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A shrinking-horizon, game-theoretic algorithm for distributed energy generation and storage in the smart grid with wind forecasting

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Abstract: We address the demand-side management (DSM) problem for smart grids where the users have energy generation and storage capabilities, and where the energy price depends on the renewable energy sources and on the aggregate electricity demand. Each user aims at reducing its economic cost by selecting the best energy schedule subject to its local preferences and global restrictions on the aggregate net demand. From a game-theoretic perspective, we model the problem as a generalized Nash equilibrium problem. We propose a shrinking-horizon semi-decentralized DSM algorithm that exploits the most recent forecast on the renewable energy sources to perform real-time adjustments on the energy usage of the users. We investigate the potential of the proposed approach via numerical simulations on realistic scenarios, where we observe improved social welfare compared to day-ahead DSM algorithms.

Keywords: Demand-side management, Game theory, Smart grid, Shrinking-horizon control

1. INTRODUCTION

Energy demand in populated areas is currently growing fast and will be growing even faster in the near future. Smart grids are envisioned to facilitate the transition to efficient and reliable user-oriented electricity supply via intelligent, distributed energy generation (DG) and distributed energy storage (DS), bi-directional information flow and control mechanisms [Gao et al. 2014]. Among the latter, pricing mechanisms represent a promising technique to implement demand-side management (DSM), namely, to incentivate prosumers to shift their flexible energy consumption to non-peak times, hence minimize the peak-to-average ratio (PAR) of the demand. In these mechanisms, at each time period, the energy price is usually designed to be proportional to the aggregated net load over the grid [Chen et al. 2014, Mohsenian-Rad et al. 2010]. To model and design DSM algorithms for the smart grid, game theory has been recently, yet extensively, adopted [Mohsenian-Rad et al. 2010, Atzeni et al. 2013a]. Each prosumer (player, agent) in the day-ahead market (aggregative game) aims to select an energy schedule (decision variable) that minimizes its economic expense (cost function), which also depends on an aggregate measure of the schedules of the other prosumers [Grammatico 2016a], [Grammatico 2016b]. Since the prosumers are self-interested parties, a usual market equilibrium is the celebrated (generalized) Nash equilibrium.

For example, in [Wu et al. 2011], a decentralized DSM program is presented to integrate wind power and to minimize the total energy cost in an isolated microgrid, showing that direct load control can reduce generation costs for the utility. In [Dave et al. 2011], the authors develop a hybrid day-ahead and real-time consumption scheduling scheme for households that participate in a DSM program, where energy storage devices are not considered. In [Wang et al. 2018], the authors consider the user discomfort in their cost functions and propose a game-theoretic algorithm to reduce the grid PAR, however without global grid constraints and the most recent forecast on renewable generation. Global coupling constraints on the grid capacity, decoupled via dual decomposition, are considered in [Deng et al. 2014], where, however, the uncertainty of renewable energy sources is not modeled. A two-stage DSM mechanism is formulated in [Atzeni et al. 2014]; first, a noncooperative day-ahead, iterative bidding mechanism is proposed; then, real-time adjustments are made on the local energy allocations to alleviate the impact of real-time deviations with respect to their day-ahead bidded schedules.

In the literature, most DSM techniques are day-ahead problems that do not directly embed real-time adaptation to recent forecasts and real-time realizations of uncertain power generation. It follows that the realized schedules are not locally or individually optimal in general. In this paper, we design a receding-horizon, game-theoretic DSM mechanism that exploits the latest information on renewable energy availability, e.g. short-time wind generation forecast. Specifically, we consider grid users with dispatachable energy generation, storage and consumption charged

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based on the aggregate load and the renewable generation. We formulate a DSM problem as a generalized Nash equilibrium problem (GNEP) and propose a shrinking-horizon, semi-decentralized solution algorithm that allows the prosumers to adjust their energy schedules based on the most recent forecast of renewable energy generation. Finally, we conduct numerical simulations with real data for wind profiles, energy consumption curves and energy prices in The Netherlands.

2. SMART GRID MODEL

We adopt the smart grid model in [Atzeni et al. 2013a], which we review in the following subsections.

2.1 Demand-side model

Let us consider a smart power grid with an energy provider that includes a wind farm and a common generator (e.g. coal or gas) and provides energy to several users. Since we focus on DSM, let us model the demand side of the smart grid. Demand-side users are all characterized by their individual per-slot energy consumption profile at time period h, i.e, $e_i(h)$. We denote the set of all demand-side users by $\mathcal{D} = \mathcal{P} \cup \mathcal{N}$, where $\mathcal{P}$ is the set of passive users and $\mathcal{N}$ is the set of active users. Passive users are regular energy prosumers, while active users have flexible schedules. Each active user is connected to the power grid and to a communication line between his smart meter and the central unit.

The active users $\mathcal{N}$ are divided in dispatchable energy producers $\mathcal{G} \subseteq \mathcal{N}$ and energy storage users $\mathcal{S} \subseteq \mathcal{N}$. Each user $i \in \mathcal{G}$ is characterized by its *per-slot energy generation* at time slot h, i.e., $g_i(h)$, while users $i \in \mathcal{S}$ by the *per-slot energy storage* at slot h i.e., $s_i(h)$, for $h = 1, \dots, H$.

The per-slot load profile $l_i(h)$ is defined as

$$l_i(h) = \begin{cases} e_i(h) & \text{if } i \in \mathcal{P} \\ e_i(h) - g_i(h) + s_i(h) & \text{if } i \in \mathcal{N}. \end{cases} \quad (1)$$

The energy consumption of each user is restricted by maximum and minimum operating power at each time period and by a maximum consumption over the horizon H, i.e.,

$$e_i^{\min}(h) \le e_i(h) \le e_i^{\max}(h), \quad \forall i, h \quad (2)$$

$$\textstyle\sum_{h=1}^{H} e_i(h) \le \xi_i^{\max}, \quad \forall i. \quad (3)$$

Let us introduce $\boldsymbol{e}_i = \mathrm{col}(e_i(1), \dots, e_i(H))$ as the collective energy consumption vector, over the horizon of length H and define the strategy set $\Omega_{\boldsymbol{e}_i}$ for each producer $i \in \mathcal{N}$ as

$$\Omega_{\boldsymbol{e}_i} = \{\boldsymbol{e}_i \in \mathbb{R}^H \mid (2), (3) \text{ hold}\}. \quad (4)$$

2.2 Energy generation model

Energy generators can use the produced power to either support their energy consumption, or to charge their batteries. The energy producers can be classified into dispatchable and non-dispatchable producers. The latter refer to generators that are always producing whenever possible, e.g. renewable sources of intermittent nature which have fixed costs and do not require a strategy for energy production. Instead, dispatchable energy producers, e.g. internal combustion engines and gas turbines, experience a variable cost, e.g. fuel cost, hence are willing to optimize their energy generation schedule. We denote by $W_i(g_i(h))$ the price for producing the amount of energy $g_i(h)$. The generation constraints are

$$0 \le g_i(h) \le g_i^{\max}, \quad \forall i, h \quad (5)$$

$$\textstyle\sum_{h=1}^{H} g_i(h) \le \psi_i^{\max}, \quad \forall i \quad (6)$$

where (5) represents a generation capacity on each time period h and (6) the maximum generation over the multi-period horizon. Next, we introduce $\boldsymbol{g}_i = \mathrm{col}(g_i(1), \dots, g_i(H))$ as the overall energy generation schedule, and we define the feasible set $\Omega_{\boldsymbol{g}_i}$ for dispatchable energy producers $i \in \mathcal{G}$ as

$$\Omega_{\boldsymbol{g}_i} = \{\boldsymbol{g}_i \in \mathbb{R}^H_{\ge 0} \mid (5), (6) \text{ hold}\}. \quad (7)$$

2.3 Energy storage model

Storage devices, e.g. batteries, of all users are characterized by charging efficiency, discharging efficiency, leakage rate, capacity, and maximum charging rate. With no loss of generality, it is possible to define the per-slot energy stored as the sum of charging and discharging, with charging and discharging inefficiencies, $0 < \beta_i^{(+)} \le 1$ and $\beta_i^{(-)} \ge 1$, respectively. Let $0 < \alpha \le 1$ be the leakage rate of the storage device. Namely, if $q_i(h)$ denotes the charge level at (the end of) time period h, then $q_i(h)$ reduces to $\alpha_i q_i(h)$ at (the end of) period $h+1$. The maximum amount of energy that the storage device i can accumulate is the capacity ζ_i. Lastly, the maximum charging rate, $s_i^{\max}$, represents the maximum amount of energy that can be charged into the device during one time period.

The charge level $q_i(h)$ is modeled as

$$q_i(h) = \alpha_i q_i(h-1) + \beta_i^\top s_i(h) \quad (8)$$

where $s_i(h) = [s_i^{(+)}(h), s_i^{(-)}(h)]^\top$ and $\beta_i = [\beta_i^{(+)}, \beta_i^{(-)}]^\top$. Since the charge level must lie in the interval $[0, \zeta_i]$, equation (8) can be rewritten as

$$-\alpha_i q_i(h-1) \le \beta_i^\top s_i(h) \le \zeta_i - \alpha_i q_i(h-1), \quad (9)$$

where we can use the closed-form solution

$$q_i(h) = \alpha_i^h q_i(0) + \textstyle\sum_{t=1}^{h} \alpha_i^{(h-t)} \beta_i^\top s_i(h). \quad (10)$$

In addition, the total stored energy must satisfy the capacity limit of the storage device, i.e.,

$$\beta_i^\top s_i(h) \le s_i^{\max}, \quad \forall i, h. \quad (11)$$

Finally, we impose that the battery charge at the end of the horizon, i.e., at the beginning of the next day, is approximately the same as the initial one:

$$|q_i(H) - q_i(0)| \le \varepsilon_i, \quad \forall i, \quad (12)$$

where $\varepsilon_i > 0$ is a small and positive constant.

In compact form, the energy storage scheduling vector reads as $\boldsymbol{s}_i(h) = [\boldsymbol{s}_i^{(+)}(h), \boldsymbol{s}_i^{(-)}(h)]^\top$, with $\boldsymbol{s}_i^{(+)} = \mathrm{col}(s_i^{(+)}(1), \dots, s_i^{(+)}(H))$ and analogously for $\boldsymbol{s}_i^{(-)}$, and the feasible strategy set $\Omega_{\boldsymbol{s}_i}$ for energy storage user $i \in \mathcal{S}$ as

$$\Omega_{\boldsymbol{s}_i} = \{\boldsymbol{s}_i \in \mathbb{R}^{2H}_{\geq 0} \mid (9), (11), (12) \text{ hold}\}. \tag{13}$$

2.4 Energy cost and pricing model

On day-ahead markets, energy prices are set by the production offers of the energy suppliers and the consumption bids by the energy prosumers. As in [Mohsenian-Rad et al. 2010],[Fang et al. 2012], [Atzeni et al. 2013b], here we consider a single price curve by aggregating the conventional generator and the wind turbine.

Let $\omega(h)$ be the power generated by the wind turbine at time period h. Next, we introduce the function C_h, namely the cost per unit of energy at time period h, that varies linearly with the difference between the total aggregate load $L(h)$ and the generated wind power [Wu et al. 2011]:

$$C_h(L(h) - \omega(h)) = K_h \cdot (L(h) - \omega(h)), \tag{14}$$

where $K_h > 0$, for $h \in \{1, \ldots, H\}$, are time-varying coefficients because the energy production varies over the horizon, and $L(h)$ is the aggregate energy load,

$$0 < L(h) = L_P(h) + \textstyle\sum_{i \in \mathcal{N}} l_i(h), \tag{15}$$

and $L_P(h) = \sum_{i \in \mathcal{P}} e_i(h)$ denotes the aggregate per-slot energy consumption of the passive users.

The overall cost for the smart grid at each time period h is then given by $C_h(L(h) - \omega(h))L(h) = K_h(L(h) - \omega(h))L(h)$, whereas the generic user i pays $C_h(L(h) - \omega(h))l_i(h)$ to purchase the load $l_i(h)$. The aggregate per-slot energy load in (15) must satisfy

$$L_{\min}(h) \leq L(h) \leq L_{\max}(h), \quad \forall h \in \{1, \ldots, H\} \tag{16}$$

where the lower bound is introduced to prevent additional costs arising from turning off some base power plant and the upper bound represents the maximum load that the grid can afford before a blackout occurs.

3. DEMAND-SIDE MANAGEMENT ON THE SMART GRID AS A GENERALIZED AGGREGATIVE GAME

3.1 Aggregative game formulation

Since the grid users are self-interested, we model the DSM problem as a noncooperative game. Each active user is a player who chooses its energy consumption, generation and storage strategies to minimize his cost function, i.e., his economic expense, given the aggregate load over the horizon and the available wind power forecast. For each user $i \in \mathcal{N}$, let us define the strategy vector $\boldsymbol{x}_i = (\boldsymbol{e}_i, \boldsymbol{g}_i, \boldsymbol{s}_i)^\top$ and the per-slot strategy profile

$$x_i(h) = (e_i(h), g_i(h), s_i(h))^\top, \quad \forall h = \{1, \ldots, H\} \tag{17}$$

In view of the feasible sets $\Omega_{e_i}, \Omega_{g_i}, \Omega_{s_i}$ in (4), (7) and (13), respectively, we define the local strategy set for a generic user $i \in \mathcal{N}$ as

$$\Omega_{x_i} = \{\boldsymbol{x_i} \in \mathbb{R}^{4H} | \boldsymbol{e_i} \in \Omega_{e_i}, \boldsymbol{g_i} \in \Omega_{g_i}, \boldsymbol{s_i} \in \Omega_{s_i}\}. \tag{18}$$

Then, by using the pricing model in (14), we define the cost function of user $i \in \mathcal{N}$ as

$$\begin{aligned} f_i(x_i, \boldsymbol{x}_{-i}) = \textstyle\sum_{h=1}^H K_h \left(\boldsymbol{l}_{-i}(h) + \delta^\top x_i(h) - \omega(h)\right) \cdot \\ \cdot \left(\delta^\top x_i(h)\right) + \textstyle\sum_{h=1}^H W_i(\delta_g^\top x_i(h)) \end{aligned} \tag{19}$$

where $\boldsymbol{l}_{-i} = \mathrm{col}(l_{-i}(1), \ldots, l_{-i}(H))$, $l_{-i}(h) = L_P(h) + \sum_{j \in \mathcal{N} \setminus \{i\}} l_j(h)$, is the aggregate per-slot energy load of the other players at time-slot h, while $\delta = (1, -1, 1, -1)^\top$ and $\delta_g = (0, 1, 0, 0)^\top$ are auxiliary vectors. In compact form, the cost function in (19) reads as

$$f_i(x_i, \boldsymbol{x}_{-i}) = (\boldsymbol{l}_{-i} + \Delta x_i - \boldsymbol{\omega}_H)\boldsymbol{K}_H(\Delta \boldsymbol{x}_i) + \Delta_g^\top W \tag{20}$$

where $\boldsymbol{K}_H = \mathrm{diag}(K_i)$, $\boldsymbol{\omega}_H = \mathrm{col}(\omega(1), \ldots, \omega(H))$, $\Delta = [I_H, -I_H, I_H, -I_H]^\top$ and $\Delta_g = [0_H, I_H, 0_H, 0_H]^\top$. Let us denote the collection of local feasible sets of all the agents as $\boldsymbol{\Omega} = \prod_{i=1}^N \Omega_{x_i}$. Moreover, the strategies of the agents are coupled not only via their cost functions but also via their feasible strategy sets. The coupling constraint in (16) can be represented by an affine function $\boldsymbol{x} \mapsto A\boldsymbol{x} - b$, where $A = [A_1, ..., A_N] = \boldsymbol{1}_N \otimes \Delta$, where $A_i = \mathrm{col}(\Delta, -\Delta), \forall i \in \mathcal{N}$ represents how user i is involved in the coupling constraints and $b = \mathrm{col}(L_{\max}(1) - L_P(1), \ldots, L_{\max}(H) - L_P(H), L_P(1) - L_{\min}(1), \ldots, L_P(H) - L_{\min}(H)$. Thus, the collective global feasible set $\boldsymbol{\mathcal{X}}$ is defined as the intersection of local and coupling constraints:

$$\boldsymbol{\mathcal{X}} = \boldsymbol{\Omega} \cap \{\boldsymbol{y} \in \mathbb{R}^{4HN} \mid A\boldsymbol{y} - b \leq \boldsymbol{0}_{2H}\}. \tag{21}$$

Next, we model that each active user $i \in \mathcal{N}$ chooses its energy consumption, production and storage strategy to minimize his cost function, i.e.,

$$\forall i \in \mathcal{N}: \begin{cases} \min\limits_{x_i \in \mathbb{R}} \ f_i(x_i, \boldsymbol{x}_{-i}) \\ \text{s.t.} \ \ (x_i, \boldsymbol{x}_{-i}) \in \boldsymbol{\mathcal{X}}. \end{cases} \tag{22}$$

The N inter-dependent optimization problems in (22) define a GNEP, that we denote in compact form by $\mathscr{G} = (\boldsymbol{\mathcal{X}}, \boldsymbol{f})$, with $\boldsymbol{\mathcal{X}}$ as in (21) and $\boldsymbol{f} = \mathrm{col}(f_1, \ldots, f_N)$. From a game-theoretic perspective, solving the problems in (22) simultaneously means computing a generalized Nash equilibrium (GNE), which is a feasible strategy profile $\boldsymbol{x}^*$ such that no single player i can benefit by unilaterally deviating from his strategy to another feasible one:

$$f_i(x_i^*, \boldsymbol{x}_{-i}^*) \leq \inf_{x_i \in \Omega_{x_i}} \{f_i(x_i, \boldsymbol{x}_{-i}^*) \mid (x_i, \boldsymbol{x}_{-i}^*) \in \boldsymbol{\mathcal{X}}\}. \tag{23}$$

3.2 Semi-decentralized GNE computation

Different algorithms are available in the literature to find a GNE of the aggregative game in (22), e.g., [Belgioioso and Grammatico 2017]. Most of these works focus on a special subclass of GNE characterized by the solutions of the variational inequality $\mathrm{VI}(\boldsymbol{\mathcal{X}}, \boldsymbol{F})$ which is the problem to find $\boldsymbol{x}^* \in \boldsymbol{\mathcal{X}}$ such that

$$(\boldsymbol{x} - \boldsymbol{x}^*)^\top \boldsymbol{F}(\boldsymbol{x}^*) \geq 0, \quad \forall \boldsymbol{x} \in \boldsymbol{\mathcal{X}}\,, \tag{24}$$

where $\boldsymbol{F}$ is the so-called *pseudo-gradient*, defined as

$$\boldsymbol{F}(\boldsymbol{x}) = \mathrm{col}\left(\nabla_{x_1} f_1(x_1, \boldsymbol{x}_{-1}), \ldots, \nabla_{x_N} f_N(x_N, \boldsymbol{x}_{-N})\right). \tag{25}$$

It follows from [Scutari et al. 2012, Lemma 4.2] that every solution to the variational inequality $\mathrm{VI}(\boldsymbol{\mathcal{X}}, \boldsymbol{F})$ is a GNE of our game $\mathscr{G} = (\boldsymbol{\mathcal{X}}, \boldsymbol{f})$. To efficiently compute a GNE in a semi-decentralized fashion, let us consider the preconditioned Forward Backward (pFB) algorithm [Belgioioso and Grammatico 2018, Alg. 1] ($\kappa \in \mathbb{N}$):

Algorithm 1 pFB

$$x_i(\kappa+1) = \mathrm{proj}_{\Omega_{x_i}} \left[\boldsymbol{x}_i(\kappa) - \gamma(\nabla_{x_i} f_i(x_i(\kappa), \boldsymbol{x}_{-i}(\kappa)) + A_i^\top \lambda(\kappa))\right]$$
$$\lambda(\kappa+1) = \mathrm{proj}_{\mathbb{R}^m_{\geq 0}} \left[\lambda(\kappa) + \gamma(2A\boldsymbol{x}(\kappa+1) - A\boldsymbol{x}(\kappa) - b)\right]$$

Algorithm 1 has a semi-decentralized structure: a central unit broadcasts the grid coefficients $\boldsymbol{K}_H$ and the wind power forecast $\boldsymbol{\omega}_H$, as well as the incentive signal λ and the aggregative load L. Specifically, at each iteration, the users update their strategies x_i by solving (22) given the current value of the grid coefficients, the aggregative load and the λ incentive, taking a gradient step of length γ, projected onto the local feasible set Ω_{x_i}. The central unit updates the incentive λ based on the expected violation of the coupling constraints. Thus, there is no exchange of information among the selfish users.

4. REAL-TIME DSM ALGORITHM

In real-time, we assume that the demand-side users have a forecast of the upcoming wind power generation, $\omega(h)$, which is more accurate than that made the day ahead. Therefore, we propose an iterative algorithm that exploits the latest information on the wind power generation. The proposed on-line DSM algorithm works in a shrinking-and-rolling horizon fashion: the DSM problem is solved on-line at each stage using the current data of the aggregated load and the latest data of the wind power forecast over the shrinking-horizon. The solution generates strategies for the remainder of the horizon, but only the current stage component is applied. More specifically, the shrinking-horizon framework is as follows:

At each stage k, $\boldsymbol{x}^k$ represents the predicted strategies over the remaining time slots of the horizon, $\{k, k+1, \dots, H\}$, and $\boldsymbol{x}_s^{*,k}\{k\} = \boldsymbol{x}^k\{k\}$ the actual strategy played by the agents at stage k, where $\boldsymbol{x}^k\{\tau\} = \mathrm{col}(x_i^k\{\tau\}, \dots, x_N^k\{\tau\})$ is used to denote the τ-th time component of $\boldsymbol{x}^k$. This means, that at each stage k, the output of the algorithm $\boldsymbol{x}_s^*$ is equal to the k-th element of the current predicted strategy $\boldsymbol{x}^k$ of the agents (which ranges from time k to time H). A graphical representation is provided in Fig. 1.

We remark that the strategies of each player $i \in \mathcal{N}$ must satisfy the local and global constraints at each stage, given the previous strategies. Thus, we define the updated global set of feasible strategies at each stage k as

$$\bar{\boldsymbol{\mathcal{X}}}_k = \boldsymbol{\mathcal{X}} \cap \bar{\Theta}_k \neq \varnothing, \tag{26}$$

where $\bar{\Theta}_k = \bigcap_{h=1}^k \Theta_h$ accounts for all the strategies already applied up to period k. At each time period k of the horizon, a new game $\mathscr{G}_k$ is defined with the updated global set and solved using Algorithm 1 with the latest wind forecast. Each user computes its strategy $x_i^{*,k}\{k\}$ to minimize his cost for the remaining time periods, $\{k, \dots, H\}$. The proposed real-time approach is formalized in Algorithm 2.

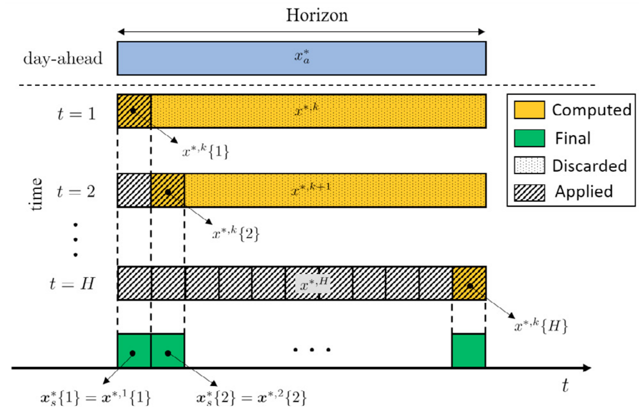

Fig. 1. Shrinking-horizon approach.

Algorithm 2 Shrinking-horizon pFB Algorithm

Initialization: Set $\Theta_1 = \mathbb{R}^{4HN}$
for $k = 1, \dots, H$ **do**
  $\bar{\Theta}_k = \bigcap_{h=1}^k \Theta_h$
  $\bar{\boldsymbol{\mathcal{X}}}_k = \boldsymbol{\mathcal{X}} \cap \bar{\Theta}_k$
  Consider the new game $\mathscr{G}_k = (\bar{\boldsymbol{\mathcal{X}}}_k, \boldsymbol{F})$
  Update wind power forecast $\boldsymbol{\omega}_H$
  Compute GNE of $\mathscr{G}_k$, $\boldsymbol{x}^{*,k}$, via Algorithm 1
  Apply $\boldsymbol{x}^{*,k}\{k\}$ and discard $\boldsymbol{x}^{*,k}\{k+1, \dots, H\}$
  $\Theta_{k+1} = \{\boldsymbol{y} \in \mathbb{R}^{4HN} \mid \boldsymbol{y}\{k\} = \boldsymbol{x}^{*,k}\{k\}\}$
end for

Algorithm 2 can be seen as a repeated Nash game: at each stage k, a new GNE $\boldsymbol{x}^{*,k}$ is computed with the most up-to-date wind information and incorporating future actions. After that, users apply strategy $\boldsymbol{x}^{*,k}\{k\}$ while discarding the rest. Next, the new global feasible set $\bar{\boldsymbol{\mathcal{X}}}_k$ is updated by considering the strategies that have been applied and should be taken into consideration for future strategies.

5. NUMERICAL SIMULATIONS

In this section, we present numerical results to test the performance of the algorithm. A smart grid with $N = 100$ demand-side users is considered, with a time horizon of $H = 24$ hours. Each demand-side user has a random energy consumption curve, i.e., $e_i(h)$ for $h = 1, \dots, H$ according to an average household demand profile in The Netherlands [EDSN, the Dutch Energy Data Hub 2014], with an average daily consumption of $\sum_{h=1}^H e_i(h) = 12$ kWh. Each user $i \in \mathcal{G}$ with power generation capabilities has a linear production cost as in [Atzeni et al. 2013a], [Walt 2004] i.e., $W_i(x) = \eta_i\, x$ for some $\eta_i > 0$.

For simplicity, we assume that all active users are subject to the same production and storage constraints (4), (7), (13) as in [Atzeni et al. 2013a],[Stephens et al. 2015], with $\eta_i = 0.039$ €/KWh. Moreover, we set $g_i^{\max} = 0.4$ kW and $\psi_i^{\max} = H \cdot 0.8\, g_i^{\max}$, $\forall i \in \mathcal{G}$. For users with storage capabilities, we set the parameters of the storage devices (Lithium-ion batteries) as in [Atzeni et al. 2013a], [Wang et al. 2018]: $\alpha_i = 0.9^{1/24}$ which represents leakage rate of 0.9 over the full day, $\beta_i^{(+)} = 0.9$, $\beta_i^{(-)} = 1.1$, $\zeta_i = 4$

kW, $s_i^{\max} = \frac{\zeta_i}{8}$, $q_i(0) = \frac{\zeta_i}{4}$. We consider a wind turbine with a rated power of 35 kW and a rotor diameter of 14 m. Moreover, we arbitrarily set the global constraint constant at each time slot h, i.e., $L_{\min}(h) = 35$ kWh and $L_{\max}(h) = 55$ kWh for all $h \in \{1, \ldots, H\}$. The pricing scheme its divided in day-time hours, i.e., from 8:00 to 00:00 and night-time, i.e., from 00:00 to 8:00. The grid coefficients are defined as in [Mohsenian-Rad et al. 2010],[Atzeni et al. 2013a], i.e., $K_{\text{day}} = 1.5K_{\text{night}}$ and so that to obtain an initial mean price of 0.17 €/KWh [Eurostat 2017].

5.1 DSM via day-ahead pFB

In the first experiment, we consider a smart grid with 30% active users equally divided in users with only dispatchable energy generators, with only storage capabilities and users with both generation and storage devices. First, we compare the final scheduling strategies obtained by Algorithm 1 versus a DSM scheme based on the proximal decomposition algorithm (PDA), which does not consider the coupling constraints on the aggregate load in (16), [Atzeni et al. 2013a, Alg. 1]. Fig. 2 shows the aggregate load during the day corresponding to the following cases: (1) no demand-side management algorithm applied (blue bars), namely, all the users are passive users with energy consumptions randomly generated according to an average household energy consumption in the Netherlands [EDSN, the Dutch Energy Data Hub 2014], (2) PDA is applied (red bars) and (3) Algorithm 1 is used (yellow bars). The main outcome is that the storage devices are charged during the night hours, i.e., when the energy is cheaper, and discharged at peak hours. Similarly, the energy generated is dispatched mostly during the day when the price of energy is the more expensive. We note that the profile generated by Algorithm 1 is the only that satisfies the constraint on the aggregate load in (16), thus ensuring secure grid operation.

5.2 DSM via Shrinking-horizon pFB

After the day-ahead DSM, we implement the real-time adjustments via Algorithm 2. We consider the same parameters as above for 10 different random scenarios on the consumption curves for reproducibility purposes. We compare the scheduling strategies obtained by Algorithm 1, i.e., x_{a}^* and by Algorithm 2, i.e., x_{s}^* with the *ideal scenario*, i.e., x_{o}^*, namely the strategy profile obtained by Algorithm 1 (day-ahead DSM) when the prosumers have a perfect, predictive knowledge of the future wind generation. The difference on the aggregate total cost and total savings for Algorithm 1 and Algorithm 2 with respect to the best-case scenario is shown in Fig. 4. By implementing the day-ahead DSM, the final aggregate cost reduces by an average of 48% compared to the case when no DSM is implemented. Furthermore, by applying Algorithm 2, the total aggregate cost can be further reduced of about 5%.

Finally, we study how the percentage of active users in the grid influences the efficacy of the proposed Algorithm 2 in terms of aggregate load and savings. Fig. 3 illustrates the aggregate load when there are 15%, 30%, 60% of active users and the total savings on the price of energy. As N grows, the increment on the overall production and storage, allows the load curve to progressively flatten and has its higher peaks when the energy is cheaper, thus increasing the savings.

6. CONCLUSION

We presented a DSM problem for smart grid users with dispathchable energy generation and storage devices where the energy price varies with the overall aggregate load and the wind power generation. Based on game theory, we have developed a real-time DSM method that exploits the latest forecast of the wind power generation in a shrinking-horizon fashion. Numerical results conducted in a real-date scenario show that the proposed real-time DSM algorithm reduces the individual economic expenses, while satisfying the constraints set on the aggregative load.

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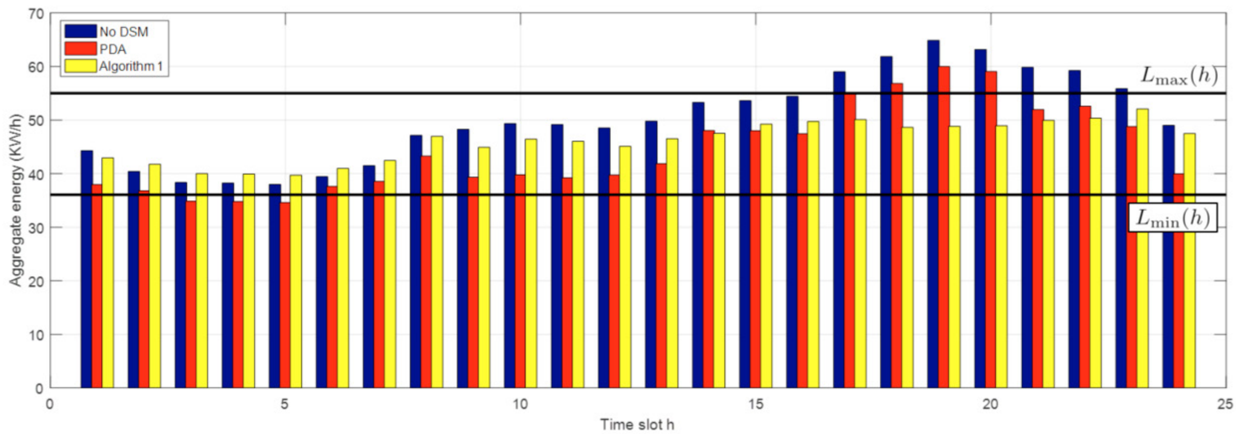

Fig. 2. Comparison on the aggregate load between a static DSM algorithm (PDA) and Algorithm 1.

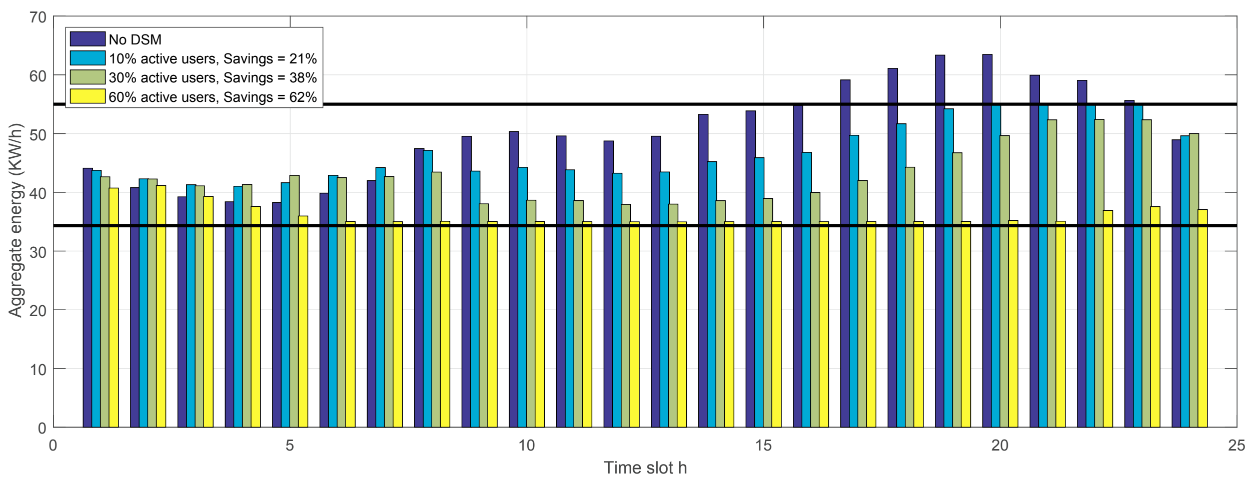

Fig. 3. Algorithm 2 applied for different percentages of active users.

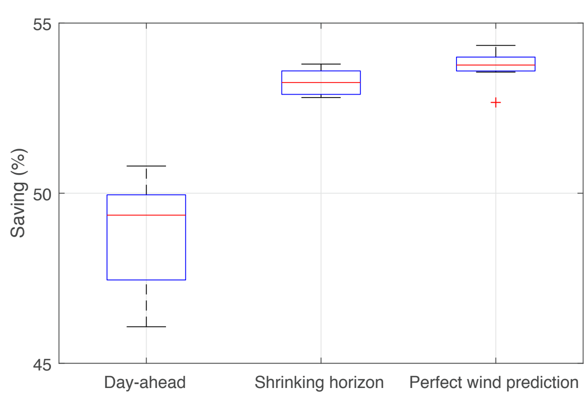

Fig. 4. Savings with respect to no DSM obtained via Algorithm 1, by Algorithm 2, and in the ideal scenario.

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