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Oscillations up to 712 GHz in InAs/AlSb resonant-tunneling diodes
The stability of the self-consistently determined current of a double-barrier resonant-tunneling diode

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Double-barrier resonant-tunneling devices belong to a novel class of nanoelectronic devices with great potential applications. In these devices, the self-consistent buildup of charge due to resonant carriers in the well may lead to bistability and hysteresis. To investigate aspects of dynamical (in)stability, a simple set of equations is derived from an extension of the static theory. These dynamic equations adequately describe small and slow (<100 GHz) deviations from the stationary state. This approach is viewed more as being more satisfactory than an equivalent-circuit analysis, but its limitations are also discussed.

I. INTRODUCTION

Double-barrier resonant-tunneling (DBRT) structures have attracted much attention, because they exhibit, among many other interesting features, intrinsic bistability in the I-V characteristic. Experimentally, this bistability is encountered as a hysteresis, whereas theoretically a voltage interval with triple-valued current (a Z-shaped I-V curve) is predicted. One way to harmonize these different data, is to declare one of the three calculated branches in the I-V characteristic unstable, whereas the other curves are assumed to be completed in a hysteresis way. Another approach is to speak of intrinsic tristability, assuming the third solution to be hard to access experimentally.

As the discrepancy between theory and experiments is still too large to justify a complete identification of the hysteresis interval with the triple-current interval, a stability criterion based on a theoretical analysis of the DBRT is desirable. In this paper, we present such a stability analysis, resulting from minor extensions to our static model. First, we release the coupling between the charge densities in the emitter and the well, thus attaining a dynamical system with two independent variables. Second, we include a model of the leads connecting the double-barrier structure to the battery, yielding a fair description of the way in which fluctuations are compensated in this part of the circuit. In the present study, only the simplest case, that of a purely Ohmic contact, is considered.

II. STATIC MODEL

In some models of the DBRT, the bistability in the I-V curve corresponds to a voltage interval where the voltage $V_x$ across the structure is related to three values of the current density $J$. Hence, there exists no single-valued function $J(V_x)$ in the bistable regime, though we can give a parametric description of the I-V curve. This is done by taking the Fermi energy $E_F$ in the emitter as a parameter: $V_x(E_F)$, $J(E_F)$, where both $V_x(E_F)$ and $J(E_F)$ are single-valued (but nonmonotonic, hence noninvertible) functions. Expressions for $V_x(E_F)$ and $J(E_F)$ result from a combined solution of the Poisson equation and the Schrödinger equation: the charge densities calculated via the wave functions from the Schrödinger equation enter the Poisson equation for the electron potential, while this potential is used in the Schrödinger equation for the wave functions. Thus, $V_x(E_F)$ and $J(E_F)$ are determined in an iterative scheme, so that on convergence the ultimate electron potential and charge density are qualified as "self-consistent."

For $V_x(E_F)$ the above method can easily be worked out. In the double-barrier structure, charge buildup takes place in the emitter, well, and collector. Since the three charge densities are interrelated due to the demand of overall charge neutrality, we need only consider two of them. Thus, the Poisson equation yields a function $V_x(\Sigma_z, \Sigma_w)$, where $\Sigma_z$ and $\Sigma_w$ are the areal charge densities in the emitter and well, respectively. Considering $\Sigma_z$ to result from impinging electrons in the energy range $0-E_F$, and $\Sigma_w$ from electrons in the same energy range tunneling into the well, we can find expressions for $\Sigma_z(E_F)$ and $\Sigma_w(E_F)$. Now we can write for the voltage over the structure $V_x(E_F) = V_x[\Sigma_z(E_F), \Sigma_w(E_F)]$, where it is the Poisson equation that relates $V_x$ to $\Sigma_z$ and $\Sigma_w$ and the time-independent Schrödinger equation that makes these charge densities in turn related to $E_F$. If we abandon the latter equation (as we should in a dynamical analysis), $V_x(E_F)$ can be generalized.

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The current density $J(E_F)$ needs a different approach. Since $J$ is defined within the stationary wave-function formalism, i.e., for self-consistent situations only, giving up the time-independent Schrödinger equation means giving up our expression for the current density. In this case the generalization, necessary for the dynamic model, is not straightforward, nor even unique. Let us start from the expression for the self-consistent current density:

$$
J(E_F) = N_c^2 \phi_0 \left( \frac{E_F - E_{\text{res}}}{kT} \right) \times g \frac{1 - R_1 (1 - R_2)}{2(1 - R_1 R_2)} \frac{\sqrt{2E_F/m}}{\omega},
$$

where $g = 2$ takes into account the spin degeneracy, $e$ is the electron charge, and $R_1$ and $R_2$ are the reflection coefficients on the first and second barrier at the resonance energy. Finally, $m$ is the effective mass of electrons in the conduction band, $w$ is the width of the well, and $E_F$ is the resonance energy with respect to the conduction band in the well. Note that $E_{\text{res}}$ and $E_0$ are different quantities that refer to the same energy level. The first factor in Eq. (1),

$$
N_0 = N_c^2 \phi_0 (E_F - E_{\text{res}})/kT,
$$

has an easy interpretation: it is the number of points in $k$ space per square meter, for which $|k|^2 < 2mE_F/h^2$ and $k_z = k_{\text{res}}$. In other words, $N_0$ is the areal density of resonant electrons in the emitter. In order to interpret the remaining factors, a little rearrangement is helpful. The second factor multiplied by $(1 + R_2)/(1 - R_2)$ yields

$$
q_0 = g \frac{1 - R_1 (1 - R_2)}{2(1 - R_1 R_2)} \frac{\sqrt{2E_F/m}}{\omega},
$$

which is the charge per resonant electron located in the well. If the second barrier were impenetrable, i.e., $R_2 = 1$, the $q_0$ would be $g \phi_0$ as expected. Thus, the reflection coefficients in Eq. (3) take into account the leaky nature of the well. The product of the first two factors, $N_0 q_0$, is the areal density of resonant electrons in the emitter and contains no new information.

III. EXTENSIONS TO STATIC MODEL

The static model of the DDRT outlined in the previous section, amounts to handling the Poisson equation and the Schrödinger equation simultaneously. Since both equations are time independent, no dynamics is included. In order to investigate the stability, however, we have to deal with time-dependent charge and current densities, so we cannot adhere to this set of equations. Nevertheless, it suffices to give up the time-independent Schrödinger equation. This is because the electron potential adjusts itself to changes in the charge distribution effectively instantaneously, i.e., with the speed of light. Instead of replacing the time-independent Schrödinger equation with its time-dependent version (which is the formally correct method), we opt for a different, approximative, and intuitive approach: the time-independent equation effectuated a coupling between $\Sigma_e$ and $\Sigma_w$ so that abandoning this equation amounts to considering $\Sigma_e$ and $\Sigma_w$ as henceforth independent variables. Since in the dynamic case the current density has nonzero divergence, we need to specify it for various positions in the structure: Introducing the current densities $J_L$ in the lead, and $J_1 (J_2)$ through the first (second) barrier (see Fig. 1), we write for the rates of areal charge density in the emitter and well

$$
\frac{d\Sigma_e}{dt} = J_L - J_1,
$$

$$
\frac{d\Sigma_w}{dt} = J_1 - J_2.
$$

The third equation, for the charge density in the collector, is equivalent to the condition of overall charge neutrality, and contains no new information.

If the simplest example of a lead, a purely Ohmic contact, is considered, we can specify the current density $J_L$ by the quotient

$$
J_L(t) = \frac{V_s(t) - V_c \left[ \Sigma_e(t), \Sigma_w(t) \right]}{\rho l},
$$

where $\rho$ is the specific resistance of the lead and $l$ is its length. $V_s(t)$ is the externally applied battery voltage, and
FIG. 1. Conduction-band minimum in the DBRT structure as a function of position \( z \). Below: schematic picture of charge buildup and current densities.

\( V_{\varepsilon}(\varepsilon_{e}(t),\varepsilon_{w}(t)) \) is the potential drop across the double-barrier structure as obtained from the Poisson equation.

In order to specify the current densities \( J_1 \) and \( J_2 \), we turn to the two generalizations of the previous section. A natural and appealing choice is

\[
J_1(t) = N_0[\varepsilon_{e}(t),\varepsilon_{w}(t)] \times I_0[\varepsilon_{e}(t),\varepsilon_{w}(t)]
\]

and

\[
J_2(t) = \frac{\varepsilon_{w}(t)}{t_d[\varepsilon_{e}(t),\varepsilon_{w}(t)]}.
\]

Equation (7b) represents the current density as seen from the emitter, and Eq. (7c) as seen from the well. In Eq. (7b) the instream of electrons from the emitter to the well is proportional to the areal density of resonant electrons in the emitter, but the current they carry depends on the actual amount of space charge in the well. In Eq. (7c) the outstream of electrons from the well to the collector is proportional to \( 2 \), but the time constant \( t_d \) depends on the actual amount of space charge in the emitter. This intuitive reasoning substitutes a formal derivation.

Equations (6) and (7) are an approximate model for the dynamics of the DBRT structure. It may seem that the dynamics is described by a set of truly nonlinear equations. However, the way in which the expressions for \( J_1 \) and \( J_2 \) are obtained, prevents us from taking Bqs. (7b) and (7c) too seriously for large deviations from the self-consistent solution. Therefore, a linearized version of Eq. (7) will be investigated in the following section.

IV. LINEARIZATION OF DYNAMIC EQUATIONS

Let us suppose that at \( t = 0 \) the DBRT structure has a self-consistent electron potential and corresponding charge distribution. A small fluctuation in the battery voltage \( V_b(t) \), e.g., a pulse function of small amplitude and finite duration, will initiate a dynamical evolution of charge and current densities, that may or may not be bounded at infinite time. If all quantities remain bounded, the system originated from a stable point in the \( I-V \) characteristic. If, on the other hand, there is an unbounded quantity, the starting point is called unstable. The boundedness is most easily checked by performing Laplace transforms of the linearized dynamic equations.

Let for all quantities the difference between the value at time \( t \) and the value at \( t = 0 \) be indicated with the corresponding lower-case notation, e.g., \( \varepsilon_e(t) = V_e(t) - V_e(0), \varepsilon_w(t) = \Sigma_w(t) - \Sigma_w(0), \) etc. In terms of these differences, and leaving out all higher-order terms, Eqs. (7) read as follow:

\[
j_L(t) = \frac{v_b(t)}{\rho l} A_e \sigma_e(t) A_w \sigma_w(t),
\]

\[
j_1(t) = B_e \sigma_e(t) + B_w \sigma_w(t),
\]

\[
j_2(t) = C_e \sigma_e(t) + C_w \sigma_w(t),
\]

where the coefficients \( A_e \) and \( C_w \) are partial derivatives in the original self-consistent solution:

\[
A_e = \frac{\partial V_s}{\partial \varepsilon_e}, \quad A_w = \frac{\partial V_s}{\partial \varepsilon_w},
\]

\[
B_e = \frac{\partial N_0}{\partial \varepsilon_e} I_0 + N_0 \frac{\partial I_0}{\partial \varepsilon_e}, \quad B_w = \frac{\partial N_0}{\partial \varepsilon_w} I_0 + N_0 \frac{\partial I_0}{\partial \varepsilon_w},
\]

\[
C_e = - \frac{\Sigma_w(0)}{t_d} \frac{\partial \varepsilon_e}{\partial \varepsilon_w}, \quad C_w = \frac{1}{t_d} + \frac{\Sigma_w(0)}{t_d^2} \frac{\partial I_0}{\partial \varepsilon_w}.
\]

In these expressions, \( t_d \) and \( \partial I_0/\partial \varepsilon_e \) are to be understood as \( t_d[\varepsilon_e(0),\varepsilon_w(0)] \) and \( \partial I_0[\varepsilon_e(0),\varepsilon_w(0)]/\partial \varepsilon_e \) etc. Substituting Eq. (8) in Eq. (6), we obtain the linearized rate equations

\[
\frac{d \sigma_e}{dt} = \frac{v_b(t)}{\rho l} + \left( - \frac{A_e}{\rho l} - B_e \right) \sigma_e(t) + \left( - \frac{A_w}{\rho l} - B_w \right) \sigma_w(t),
\]

\[
\frac{d \sigma_w}{dt} = (B_e - C_e) \sigma_e(t) + (B_w - C_w) \sigma_w(t),
\]

which are most easily solved using the Laplace transform. Denoting the Laplace transform of \( \sigma_e(t) \) by \( \bar{\sigma}_e(s) \), etc., we can cast Eq. (9) in matrix form:

\[
\begin{pmatrix}
1 + \frac{A_e}{\rho l} + B_e + \frac{A_w}{\rho l} + B_w \\
\frac{B_e - C_e}{s - B_w + C_w}
\end{pmatrix}
\begin{pmatrix}
\bar{\sigma}_e(s) \\
\bar{\sigma}_w(s)
\end{pmatrix}
= \begin{pmatrix}
\bar{v}_b(s) \\
0
\end{pmatrix},
\]

which is solved by inverting the matrix. Thus we find

\[
\bar{\sigma}_e(s) = \frac{s - B_w + C_w}{(s - s_1)(s - s_2)} \bar{v}_b(s),
\]

\[
\bar{\sigma}_w(s) = \frac{B_e - C_e}{(s - s_1)(s - s_2)} \bar{v}_b(s),
\]

where \( s_1 \) and \( s_2 \) obey

\[
s_1 + s_2 = -(A_e/\rho l) - B_e + B_w - C_w.
\]
Equation (12) is of utmost importance for the stability, as can be seen from the inverse transforms of Eq. (11):

\[ \sigma_a(t) = \int_0^t \left( s_1 - B_w + C_w e^{st} + \frac{B_w - C_w - s_2}{s_1 - s_2} e^{st} \right) \frac{v_B(t - t')}{pl} dt' \]

or

\[ \sigma_w(t) = \int_0^t \frac{B_w - C_w - s_2}{s_1 - s_2} e^{s_2(t - t')} - e^{s_1(t - t')} \frac{v_B(t - t')}{pl} dt' \]

If \( s_1 \) or \( s_2 \) have a positive real part, the exponential functions in the integrands in Eq. (13) increase unboundedly as \( t \) goes to infinity. Hence, the criterion for stable charge densities \( \sigma_a(t) \) and \( \sigma_w(t) \) must be that both \( \text{Re}(s_1) < 0 \) and \( \text{Re}(s_2) < 0 \), or, since the sum and product of \( s_1 \) and \( s_2 \) are real, that \( s_1 + s_2 < 0 \) whereas \( s_1 \cdot s_2 > 0 \). Combining this with Eq. (12) yields the stability criterion

\[ \left( \frac{A_w}{pl} + B_w - B_e + C_w \right) > 0, \quad \frac{A_w(B_e - C_e) - A_e(B_w - C_w)}{pl} + (B_e C_w - B_w C_e) > 0. \]

In the following section, the consequences of this stability result for the \( I-V \) characteristic will be investigated.

V. STABILITY AND IMPEDANCE

In order to appreciate the internal stability conditions (14), we will relate them to the impedances of the circuit. For this purpose, we derive some useful relations between the coefficients from \( A_e \) and \( C_w \) in Eq. (8).

Since we assumed that at \( t = 0 \) the system was in a self-consistent point of the \( I-V \) curve, the \( t = 0 \) charge densities \( \Sigma_e(0) \) and \( \Sigma_w(0) \) are interrelated in such a way that \( J_1 = J_2 \) [see Eqs. (7a) and (7b)]. We use this equation to derive a relation for the \( B \) and \( C \) coefficients in Eq. (8). At \( t = 0 \), we have \( N_0 t_d = \Sigma_w / t_d \) or \( \Sigma_w = N_0 t_d \). Then for the total derivative \( d\Sigma_w / d\Sigma_e \) we find

\[ d\Sigma_w \frac{\partial}{\partial \Sigma_e} (N_0 t_d) + d\Sigma_e \left( \frac{\partial}{\partial \Sigma_w} \right) (N_0 t_d) \]

or

\[ d\Sigma_w \frac{\partial}{\partial \Sigma_e} (N_0 t_d) \left[ 1 - \frac{\partial}{\partial \Sigma_e} (N_0 t_d) \right]. \]

The partial derivatives can be worked out and replaced by the \( B \) and \( C \) coefficients, yielding

\[ d\Sigma_w \frac{\partial}{\partial \Sigma_e} = - \frac{B_e - C_e}{B_w - C_w}. \]

The total derivative in Eq. (15) describes the change of \( \Sigma_w \) with \( \Sigma_e \) along the self-consistent curve. With the help of this result, two more identifications can be made. Introducing the abbreviations

\[ \Delta_1 \equiv B_w C_e - B_e C_w, \]

\[ \Delta_2 \equiv A_e (B_w - C_w) - A_w (B_e - C_e), \]

\[ \Delta_3 \equiv - B_e + B_w - C_w, \]

we can express the changes in \( V_e \) and \( J_1 \) along the self-consistent curve as

\[ \frac{dV_e}{d\Sigma_e} = \frac{\partial V_e}{\partial \Sigma_e} + \frac{d\Sigma_w}{d\Sigma_e} \frac{\partial V_e}{\partial \Sigma_w} = A_e - \frac{B_w - C_w}{B_w - C_e} A_w \]

\[ \frac{dJ_1}{d\Sigma_e} = \frac{\partial J_1}{\partial \Sigma_e} + \frac{d\Sigma_w}{d\Sigma_e} \frac{\partial J_1}{\partial \Sigma_w} = \frac{B_w C_e - B_e C_w}{B_w - C_m} - \frac{B_w - C_w}{B_w - C_m}. \]

As a matter of course, \( dJ_2 / d\Sigma_e \) is identical with \( dJ_1 / d\Sigma_e \). The quotient of Eqs. (16a) and (16b),

\[ \frac{dV_e / d\Sigma_e}{dJ_1 / d\Sigma_e} = \frac{\Delta_2}{\Delta_1}, \]

is the differential resistance of the DBRT structure (times unit area). We see that the top of the \( I-V \) curve \( (P_1 \) in Fig. 2) corresponds to \( dJ_1 / dV_e = 0 \), hence \( \Delta_1 = 0 \). The point on the \( I-V \) curve where the tangent is vertical \( (P_2 \) in Fig. 2) corresponds to \( dJ_1 / dV_e = \infty \), hence \( \Delta_2 = 0 \). Between \( P_1 \) and \( P_2 \) we have \( \Delta_1 > 0 \) and \( \Delta_2 < 0 \), so that \( r_d \) is negative here (NDR region). The negative differential resistance \( r_d \) is present in the rewritten stability conditions (14):

\[ 1 - \rho I A_e > 0, \]

\[ \frac{\Delta_2}{A_e} + \rho I \frac{\Delta_1}{A_e} = \frac{\Delta_3}{A_e} (r_d + \rho I) < 0. \]

FIG. 2. \( I-V \) characteristic of the asymmetric DBRT structure (for structure parameters see Sec. VI), and the loadline \( (\rho l = 10^3 \Omega \, \text{m}^2) \). Between \( P_1 \) and \( P_2 \) the differential resistance is negative, \( P_1 \) is the point of contact between the \( I-V \) curve and the loadline, and the turning point from stability to instability.
which can now be easily related to the impedance of the circuit.

We define the small signal impedance $Z(s)$ as the quotient of $b(s)$ and $f(s)$. This is obtained by combining Eqs. (8a) and (12) yielding

$$Z(s) = \frac{b(s)}{f(s)} = pI + \frac{A_0^2 - A_2}{s^2 - A_2 - A_3}.$$  \hspace{1cm} (18)

The first term of this equation is the contribution of the leads. Since we have assumed purely Ohmic contacts, it is simply $pI$. For more realistic leads, it is to be replaced by some $Z_{leads}(s)$. The second term of Eq. (18) is the impedance of the "bare" DBRT structure, $Z_{DBRT}(s)$. It has a simple zero $s = A_2/A_0$ so that stability is ensured if $A_2/A_0 < 0$. Since $A_2$ changes sign at $P_2$ (see Fig. 2), a stable solution for the bare DBRT can be found throughout the NDR region. This agrees fully with the conditions (17) for $pI = 0$, thus reading $A_2/A_0 < 0$ [Eq. (17a) being trivial for $pI = 0$].

For $pI = 0$, i.e., for the DBRT in circuit, the total impedance must be considered. An extra zero is introduced, and the two stability criteria of Eq. (17) result again. It is clear, that for $pI$ not too large, the second criterion (17b) is decisive. For points on the $I$-$V$ curve that are before $P_1$ (see Fig. 2), Eq. (17b) is satisfied since here $A_1 < 0$ and $pI + r_d > 0$. Just after $P_1$, Eq. (17b) is still satisfied because both factors change sign at the top of the $I$-$V$ curve. However, at $P_2$ we have $pI + r_d = pI > 0$ whereas still $A_1 > 0$, so that the stability criterion is no longer met. Therefore, the turning point from stability to instability must be between $P_1$ and $P_2$, in the NDR region. A more precise position can be specified. Since the factor responsible for the change of sign is $(pI + r_d)$, the end of the stability interval can be characterized by $r_d = - pI$. The slope of the load line $J_L(V_L)$ being $- 1/pI$ [see Eq. (7a)], the turning point is found to be the point of contact between the $I$-$V$ curve and this load line.

From Eq. (18) a resistance and reactance for the bare structure are defined as the real and imaginary part of $Z_{DBRT}(i\omega)$:

$$R_{DBRT}(\omega) = \text{Re}[Z_{DBRT}(i\omega)] = \frac{A_0^2 - A_2}{(A_0 + \omega^2)^2 + \omega^2A_2}.$$  \hspace{0.5cm} (19a)

$$X_{DBRT}(\omega) = \text{Im}[Z_{DBRT}(i\omega)] = - \frac{A_0(A_0 + \omega^2)^2 + \omega^2A_2}{(A_0 + \omega^2)^2 + \omega^2A_2}.$$  \hspace{0.5cm} (19b)

From the condition $R_{DBRT}(\omega_\infty) = 0$, we define for the bare structure a cutoff frequency $\omega_\infty = - A_1/(A_2 - A_3 - A_4)$. Analogously, a self-resonance frequency $\omega_\omega^2 = - A_1 - A_3A_4/A_2$ is found from the demand that the reactance $X_{DBRT}(\omega_\omega^2) = 0$. The cutoff frequency is positive in the NDR region, as expected. Here, the DBRT structure can be used as an oscillator for frequencies up to $\omega_\infty$. The series resistance $pI$ will lower the cutoff frequency. Since our model does not contain any inductive elements, only the addition of an external inductance can yield a positive $\omega_\omega^2$. At low frequencies the impedance of the structure becomes purely resistive and approaches the differential resistance $r_d = \Delta_2/\Delta_1$.

VI. NUMERICAL RESULTS FOR ASYMMETRIC GaAs-AlGaAs STRUCTURE

The above theory has been worked out numerically for a GaAs-Al$_x$Ga$_{1-x}$As DBRT device with different widths for the two barriers. For the purpose of calculations we choose $x = 0.4$, barrier height 0.44 eV, well width 5 nm, and barrier widths 5.6 and 8.4 nm, respectively. The effective mass of GaAs has been used for all the layers. Assuming contact layers of 50 nm (doping $2 \times 10^{16}$) plus 1 mm (doping $2 \times 10^{18}$) on both sides of the DBRT structure, we can estimate $pI$ to be of the order $10^{-10}$ $\Omega$ m$^2$.

In Fig. 3, the real and imaginary part of the impedance is shown as a function of the Fermi level $E_F$ in the left-hand-side reservoir, for $\omega = 10^9$ rad/s. We know that for $\omega = 0$ we have $R_{DBRT}$ and $X_{DBRT} = 0$, so that the impedance at zero frequency is merely the derivative of $V_s(J_1)$ shown in the $I$-$V$ curve of Fig. 2. Hence, $R_{DBRT}(E_F)$ has a vertical asymptote when $A_1(E_F) = 0$, and a zero when $A_2(E_F) = 0$. As we see from Fig. 3(a) this picture is not changed very much for frequencies up to a few GHz. The vertical asymptote of $R_{DBRT}(E_F)$ now corresponds to $(A_1 + \omega^2)^2 + \omega^2A_2 = 0$, and is accompanied by a sharp negative peak in the reactance $X_{DBRT}(E_F)$. The zero of $R_{DBRT}(E_F)$ is shifted too [see Eq. (19a)], but both shifts are small, so that $R_{DBRT}(E_F)$ still resembles the differential resistance $r_d$ very well.

Figure 4 shows the impedance as a function of frequency at fixed value of $E_F$ $(E_F = 0.1$ eV). As expected from Eq. (19), $X_{DBRT}(\omega)$ is zero for $\omega = 0$ and $\omega \rightarrow \infty$, whereas $R_{DBRT}(\omega)$ is $r_d$ for $\omega = 0$ and zero for $\omega \rightarrow \infty$. Around $\omega/2\pi \approx 0.3$ GHz, the imaginary part shows a peak where the real part increases to zero. This is the cutoff frequency $\omega_\infty$ that separates the amplified oscillations from those that are damped by the DBRT structure. The cutoff frequency is real for $E_F$ values in the NDR region only. For the bare structure, $\omega_\infty$ is zero at $P_1$ and $P_2$ (see Fig. 2), and increases to infinity when $A_2 - A_3A_4 = 0$. Its typical value is found to be $\omega_\infty/2\pi \approx 5$ GHz. Decreasing the barrier width by 10% leads to an increase of this value to $\approx 15$ GHz. Both tendency and magnitude agree quite well with other models.

VII. CONCLUSION

Comparison of our results with experimental findings is complicated by the fact that most impedance measurements were done on symmetric structures showing no bistability. Besides, our simple model yields a negative differential conductance that decreases from 0 to $-\infty$, whereas in real devices (due to nonresonant tunneling) the conductance remains finite. Nevertheless, we find a qualitative agreement between our calculated impedance as a function of bias and frequency (Figs. 3 and 4) and the measured curves presented in Refs. 9–11.
Zarea et al. studied a GaAs-AlGaAs structure with a 5-nm well and 5.6-nm barriers (cf. Sec. VI), for which they present the real and imaginary part of $Z$ as function of bias. These curves show the same trends as our Fig. 3. To be able to compare the horizontal scales we use $V_s \sim 4E_F/e$ (see Fig. 2), giving a nice agreement. For the comparison of the vertical scales the impedances of Ref. 9 are multiplied by the mesa area. It is then found that the scales differ by a factor of 10. This is, however, due to our static model (cf. Fig. 2 and Fig. 1 of Ref. 9) which overestimates the current, and not to the dynamical extensions.

The cutoff frequency for the device of Ref. 9 was measured to be 1.10 GHz. Gering et al. found a value of 88.2 GHz for a similar structure with slightly smaller barriers (5 nm). The difference is mainly due to a difference in series resistance (4 and 0.17 Ω, respectively). Our calculated typical value of 5 GHz has the correct order of magnitude. Moreover, we find an exponential decrease of $\omega_0$ with barrier thickness, in agreement with the results of Ref. 10.

Lippens and Mounaix measured the impedance of a structure similar to the one of Ref. 10 as a function of frequency, corresponding to our Fig. 4 (but with linear horizontal scale). Again we see the same trends in the curves. The real part starts at the negative differential resistance to become quite small at higher frequencies; the imaginary part shows a negative peak. The inductive behavior at higher frequencies is, of course, not reproduced by our calculations. Comparing the horizontal scale we find that our frequencies are smaller by a factor of 3.

We conclude that our dynamical equations are a fair description of the ac behavior of DBRT structures in the GHz regime. This is partly due to the fact that the impedance is strongly related to the differential resistance. Hence a reasonable $I$-$V$ characteristic will yield a reasonable impedance. Thus the agreement with experiments is partly owing to our static model. The essentially new element is the generalized expression for the current density [Eqs. (7b) and (7c)], making it a time- and position-dependent quantity. This distinguishes our approach from experimental equivalent-circuit fits, and enables a purely theoretical analysis of the DBRT dynamics.

Yet one should bear in mind that this double-current definition poses some limitations to the applicability of the model. Its close relation with the static description leads to a restriction in the frequency domain to values of $\omega$ for which the adiabatic approach is valid. A simple estimation shows that the THz regime is beyond the scope of our approach. Also, only small-amplitude oscillations in the applied voltage $V_s$ are allowed, thus excluding interesting...
nonlinear behavior. Finally, Eqs. (7) cannot describe any inductive effects of the DBRT structure that may nevertheless be present.

In fact, Eqs. (7) amount to a dynamic description of the DBRT structure in terms of a couple of shunted capacitors. If there is some charge buildup in the emitter and well, we expect a connection between the time constants $1/|s_1|$ and $1/|s_2|$ of Eq. (12), and the typical times for charging these two regions, $\lambda_\perp E_F$ and $t_d$, respectively. Here, $\lambda_\perp$ is the emitter screening length, $v_F$ is the Fermi velocity, and $t_d$ the dwell time already discussed in Eq. (4). Indeed, where the coupling is small, $s_1$ and $s_2$ have the correct order of magnitude. In the general case, the complete matrix equation (10) must be solved, thus making the task nontrivial.

In summary, we have been able to describe the low-frequency small-signal part of the DBRT dynamics by minimal extensions of the static model. From a theoretical point of view, this approach is an improvement with regard to the equivalent-circuit method. A more complete account of the dynamics requires the consideration of the time-dependent Schrödinger equation and the nonlinear aspects.

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