{\mathcal{E}}$$

Resulting linear system:

-Solve for the points $n_{i,j}$

-25,921 equations with 25,921 unknowns

3. Matrix Market results

Every linear system Ax=b with rhs from the Matrix Market [3] database

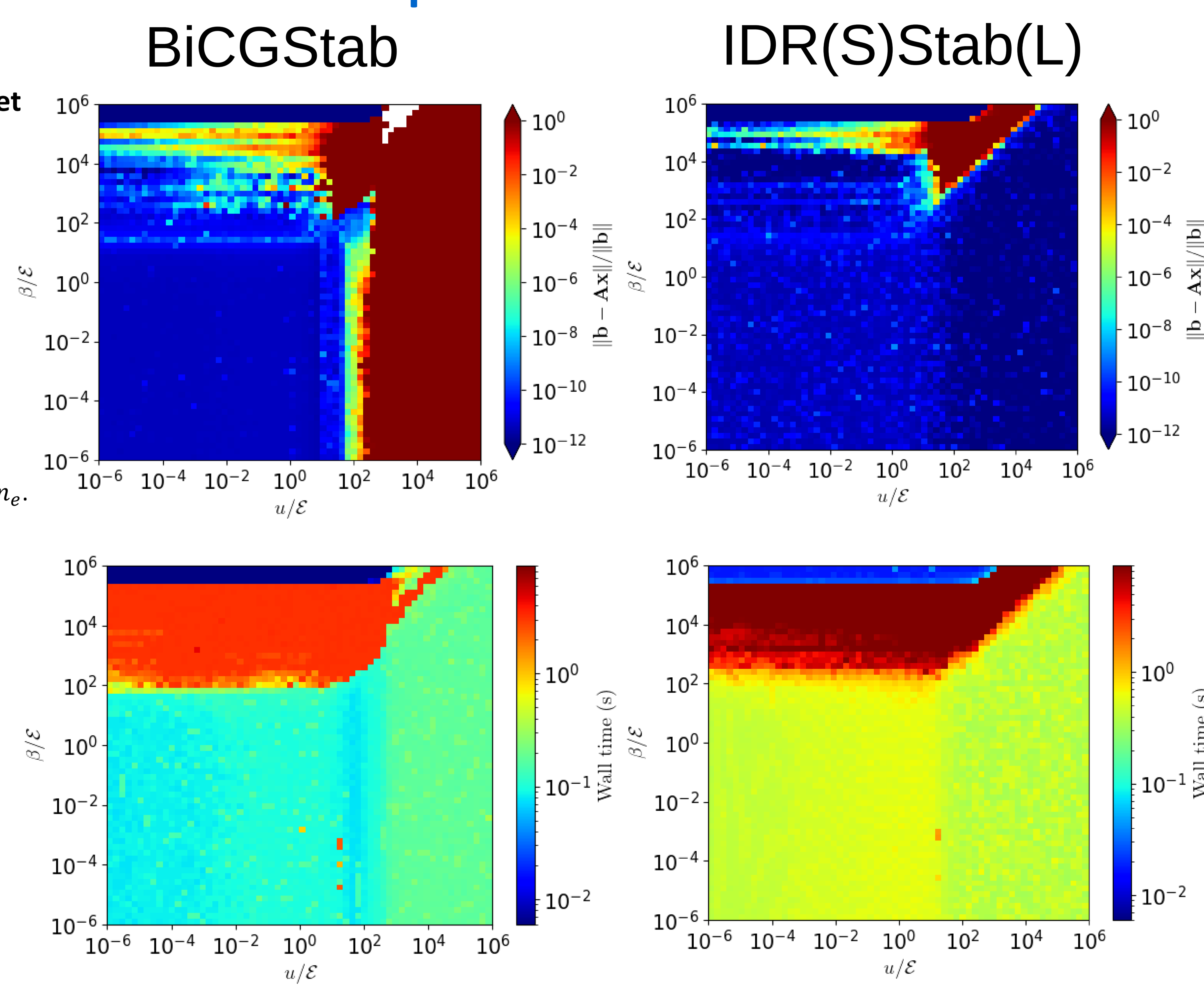
Tolerance: 1e-12, max iterations: 2000 IDR(S)Stab(L), 13000 BiCGStab

Total: 113 linear systems

BiCGStab converges for 37

IDRStab converges for 47

4. 2D model problem results



5. Conclusion

1. ***IDR(S)Stab(L) can solve problems Eigen-BiCGStab implementation cannot***
2. ***IDR(S)Stab(L) uses less matrix-vector products to reach the tolerance, however***
3. ***current IDR(S)Stab(L) version generally not faster.***

6. Outlook

1. ***Improve IDR(S)Stab(L) speed***
2. ***Propose this IDR(S)Stab(L) implementation for the Eigen-Library***

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