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Probabilistic fatigue resistance model for steel welded details under variable amplitude loading – Inference and uncertainty estimation

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\textbf{ABSTRACT}

This paper presents a probabilistic fatigue life prediction model for welded details subjected to variable amplitude random loading, using the nominal stress approach. The proposed model makes use of CA S-N curves in combination with the relative damage rule which allows to globally account for load interaction effects, and the damage limit concept that permits the estimation of the damage for the stress ranges lower than the fatigue limit and is based on linear elastic fracture mechanics. Epistemic (sampling) and aleatory (statistical) uncertainties are estimated in a statistical framework, making the model applicable for structural reliability analyses.

1. Introduction

Performing fatigue tests under Constant Amplitude (CA) loading is of primary importance to characterize the fatigue resistance of a material or a structural component. As proposed by Wöhler, the test data are plotted relating either the amplitude or the range of the fluctuating stress, \( \Delta \sigma /2 \), as function of the number of cycles to failure, \( N \), the latter in log-scale. Since then, many S-N models were proposed by different authors. However, not all of them reflect the frequently observed trends of the fatigue life vs. the stress range \([1]\): (1) the increasing variability of the logarithm of the fatigue life with decreasing stress range, (2) the smooth transition between infinite and finite life, and (3) the presence of the endurance strength, intended as the threshold stress for small surface crack propagation, hereby referred to as fatigue limit. Statistical analysis of fatigue test data was proposed by scientists since the very early works \([2,3]\). Recent contributions allowed the development of advanced statistical models and techniques to infer CA fatigue test data \([4-8]\). In some of these models, the fatigue limit is modelled as a random variable, addressing all the three points listed above, whereas in some cases solely the first two previously mentioned points are addressed. The Six Parameters Random Fatigue Limit Model (6PRFLM) was proposed by the present authors and it allows a more accurate inference of CA fatigue test data at relatively low stress ranges, when compared to other models having similar features \([7]\).

However, a resistance model obtained under CA loading is rarely applicable for service loading. This is ascribed to the different nature of the applied load in real conditions. The load might fluctuate resulting in stress cycles of variable mean stress and amplitude, i.e. variable amplitude (VA) loading, and the sequence of the load cycles is often nondeterministic, i.e. random. Bridge structures, for example, are loaded as a result of the traffic, which includes various vehicles with different (axle) weights crossing the bridge \([9]\). The resulting load history is often condensed in a stress spectrum obtained by a cycle counting procedure, such as the Rainflow counting procedure \([10]\). Due to the stochastic nature of the occurrence of a certain stress range, it follows that, independently from the counting procedure that is employed, the stress spectrum must provide information about the frequency of occurrence of a certain stress range or, alternatively, the frequency of exceeding a certain stress range \([11]\). Different types of VA stress history influence the fatigue performance of the structural component because of load interaction and sequence effects. The Load interaction effect is caused by the presence of underloads and overloads \([12,13]\), affecting both the crack initiation and the crack propagation life. In the crack initiation phase the formation of dislocations and persistent slip bands is altered \([14-16]\), whereas in the crack propagation phase, crack tip blunting/sharpening, crack closure and residual stresses in the wake of the crack influence the rate of crack propagation \([12,17]\). The load sequence effect is caused by the same phenomena resulting in the load interaction effect, but it is usually identified for variable amplitude block loading and concerns the effect of the order in which blocks of different load amplitudes are applied \([1]\). It should be noted that load sequence is not present in random block loading. In other words, the effect of a stress range in the damage accumulation depends on the stress history which precedes its application. Several experimental
studies showed the load sequence and load interaction effect of the VA load type on the fatigue life with random sequence of individual cycles, see [18,19], among others.

The nominal stress approach for fatigue design requires inferring experimental data in order to characterize the fatigue resistance of a welded detail. The statistical inference allows estimating the properties of a population, e.g. the fatigue resistance of a welded detail, by analysing test data achieved from a finite number of samples. The inferential process estimates the parameters of the model, but also the uncertainty underlying the estimation. With respect to the definition of uncertainty, reference is made to two types [20–22]: (1) aleatory uncertainty, to indicate the intrinsic variability of a certain quantity, e.g.

<table>
<thead>
<tr>
<th>Symbols</th>
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<tr>
<td>$\alpha$</td>
<td>geometry dependent constant for $M_0$</td>
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</tr>
<tr>
<td>$\beta_0$</td>
<td>parameter of the regression models</td>
<td>$\sigma_0$</td>
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<td>$\beta_1$</td>
<td>parameter of the regression models</td>
<td>$\sigma_s$</td>
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<tr>
<td>$\Delta$</td>
<td>nominal stress range</td>
<td>$\sigma_{\text{cr}}$</td>
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<tr>
<td>$\Delta N_{\text{th}}$</td>
<td>fatigue limit</td>
<td>$D$</td>
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<tr>
<td>$\Delta N_{\text{th}}$, $\Delta N_{\text{th},\text{eff}}$</td>
<td>threshold stress range for fatigue damage accumulation</td>
<td>$E$</td>
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<tr>
<td>$\Delta K_{\text{th}}$</td>
<td>threshold Stress Intensity Factor range for small cracks</td>
<td>$F_{\text{F}}$</td>
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<td>$\Delta K_{\text{th},\text{eff}}$</td>
<td>threshold Stress Intensity Factor range for long cracks</td>
<td>$f_v$</td>
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<tr>
<td>$\Delta d$</td>
<td>dispersion parameter of the Rayleigh distribution</td>
<td>$F_{\text{W}}$</td>
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<tr>
<td>$\Omega_j$</td>
<td>location parameter of the Rayleigh distribution</td>
<td>$f_w$</td>
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<tr>
<td>$\Phi$</td>
<td>cumulative probability function</td>
<td>$G_w$</td>
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<td>$\phi$</td>
<td>probability density function</td>
<td>$h$</td>
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<td>$\psi_{\text{nc}}$</td>
<td>normalised frequency for the spectrum to be higher than the fatigue limit</td>
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<tr>
<td>$\sigma_{\text{fr}}$</td>
<td>standard deviation of the critical damage</td>
<td>$j$</td>
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<td>$\sigma_{\text{fr}}$</td>
<td>standard deviation of the fatigue life</td>
<td>$k$</td>
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<td>$\hat{\Theta}_i$</td>
<td>vector of the parameters of the CA model</td>
<td>$m_1$</td>
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<td>$\hat{\Theta}_{\text{VA}}$</td>
<td>vector of the parameters of the VA model</td>
<td>$m_{\text{CA}}$</td>
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<td>vector of the parameters of the VA model</td>
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<td>$a$</td>
<td>crack depth</td>
<td>$n_i$</td>
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<tr>
<td>$a_{\text{fit}}$</td>
<td>fitting parameter for El-Haddad model</td>
<td>$n_{\text{VA}}$</td>
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<td>$a_{\text{fit}}$</td>
<td>crack depth related to $N = N_k$</td>
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<td>$a_{\text{ini}}$</td>
<td>initial crack depth</td>
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<td>$a_k$</td>
<td>crack depth at $N = N_k$</td>
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<td>$\bar{\mu}_{\text{V}}$</td>
<td>mean of $V$</td>
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<td>$\bar{\mu}_{\text{V}}$</td>
<td>mean of $V$</td>
<td>$W$</td>
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<td>$\nu$</td>
<td>base 10 logarithm of the fatigue limit</td>
<td>$x$</td>
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<td>$\mu_{\text{V}}$</td>
<td>mean of $V$</td>
<td>$y$</td>
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<td>$\sigma$</td>
<td>standard deviation of the critical damage</td>
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<td>$\sigma_{\text{fr}}$</td>
<td>standard deviation of the fatigue life</td>
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the observed scatter of the fatigue life; (2) epistemic uncertainty, or stochastic uncertainty, to indicate the uncertainty about a model parameter and the random outcome, e.g. the confidence distribution, of which the width is strongly dependent on the amount of data available. The epistemic uncertainty is further conditional to the assumed model [22]. The identification of models and methods to estimate the aleatory and epistemic uncertainty in CA and VA fatigue test data analysis has been object of several works, using both frequentist [6,7,23,24], and Bayesian [7,23,25,26] approaches.

In [27] a fatigue reliability framework for welded details based on the nominal stress approach is presented. The proposed fatigue resistance model is based on the deterministic fatigue resistance model used in Eurocode 3, where the effect of the stress ranges lower than the fatigue limit is accounted by extending the CA S-N curve below the fatigue limit using a modified slope. The estimated variability of the fatigue life is assumed based on information from guidelines for fatigue design and is not based on direct estimation from data. By using a similar approach as in Eurocode 3, a probabilistic model to perform the calibration of the second slope was defined in [6]. CA and VA fatigue test data related to the welded cover plated steel beam detail were used to estimate the parameters of the model proposed and their statistical uncertainty was estimated using the Maximum Likelihood Method. The probabilistic VA fatigue resistance models proposed in [6,27] determine a constant damage rate function, independently of the cumulated damage. The cumulated damage is a linear function of the number of cycles to failure. The available VA fatigue prediction models that make use of nominal S-N curves are such that the stress ranges lower than the fatigue limit are considered either as not contributing to the accumulation of the fatigue damage, as postulated by Miner [28], or less/equally damaging as the stress ranges above the fatigue limit, but independently of the damage level, i.e. the crack size, [9,29,30]. Therefore, these models are lacking a phenomenological foundation.

A phenomenological explanation of the damage produced by the stress ranges below the fatigue limit is given in [9], and depicted in Fig. 1. The author defines the threshold stress range, \( \Delta \sigma_{0} \), as the lowest stress range leading to crack propagation. Therefore, crack propagation occurs if \( \Delta \sigma > \Delta \sigma_{0} \), see shaded stress ranges in Fig. 1. A larger crack leads to a lower threshold stress range. As a consequence of the analogy between crack size and damage, [31], the threshold stress range decreases with increasing cumulated damage, \( D \). When a fatigue crack has nucleated, the threshold stress range, \( \Delta \sigma_{0} \), can be considered as the damage limit instead of the CA fatigue limit, \( \Delta \sigma \). The threshold stress range for fatigue damage accumulation is a function of the number and magnitude of the applied stress cycles and it is characterized by a decreasing trend, see Fig. 1. Its value depends on the previous stress history, similarly to the cumulated fatigue damage. Based on these assumptions, an analytical damage limit model was proposed by Kunz [32]. According to it, the threshold stress range, see Fig. 1, can be expressed as function of the damage in the following form:

\[
\Delta \sigma_{th}(D) = \Delta \sigma_{0} f(D)
\]  

where the function \( f(D) \) was obtained using considerations based on Linear Elastic Fracture Mechanics (LEFM).

The current paper is intended to formulate a sound probabilistic model to estimate fatigue life of welded details for random block VA loading using the nominal stress approach. The model makes use of two elements. The first is a characterization of the fatigue resistance under CA loading, i.e. an S-N curve for the considered welded detail. The second is a damage rule in order to consider the effect of the application of different stress ranges, above and below the fatigue limit. The formulation of the proposed model is presented in Section 3. Successively, the method used to infer fatigue test data is described in Section 4. This includes the estimation of the model parameters and the epistemic uncertainty. In Section 5, the results of the inferential process are presented for a certain welded detail. A comparison is made with other VA fatigue prediction models. Further results related to other welded details are presented Appendix A. It appears that proposed model is able to better interpret the fatigue damage phenomenon in welded details, when compared to other models based on CA S-N curves and the nominal stress approach. Therefore it represents a valuable tool for fatigue reliability analysis of welded details.

2. Fatigue under VA loading using fatigue resistance curves

As mentioned in the introduction, the nominal stress approach for fatigue design requires inferring experimental data in order to characterize the fatigue resistance of a welded detail. In the case of VA loading, in order to infer useful information for the purpose of designing against fatigue, the stress history to be applied in testing should be based on in-service measurements [33]. This concept was introduced by Gassner [34,35], and it is known as operational fatigue strength, intended as dimensioning a structural component for the design life under variable amplitude loading, based on in-service measurements. It was suggested to standardize the shape of the load spectrum, defining load spectra dependent on the field of application. VA fatigue test results could be plotted using the Gassner curve, which is similar to a S-N curve but derived for a determined type of stress spectrum. The experimental results are presented in log–log scale with the maximum stress range of the spectrum on the ordinate axis and the total number of cycles to failure, \( N \), on the abscissa. This is a generalization of the concept of S-N curves: a Gassner curve obtained for a certain stress spectrum corresponds to the S-N curve shifted towards longer life by a certain factor that is depending on the type of load spectrum [29], which can be estimated by phenomenological approaches [36] or evaluated by performing tests [37]. Therefore, the procedure to test and design a structural component against VA fatigue is related to the type of stress history and the amount of the acceptable uncertainty required [1]. For example, structural components subjected to a predetermined

![Fig. 1. Effect of changing the threshold stress range due to crack propagation, adapted from [9]: (1) Before crack initiation at the notch root, (2) a crack has initiated, (3) the crack has propagated. The shaded portion of the stress spectrum denotes the stress ranges contributing to the propagation of the crack, since \( \Delta \sigma > \Delta \sigma_{0} \). According to LEFM, with increasing the crack size, the threshold stress range decreases, starting from the fatigue limit.](image-url)
load history can be tested considering the actual load history to be faced during the service life, allowing an experimental determination of the fatigue life and its variability. In other cases, when the load history is not known a priori, it is not possible to test the structural component by applying a deterministic load history. The stress spectrum is obtained by measuring service loading and applied to the specimen by repeated blocks of the simulated stress history having a predetermined length, as done in [38].

Without using Gassner curves, the VA fatigue life can be estimated based on the fatigue resistance obtained for CA loading. Therefore it is necessary to use the CA S-N curve, which is often modified below the fatigue limit, and a damage rule. More than 50 alternative fatigue damage accumulation rules were presented in the literature up to the end of the previous century [39]. Since then, many others have been proposed by the scientific community. In the civil engineering field, where the nominal stress approach is widely used, it is common practice to estimate the VA fatigue life using an S-N curve for each welded detail and the linear damage rule (LDR) proposed by Palmgren and Miner [28,40]:

$$D = \sum \frac{n_i}{N_i} \quad (2)$$

where $D$ is the damage caused by the load spectrum, $n_i$ and $N_i$ are the number of occurrences of a certain spectral stress range and the predicted number of cycles to failure at that same stress range, $\Delta\sigma$, respectively. Failure is deemed to occur when $D = 1$. The LDR assumes that the fatigue damage is linearly related to the cycle ratio $n_i/N_i$, neglecting the effects of load sequence and load interaction. In order to deal with load sequence and interaction effects, the relative linear damage rule (RLDR) was proposed [41]. The formulation of the fatigue damage, Equation (2), remains unchanged but the critical damage $D_{cr}$ is set based on the experience gained from VA fatigue tests and is meant to be related to a specific type of load history. In this way, the load sequence effect and/or load interaction effects are globally accounted for, based on the results of experiments.

With reference to steel bridge infrastructures, guidelines and standards for fatigue design, such as EN 1993–1-9 (Eurocode 3) [42], or the AASHTO specifications, use similar approaches to account for VA loading: the CA S-N curve is extended to stress ranges lower than the fatigue limit by a linear log–log relation and the damage is calculated according to the LDR. Therefore, $D_{cr}$ is not calibrated and there is no direct reference to the use of the Gassner curves. The S-N curve is modified in such a way that $D_{cr} = 1$, in case of VA loading. For the extended branch of the S-N curve, the value of the negative inverse slope, $m_1$, is equal or lower than for the first branch $m_0$, determining a shallower linear relation in the log–log scale, see Fig. 2. In particular, the S-N curve in the Eurocode 3 is extended using the value $m_2 = 2m_0 - 1$, i.e. the Haibach rule [29], see Fig. 2, and verified for welded details by Huther [30] by using linear elastic fracture mechanics (LEFM). In addition, a cut-off limit is applied for stress ranges which are associated to a fatigue limit higher than 100 million cycles, as their contribution to the cumulated damage is assumed negligible. This fatigue resistance model has often been used for comparison with test results [19,43,44], showing that it provides a reasonable lower bound prediction. Instead, the AASHTO standard prescribes a constant slope in combination with the Palmgren-Miner damage accumulation rule, Equation (2). This is obtained by the good correspondence between the CA S-N curve and the results of the VA fatigue tests, i.e. the experimental VA fatigue life plotted against the root mean cube of the stress spectrum [45,46]. Both the AASHTO and Eurocode 3 suggest values of $D_{cr} = 1$. However, other recommendations suggest different values for some specific cases. The International Institute of Welding [47] suggests a value of $D_{cr} = 0.5$ for welded joints subjected to multiaxial loading or even $D_{cr} = 0.2$ if the stress spectrum is characterized by high mean stress fluctuations. The current analysis, however, is not related to such load conditions and therefore either uses $D_{cr} = 1$ or determines the value of $D_{cr}$ based on test data.

3. Proposed model for variable amplitude fatigue prediction

In this section, the proposed model is presented. To consider the effect of the stress ranges lower than the fatigue limit, the load sequence effect and the damage accumulation, the model consists of three parts: (1) a CA S-N curve with fatigue limit, which is described in Section 3.1, (2) the RLDR, and (3) the damage limit concept, which are described in Section 3.2. The model is intended to estimate the fatigue life of welded details under random blocks of VA loading. Section 3.3, explains how to evaluate the fatigue life using the proposed model.

3.1. The 6 parameters random fatigue limit model

Among the several probabilistic S-N curves proposed in the literature, the 6PRFLM is used because of its phenomenological foundation based on LEMF, and the better agreement with test data, when compared to other fatigue resistance models for CA loading in which the fatigue limit is modelled as a random variable, see [7]. The equation of the S-N curve of the 6PRFLM is:

$$w = \beta_0 + \beta_1x - p\log_{10}\left(1 - \frac{10^v}{10^x}\right) \text{ for } x > v \quad (3)$$

where $w = \log_{10}(N)$ is the logarithm of the number of stress cycles to failure, $N$, which is the dependent variable, and $x = \log_{10}(\Delta\sigma)$ is the logarithm of the stress range, $\Delta\sigma$, which is the independent variable. $v = \log_{10}(\Delta\alpha_0)$ is the the logarithm of the fatigue limit, $\Delta\alpha_0$. The parameter $\beta_0$ controls the location of the curve, $\beta_1$ controls the slope of the finite life region, and both the parameters $\beta_0$ and $p$ affect the curvature of the S-N curve between the finite and the infinite life. The logarithm of the number of stress cycles to failure, $w$, is assumed to be a random variable with the following probability density and cumulative distribution functions:

$$f_{w}(w; x, \Delta\alpha_0) = \int_{-\infty}^{w} f_{w}(w; x, v, \Delta\alpha_0) f_{v}(v; \mu_v, \sigma_v) dv \quad (4)$$

$$F_{w}(w; x, \Delta\alpha_0) = \int_{-\infty}^{w} F_{w}(w; x, v, \Delta\alpha_0) f_{v}(v; \mu_v, \sigma_v) dv \quad (5)$$

where $f_{w}(w)$ is the probability density function of the log-fatigue limit, and $f_{w}(w)$ and $F_{w}(w)$ are the conditional probability density and cumulative distribution functions of the log-fatigue life given the log-fatigue limit, respectively. The logarithm of the fatigue limit, $v$, is modelled as a random variable $V$ having location parameter $\mu_v$, and scale parameter $\sigma_v$. The location parameter of $W/V$ results from Eq. (3), and the scale parameter is $\sigma$. In summary, the model consist of 6 parameters $\Delta\alpha_0 = [\beta_0, \beta_1, \sigma_v, \mu_v, \sigma_v, p]$. A more detailed description can be found in [5,7]. The proposed model is based on the assumption that the two random variables $W/V$ and $V$ are assumed to be uncorrelated.
3.2. The damage limit function and the relative linear damage rule

The damage limit concept, introduced in [32], is here used to account for the stress ranges lower than the fatigue limit into the quantification of the damage. The damage limit concept is based on the definition of the threshold stress range, \( \Delta \sigma_{th} \), above which the stress ranges contribute to damage, whereas those below do not, see Fig. 1. The definition of the threshold stress range results from fracture mechanics considerations [9,32]. Kunz [32] related the threshold stress range to the crack size, \( a \),:

\[
\Delta \sigma_{th} = \Delta \sigma_0 - \frac{Y(a_{ini})}{Y(a)} \left( \sqrt{2m} \right) \tag{6}
\]

where \( a_{ini} \) is the initial crack size, and \( Y(a) \) is the geometry correction factor. Equation (6) is based on the assumption that \( \Delta \sigma_{th} = \Delta \kappa_{th,L}/(Y(a_{ini})/\sqrt{2m}) \), where \( \Delta \kappa_{th,L} \) is the threshold stress intensity factor range for long fatigue crack propagation, and that for a generic crack the threshold condition is given by:

\[
\Delta \kappa_{th,L} = Y(a) \Delta \sigma_{th} \sqrt{2m} \tag{7}
\]

In order to express the crack size, \( a \), and therefore the threshold stress range, \( \Delta \sigma_{th} \), as a function of the number of cycles, the Paris law [48] was used. It relates the the crack growth increment, \( da/dn \), to the stress intensity factor range, \( \Delta \kappa \), in the Stage II crack growth regime, see Fig. 3:

\[
\frac{da}{dn} = C \Delta \kappa^m \tag{8}
\]

where \( C \) and \( m \) are material constants. By further assuming that \( Y(a) \) is constant, i.e. independent of the crack size, it results that [32]:

\[
a = a_{ini} (1 - D)^{1/(2 + m)} \tag{9}
\]

where the damage \( D \) is defined as in Eq. (2). Therefore, considering Eq. (6), the following was obtained:

\[
\Delta \sigma_{th} = \Delta \sigma_0 (1 - D) \tag{10}
\]

Given the fact that for structural steels \( m = 3 \) the previous equation was further simplified in:

\[
\Delta \sigma_{th} = \Delta \sigma_0 (1 - D) \tag{11}
\]

which establishes a decreasing linear relation between the normalised threshold stress, \( \Delta \sigma_{th}/\Delta \sigma_0 \), and the cumulated damage.

As expressed in Eq. (10), the damage limit model:

1. Does not consider that the propagation of fatigue cracks under relatively small stress ranges occurs in the near threshold region, where the Paris law, Eq. (8), is not valid, see Fig. 3.
2. Does not consider the difference between the threshold condition for small and long cracks. The threshold stress intensity factor range for long cracks, \( \Delta \kappa_{th,L} \), is assumed to be valid also in the early stages of the fatigue life, when the threshold stress is identified by the fatigue limit, see Eq. (7). Kitagawa and Takahashi [49] observed that the threshold stress intensity factor for crack propagation, \( \Delta \kappa_{th} \), is also dependent on the crack size, and not only on the applied stress ratio. This behaviour was later observed and modelled by other authors [50–54].
3. Assumes that \( Y(a_{ini}) = Y(a) = \text{constant} \). The value of \( Y(a) \) increases for crack depths approximately larger than 0.3T, where \( T \) is the thickness of the plate. However, it must be noted that most of the propagation cycles already occurred when this condition is not any more a reasonable approximation. Therefore, this is considered here as an acceptable simplification.

In other words, the damage limit model expressed as in the Equation (10) is valid under the condition that the crack is a long fatigue crack, and that the stress range level is high enough for this crack to propagate in the Paris regime, i.e. the stage II of the fatigue crack growth rate curve, see Fig. 3, which might be in contrast with the assumption that \( Y(a) \) is constant, making Equation (11) valid in very specific cases. Under these conditions, Equation (10) allows taking into account the contribution to the damage of the stress ranges lower the threshold stress, which is a function of the cumulated damage and the fatigue limit. For welded details, where the fatigue life can be assumed to be dominated by fatigue crack growth of a physically small crack, the assumptions underlying Equation (10), and discussed in the points (1)–(3) above, are weak for at least two reasons. The first is that the geometric correction factor, \( Y(a) \) is magnified by the SIF intensification factor, \( M_a(a) \), which takes into account the effect of the weld toe geometry, and the severity of the weld. The second is that the initial crack \( a_{ini} \) is small enough that the extrinsic effect, i.e. crack closure phenomena, are not fully developed yet, but big enough for the effect of the micro-structure to be negligible, i.e. the initial crack can be addressed as a physically small crack. It has been widely shown in the literature that a short crack grows faster than large crack subjected to the same stress intensity factor range, \( \Delta \kappa \), and that for a short crack the threshold condition differs from that of long cracks [51,52]. However, the FGCR can still be described as a function of the stress intensity factor range [53–55], despite the loss of simultitude.

For the purpose of this paper, \( M_a(a) \) is considered as in [56]:

\[
M_a(a) = \max \left( C_a \left( \frac{Y(a)}{T} \right)^{1/2}, 1 \right) \tag{12}
\]

where \( T \) is the plate thickness, and the parameters \( C_a \) and \( \alpha \) solely depend on the geometry of the weld, i.e. the severity of the notch, and are often obtained trough fitting of numerical solutions. It follows that the stress intensity factor range for a crack growing at the weld toe is:

\[
\Delta \kappa = M_a(a) Y(a) \Delta \sigma \sqrt{2m} \tag{13}
\]

Based on the trend of experimental observations produced by Kitagawa and Takahashi [49], El-Haddad [50] proposed a model for correcting the value of \( \Delta \kappa_{th,L} \), for physically small cracks, making explicit the dependence on the crack size. According to El-Haddad, for a generic crack \( a \) growing at the toe of the weld the threshold condition can be modelled as:
\[ \Delta K_{th} = M_o(a)Y(a)\Delta x_0 \sqrt{\pi(a_0 + a)} \]  

(14)

where \( a_0 \) is denoted as the intrinsic crack size, a material dependent parameter, which allows considering the effect of the development of crack closure on the threshold condition, but it is not a physical quantity [54]. By assuming that welding induces crack-like defects that can be assumed as the initial crack size, for a stress range equal to the fatigue limit the following holds:

\[ \Delta K_{th} = M_o(a_{ini})Y(a_{ini})\Delta x_0 \sqrt{\pi(a_0 + a_{ini})} \]  

(15)

For \( a > a_{ini} \) and \( a > a_0 \), it results that \( \Delta K_{th} \rightarrow \Delta K_{th,k} \). Therefore, according to the presented modelling of the threshold condition for fatigue crack growth, different from Equation (6), the threshold stress range is rewritten as:

\[ \Delta \sigma_{th} = \Delta \sigma_{th,k} = \frac{M_o(a_{ini})Y(a_{ini})\sqrt{\pi(a_0 + a_{ini})}}{M_o(a)Y(a)\sqrt{\pi(a_0 + a)}} \]  

(16)

An approximate relation for \( f(D) \) in Eq. (1) is to be derived based on linear elastic fracture mechanics. The derivation follows a similar scheme as in Kienzle [32], but it accounts for the SIF intensification for a crack growing at a weld detail and it considers the RLDR. The aim is not to provide a closed form and exact relationship, but an approximation instead. As it can be observed from Fig. 3, the logarithm of the fatigue crack growth rate, \( da/dn \), is related to the logarithm of the stress intensity factor, \( \Delta K \), in the stage II of the fatigue crack growth, by using Eq. (8). Many other equations have been proposed to relate these two quantities. Among these, the Forman-Mettu relation [57], which is able to describe this relation both in the near-threshold regime, and in the critical regime. Under some assumptions, see [7], the fatigue crack growth rate expressed by the Forman-Mettu relation can be modified to consider only the near-threshold crack growth rate, and for a fixed stress ratio:

\[ \frac{da}{dn} = C\Delta K^{m}(1 - \frac{\Delta K_{th}}{\Delta K})^{p} \]  

(17)

The analytical solution of the differential equation, Eq. (17), is non-trivial for values of \( p > 0 \). For this reason, the analytical solution is given here only for \( p = 0 \). By substituting Eq. (13) in Eq. (17), the following results:

\[ \frac{da}{dn} = C\Delta a^{m}M_o(a)y^{m}(a^{m}/a^{m/2})^{m/2} \]  

(18)

Assuming that: (1) the stress intensity magnification factor is given by Eq. (12), (2) \( Y(a) = Y \) is constant, and (3) the critical condition occurs when \( M_o(a) > 1 \), i.e. \( a_{cr} \leq C^*_{x}^{1/\alpha}T \), the integration of both terms of Eq. (18) leads to:

\[ C^*_x^{-m}T^{mn}\int_{a_0}^{\alpha_{cr}}a^{-m(1/2)}da = B\int_{\alpha_{cr}}^{\alpha}dn \]  

(19)

where \( B = C\Delta a^{m}M_o^{m}a^{m/2} \), and the subscript \( k \) denotes an arbitrary moment in time, as in Fig. 1. The general solution of Eq. (19), expressed as the crack size, \( a_k \), as a function of the number of applied stress cycles, \( N_k \), is:

\[ a_k = \left( \frac{B}{C^*_x^{-m}T^{mn}N_k[1 - m(\alpha + 1/2)] + a_0} \right)^{1/[1 - m(1/2)]} \]  

(20)

The critical condition consists in determining a finite number of cycles, \( N_k \) for which fracture occurs, i.e. the crack growth becomes unstable and the crack size, \( a_k \), goes to infinity. Therefore, Eq. (19) can be rewritten by changing the extremes of the integration interval:

\[ C^*_x^{-m}T^{mn}\int_{a_0}^{a_{cr}}a^{-m(1/2)}da = B\int_{0}^{N_k}dn \]  

(21)

where the indefinite integral on the left side of the Eq. must be finite. In order for this to happen, \( a^{-m(1/2)} \rightarrow 0 \) faster than \( a^{-1} \), for \( a \rightarrow \infty \). This occurs if \( -m(\alpha + 1/2) < -1 \). The solution of Eq. (21), leads to:

\[ \frac{\Delta \sigma_{th}}{\Delta \sigma_0} = \frac{1}{1 - m(\alpha + 1/2)} \left[ (1 - D)^{-1/\alpha} \right]^{m(1/2)} \]  

(22)

Substituting Eq. (23) into Eq. (20), the following is obtained:

\[ \frac{a}{a_{ini}} = \left( \frac{1 - N_k}{N_k} \right)^{1/[1 - m(1/2)]} = (1 - D)^{-1/[1 - m(1/2)]} \]  

(24)

On the other hand, the first order approximation for the relation between the threshold stress range and the fatigue limit, Eq. (16), is:

\[ \Delta \sigma_{th} = \Delta \sigma_{0} \frac{M_o(a_{ini})Y(a_{ini})\sqrt{\pi a_{ini}}}{M_o(a)Y(a)\sqrt{\pi a}} \]  

(25)

By substituting Eq. (12) into Eq. (25), and considering \( Y(a) = Y(a_{ini}) \), i.e. \( Y(a) \) is constant, the latter Eq. becomes:

\[ \Delta \sigma_{th} = \Delta \sigma_{0} \frac{a_{ini}}{a} \]  

(26)

Finally, by substituting the Eq. (24) in Eq. (26) results in the relation between the threshold stress and the damage:

\[ \Delta \sigma_{th} = \Delta \sigma_{0}(1 - D)^{a_{ini}/a} \]  

(27)

Hence, an approximation of the function \( f(D) \) in Eq. (1) has been obtained:

\[ f(D) = \frac{\Delta \sigma_{th}}{\Delta \sigma_{0}} = \left( 1 - \frac{D}{D_{cr}} \right)^{\zeta} \]  

(28)

where

\[ \zeta = f(\alpha, m) = \frac{\alpha + 1/2}{1 - m(\alpha + 1/2)} \]  

(29)

is a function of the parameters \( m \) and \( \alpha \). Eq. (28) is plotted in Fig. 4, where the dependency on \( \zeta \) is shown.

The first order approximation of the relation proposed in Eq. (16), Eq. (25), is considered sufficient because the scope of the present paper is to obtain a simple fracture-mechanics based solution for the damage limit function to demonstrate that the notch geometry affects the trend of the threshold stress range. The second order approximation of the terms \( \sqrt{a_{ini}} + a_{ini} \) and \( \sqrt{a_{ini}} + a \) in Eq. (16) is evaluated for the purpose of checking if the overall trend of the damage is affected. The implementation of the second order approximation of \( \sqrt{a_{ini}} + a_{ini} \) and \( \sqrt{a_{ini}} + a \) in Eq. (16) leads to:

Fig. 4. Normalised threshold stress as function of the normalised cumulated fatigue damage, as expressed by the damage limit model for different values of \( \zeta \), see Eq. (28).
\[
\Delta \sigma_{th} = \Delta \sigma_{th} - M_{th}(a_{th}) Y(a_{th}) \sqrt{a_{th}} \left( 1 + \frac{a_{th}}{a_{ini}} \right)
\]

the term \((1 + a_{th}/(2a_{ini}))\) is a constant, and despite \((1 + a_{th}/(2a))\) being a decreasing function of \(a\), the general trend of damage limit function is not affected because \(\sqrt{a}\), and eventually \(M_{th}\), dominate. Therefore, given our goal, it is not necessary to derive the Eq. of the damage limit function based on a second order approximation of Eq. (16).

In addition to the findings of Kunz, it is found that the dependency between \(\alpha\), \(m\) and \(\zeta\) demonstrates that there is an effect of the type of welded detail on the trend of the damage limit function, indicating that a difference must be expected between welded details, with respect to their fatigue strength. It is expected that the value of \(\zeta\) is not only dependent on the material, as found in [32], but also on the fatigue strength of the detail, as \(\alpha\) depends on both the type of detail and the weld toe geometry [56]. The more severe the welded detail is, the smaller \(\alpha\) is, and the lower \(\zeta\) is. It must be noted that the present formulation for \(\zeta\) is based on some assumptions and limitation. First of all, those related to the determination of the critical condition, Eq. (21), and the fact that CA loading is applied. Therefore, it is not expected that Eq. (29) is exact. But, it provides that \(\zeta\) is dependent of the structural detail, because of the (geometrical) notch effect. The effect of the residual stress state is neglected.

The influence of the model parameters \(\mu_{D\sigma}, \sigma_{D\sigma}\), and \(\zeta\) on the fatigue life prediction for VA loading is qualitatively presented in terms of Gassner curve in Fig. 5:

1. By increasing the mean value of the critical damage, the resulting Gassner curve is shifted towards longer life, with no difference in the coefficient of variation. In a similar way, the effect on the extended S-N curve method is a shift towards longer lives.
2. By increasing the scale parameter of the critical damage, an increase in the scatter of the predicted life is encountered in both cases.
3. By increasing \(\zeta\) a generic stress range \(\Delta \sigma < \Delta \sigma_{th}\) contributes to the fatigue damage accumulation at a lower value of the damage ratio, \(D/D_{th}\). Regarding the Gassner curve, this affects the transition between finite and infinite life, the extent of which (in terms of stress range values) depends on the dispersion of the stress spectrum, i.e. the difference between \(\Delta \sigma_{max}\) and \(\Delta \sigma_{min}\). Instead, the effect on the extended S-N curve method is a decrease in the absolute value of the inverse slope of the extension of the S-N curve, \(m_{n}\), which is bounded on one side by the value of \(m_{i}\).

Whereas the values of \(\mu_{D}\) and \(\sigma_{D}\) affect the predicted life, independent of \(\Delta \sigma_{max}\), \(\zeta\) affects the fatigue life prediction the more the spectrum is characterised by a lower normalised frequency that the stress spectrum exceeds the fatigue limit, \(\bar{\sigma}_{\text{max}}\):

\[
\Phi_{\text{max}} = \sum_{i} \frac{Pr(\Delta \sigma > \Delta \sigma_{i}) n_{i}}{\sum n_{i}}
\]

where, \(Pr(\Delta \sigma > \Delta \sigma_{i}) = \mathcal{N}(\log \sigma_{i}; \hat{\mu}_{D}, \hat{\sigma}_{D})\) because in [7] the logarithm of the fatigue limit is assumed to follow a normal distribution, and \(i\) indicates the \(i\) -th stress range in the stress spectrum. \(\Phi_{\text{max}}\) is given as the weighted average of the probability that a stress range is higher that the fatigue limit, of which the base-10 logarithm is the random variable \(V\). This definition differs from those in [38,45,46] because the fatigue limit is here defined as a random variable and not a deterministic quantity.

Different from the model proposed by Kunz [58], the present model makes use of the RLDRO, and the critical damage is modelled using a random variable. The RLDRO is selected in order to globally consider load sequence and load interaction effects typical for the considered stress history. Moreover, the critical damage \(D_{th}\) is modelled as a random variable having mean and coefficient of variation equal to \(\mu_{D}\) and \(\sigma_{D}\), respectively. With respect to this, similar to [6,59] it is assumed that the logarithm of the critical damage is distributed according to a Normal distribution.

3.3. Damage integration and fatigue life estimation

By assuming that the load history is an ergodic process, the fatigue life \(N\) is obtained by the following:

\[
N = \int_{0}^{D_{th}} E[D]dD
\]

where \(D_{th}\) is the critical damage, and \(E[D]\) is the expected damage rate. Using the proposed model, the expected damage rate can be evaluated by:

\[
E[D] = \sum_{i} n_{i} \frac{n_{i}}{N} \sum n_{j}
\]

where \(\Delta \sigma_{th}\) is expressed by Eq. (28), and \(i\) indicates the \(i\) -th stress range in the stress spectrum. Eq. (32) is a recursive integral equation. By substituting Eqs. (28) and (34) into Eq. (33), the numerical sequence used to evaluate Eq. (32) is:

\[
N = \sum_{h=1}^{h_{max}} \left[ \sum \frac{n_{i}}{10^{\mu_{D}\theta_{D}+\sigma_{D}\theta_{D}}} \log^{0} \left( \frac{\sigma_{D} - \sigma_{D}}{\mu_{D}} \right) \sum n_{j} \right]
\]

where \(h_{max}\) is the number of integration steps, and the damage evaluated at \(h - 1\) is used to estimate the threshold stress range at the step \(h\), i.e. \(\Delta \sigma_{th,h} = \Delta \sigma_{th}(1 - D_{th,h-1}/D_{th})\), and \(D_{th,h} = 0\). From Eq. (35) it follows that if \(\Delta \sigma_{max} < \Delta \sigma_{th}\) no damage is cumulated, leading to the threshold

\[
\Delta \sigma_{max} = \text{Median} - \text{Prediction Bound}
\]

\[
\mu_{D} = \sigma_{D}
\]

\[
\Delta \sigma = \text{CA S-N curve}
\]

\[
\text{Gassner curve}
\]

\[
\text{VA model curve}
\]

\[
\text{Extended S-N curve}
\]

Fig. 5. Effect of an increase of the model parameters on: (a) the resulting Gassner curve, and (b) the extended S-N curve model.
condition for VA loading, sometimes referred to as to VA fatigue limit [60].

4. Inference of VA fatigue test data and model assessment

Hereafter, it is verified if the derived dependency of $\xi = f(\alpha, \mu)$ is indeed appropriate by inferring VA fatigue test data of welded details with different notch severity, i.e. different $M_n$ (and therefore different $\alpha$). For this purpose, Eq. (27) is not considered to determine the fatigue life. Instead, Eq. (28) is used. It results that $\xi$, $\mu$, and $\sigma_0$ are the parameters of the proposed model. Applying the proposed model requires two datasets. Firstly, a set of CA test results, from which the model parameters of the 6PRFLM, $\hat{\theta}_{CA}$, can be estimated [7]. Secondly, a set of VA test results obtained with a selected type of stress spectrum, from which the model parameters $\mu_d$, $\sigma_0$, and $\xi$ are estimated by using the maximum likelihood method (MLM). The likelihood is a function of the model parameters given the experimental data [22]:

$$L(\theta_{CA}; \text{data}) = \sum_j \log(L_j(\theta_{CA}; w_j, \Omega_j))$$  \hspace{1cm} (36)

where $j = 1..m_{CA}$ indicates the $j$-th VA fatigue test data, $L_j$ is the likelihood of the parameters for a single data, $w_j = \log_{10}(N_j)$ is the logarithm of the number of cycles to failure, $N_j$ obtained experimentally by applying the stress spectrum $\Omega_j = (\Delta_{0j}, n_j)$, and $\theta_{CA} = (\mu_d, \sigma_0, \xi)$ is the vector containing the model parameters. The likelihood evaluated for a each experimental data is a function of the probability density function (pdf), $\phi$, and the cumulative distribution function (cdf), $\Phi$, of the model response:

$$L_j(\theta_{CA}; w_j, \Omega_j) = [\phi(w_j; \theta_{CA}, \Omega_j)]^{1-\delta}[1 - \Phi(w_j; \theta_{CA}, \Omega_j)]^{-\delta}$$  \hspace{1cm} (37)

$\delta$ is the failure indicator ($\delta = 1$ for failure data, or $\delta = 0$ for runout data). Therefore, the present model formulation can accommodate VA fatigue runout data, i.e. censored data, in the inference procedure.

The procedure to estimate the likelihood of the VA model parameters, $\theta_{VA}$, for a set of experimental VA fatigue data is depicted in Fig. 6. The procedure, divided in 3 steps, aims to estimate the likelihood of $\theta_{VA}$ by approximating the model response. This is because, similar to [6], the distribution of the model response, i.e. of the fatigue life, is not described by a closed-form equation. However, it can be approximated by a number of random Monte Carlo samples, as a result of a Monte Carlo simulation.

Input and variables The procedure requires as input the parameter of the 6PRFLM, $\theta_{CA}$, and the experimental VA fatigue data, $(\Omega, N)$. Furthermore, a value of $\delta_{VA} = (\mu_d, \sigma_0, \xi)$, the independent variable, is required to start the procedure, and is randomized within a certain boundary domain.

Step A - Monte Carlo Simulation Each Monte Carlo sample of the fatigue life is obtained by the following four inputs: (1) A random sample of the CA S-N curve, which is obtained by uncorrelated sampling of $V$, and $W/V$, and the 6PRFLM, Eq. (3). (2) A random sample of $D_{0j}$, which is obtained using $\mu_d$, and $\sigma_0$, considering that the logarithm of the critical stress is a random variable. (3) The damage limit function, Eq. (28), which is determined given the value of $\xi$. (4) The stress spectrum $\Omega_j = (\Delta_{0j}, n_j)$, in the first iteration, the values of $\mu_d$, $\sigma_0$, and $\xi$ are those of the initial point. Successively, the values of $\delta_{CA}$ is determined by the maximization algorithm. Given these four inputs, Eq. (35) can be solved and a random Monte Carlo sample of the fatigue life is obtained. Many samples are required to approximate the model response: the higher the number of samples that is simulated is, the more accurate the approximation of the model response is. The procedure stops when a sufficiently high number of failure samples is obtained, i.e. $\Delta_{max} > \Delta_{0j}$.

Step B - Parametric Model Response If the simulated samples of the fatigue life are directly used to estimate $\phi$ and $\Phi$, and therefore the likelihood by Eqs. (36) and (37), the resulting likelihood function is not a $C^2$ function of the model parameter, $\theta_{VA}$, i.e. a function with continuous second derivatives. This makes it difficult to maximise the likelihood function using maximization algorithms based on gradient methods and finite differences. Therefore, in order to not to have numerical difficulties in the maximization procedure, a parametric formulation of the distribution of the model response, i.e. of the fatigue life, is used. For this reason, a parametric random variable is used to fit the random samples of the Monte Carlo samples of the fatigue life. The probability function of the fatigue life is assumed to be of the type as formulated in the Random Fatigue Limit Model [5] having probability density and cumulative probability functions, respectively, given by:

$$g_{W}(w; \Omega, \theta_{FL}) = g_{W}(w; \mu_w, \sigma_{w}; F_V(\Omega; \mu_v, \sigma_v))$$  \hspace{1cm} (38)

$$G_{W}(w; \Omega, \theta_{FL}) = G_{W}(w; \mu_w, \sigma_w; F_V(\Omega; \mu_v, \sigma_v))$$  \hspace{1cm} (39)

where $\theta_{FL} = (\mu_w, \sigma_w; \mu_v, \sigma_v)$ is the vector of the parameters. The functions $g_{W}$, $G_{W}$, and $F_{V}$ are assumed to be normal random variables. Therefore, the parameters $\mu_w$, $\sigma_w$, $\mu_v$, $\sigma_v$ are estimated as the mean and the standard deviation of the log-fatigue life of those Monte Carlo samples of the fatigue life that result in a finite life. Fig. 7 depicts some of the characteristics of $g_{W}$ and $G_{W}$, in particular, it should be noted that the horizontal asymptote of $G_{W}$ for $w \to \infty$ is not necessarily located at $G_{W} = 1$; its value depends on the distribution of the fatigue limit, see Eq. (39), since conditional probabilities are involved. This means that the area underlying $g_{W}$ is also not necessarily one, since right censored data are associated to an infinite life. Therefore, the use of the random variables $g_{W}$ and $G_{W}$, of which the model parameters are $\theta_{FL}$, allows a parametric evaluation of the response of the proposed model, i.e. a parametric evaluation of the distribution of the VA fatigue life resulting by a selection of the model parameters $\mu_d, \sigma_0$, and $\xi$, given a stress spectrum $\Omega_j$ and $\delta_{CA}$.

Step C - Likelihood Estimation The likelihood of $\theta_{VA}$ given a single experimental fatigue life, Eq. (37), can be rewritten considering Eqs. (37) and (38):

$$L_j(\theta_{VA}; w_j, \Omega_j) = [g_{W}(w_j; \Omega_j, \delta_{CA})]^{1-G_{W}(w_j; \Omega_j, \delta_{CA})}$$  \hspace{1cm} (40)

Fig. 6. Procedure for estimating the likelihood function.
The maximum likelihood estimator (MLE) of $\theta_{V_A}$ is obtained by maximizing the likelihood function, therefore iterating the procedure. Following a convergence analysis on the likelihood function, it has been ensured that if $10^5$ failures are simulated as a result of the Monte Carlo procedure, the estimation of the likelihood function converges, and the estimators are not affected by the sampling error, which is one order of magnitude lower than the tolerance of the maximization procedure.

The estimation of statistical (or equivalently, epistemic) uncertainty is performed in a parametric way by estimating the Hessian matrix of the log-likelihood function at the MLE, see [6,7,22]. This procedure estimates the covariance matrix of the long-run multivariate normal distribution of the model parameters of $\theta_{V_A}$.

5. Results and discussion

In this section, the statistical inference of the data resulting from the application of the proposed model is presented. The model is used to infer four datasets of variable amplitude fatigue test data respectively belonging to (a,b) cover plated steel beam detail, (c) cover plated steel detail, (d) welded beam detail, and (e) non-load carrying cruciform joint detail, see Fig. 8. The difference between (a,b) with (d) is meant to give an indication of the effect of the specimen scale. The selection of (a,b) and (d) (or (c) and (e)) is made to show that the model is able to interpret the fatigue behaviour of welded details associated to a relatively low and a relatively high fatigue resistance, which is associated to different values of $M_w$, and therefore $\alpha$. It will be investigated if the trend of $\zeta$ following from Section 3.2 is confirmed by actual fatigue test data, i.e. $\zeta$ reduces with increasing severity of the detail. The analysis of the cover plated steel beam dataset is reported in the following section, whereas the inference of the other details is reported in Appendix A. In any case, the datasets of VA fatigue data composed of tests performed in the past years and they are available in the scientific literature. All the tests reported here were conducted by applying random block loading, where the frequency of occurrence of the stress ranges follows a Rayleigh distribution function, truncated at a maximum value. In order to determine this distribution, measurement data of in-service loading acquired from bridges were analysed in [38], and provided the basis for all the VA fatigue test data considered in the present work. A summary of the properties of this distribution is reported in Section 5.1.

5.1. The truncated Rayleigh stress spectrum

The investigations performed in [38,45,46] make use of normalised stress spectra following a truncated Rayleigh distribution. The probability density function of the Rayleigh distribution is formulated as:

$$f(z|b_R) = \frac{z}{b_R^2} e^{-\frac{z^2}{2 b_R^2}}$$

(41)

where $b_R$ is the scale parameter, which is set equal to 1. The truncation value $z_{\text{max}}$ is defined at different levels, depending on the investigation. Therefore, the area below the truncated curve has to be normalised. The truncation introduces a constant $C_k$ that multiplies $f(z|b_R)$, and it is obtained by solving the following:

$$\int_0^{z_{\text{max}}} C_k z^{a-k} e^{\frac{-z^2}{2 b_R^2}} dz = 1$$

(42)

Therefore, the value of $C_k$ can be directly obtained by

$$C_k = \frac{1}{F(z_{\text{max}}|b_R)} = 1$$

(43)

where $F(z_{\text{max}}|b_R) = 1$ is the standard Rayleigh cumulative distribution function evaluated at $z = z_{\text{max}}$. The values of $C_k$ are in Table 1. To generate a load history of $z_{\text{tot}}$ individual loads, the area below the curve is divided in segments of equal area. The mid-width of each bar corresponds to a non-dimensional stress range, $z$, with a frequency of occurrence equal to $1/z_{\text{tot}}$. The generic stress range $\Delta z$ is related to the non-dimensional stress range, $z$, by:

$$\Delta z = \Delta z_{\text{max}} + z \Delta \Delta z$$

(44)

where

$$\Delta \Delta z = \frac{\Delta z_{\text{max}} - \Delta z_{\text{min}}}{z_{\text{max}}}$$

(45)

The modal stress range is:

$$\Delta z_{\text{m}} = \Delta z_{\text{min}} + \Delta \Delta z$$

(46)

5.2. Cover plated steel beam detail

As mentioned in Section 4, the model requires CA and VA data to estimate the model parameters and the uncertainty. Both datasets are

Fig. 8. Geometry of the considered welded details.
presented in the following two sections. Successively, the CA data are used to define two VA fatigue resistance models based on current standards, i.e. the Eurocode 3 and the AASHTO. The prediction obtained with these two models are compared with the experimental results. In the last section, the data are inferred using the proposed model. The estimation of the epistemic and aleatory uncertainty is presented on the basis of the datasets. Comparison is made with an existing probabilistic fatigue life prediction model based on the nominal stress approach, formulated in [6].

5.2.1. Constant amplitude fatigue test data

In [7] CA fatigue test data for the cover plated beam detail were inferred using the 6PRFLM. Table 2 reports the MLE of the 6PRFLM for the large dataset analysed in [7]. The same dataset used in [7] is used here to estimate the best fitting Basquin [61] relation:

\[
\log_{10}(N) = a_1 + m_1 \log_{10}(\Delta \sigma)
\]  

(47)

where \(m_1 = -3\) and \(a_1\) are the model parameters. \(a_1\) is estimated using the least square method, by considering the distribution of the failure data analysed in [7]. In particular, since \(m_1\) is selected a priori, the value of \(a_1\) can be estimated in a closed form:

\[
a_1 = -m_1 E[x] + E[w]
\]  

(48)

where \(j = 1,\ldots,m_1\), \(\Delta \sigma_j = 1\) indicates the \(j\)th failed fatigue test contained in the CA dataset, which is composed of \(m_{CA}\) data. It resulted that \(a_1 = 11.7\), and the standard deviation of the residuals is \(\sigma_{res} = 0.155\).

5.2.2. VA fatigue test data

The data available for VA loading are summarised in Table 3. Out of these data, the data belonging to the cover plated detail type A have been excluded from the analysis because the welding sequence adopted determined a different fatigue life than that one usually observed in the other analyses [38,45,46]. This was attributed to a different residual stress state. The considered VA fatigue test dataset contains 94 experimental results, of three different steel grades: A36, A514 and A588. The stress spectrum was of Rayleigh type with different and \(\Delta \sigma/\Delta t_0\) ranging from 0.25 to 1.00, see Section 5.1 for more information about the Rayleigh spectrum.

In Table 3 the VA fatigue test data are described by the respective Gassner curve, reporting also the standard error for the estimator of the curve parameters. Only failure data are inferred. When the data are produced at the same \(\Delta \sigma_{max}\), the data cannot be inferred by the Gassner curve. For this reason the estimators of the parameters are not reported. Instead, the mean value of the logarithm of the number of cycles to failure, \(E[w]\), is reported in Table 3. In some cases, no value is reported. This is because only runout data result from the experiments. Therefore, the value of \(\Delta \sigma_{max}\) is reported. The datasets from [45] were also produced using the Rayleigh spectrum. However, the block of loading was also characterised by a single stress range peak, \(\Delta \sigma_{sp}\), higher than the maximum stress range, \(\Delta \sigma_{max}\), defined at the truncation level. These data are considered as well.

5.2.3. Fatigue resistance under VA loading using current standard approach

Given the Basquin relation, Eq. (47), fitting the CA data, it is possible to construct two fatigue resistance models for VA loading, one based on Eurocode 3 (Model 1) and one based on AASHTO (Model 2). In both cases, the fatigue limit is defined as the stress range at a certain number of cycles, and VA loading is taken into account by extending the CA S-N curve below the fatigue limit without the need of VA fatigue test data. The goal is to highlight the limitation of this approach. In Model 1, the fatigue limit is located at \(N_0 = 5\) million cycles. For considering the effect of the stress ranges lower than the fatigue limit, the S-N curve is extended below the CA fatigue limit with an inverse slope \(m_2 = -5\), following from the Haibach rule [29], until the fatigue life reaches \(10^6\) cycles, where a cut-off limit is applied, see Fig. 2. In Model 2, the fatigue limit is located at \(N_0 = 10\) million cycles. In this case, for considering the effect of the stress ranges lower than the fatigue limit, the curve is extended without changing the slope, i.e. \(m_1 = m_1 = -3\). In this model, no cut-off limit is defined, see Fig. 2. For both Model 1 and Model 2 the value of the critical damage is 1. The fatigue life prediction of these two fatigue resistance models, predominate defined using CA test data and conventions, are compared to VA test results, in Fig. 9. The mean response of the model prediction is on the horizontal axis, whereas the experimental fatigue life is on the vertical axis. If the points are above the bisection line, \(N_{exp} > N_{test}\). The black points represent the experimental failure data, and the red points represent the runout data. If the predicted fatigue life is finite, the resulting point is a circle, otherwise it is a triangle positioned at \(10^6\) cycles. For relatively long lives not only the distribution on the data around the bisection line matters, but also an unbalanced distribution of failure data predicted as runouts (black triangles) and runout data predicted as failures (red circles) denotes either a too optimistic prediction or a too conservative one. By analysing Fig. 9a, it appears that Model 1 predicts conservative lives up to approximately \(10^7\) cycles. For longer lives, the predicted number of cycles to failure is consistently higher than the observed ones. In addition, the model predicts infinite life for a substantial number of failure data. The use of Model 2 determines a generally conservative fatigue life prediction because Fig. 9b demonstrates that most data points fall between the bisection line and the upper dashed line. However, a better agreement is obtained in comparison with Model 1, especially at relatively long lives.

5.2.4. Inference of VA fatigue test data and estimation of the uncertainty

The variable amplitude test data presented in Table 3 are inferred using the proposed model, following the procedure described in Section 4, considering the values of \(a_{VA}\) as reported in Table 2. As a result of the inferential process, the MLE of \(a_{VA}\) is obtained, see Table 4. The mean value of the critical damage is greater than one, with a coefficient of variation of 0.41. The relatively high mean value is ascribed to the load sequence effects typical of the applied stress history. The relatively large scatter of the critical damage is likely to arise from the circumstance that the dataset used for VA loading is composed of several datasets pooled together. It is believed that the differences in the execution of the tests, i.e. the applied stress histories, the small differences in the geometry, and the fabrication of the beams, are the cause of the large scatter. The distribution of the critical damage with and without the contribution of epistemic uncertainty is depicted in Fig. 10, where it is compared with the distribution proposed in the JCSS [59], suggesting \(\mu_0 = 1 \) and \(\sigma_0 = 0.3\) which is an estimate of the critical damage

Table 2

<table>
<thead>
<tr>
<th>MLE</th>
<th>log((L(\theta)))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\hat{a}_i)</td>
<td>1.11E + 01</td>
</tr>
<tr>
<td>(\hat{b}_i)</td>
<td>-2.78E + 00</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>1.32E-01</td>
</tr>
<tr>
<td>(\mu_0)</td>
<td>1.54E + 00</td>
</tr>
<tr>
<td>(\delta_0)</td>
<td>7.27E-02</td>
</tr>
<tr>
<td>(\hat{\beta})</td>
<td>4.74E-01</td>
</tr>
</tbody>
</table>
The mean fatigue life prediction of the current model is compared in Fig. 11 with the experimental data. Where as the results of Model 1 shown in Fig. 9a significantly differ from those of Fig. 11, there is reasonable agreement for Model 2, Fig. 9b, and the best agreement is obtained with the proposed model, Fig. 11, where the MLE of $\hat{\phi}_1$ for the dataset is considered. It can be deduced that the fatigue life prediction of the proposed model using $\zeta = 18.7$ is roughly equivalent to extending the CA S-N curve with branch having $m_1$: $m_2$.

The model proposed in [6] has been used for inferring the presented CA and VA fatigue test data. In this way, it is possible to compare the proposed model with a similar fatigue resistance model. All the models are based on nominal stresses. The Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) are used to weigh the accuracy of the different models. The AIC and the BIC statistics are respectively given by [62]:

$$\text{AIC} = -2\log L(\hat{\phi}_{MLE}) + 2n$$  \hspace{1cm} (49)

$$\text{BIC} = -2\log L(\hat{\phi}_{MLE}) + n\log (m_{m0})$$  \hspace{1cm} (50)

where $n$ is the number of model parameters and $m_{m0}$ is the number of VA fatigue tests. The smaller the AIC and BIC values are, the more efficient the fit is since both statistics penalize the number of model parameters $n$, but for $\log (m_{m0}) > 2$, the BIC statistic penalizes it more strongly. Since the same dataset is inferred and both models contain three parameters for modelling VA loading, the AIC and the BIC statistics do not give additional information over the use of the loglikelihood for model adequacy and selection. They are reported anyway for completeness.

Based on the standard approach for considering VA loading, i.e. $m_2 = 2 \cdot m_1 + 1$ and $m_2 = m_1$ used for Model 1 and Model 2, two alternative prediction models have been constructed: Model 1b and Model 2b. Both models have been constructed as for Model 1 and 2, respectively. The only difference consists in the fact that in this case the parameter $m_1$ is not set equal to $-3$, as it happens for Models 1 and 2, but is determined by CA fatigue test data. For Models 1b and 2b it is obtained that $a_1 = 12.2$, $m_1 = -3.29$, and the standard deviation of the residuals is $\sigma_e = 0.147$.

The values of the loglikelihood, AIC, and BIC statistics for Models 1, 2, 1b and 2b based on both the CA and VA dataset are calculated. The results of the inference are shown in Table 5. If the number of parameters of the model and the inferred data are the same, a lower maximum value of the loglikelihood function determines higher uncertainty underlying the estimation. Therefore, the model proposed in [6] would determine a more uncertain prediction than the proposed model.

The Models 1, 2, 1b and 2b are not used to infer the VA data. For this reason, the loglikelihood value also gives an indication about the accuracy, and not only about the extent of the uncertainty. It can be deduced from Fig. 9, that model 2 has an higher accuracy than model 1. However, as reported in Table 5, the loglikelihood value for model 2 is lower. This can be attributed to two circumstances. The first one is that the current dataset for VA loading is characterized by a relatively large scatter, when compared to the CA fatigue tests available. This can be also deduced by the fact that the estimator of $\phi_0$ is relatively high, see Table 4. The second circumstance is that model 2 predicts for VA loading the same scatter as for CA loading. Instead, model 1 predicts a larger scatter. The larger scatter is caused by $m_{1} > |m_{0}|$, and the fatigue limit defined at a fixed number of cycles. Because, $\sigma_{e} = 0.155$ in the CA branch of the resistance model, see Section 5.2.1, the scatter in the

Table 3  
Summary of VA fatigue test data for the cover plated beam detail with welded and unwelded ends. Statistical description and inference using the Gassner curve.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Material</th>
<th>Detail type</th>
<th>$\delta_{min}$</th>
<th>$\delta_{c}/\delta_{min}$</th>
<th>Failure</th>
<th>Runout</th>
<th>$\hat{\phi}_0$</th>
<th>$\sigma(\phi_0)$</th>
<th>$\hat{\phi}_1$</th>
<th>$\sigma(\phi_1)$</th>
<th>$\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[38]</td>
<td>A514 (WE)/C</td>
<td>68.95</td>
<td>0.5</td>
<td>9</td>
<td>0</td>
<td>12.083</td>
<td>0.544</td>
<td>2.811</td>
<td>0.237</td>
<td>0.087</td>
<td></td>
</tr>
<tr>
<td>[38]</td>
<td>A514 (WE)/C</td>
<td>103.4</td>
<td>1</td>
<td>8</td>
<td>0</td>
<td>$E[\phi] = 7.302$ at $\Delta_{max} = 62.055$MPa</td>
<td>0.051</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[38]</td>
<td>A514 (WE)/C</td>
<td>68.95</td>
<td>1</td>
<td>8</td>
<td>1</td>
<td>12.816</td>
<td>0.570</td>
<td>3.015</td>
<td>0.263</td>
<td>0.245</td>
<td></td>
</tr>
<tr>
<td>[38]</td>
<td>A514 (WE)/C</td>
<td>0</td>
<td>1</td>
<td>8</td>
<td>0</td>
<td>$E[\phi] = 5.87t$ at $\Delta_{max} = 206.9$MPa</td>
<td>0.043</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[38]</td>
<td>A36 (WE)/C</td>
<td>0</td>
<td>0.25</td>
<td>8</td>
<td>0</td>
<td>11.454</td>
<td>0.623</td>
<td>2.746</td>
<td>0.104</td>
<td>0.039</td>
<td></td>
</tr>
<tr>
<td>[38]</td>
<td>A36 (WE)/C</td>
<td>0</td>
<td>0.5</td>
<td>9</td>
<td>0</td>
<td>11.379</td>
<td>0.661</td>
<td>2.613</td>
<td>0.288</td>
<td>0.106</td>
<td></td>
</tr>
<tr>
<td>[38]</td>
<td>A36 (WE)/C</td>
<td>68.95</td>
<td>0.5</td>
<td>3</td>
<td>0</td>
<td>$E[\phi] = 5.734$ at $\Delta_{max} = 138$MPa</td>
<td>0.027</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[38]</td>
<td>A36 (WE)/C</td>
<td>0</td>
<td>0.5</td>
<td>6</td>
<td>3</td>
<td>11.977</td>
<td>1.254</td>
<td>2.576</td>
<td>0.495</td>
<td>0.107</td>
<td></td>
</tr>
<tr>
<td>[38]</td>
<td>A36 (WE)/C</td>
<td>0</td>
<td>1</td>
<td>8</td>
<td>0</td>
<td>14.230</td>
<td>1.303</td>
<td>3.414</td>
<td>0.527</td>
<td>0.194</td>
<td></td>
</tr>
<tr>
<td>[38]</td>
<td>A36 (WE)/C</td>
<td>68.95</td>
<td>0.5</td>
<td>9</td>
<td>0</td>
<td>12.104</td>
<td>0.473</td>
<td>2.737</td>
<td>0.206</td>
<td>0.076</td>
<td></td>
</tr>
<tr>
<td>[38]</td>
<td>A36 (WE)/C</td>
<td>275.8</td>
<td>0.5</td>
<td>8</td>
<td>0</td>
<td>12.068</td>
<td>0.338</td>
<td>2.829</td>
<td>0.141</td>
<td>0.083</td>
<td></td>
</tr>
<tr>
<td>[38]</td>
<td>A36 (WE)/C</td>
<td>68.95</td>
<td>0.25</td>
<td>8</td>
<td>0</td>
<td>13.606</td>
<td>0.369</td>
<td>3.651</td>
<td>0.170</td>
<td>0.063</td>
<td></td>
</tr>
<tr>
<td>[46]</td>
<td>A588 (WE)/-</td>
<td>–</td>
<td>–</td>
<td>0</td>
<td>8</td>
<td>$\Delta_{max} = 24.6$ MPa (a) (c)</td>
<td>–</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[46]</td>
<td>A588 (WE)/-</td>
<td>–</td>
<td>–</td>
<td>0</td>
<td>4</td>
<td>$\Delta_{max} = 19.2$ MPa (a) (c)</td>
<td>–</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[46]</td>
<td>A588 (WE)/-</td>
<td>–</td>
<td>–</td>
<td>4</td>
<td>4</td>
<td>$\Delta_{max} = 8.04t$ at $\Delta_{max} = 38.3$MPa (b) (c)</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[46]</td>
<td>A588 (WE)/-</td>
<td>–</td>
<td>–</td>
<td>1</td>
<td>3</td>
<td>$\Delta_{max} = 8.02t$ at $\Delta_{max} = 23.0$MPa (b) (c)</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[45]</td>
<td>A588 +/-</td>
<td>unknown</td>
<td>0.34 (d)</td>
<td>4</td>
<td>0</td>
<td>21.6</td>
<td>3.36</td>
<td>8.78</td>
<td>2.07</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>[45]</td>
<td>A588 +/-</td>
<td>unknown</td>
<td>0.40 (e)</td>
<td>3</td>
<td>1</td>
<td>$E[\phi] = 6.92$ at $\Delta_{max} = 47.6$MPa</td>
<td>–</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[45]</td>
<td>A588 +/-</td>
<td>unknown</td>
<td>0.50 (f)</td>
<td>4</td>
<td>4</td>
<td>$E[\phi] = 8.14$ at $\Delta_{max} = 44.1$MPa</td>
<td>–</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Detail Type A: the cover plates were welded to the flange by longitudinal fillet welds only. The assembly was then fillet welded to the web. Detail Type B: the flange was fillet welded to the web. Successively, the cover plates were welded to the flange by longitudinal fillet welds only. Detail Type C: transverse fillet welds were added to the ends of the cover plates. Exact welding sequence unknown.

(a) The welding sequence is not specified. The stress spectrum is reported in the reference. The results from the girders 1 N,1S,and 3E are considered.

(b) The welding sequence is not specified. The stress spectrum is reported in the reference. Only the results from the girders 2 N,2S, and 3 W are considered, excluding those from girder pair 4 due to the different type of stress spectrum used.

(c) The stress spectrum follows a Rayleigh distribution, as described in Section 5.1 however, in this case, single spikes at a higher stress range, $\Delta_{sp}$, were applied in the random block.

(d) The stress spectrum follows a Rayleigh distribution, as described in Section 5.1 with $W = 3.75$.

(e) The stress spectrum follows a Rayleigh distribution, as described in Section 5.1 with $W = 4.00$.

(f) The stress spectrum follows a Rayleigh distribution, as described in Section 5.1 with $W = 4.75$.
For Models 1 and 2, where the slope $m$ is calculated based on CA fatigue test data, the value of the loglikelihood is significantly lower than for Models 1 and 2. This is attributed to two factors: (1) the CAS-N curve obtained for constructing Models 1 and 2 is shallower than that one obtained for constructing Models 1 and 2, and (2) for this detail it appearsthat the stress ranges below the fatigue limit are damaging as those above.

Fig. 11 allows to visualise the difference between the fatigue resistance curves derived for Model 1, Model 2, the proposed model, and the model in reference [6]. In this figure, failure and runout data are respectively indicated by circles and triangles. The value of the critical damage is reported for each subfigure, and the location of the fatigue limit is indicated by a dashed line. The CAS-N curve is indicated by a black line, whereas the fatigue resistance curve for considering VA loading is depicted in red. The figures show that the estimated value of the fatigue limit in Models 1 and 2 is biased with respect to the distribution of failure and runout data. Instead, the proposed model and the model in reference [6] show the location of the fatigue limit to be identical. The difference between Models 1 and 2 in the extension of the S-N curve for VA loading is obvious. For the proposed model, since the threshold stress range $\Delta d_0$ decreases with increasing the damage, the fatigue resistance curve is plotted for four values of the ratio $\Delta d_0/D_{cr}$. For $\Delta d_0/D_{cr} = 0.1$ the resulting fatigue resistance curve is approximately a straight line, as for Model 2. However the fatigue resistance curve is, in this case, used in conjunction with a significantly higher value of $D_{cr}$. The fatigue resistance curve resulting from the model in [6] is characterised by two slopes: $-3.28$ and $-3.68$, for the first and the second branch.

$$\sigma_{d} = m_2 \frac{D_{cr}}{m_1} = 0.258$$

for model 1, i.e. 1.66 times larger than in the first branch. For Models 1b and 2b, where the slope $m$ is calculated based on CA fatigue test data, the value of the loglikelihood is significantly lower than for Models 1 and 2. This is attributed to two factors: (1) the CA S-N curve obtained for constructing Models 1b and 2b is shallower than that one obtained for constructing Models 1 and 2, and (2) for this detail it appears that the stress ranges below the fatigue limit are as damaging as those above.

Fig. 12 allows to visualise the difference between the fatigue resistance curves derived for Model 1, Model 2, the proposed model, and the model in reference [6]. In this figure, failure and runout data are respectively indicated by circles and triangles. The value of the critical damage is reported for each subfigure, and the location of the fatigue limit $\Delta d_0$, is indicated by a dashed line. The CA S-N curve is indicated by a black line, whereas the fatigue resistance curve for considering VA loading is depicted in red. The figure shows that the estimated value of the fatigue limit in Models 1 and 2 is biased with respect to the distribution of failure and runout data. Instead, the proposed model and the model in reference [6] show the location of the fatigue limit to be identical. The difference between Models 1 and 2 in the extension of the S-N curve for VA loading is obvious. For the proposed model, since the threshold stress range $\Delta d_0$ decreases with increasing the damage, the fatigue resistance curve is plotted for four values of the ratio $D_{cr}/D_{cr}$. For $D_{cr}/D_{cr} = 0.1$ the resulting fatigue resistance curve is approximately a straight line, as for Model 2. However the fatigue resistance curve is, in this case, used in conjunction with a significantly higher value of $D_{cr}$. The fatigue resistance curve resulting from the model in [6] is characterised by two slopes: $-3.28$ and $-3.68$, for the first and the second branch.
branch of the curve, respectively. This determines a fatigue resistance curve similar as for Model 2. However, also in this case, the value of the critical damage to be used is higher than for Model 2.

In order to quantify the difference between the proposed model and the model in [6], the predicted fatigue life distributions are evaluated considering epistemic uncertainty and compared. Additionally, for the proposed model, the fatigue life distribution has been computed with and without epistemic uncertainty, in order to quantify its effect. The prediction of the fatigue life has been obtained considering a stress spectrum following a Rayleigh distribution as defined in [38], see Section 5.1, having a dispersion parameter, \( \Delta \sigma_0 \), equal to the modal stress range, \( \Delta \sigma_m \), the latter being equal to the mean of the fatigue limit \( \Delta \sigma_0 = 10^\mu \). The cumulative distributions of the predicted fatigue life are shown in Fig. 13.

For the proposed model, it results that taking epistemic uncertainty into account results in a more scattered fatigue life. However, this effect is negligible for cumulative probability higher than 0.1 and becomes slightly more relevant in the tail of the distribution. The small effect is ascribed to the relatively large number of fatigue test data analysed. The distribution resulting from the application of the model in [6] is heavier in the tails, and the prediction is characterised by a significantly more scattered distribution. This results in a shallower slope of the cumulative distribution, and determines a significant difference in the predicted fatigue life for cumulated probability less than 0.1. An implication of having larger uncertainty is that a larger failure probability is predicted for a certain fatigue life required by the design. This means that a shorter target life is required in order to satisfy a predetermined safety level.

The one-sided 95% tolerance lower bound (75% confidence) has been estimated from the proposed model, by using the Monte Carlo method. The numerical procedure involves the following steps:

1. A Rayleigh stress spectrum is defined;
2. A sample of \( \theta_{\text{CA}} \) is drawn from its distribution, the same is done for \( \theta_{\text{VA}} \);
3. The model response, i.e. the distribution of the fatigue life, is evaluated using the Monte Carlo method, using Eq. (35);
4. The 5% lower prediction bound is evaluated;
5. The steps (2)–(4) are repeated iteratively, leading to a set of 5% lower prediction bound of the fatigue life;
6. The 75% lower confidence level is quantified by selecting the (1–0.75) percentile of the set of 5% lower prediction bound.

The Eurocode 3 - part 9 is meant to provide a fatigue life prediction.
that statistically corresponds to a one-sided 95% tolerance lower bound (75% confidence). For this reason, the prediction of such tolerance lower bound is compared to the prediction obtained using the S-N curve of this detail as in the Eurocode 3, for a cover plated beam having the thickness of the flange between 20 and 30 mm, and the thickness of the cover plated smaller than that of one of the flange. The difference with Model 1 is that the S-N curve prescribed in the Eurocode 3 is used here, and not the one obtained by the inference of the test data. The two predictions using the Eurocode 3 and using the proposed model are compared in Fig. 14. It results that if \( r = -5 \), and \( D_0 = 1 \) are selected, the resulting prediction reasonably approximates the selected lower confidence bound for a fatigue life up to \( 10^6 \) cycles. Instead, the prediction using the resistance model provided in the Eurocode 3 becomes optimistic with, in the worst case, a factor of 2, for a fatigue life higher than \( 10^6 \) cycles.

5.3. Discussion

The results for the cruciform joint, the cover plated specimen and the welded beam, see Fig. 8, are reported in the Appendix A, in Table A.10.

The mean value of the critical damage, \( \mu_D \), obtained from the inference for the non-load carrying cruciform joint detail is similar to that one obtained from the cover plated beam detail, see Table 4. Whereas, the value obtained for the cover plated specimen is similar to that one obtained for the welded beam. This appears to be associated to the type of weld. The cover plated specimen and the welded beam are characterised by welds longitudinal to the loading direction. Whereas, the non-load carrying cruciform joint detail and most of the cover plated beams tested under CA loading have transverse welds.

The difference obtained for the estimator of the standard deviation of the critical damage, \( \sigma_D \), is attributed to the number of datasets involved for the estimation of the fatigue resistance under both constant and variable amplitude loading. In particular, it can be observed that the estimators of \( \sigma_D \) obtained for the cruciform joint and for the cover plated specimen are in the same order of magnitude, with the value for the cruciform joint being approximately 1.5 times higher than for the cover plated specimen. The fact that the values are in the same order of magnitude is associated with the circumstance that the test data performed for both CA and VA loading are associated to specimens fabricated and tested of the same material and within the same experimental campaign. The larger value encountered for the cruciform joint can be associated to a lower severity of the detail, leading to multiple initiation points, and imperfections, such as misalignment of the external plates. The value obtained for the welded beam detail is one order of magnitude smaller than for the cruciform joint and the cover plated specimen. This is attributed to the circumstance that several datasets from different experimental research works and different batches of material are used to characterise the fatigue resistance under CA loading. Instead, for VA loading the datasets are from one source only, despite being from two different materials. However this appears not enough to cover the large scatter found in CA test data due to the presence of test data from different sources. The value of \( \sigma_D \) estimated for the cover plated beam detail is the largest. This is attributed, in line with what was stated for the other details, to the large amount of datasets used for both CA and VA loading. In addition, the datasets used for VA loading, despite being still determined by applying a stress spectrum following a Rayleigh distribution, are subjected to spectra that have some differences in: (1) the length of the random block of loading ranges from 500 to 1024 cycles, (2) the truncation level of the spectra is not constant, as it happens for the other datasets, (3) the tests presented in [46] were obtained by applying a single overload, \( \Delta e_p > \Delta e_{\max} \) within the random block, or by truncating the lower tail of the spectrum.

The values obtained for \( \zeta \) are similar for the three details examined in the appendix, and one order of magnitude lower than the value observed for the cover plated beam. The largest difference with the value determined by Kunz [32], i.e. \( \zeta = 1 \), is estimated for the welded beam detail. The large difference obtained for the cover plated beam detail is attributed to the different types of weld dominant in the datasets used for VA and CA loading. The CA fatigue tests examined in [7] are obtained using mostly cover plated beams with welded ends. Instead, the number of cover plated beams with unwelded ends dominates for the VA fatigue test dataset. Furthermore, among the VA data from [38], only those related to a welding sequence that was inducing the shortest fatigue life have been considered. These considerations suggest that the fatigue performance, that most of the specimens belonging to the dataset used for VA loading would have under CA loading, is lower than estimated for CA loading.

The value of \( \zeta \), which has been here found to be dependent also on the geometry of the welded detail, is observed to be in every case higher than 1, that is the upper value given in [32]. The relation between \( \zeta \) and the geometry of the welded detail found in Section 3.2, see Eq. (29), determines that with decreasing \( \sigma \) (i.e. increasing its absolute value, being \( \sigma < 0 \)) \( \zeta \) decreases. It is observed that this trend appears to be verified for three out of the four considered details. By comparing the results of the cruciform joint, the welded beam, and the cover plated specimen, see Table A.10, it results that the more severe detail, i.e. the cover plated specimen, is associated to a lower value of \( \zeta \), and this value increases with decreasing the severity of the detail, quantified using the mean value of the fatigue limit, \( \mu_D \), as determined using linear elastic fracture mechanics, Section 3.2. The selection of \( \mu_D \) over \( \mu_D \), which locates the finite life region of the 6PRFLM, is made because \( \mu_D \) is strongly correlated with \( \mu_D \), for which the estimators significantly differ among the selected details, ranging from \( \sim 3.99 \) to \( \sim 2.57 \). The lack of correspondence found for the cover plated beam is associated with the aspects stated above. An analytical derivation of \( \zeta \) using fracture mechanics remains a challenging task. The relation found, Eq. (29), is limited to the following simplifying assumptions: constant amplitude loading, the value of \( \mu_D \) in Eq. (17) being zero, the critical condition occurring when \( \mu_D > 1 \), and the approximation of the small crack behaviour modelled using a first order approximation of Eq. (16).

6. Conclusions

In the present work, a model for fatigue life prediction under variable amplitude loading has been proposed and applied to welded details subjected to Rayleigh-based random block loading. The model makes use of the S-N curve to modelling the constant amplitude fatigue resistance, the relative linear damage rule, and the damage limit concept. The 6 Parameters Random Fatigue Limit Model has been selected as S-N curve to model the fatigue resistance under CA loading. Similarly to the models in which the S-N curve is extended below the fatigue limit with a modified slope, in which the parameter \( m \) has to be calibrated.
the proposed model requires the calibration of a model parameter, namely $\zeta$. Based on linear elastic fracture mechanics, $\zeta$ is found to be dependent on the severity of the weld notch geometry.

The model has been used for inferring VA fatigue test data. Following the inference, the epistemic uncertainty has been estimated. Two large scale and two small scale details have been selected for the analysis: the cover plated steel beam, as a welded detail with a relatively low fatigue resistance, and the welded beam, which instead is characterised by a relatively high fatigue resistance, are the large scale details. The cover plated specimen and the non-load carrying cruciform joint are selected as small scale specimens. The proposed model appears to be more adequate in predicting the fatigue life than currently available models used for welded details because compared to other models the uncertainty is minimised.

The difference in the mean value of the critical damage, $\mu_D$, estimated from inferring different structural details seems to be related to the type of weld, longitudinal or transversal to the loading direction. This might be associated with load sequence effects, which differ because of the different crack propagation mechanisms/paths in these type of welds. It is found that the variability associated with the critical value of the fatigue damage parameter, $\sigma_D$, is strongly dependent on the source and fabrication of the specimens. Special attention should be paid to selecting the value most suitable to applications for real structures. The value of the parameter controlling the effect of the stress ranges lower than the fatigue limit, $\sigma_f$, is found to be higher than the value previously suggested in the literature. In particular, a value comprised between 1.0 and 1.5 seems reasonable for the type of spectrum used in the investigation. Other types of stress spectra, different from a Rayleigh distribution, might be associated with different values. By comparing the results of the cruciform joint, the welded beam, and the cover plated specimen, it results that $\sigma_f$ increases with decreasing the severity of the detail, as determined using linear elastic fracture mechanics. The significantly larger estimator found the cover plated beam is associated with the different type of weld (either longitudinal or transverse) dominant in CA and in VA fatigue test data.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

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Appendix A. Fatigue test data

In this Appendix, the fatigue test datasets for the non-load carrying cruciform joint, the cover plated specimen, and the welded beam are presented for both constant and variable amplitude loading.

Non-load carrying cruciform joint

A large number of the test datasets is present in the scientific literature to characterise the fatigue behaviour of the non load carrying cruciform joint under CA loading [63–67]. However, a large scatter was found when analysing the pooled dataset. It was attributed to the circumstance that dataset by dataset, the tests are executed on specimens with different materials, different production processes (i.e. welding), and with different loading conditions. Moreover, misalignment plays a role for this type of welded detail. The dataset presented in [65] is here considered as a reference because the VA test data considered here are also from [65], therefore the specimens fabrication and test control are assumed to be similar.

The variable amplitude dataset considered in this study was first presented in [65]. The dataset consists of 10 test data generated by applying a random block stress history sampled from a Rayleigh distribution. In this case, the spectrum is characterised by $\Delta\sigma_{\text{min}} = 0$, $\Delta\sigma_1 = \Delta\sigma_2$, and $\Delta\sigma_{\text{max}} - \Delta\sigma_{\text{min}} = 3\Delta\sigma_1$.

Cover plated specimen

The dataset presented in [38] is here considered for characterizing the fatigue resistance of the cover plated specimen for CA loading. Different from the cover plated beam detail, the cover plate is welded on one side of the specimen. Moreover, the specimens are axially loaded. This determines that a bending moment is induced because of the presence of the cover plated.

The variable amplitude dataset considered in this study was first presented in [38]. The dataset consists of 54 test data generated by applying a random block stress history sampled from a Rayleigh distribution, as described in Section 5.1. In this case, the spectrum is characterised by: $\Delta\sigma_{\text{min}} = 0$, $\Delta\sigma_{\text{max}} - \Delta\sigma_{\text{min}} = 3\Delta\sigma_1$, and $\Delta\sigma_1 = \Delta\sigma_2$.

<table>
<thead>
<tr>
<th>Detail</th>
<th>Ref</th>
<th>$\sigma_{\text{min}}$</th>
<th>Weld</th>
<th>Base steel</th>
<th>Data</th>
<th>Runout $\beta_0$</th>
<th>Runout $\beta_1$</th>
<th>$\sigma_{\text{u}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cruciform joint</td>
<td>[65]</td>
<td>13.8</td>
<td>SAW</td>
<td>ASTM A572</td>
<td>29</td>
<td>9</td>
<td>15.0 (14.5, 15.5)</td>
<td>-4.07 (-4.28, -3.85)</td>
</tr>
<tr>
<td>Cover plated specimen</td>
<td>[38]</td>
<td>0</td>
<td>SAW</td>
<td>ASTM A514</td>
<td>4</td>
<td>1</td>
<td>13.1 (12.2, 13.9)</td>
<td>-3.33 (-3.71, -2.94)</td>
</tr>
<tr>
<td></td>
<td>[38]</td>
<td>68.95</td>
<td>SAW</td>
<td>ASTM A514</td>
<td>9</td>
<td>0</td>
<td>11.6 (11.3, 11.8)</td>
<td>-2.69 (-2.80, -2.58)</td>
</tr>
<tr>
<td></td>
<td>[38]</td>
<td>275.8</td>
<td>SAW</td>
<td>ASTM A514</td>
<td>6</td>
<td>0</td>
<td>12.1 (11.6, 12.5)</td>
<td>-2.89 (-3.11, -2.66)</td>
</tr>
<tr>
<td>Pooled dataset</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>19</td>
<td>1</td>
<td>11.9 (11.6, 12.2)</td>
<td>-2.82 (-2.95, -2.70)</td>
</tr>
<tr>
<td>Welded beam</td>
<td>[38]</td>
<td>0</td>
<td>SAW</td>
<td>ASTM A514</td>
<td>9</td>
<td>0</td>
<td>14.2 (12.6, 15.7)</td>
<td>-3.6 [-4.3, -2.9]</td>
</tr>
<tr>
<td></td>
<td>[38]</td>
<td>0</td>
<td>SAW</td>
<td>ASTM A36</td>
<td>6</td>
<td>0</td>
<td>14.3 (11.4, 17.3)</td>
<td>-3.6 [-4.9, -2.3]</td>
</tr>
<tr>
<td></td>
<td>[38]</td>
<td>68.5</td>
<td>SAW</td>
<td>ASTM A36</td>
<td>9</td>
<td>0</td>
<td>12.9 (11.5, 14.2)</td>
<td>-3.0 [-3.6, -2.4]</td>
</tr>
<tr>
<td>[68]</td>
<td></td>
<td></td>
<td>SAW</td>
<td>ASTM A36</td>
<td>14</td>
<td>1</td>
<td>13.5 (11.5, 15.5)</td>
<td>-3.7 [-4.2, -2.4]</td>
</tr>
<tr>
<td></td>
<td>[68]</td>
<td></td>
<td>SAW</td>
<td>ASTM A514</td>
<td>20</td>
<td>0</td>
<td>11.6 (10.8, 12.3)</td>
<td>-2.4 [-2.8, -2.1]</td>
</tr>
<tr>
<td></td>
<td>[68]</td>
<td></td>
<td>SAW</td>
<td>ASTM A414</td>
<td>20</td>
<td>0</td>
<td>13.4 (12.3, 14.6)</td>
<td>-3.3 [-3.8, -2.8]</td>
</tr>
<tr>
<td>Pooled dataset</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>78</td>
<td>1</td>
<td>13.2 (12.7, 13.8)</td>
<td>-3.2 [-3.4, -2.9]</td>
</tr>
</tbody>
</table>
The welded beams are produced from three types of steels, either A514, or A414, or A36, therefore a wide range of structural steel grades is covered. Test data are from [38,68]. The beams were produced by first tack welding the flanges to the web. Successively, a continuous weld was laid down. The VA fatigue test data used here are from [38], and were produced in a similar fashion as those used for CA loading. In most of the specimens, during the execution of the tests, fatigue cracks were observed to nucleate at the tack welds, for both CA and VA loading. In most cases, more than one crack were found to be present at failure.

### Table A7
MLE and Likelihood value for the 6PRFLM, and parameters of the Basquin relation ($m = -3$) fitting the considered CA datasets.

<table>
<thead>
<tr>
<th>Detail</th>
<th>MLE</th>
<th>log $[L(0)]$</th>
<th>MLE</th>
<th>log $[L(0)]$</th>
<th>MLE</th>
<th>log $[L(0)]$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\hat{\beta}_0$</td>
<td></td>
<td>$\hat{\beta}_0$</td>
<td></td>
<td>$\hat{\beta}_0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+1.48E + 01</td>
<td>14.875</td>
<td>+1.13E + 01</td>
<td>17.769</td>
<td>+1.20E + 01</td>
<td>36.129</td>
</tr>
<tr>
<td></td>
<td>$\hat{\beta}_1$</td>
<td>-3.99E + 00</td>
<td>-2.57E + 00</td>
<td>-2.69E + 00</td>
<td>+1.98E + 00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+7.33E - 02</td>
<td></td>
<td>+4.98E - 02</td>
<td></td>
<td>+1.39E - 01</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+2.10E + 00</td>
<td></td>
<td>+1.69E + 00</td>
<td></td>
<td>+1.98E + 00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+3.83E - 02</td>
<td></td>
<td>+1.97E - 01</td>
<td></td>
<td>+8.04E - 02</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+4.36E - 08</td>
<td></td>
<td>+3.67E - 01</td>
<td></td>
<td>+4.20E-01</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+1.26E + 01</td>
<td></td>
<td>+1.26E + 01</td>
<td></td>
<td>+1.26E + 01</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+2.11E - 01</td>
<td></td>
<td>+1.09E - 01</td>
<td></td>
<td>+1.26E - 01</td>
<td></td>
</tr>
</tbody>
</table>

### Table A8
Epistemic uncertainty estimated for the MLE of the 6PRFLM fitting the datasets.

<table>
<thead>
<tr>
<th>Detail</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\hat{\mu}_v$</td>
</tr>
<tr>
<td>Cruciform joint</td>
<td>4.38E - 01</td>
</tr>
<tr>
<td>Cover plated specimen</td>
<td>4.74E - 01</td>
</tr>
<tr>
<td>Welded Beam</td>
<td>8.92E - 01</td>
</tr>
</tbody>
</table>

### Table A9
Summary of VA fatigue test datasets, statistical description and inference using the Gassner curve.

<table>
<thead>
<tr>
<th>Detail</th>
<th>Ref</th>
<th>$\Delta \sigma / \Delta \sigma_m$</th>
<th>$\sigma_{min}$</th>
<th>Material</th>
<th>Failure</th>
<th>Runout</th>
<th>$\hat{\sigma}_0$</th>
<th>$\hat{\sigma}_1$</th>
<th>$\Delta \sigma$</th>
<th>$\sigma_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cruciform joint</td>
<td>[65]</td>
<td>1</td>
<td>13.97</td>
<td>ASTM A 572</td>
<td>10</td>
<td>0</td>
<td>18.0 [15.7, 20.2]</td>
<td>-4.84 [ -5.81, -3.87]</td>
<td>0.131</td>
<td></td>
</tr>
<tr>
<td>Cover plated specimen</td>
<td>[38]</td>
<td>0.5</td>
<td>68.95</td>
<td>ASTM A514</td>
<td>6</td>
<td>0</td>
<td>12.3 [11.5, 13.2]</td>
<td>-2.69 [-3.04, -2.34]</td>
<td>0.126</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[38]</td>
<td>0.5</td>
<td>68.95</td>
<td>ASTM A514</td>
<td>6</td>
<td>0</td>
<td>12.3 [11.5, 13.2]</td>
<td>-2.69 [-3.04, -2.34]</td>
<td>0.126</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[38]</td>
<td>0.5</td>
<td>275.8</td>
<td>ASTM A514</td>
<td>30</td>
<td>0</td>
<td>13.0 [12.5, 13.4]</td>
<td>-2.84 [-3.00, -2.68]</td>
<td>0.0774</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[38]</td>
<td>1.0</td>
<td>68.95</td>
<td>ASTM A514</td>
<td>6</td>
<td>0</td>
<td>12.3 [12.0, 12.5]</td>
<td>-2.57 [-2.66, -2.48]</td>
<td>0.0322</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[38]</td>
<td>0.5</td>
<td>11.7 [11.4, 12.0]</td>
<td>54</td>
<td>0</td>
<td>-2.39 [-2.52, -2.27]</td>
<td>0.144</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Welded Beam</td>
<td>[38]</td>
<td>0.5</td>
<td>68.5</td>
<td>ASTM A36</td>
<td>3</td>
<td>0</td>
<td>12.0 [11.2, 12.9]</td>
<td>-2.4 [-2.7, -2.1]</td>
<td>0.188</td>
<td></td>
</tr>
</tbody>
</table>

**Welded beam**

The welded beams are produced from three types of steels, either A514, or A414, or A36, therefore a wide range of structural steel grades is covered. Test data are from [38,68]. The beams were produced by first tack welding the flanges to the web. Successively, a continuous weld was laid down. The VA fatigue test data used here are from [38], and were produced in a similar fashion as those used for CA loading. In most of the specimens, during the execution of the tests, fatigue cracks were observed to nucleate at the tack welds, for both CA and VA loading. In most cases, more than one crack were found to be present at failure.
The considered datasets are summarised in Table A.6. The MLE of the parameters of the 6PRFLM are reported in Table A.7, together with the value of the log-likelihood function, and the estimators of the parameters of the Basquin relation, Eq. (47), according to the standard procedures, where \( m_0 = -3 \). The epistemic uncertainty resulting from the inference of the 6PRFLM to the CA fatigue test data is presented in Table A.8. The datasets considered for VA loading are summarised in Table A.9. The estimators for the proposed model and the results of the inference, i.e. the estimation of the epistemic uncertainty, are reported in Table A.10.

### Table A10

Model parameters and uncertainty estimated using the MLM for the proposed model using the datasets, given \( \sigma_{av} \) as in Table A.7.

<table>
<thead>
<tr>
<th>Cruciform joint</th>
<th>( \beta_0 )</th>
<th>( \sigma_{av} )</th>
<th>( \xi )</th>
<th>( \log L(\theta) )</th>
<th>( \kappa )</th>
<th>( \log L(\theta) )</th>
<th>( \delta )</th>
<th>( \sigma_{av} )</th>
<th>( \log L(\theta) )</th>
<th>( \delta )</th>
<th>( \sigma_{av} )</th>
<th>( \log L(\theta) )</th>
<th>( \delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_0 )</td>
<td>1.61E + 00</td>
<td>1.62E - 01</td>
<td>1</td>
<td>2.49E - 01</td>
<td>2.43E - 02</td>
<td>2.98E - 01</td>
<td>2.81E - 02</td>
<td>2.79E - 02</td>
<td>2.68E - 02</td>
<td>2.63E - 02</td>
<td>2.58E - 02</td>
<td>2.53E - 02</td>
<td>2.48E - 02</td>
</tr>
</tbody>
</table>

**References**


[31] Fisher JW. Resistance of welded details under variable amplitude long-life fatigue
loading. Transportation Research Board; 1993.


