

Stability of a resonant tunneling diode

Citation for published version (APA):

Noteborn, H. J. M. F., Joosten, H. P., Lenstra, D., & Kaski, K. (1992). Stability of a resonant tunneling diode. *Proceedings of SPIE - The International Society for Optical Engineering*, 1675, 57-66.
<https://doi.org/10.1117/12.137631>

DOI:

[10.1117/12.137631](https://doi.org/10.1117/12.137631)

Document status and date:

Published: 01/01/1992

Document Version:

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
- The final author version and the galley proof are versions of the publication after peer review.
- The final published version features the final layout of the paper including the volume, issue and page numbers.

[Link to publication](#)

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license above, please follow below link for the End User Agreement:

www.tue.nl/taverne

Take down policy

If you believe that this document breaches copyright please contact us at:

openaccess@tue.nl

providing details and we will investigate your claim.

PROCEEDINGS OF SPIE

[SPIDigitalLibrary.org/conference-proceedings-of-spie](https://spiedigitallibrary.org/conference-proceedings-of-spie)

Stability of a resonant tunneling diode

Noteborn, Harry, Joosten, H., Lenstra, Daan, Kaski, K.

Harry J. M. F. Noteborn, H. P. Joosten, Daan Lenstra, K. Kaski, "Stability of a resonant tunneling diode," Proc. SPIE 1675, Quantum Well and Superlattice Physics IV, (3 September 1992); doi: 10.1117/12.137631

SPIE.

Event: Semiconductors '92, 1992, Somerset, NJ, United States

Stability of a resonant tunneling diode

H.J.M.F. Noteborn¹, H.P. Joosten², D. Lenstra³, and K. Kaski²

¹Department of Physics, Eindhoven University of Technology,
P.O.Box 513, 5600 MB Eindhoven, The Netherlands

²Department of Electronics, Tampere University of Technology,
P.O.Box 527, SF-33101 Tampere, Finland

³Department of Physics and Astronomy, Vrije Universiteit Amsterdam,
De Boelelaan 1081, 1081 HV Amsterdam, The Netherlands

ABSTRACT

Space charge build-up in the well is shown to be the cause of the inductive effects in double-barrier diodes. A new impedance model for the diode is presented, built on a static model of coherent tunneling in a selfconsistent electron potential. The corresponding equivalent circuit is made up of two capacitances – related to the charge accumulations in the emitter and in the well –, and two conductances – one for each barrier. The numerical results of this circuit model are in qualitative agreement with experimental data. The success of the earlier quantum inductance model of Brown *et al.*¹ is explained in terms of the presented model, without the need of introducing such a quantum inductance.

1. INTRODUCTION

Recent literature shows a shift in the interest of both experimentalists and theorists from the static description of Double-Barrier Resonant-Tunneling (DBRT) structures to the dynamical behaviour of these diodes¹⁻¹⁰. This has yielded a number of equivalent circuit models²⁻⁶, most of which use two elements, the differential conductance G and the diode's capacity C , to describe the experimental data. Recently, Brown *et al.*¹ have suggested that the addition of a third element, a 'quantum inductance' L_q , to such models improves the agreement with experimental results considerably. Important point of comparison is the cutoff frequency, that is decreased by the inductance. However, a fundamental theoretical derivation of the quantum inductance is lacking. The delay of current with respect to voltage is intuitively expected to be related to 'the buildup or decay of the wave function in the quantum well'¹. The magnitude of L_q is therefore taken to be t_0/G , where t_0 is the life time of the resonant state. An elaboration of this model has been given

by Boudreau and Liu¹⁰.

In this paper a new equivalent circuit for the DBRT–diode is proposed, based on a static model of coherent electron tunneling in a selfconsistent potential. Because of the selfconsistency of the calculations, the charge buildup in the well is already taken into account. In the equivalent circuit this buildup is encountered as an extra capacitance. Our model is able to reproduce the cutoff frequency shifts without the need of introducing an inductive element.

The static model which serves as a starting point for our calculations, is described in Section 2. The equivalent circuit model is then introduced in Section 3, and compared with the inductance model. Section 4 discusses the stable dc biasing of the DBRT–diode. Finally, Section 5 gives the concluding remarks.

2. STATIC MODEL

The operation of the DBRT–diode is based on the phenomenon of quantummechanical tunneling through a potential barrier. In the familiar GaAs–Al_xGa_{1-x}As diode, the barriers are formed by the two undoped layers of Al_xGa_{1-x}As, enclosing an undoped GaAs well, and sandwiched by Si–doped GaAs layers (see Table 1). Since the band gap energy of GaAs is smaller than that of Al_xGa_{1-x}As, both the conduction and valence band do not line up. It is the offset of the conduction band minimum that serves as a barrier to the conduction electrons, provided by the ionized doping.

Table 1.

layer		donor concentration (cm ⁻³)	width (nm)
n ⁺ layer	GaAs	2.10 ¹⁸	
n layer	GaAs	2.10 ¹⁶	101.76
barrier	Al _{0.42} Ga _{0.58} As	–	5.091
well	GaAs	–	5.088
barrier	Al _{0.42} Ga _{0.58} As	–	5.091
n layer	GaAs	2.10 ¹⁶	50.88
n ⁺ layer	GaAs	2.10 ¹⁸	
lateral diameter: 26 μm			

To find at a certain bias V_b the electron potential, or equivalently the conduction band edge $E_c(z)$, where z is the coordinate in the tunneling direction, we divide the structure into several layers, from emitter to collector (see Fig.1 and Table 1), and solve in each layer the Poisson equation:

$$\nabla(\epsilon_r(\nabla E_c)) = e \cdot \rho \quad (1)$$

with boundary conditions $E_c=0$ in the emitter and $E_c=-e \cdot V_b$ in the collector. The charge density ρ in the accumulation layer in front of the first barrier is calculated in the Thomas–Fermi screening approximation¹¹:

$$\rho = e \cdot (N_d - N_c \mathcal{F}_{1/2}(\frac{E_F - E_c}{kT})) \quad (2)$$

where N_d is the donor concentration, N_c the effective number of electron states in the conduction band and kT the thermal energy. E_F is the Fermi energy in the charge neutral emitter contact, and \mathcal{F}_j is the j -th order Fermi–Dirac integral¹¹. A similar equation applies to the charge density in the depletion layer after the second barrier. The charge density in the well is the result of quantummechanical tunneling. Electrons in the accumulation region can tunnel to the depletion region, and therefore have a certain probability density to be found in the well. If $F_n(z)$ is the wave function of the electrons with incoming energy E_n , the charge density in the well is:

$$\rho = -e \sum_n |F_n|^2 f(E_n) \quad (3)$$

where $f(E)$ is the Fermi–Dirac distribution function. The wave functions F_n and energies E_n can be found by solving the time–independent Schrödinger equation:

$$-\frac{\hbar^2}{2} \nabla \left(\frac{1}{m} (\nabla F_n) \right) + E_c F_n = E_n F_n \quad (4)$$

Since m is the effective mass of electrons in the conduction band, Eq.(4) is called the effective mass equation, and F_n the envelope function. The Hamilton operator on the lhs of (4), known as the Ben Daniel–Duke hamiltonian, is hermitian even with a position–dependent mass¹². The index n of the electron wave functions and energies denotes the good quantum numbers in the tunneling problem, viz. the lateral momentum and the total energy, and is in fact continuous. The integration over n includes all incoming energies $E > E_{c,ac}$ where $E_{c,ac}$ is the average potential in

Fig.1. Spatial variation of conduction band edge in DBRT structure under bias. Dashed lines indicate average value per layer.

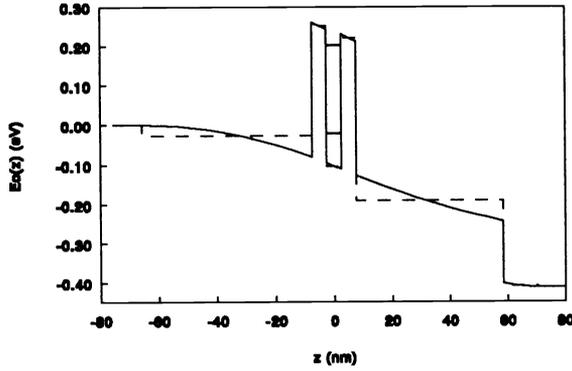
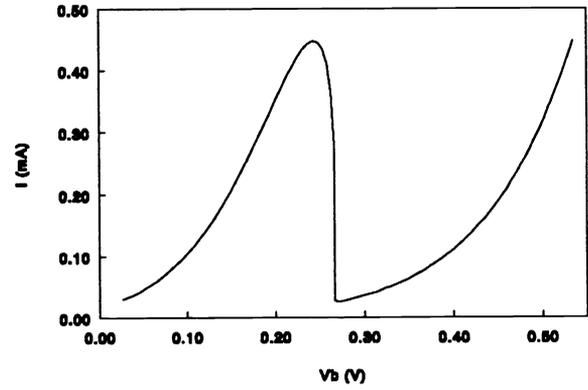


Fig.2. Current-voltage characteristic for diode of table 1 at T = 300 K.



the accumulation region.

Eqs.(1-4) form a coupled set of equations for the band edge E_c and the charge density ρ , that can be solved iteratively. With the so found selfconsistent potential, we determine the tunnel current density, using a formula similar to (3):

$$J = -e \sum_n |F_n|^2 \frac{\hbar}{m} \nabla \arg(F_n) f(E_n) \quad (5)$$

In Fig.2 the room-temperature $I-V_b$ characteristic for the DBRT-diode of Table 1 is plotted. Parameters are chosen in agreement with Refs. 1 and 13. The selfconsistent charge densities in the accumulation layer, the well, and the depletion layer are plotted in Fig. 3 as a function of bias voltage V_b .

From multiple reflections within the well between the barriers a resonance or quasibound state originates, corresponding to a particular incoming momentum p_z along the growth axis. As a consequence, only electrons with p_z near the resonant momentum contribute substantially to the summation in (3) and (5). We can therefore approximate the charge and current density by¹⁴:

$$\rho = -e \cdot N_c^{2/3} \mathcal{F}_0\left(\frac{E_F - E_r}{kT}\right) \frac{P_1}{P_1 + P_2} \cdot \frac{1}{w} \quad (6)$$

$$J = -e \cdot N_c^{2/3} \mathcal{F}_0\left(\frac{E_F - E_r}{kT}\right) \frac{1}{2} \frac{P_1 P_2}{P_1 + P_2} \cdot \frac{w}{v_r} \quad (7)$$

Fig.3. Charge densities in depletion layer, well and accumulation layer vs. applied voltage.

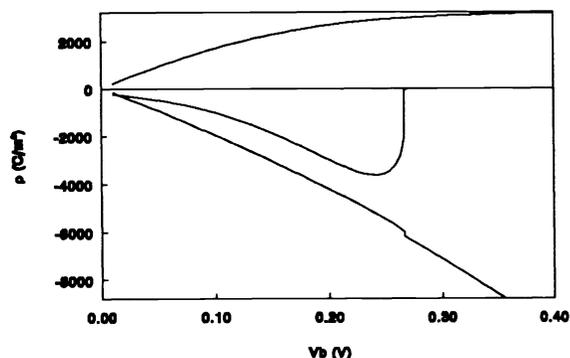
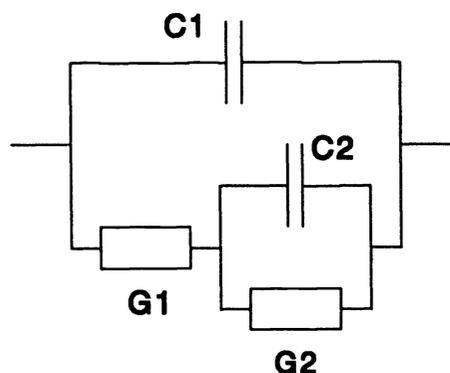


Fig.4. Equivalent Circuit for DBRT-diode.



where E_r is the resonance energy with respect to the emitter band edge, $v_r = \sqrt{\frac{2(E_r - E_{c,w})}{m}}$ is the resonant velocity in the well ($E_{c,w}$ being the average potential in the well), and P_1 (P_2) is the transmission probability through the first (second) barrier for electrons in the resonant state. This approximation is valid if both P_1 and P_2 are much smaller than unity, which is always the case in DBRT structures. An expression for the dwell time t_d of the electrons in the well, defined by the quotient of $\rho w/J$, is now found to be $\frac{2}{P_2} \cdot \frac{w}{v_r}$. For the biased structure, this dwell time is approximately equal to the resonance life time t_0 , defined as the reciprocal width of the resonance peak, $t_0 \equiv \hbar/\Gamma = \frac{2}{P_1 + P_2} \cdot \frac{w}{v_r}$. A necessary condition for $t_0 \approx t_d$ is that $P_2 \gg P_1$, a condition which is easily fulfilled when the structure is biased. Both t_0 and t_d will be met again in the next section on the dynamical behaviour.

3. IMPEDANCE MODEL

There are several approaches to deducing an impedance model starting from a static description of the diode behaviour. In Ref. 15 dynamical equations for the charge densities in the accumulation layer and the well are introduced. The diode impedance can be expressed in the coefficients of the linearized, Fourier-transformed version of these dynamical equations. Finally this impedance expression is related to the equivalent circuit of Fig. 4. In this paper, we go in opposite direction, taking the equivalent circuit of Fig. 4 as our starting point. This choice is supported by the direct and natural connection of this circuit to the physics of the electron tunneling: an incoming electron from the emitter can either stay in the accumulation layer

Fig.5. Real (solid curve) and imaginary part of diode admittance vs. bias for $f=20\text{GHz}$.

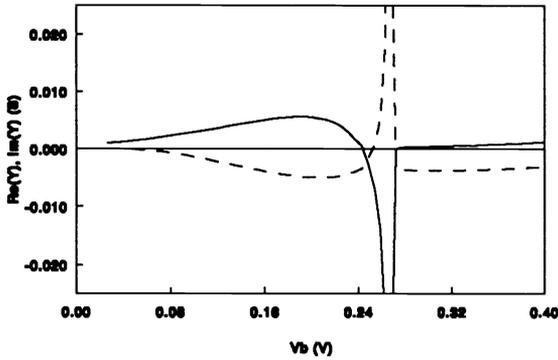
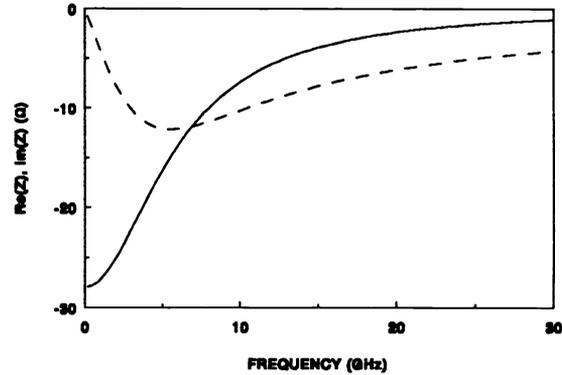


Fig.6. Real (solid curve) and imaginary part of impedance vs. frequency at $V_b=0.2661\text{V}$.



(contributing to C_1) or pass the first barrier and reach the well (contributing to G_1). In the well, the electron has again the possibility of staying (C_2) or tunneling into the collector (G_2). The four elements of the circuit are easily related to the static $I-V_b$, $Q_{ac}-V_b$ and Q_w-V_b characteristics, where Q_{ac} (Q_w) is the total charge in the accumulation region (well). If V_1 is the potential drop from emitter to well, and V_2 the potential drop from well to collector, so that $V_1+V_2=V_b$, we define:

$$G_1 = \frac{dI}{dV_1}, \quad G_2 = \frac{dI}{dV_2}, \quad C_1 = \frac{dQ_{ac}}{dV_b}, \quad C_2 = \frac{dQ_w}{dV_2} \quad (8)$$

with the derivatives taken along the selfconsistent curve. The complex impedance corresponding to the circuit of Fig. 4 is:

$$Z(\omega) = \frac{1 + i\omega\tau}{G - \omega^2 C_1 \tau + i\omega(C_1 + \tau G_1)} \quad (9)$$

where $G = G_1 G_2 / (G_1 + G_2) = dI/dV_b$, the differential conductance, and the time constant $\tau = C_2 / (G_1 + G_2)$. For small ω the impedance is $1/G$, whereas in the high frequency limit it becomes capacitive, $Z(\omega) \rightarrow 1/i\omega C_1$. The time constant τ can be related to the dwell time t_d . Using (8) we find $\tau = \frac{dQ_w}{dI} \cdot \frac{dV_1}{dV_b}$. Since $Q_w \approx I \cdot t_d$ and t_d is almost constant, and since $\frac{dV_1}{dV_b} \approx \frac{1}{2}$, the two times τ and t_d are of the same order of magnitude. In Figs. 5 and 6 the real and imaginary part of the impedance (or its reciprocal the admittance) are shown as function of applied voltage, and of frequency, respectively. The latter concerns a point in the region of negative differential resistance

Fig.7. Oscillation power vs. frequency for RC-(dotted), L_q -(dashed) and present model (solid curve).

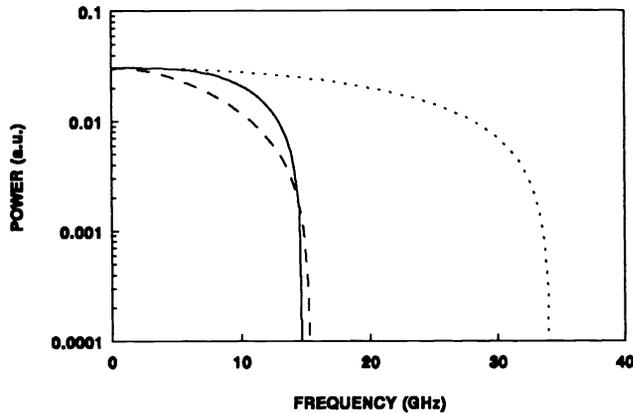
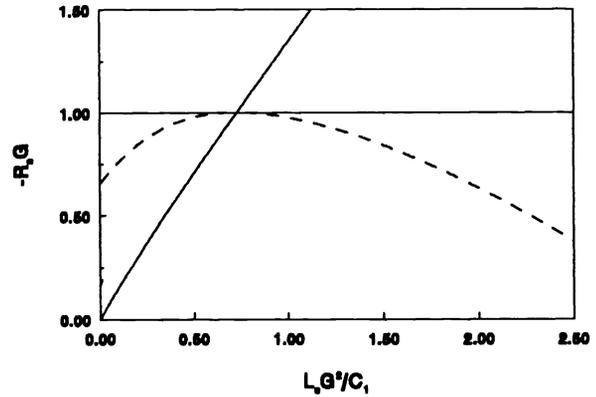


Fig.8. Stability chart for $V_b=0.2661V$. For explanation see Sect. 4.



(NDR), as can be seen from the negative offset of $\text{Re}(Z(\omega))$. The NDR region is interesting since here the circuit is capable of oscillating at frequencies ω for which $\text{Re}(Z(\omega)) < 0$.

The impedance expression (9) can be compared to the impedance in the quantum-inductance model of Brown *et al.*¹ There, the capacitance C_2 parallel to G_2 is substituted by a quantum inductance L_q in series with G_2 , and of magnitude t_0/G , where t_0 is the resonance life time. The resulting impedance:

$$\frac{1 + i\omega t_0}{G - \omega^2 C t_0 + i\omega C}$$

agrees with (9) if $\tau \approx t_0$ and $C_2 \frac{G_1}{G_1 + G_2} \ll C_1$, in which case the three elements of this model, G , C and L_q , can be related to the four of our model in the following way: $G = G_1 G_2 / (G_1 + G_2)$, $C = C_1 + C_2 G_1 / (G_1 + G_2)$ and $L_q = C_2 / G_1 G_2$. Since τ and t_d are of the same order of magnitude, as we have seen above, and $t_d \approx t_0$, as was shown in the previous section, we find that the two time constants concur indeed. Also the second condition is met in most points of the NDR interval. The graphical counterpart of this formal resemblance of the two models is Fig. 7, where the oscillation power P is plotted as a function of frequency. This power P , delivered to a load impedance by the diode when oscillating, can be shown to be proportional to $-\text{Re}(R_s + Z_{\text{dbrt}}) / |-Z_{\text{dbrt}}|^2$, where R_s is a series resistance that comprises ohmic loss outside the active region¹. From Fig. 7 it is seen that the inductance model and the present model agree on the magnitude of the cutoff frequency at which $\text{Re}(R_s + Z_{\text{dbrt}})$ or P vanishes, and both differ in

this respect from the usual RC-model.

4. STABILITY OF DBRT-DIODE

A point of operation in the NDR region of the $I-V_b$ curve, representing a selfconsistent solution of the time-independent Schrödinger and Poisson equation, is called 'stable' in this section, if the diode can be stably dc biased in this point. Stability in this sense is related to the zeros of the total impedance Z_{tot} , the sum of the diode impedance and that of the biasing circuit. With $i\omega$ in Z_{tot} replaced by the complex s , the equation $Z_{tot}(s) = 0$ yields complex zeros s_j , $j=1,2,3$, all of which must have a negative real part in order for the point of operation to be stable. If the imaginary parts of s_j are zero, small fluctuations around the stable point will damp out exponentially. With nonzero imaginary part the damping will be oscillatory^{15 16}.

The biasing circuit is represented by a series resistance R_s and a series inductance L_s thought to be due to contacts and leads⁴. In a certain point of operation the stability will now depend on the values of R_s and L_s . A plot of the stable regions in the R_s-L_s plane is called a stability chart^{10 15 16}. In Fig. 8 such a stability chart is drawn, using the dimensionless resistance $-R_s G$ and the dimensionless inductance $L_s G^2 / C_1$. The region of stability is the solid triangle bounded by $L_s = 0$ and $-R_s G = 1$. Below the dashed curve stability corresponds to damped oscillations and instability to growing oscillations. Above this dashed curve the damping or growing is exponential. From Fig. 8 it is seen that stabilizing a DBRT-diode amounts to minimizing L_s . The stability is then limited by the demand that $R_s < 1/|G|_{max}$. How the values of L_s and R_s are related to the structural parameters was studied in Ref. 4.

5. CONCLUSION

Since some of the features of the static model are inherited by the impedance model, we will first evaluate the former. Comparison with experimental results¹³ shows that the shape of the $I-V$ characteristic is reproduced well, but that both the current and voltage scale are wrong. In the case of the voltage scale, this is due to potential drops in the contacts and leads, that are not described by our model. In the case of the current, the difference is caused by the Fermi screening expression for the accumulation charge. For high temperatures and low donor concentration, this approach tends to overestimate $E_{c,ac}$ and thus yield too small an energy window for incoming tunneling electrons. It is the width of this energy window that largely determines the peak current. A second remark concerns the fact that at high mole-fractions of Al in the barriers, tunneling through the X minimum becomes important, whereas in our model only the direct Γ

tunneling is taken into account. Finally, no (inelastic) scattering of tunneling electrons is considered. These processes increase the valley current, wherefore a peak to valley ratio predicted by a coherent-tunneling model is always unrealistically large. We find a ratio of 17, whereas the experiment says 3.9¹³.

The electrostatic feedback of the charge in the well yields a voltage region of current bistability. Although this bistability is only of importance in asymmetric structures, theoretically it is always present and it causes the differential conductance to go to $-\infty$ at the end of the NDR region. As a result, our impedance model predicts a voltage region of instability for every DBRT-diode. In experiments however, scattering processes keep the differential conductance finite, and stable dc biasing can in principle be achieved for the complete NDR region. With respect to Figs. 6 and 7 it is remarked that the frequency scale cannot be expected to be better than the voltage or current scale.

The above draw-backs are in fact common to most DBRT models. The advantages of the present model are that all space charges are taken into account selfconsistently, while the coherent tunneling allows of analytical expressions. Furthermore, our main point, that the inductive effects can be explained within a selfconsistent model, is in no way dependent on the particular static model that is used for the numerical calculations. The involved time constant is shown to be of the same magnitude as the dwell time, which in turn is approximately equal to the resonance life time. Thus a theoretical basis is given to an ad hoc circuit adjustment.

Summarizing, we have presented a new impedance model for the DBRT-diode derived from a static theory that takes into account the effect of the charge buildup in the well. The selfconsistency of the underlying static model suffices to explain the experimentally found inductive effects.

6. AOKNOWLEDGMENTS

This work is part of the research programme of the Foundation for Fundamental Research on Matter (FOM), which is financially supported by the 'Nederlandse Organisatie voor Wetenschappelijk Onderzoek' (NWO).

7. REFERENCES

1. E.R. Brown, C.D. Parker and T.C.L.G. Sollner, "Effect of quasibound-state lifetime on the

- oscillation power of resonant tunneling diodes", *Appl. Phys. Lett.* 54 (10), pp. 934–936, March 1989; E.R. Brown, T.C.L.G. Sollner, C.D. Parker, W.D. Goodhue and C.L. Chen, "Oscillations up to 420 GHz in GaAs/AlGaAs resonant tunneling diodes", *Appl. Phys. Lett.* 55 (17), pp.1777–1779, 1989.
2. T.C.L.G. Sollner, P.E. Tannenwald, D.D. Peck and W.D. Goodhue, "Quantum well oscillators", *Appl. Phys. Lett.* 45 (12), pp.1319–1321, 1984.
 3. S.K. Diamond, E. Ozbay, M.J.W. Rodwell, D.M. Bloom, Y.C. Pao and J.S. Harris, "Resonant tunneling diodes for switching applications", *Appl. Phys. Lett.* 54 (2), pp.153–155, 1989.
 4. J.M. Gering, D.A. Crim, D.G. Morgan, P.D. Coleman, W. Kopp and H. Morkoç, "A small-signal equivalent-circuit model for GaAs–Al_xGa_{1-x}As resonant tunneling heterostructures at microwave frequencies", *J. Appl. Phys.* 61 (1), pp.271–276, 1987.
 5. D. Lippens and P. Mounaix, "Small-signal impedance of GaAs–Al_xGa_{1-x}As resonant tunneling heterostructures at microwave frequency", *Electronics Letters* 24 (18), pp.1180–1, 1988.
 6. A. Zarea, A. Sellai, M.S. Raven, D.P. Steenson, J.M. Chamberlain, M. Henini and O.H. Hughes, "Impedance analysis of GaAs/Al(Ga)As resonant tunneling diodes in the NDR region", *Electronics Letters* 26 (18) pp.1522–3, 1990.
 7. C. Kidner, I. Mehdi, J.R. East and G.I. Haddad, "Bias circuit instabilities and their effect on the d.c. current voltage characteristics of double-barrier resonant tunneling diodes", *Solid State Electron.* 34, No. 2, pp.149–156, 1991.
 8. W.R. Frensley, "Quantum transport calculation of the small-signal response of a resonant tunneling diode", *Appl. Phys. Lett.* 51, pp.448–450, 1987.
 9. H.C. Liu, "Time-dependent approach to double-barrier quantum well oscillators", *Appl. Phys. Lett.* 52, pp.453–455, 1988.
 10. M.G. Boudreau and H.C. Liu, "Effects of the quantum inductance on the equivalent circuit model of a resonant tunneling diode", *Superlattices and Microstructures* 8, No. 4, pp.429–432, 1990.
 11. W.R. Frensley, "Quantum kinetic theory of nanoelectronic devices", Nanostructure physics and fabrication, eds. M.A. Reed and W.P. Kirk, Academic Press, San Diego, 1989; J.S. Blakemore, Semiconductor Statistics, Pergamon Press, Oxford, 1962.
 12. G. Bastard, Wave mechanics applied to semiconductor heterostructures, John Wiley and Sons, New York, 1988.
 13. C.I. Huang, M.J. Paulus, C.A. Bozada, S.C. Dudley, K.R. Evans, C.E. Stutz, R.L. Jones and M.E. Cheney, "AlGaAs/GaAs double barrier diodes with high peak-to-valley current ratio", *Appl. Phys. Lett.* 51 (2), pp.121–123, 1987.
 14. H.P. Joosten, H.J.M.F. Noteborn and D. Lenstra, "Numerical study of coherent tunnelling in a double-barrier structure", *Thin Solid Films* 184, pp.199–206, 1990; H.J.M.F. Noteborn, H.P. Joosten and D. Lenstra, "Selfconsistent study of coherent tunneling through a double barrier structure", *Physica Scripta T33*, pp.219–226, 1990.
 15. H.P. Joosten, H.J.M.F. Noteborn, K. Kaski and D. Lenstra, "Stability of the selfconsistently determined current of a double-barrier resonant-tunneling diode", *J. Appl. Phys.* 70 (6), pp.3141–3147, 1991; H.P. Joosten, H.J.M.F. Noteborn, K. Kaski and D. Lenstra, "Effect of a parallel capacitor on the stability of the DC-configurations of a DBRT-structure", *Physica B* 175, pp.297–300, 1991.
 16. J.O. Scanlan, Analysis and Synthesis of Tunnel Diode Circuits, John Wiley & Sons, New York, 1966.