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# Manufacturer Competition in the Nanostore Retail Channel

Jiwen Ge

Institute of Supply Chain Analytics, Dongbei University of Finance and Economics

jiwenge@dufe.edu.cn

Dorothee Honhon

Naveen Jindal School of Management, University of Texas at Dallas

Dorothee.Honhon@utdallas.edu

Jan C. Fransoo\*

Department of Logistics, Kuehne Logistics University

School of Industrial Engineering, Eindhoven University of Technology

Jan.Fransoo@the-klu.org

Lei Zhao

Department of Industrial Engineering, Tsinghua University

lzhao@tsinghua.edu.cn

## Abstract

In emerging markets, a significant share of the revenue of Consumer Packaged Goods (CPG) manufacturers comes from the traditional retail channel, composed of millions of independent family-owned *nanostores*. Nanostore owners typically have limited cash flow and are driven by the modest goal of making a living. It is common practice for manufacturers to dispatch sales representatives to visit nanostores directly in order to drive product sales. We study the sales visit and pricing decisions of manufacturers supplying to a nanostore over an infinite time horizon. We first consider the case of a single manufacturer and show that the manufacturer should price the product such that the nanostore can earn enough to pay for his subsistence spending. Such a supplier-retailer mutual reliance

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\*corresponding author

relationship continues to hold for the two-manufacturer model where the manufacturers compete for the nanostore's cash resources under shelf space limitations. Further, under some conditions, the two manufacturers can mutually benefit, that is, instead of jeopardizing each other through competition, they contribute collectively to satisfy the nanostore family's subsistence needs such that nanostores are more likely to survive; besides, each can earn more profit than in a single-supplier setting. The results can help us understand the current industry dynamics in this vital sector.

**Keywords:** retailing, nanostore, cash competition, subsistence retail

## 1 Introduction

The current retail landscape in emerging-market megacities is highly fragmented. Before the modern multinational retailers such as Walmart and Tesco started operating in these cities in the 1990s, the retail landscape was highly dominated by small family-operated stores that Fransoo et al. (2017) refer to as *nanostores*. It was commonly expected that the entry of modern supermarkets would drive these nanostores out of the market (Child et al. 2015). However, after more than two decades of presence, the big-box retailers' market share in developing markets still lags far behind their share in developed markets. At the same time, nanostores continue to hold a significant – sometimes even dominant – share of retail sales in emerging economies (Nielsen 2015), with tens of thousands of nanostores being present in many developing megacities. In relatively developed areas, such as South East Asia or Latin America, this channel still contributes more than 40% of total Consumer Packaged Goods (CPG) sales in cities (Nielsen 2015). In less developed regions like South Asia and many countries in Africa, the market share of nanostores can be as high as 90% of CPG sales (Nielsen 2015).

It is generally more profitable for CPG manufacturers to serve nanostores than to serve big-box retailers because of their dominant position in the business relationship. In general, manufacturers can increase their revenue by 5-15% and enhance their profit by 10-20% by penetrating the traditional channel (Diaz et al. 2012). A detailed study by Díaz et al. (2007) across four product categories shows that, by selling directly to nanostores in Latin America, large CPG manufacturers realize, on average, over 10% more profit than by selling to large modern grocery chains. As a result, the traditional retail channel is very competitive as multinationals intensify their market penetration, and local players try to defend their existing market share.

Nanostores are typically very small-sized retail outlets with limited available shelf space.

Typically, they carry inventory from just one or two brand manufacturers for each product category (Fransoo et al. 2017). It is common practice for CPG manufacturers, including Unilever, Danone, AB Inbev, and Nestle, to dispatch sales representatives to visit nanostores frequently to sell and deliver their products (Ge et al. 2019). By direct distribution, these manufacturers ensure that their products remain available on the store shelf, and execute regular shelf replenishment by themselves.

Unlike supermarkets, nanostores owners are generally not sophisticated profit-maximizers but pursue the modest goal of making a living under severe cash limitations. The revenues they generate from sales are used to replenish new inventory and satisfy their family's subsistence needs. Insufficient revenues generally mean that nanostores go out of business and that the owners must find another means of subsistence. Strategically, this implies that the long-term profitability of CPG manufacturers is dependent on the survival of nanostores. This incentivizes manufacturers to sustain the operations of nanostores. Objectively, the direct distribution practice of manufacturers serves this purpose as visiting the nanostores regularly guarantees that authentic products are available for consumers, reduces the nanostore replenishment costs, and limits the occurrence of lost sales. In practice, manufacturers may even exert additional effort to help nanostores improve operations. According to a large-scale empirical study conducted in India (Joseph et al. 2008), manufacturers even help nanostores upgrade services, systems, and operations in order to mitigate the impact of the increasing presence of supermarkets on nanostores.

Operationally, the sales of a CPG manufacturer are determined by the nanostores' cash position at the time of sales visits. Due to the high risk of default and the free-rider problem, CPG manufacturers rarely offer credit to nanostores and often cancel pre-scheduled order deliveries in case of low cash balance, as demonstrated in a large-scale empirical study (see Chapter 4 of Fransoo et al. 2017) conducted in large cities in Morocco. In practice, a significant fraction of sales visits fails to generate sales because the cash position at nanostores is exhausted by prior sales visits from other manufacturers (Fransoo et al. 2017). For this reason, manufacturers may choose to visit nanostores more frequently in the hope of securing access to cash and selling more; this, however, results in higher sales visit costs because of higher logistics costs. Alternatively, manufacturers can also choose to lower their wholesale price to sell more during each visit, which decreases the necessity for frequent sales visits but reduces the gross margin per unit sold to nanostores.

In this paper, we study the sales visit and pricing strategies of two manufacturers selling to a nanostore. We are interested in the following research questions.

1. What is the manufacturer-retailer relationship like when two manufacturers compete for access to the cash resources of a subsistence-driven nanostore?
2. What is the relationship between manufacturers like when they compete to sell to a subsistence-driven nanostore?
3. Does the nanostore benefit from manufacturer competition?

We study a non-cooperative game where two CPG manufacturers simultaneously optimize their wholesale price and sales visit strategies given dedicated shelf space for their product at the nanostore. In order to study the impact of competition, we then study a single-manufacturer model, and compare the results between the two models.

We identify a mutually beneficial relationship between the nanostore and manufacturer(s) in the two-manufacturer model as well as in the single-manufacturer model. We find that, when the manufacturers enter into business with the nanostore, they price their products such that the nanostore earns exactly enough to cover his family's subsistence needs. In some cases, a single manufacturer may be unable to sustain the subsistence needs of the nanostore; however, two of them can do so as they can split the responsibility of providing for his subsistence needs. Our model provides an analytical justification for the existence and relative prosperity of multi-brand nanostores in emerging-market megacities, and the strategies of manufacturers supplying them.

The rest of this paper is structured as follows. We review the literature in Section 2. We introduce the (two-manufacturer) model in Section 3, discuss the model in Section 4, and present the results in Section 5. We then present the single-manufacturer model in Appendix B, and compare the two models in Section 6. Finally, we conclude in Section 7. All proofs can be found in the appendix.

## 2 Literature review

Our paper relates to the literature on *price and shelf space competition* and *price and delivery frequency competition*.

The existing research on *price and shelf space competition* considers multiple manufacturers optimizing their respective wholesale prices to compete for the shelf space of a profit-maximizing retailer. The underlying assumption is that the demand for the manufacturers is increasing in the amount of shelf space allocated to their products, albeit in a diminishing manner. This problem has been extensively studied when the manufacturer wholesale prices and retail assortment are exogenously given (e.g., Anderson and Amato 1974, Corstjens and Doyle 1981, Bultez and Naert 1988, Reyes and Frazier 2007, Hariga et al. 2007, Irion et al. 2012). To resolve the conflicting objectives of the retailer and manufacturers, Wang and Gerchak (2001) propose an inventory holding cost subsidy together with a wholesale price contract to effectively coordinate a supply chain consisting of one manufacturer and one retailer. For a supply chain with multiple competing manufacturers, Martinez-de-Albeniz and Roels (2011) show that the manufacturer who contributes the most profit to the retailer obtains more shelf space in equilibrium when the demand is sensitive to the shelf space. Considering two competing manufacturers, Martin-Herran et al. (2006) prove that a manufacturer's shelf space share positively correlates with the manufacturer's relative profitability and negatively correlates with his wholesale price.

Optimizing delivery frequency is another way by which manufacturers can obtain a competitive advantage when satisfying their customers' demand (e.g., Jang et al. 2013, Berling and Eng-Larsson 2016). While the price-time competition research with delivery speed as a lever has been extensively studied (e.g., So 2000, Cachon and Harker 2002), the question of the optimal delivery frequency is a relatively under-explored one. Kraemer and Kraemer (2010) and Shah and Brueckner (2012) both model multiple customers, who select one exclusive manufacturer based on their proposed offering on price and delivery frequency. Specifically, Kraemer and Kraemer (2010) consider two manufacturers, one of which offers delivery frequency flexibility and show that the flexible manufacturer realizes less (more) profit than its counterpart if the customer inventory holding cost is low (high). Shah and Brueckner (2012) consider multiple manufacturers and show that the equilibrium delivery frequency increases in the number of customers and decreases in the number of competing manufacturers. Ha et al. (2003) study a supply chain with one customer replenishing from two competing manufacturers with different delivery frequency where the customer optimizes the delivery sequence to minimize the inventory holding cost. They show that the delivery frequency competition intensifies price competition in a way which is beneficial to the customer.

All papers listed above assume that the retailer is not constrained by limited cash resources and maximizes profit through demand or shelf space allocation. Our paper, however, considers a cash-constrained retailer whose objective is to make a living rather than to maximize profit and who has pre-allocated dedicated shelf space for each product. In our setting, since CPG manufacturers take dominant positions against small-sized nanostores, the retailer passively accepts the manufacturers' pricing and delivery frequency decisions. Hence, we focus on the interplay between manufacturers' actions. To the best of our knowledge, this has not been investigated in the existing literature. Our model and analysis provide insights which differ from previous work with profit-maximizing retailers.

Our paper is also related to a recent stream of pioneering studies on traditional retail operations in developing markets. Using a competitive model, Jerath et al. (2016) show that the entry of an organized retailer may reduce the number of unorganized retailers in equilibrium. Wan et al. (2018) propose a demand estimation model in a nanostore setting where store substitution is common. Zhang et al. (2017) study the distribution channel selection of two micro-retailers where one can act as a wholesaler to her rival. Ge et al. (2019) model the distribution channel choice of a manufacturer supplying to a cluster of nanostores in a region. Ge et al. (2018) study the sales effort strategy of manufacturers selling to nanostores directly.

### 3 The model

We use the following notation in this paper: let  $x^+ = \max\{x, 0\}$  and let  $t^-$  and  $t^+$  denote the instant right before and after time  $t$ , respectively. Also, let  $\mathbb{I}_B$  denote the indicator function of event  $B$ , which is 1 if event  $B$  is true and zero otherwise.

We consider two manufacturers who each produce a non-perishable product  $i = 1, 2$  at a cost of  $c_i$ , and sell it to a (common) nanostore at a wholesale price of  $w_i$ . The nanostore sells product  $i$  to its customers at a price of  $p_i$ . The demand of Product  $i$  from nanostore customers is deterministic with rate  $\lambda_i$ . The two products are from distinct categories, so they are not substitutable, and unsatisfied demand for either product is lost. The nanostore sells only these two products and divides his total shelf space of  $L$  between them: let  $l_i$  be the shelf space allocated to Product  $i = 1, 2$ , that is,  $l_1 + l_2 = L$ . We assume that the nanostore does not have a backroom so that all inventory units at the nanostore are directly available for purchase by the customers. So,  $l_i$  is the maximum quantity of product  $i$  which can be stocked in the store.

Let  $C(t)$  denote the cash balance of the nanostore at time  $t$ . The nanostore owner spends money for his family's subsistence needs (such as food, housing, clothing) from this product's sales at a rate of  $\gamma$  per time unit as long as the cash balance is positive. We refer to  $\gamma$  as the nanostore's *rate of subsistence spending*. We assume  $p_1\lambda_1 + p_2\lambda_2 > \gamma$  as a necessary condition for the nanostore to be able to cover his family's subsistence needs. However, we do not necessarily impose that  $p_i\lambda_i > \gamma$  for  $i = 1, 2$ , that is, it is possible that selling only one of the manufacturers' products would not generate a revenue which covers the nanostore's subsistence needs. We also assume that each manufacturer charges the nanostore a wholesale price which is lower than their product's selling price, i.e.  $w_i < p_i$  for  $i = 1, 2$  as the nanostore would otherwise not accept to buy the product.

Let  $I_i(t)$  denote the inventory of product  $i$  at the nanostore at time  $t$ . Let  $A(t)$  be the nanostore's total liquid assets at time  $t$ , which is defined as the cash balance, plus the on-hand inventory evaluated at the wholesale price (as is customary in accounting practices), i.e.,  $A(t) = C(t) + w_1I_1(t) + w_2I_2(t)$ .

The sales representatives from the two manufacturers visit the nanostore to replenish the inventory of their own product at discrete moments in time. If, at time  $t$ , there is no such visit, we have:

$$I'_i(t) = \begin{cases} -\lambda_i & \text{if } I_i(t) > 0 \\ 0 & \text{if } I_i(t) = 0 \end{cases} \quad i = 1, 2$$

$$C'(t) = \begin{cases} p_1\lambda_1 + p_2\lambda_2 - \gamma & \text{if } I_1(t) > 0 \text{ and } I_2(t) > 0 \\ p_1\lambda_1 - \gamma & \text{if } I_1(t) > 0 \text{ and } I_2(t) = 0 \\ p_2\lambda_2 - \gamma & \text{if } I_1(t) = 0 \text{ and } I_2(t) > 0 \\ -\gamma & \text{if } I_1(t) = I_2(t) = 0 \text{ and } C(t) > 0 \\ 0 & \text{otw.} \end{cases}$$

If Manufacturer  $i$  is the only one to pay a visit to the nanostore at a given time, we assume that the nanostore buys the maximum quantity he can afford given his current cash balance, up to the shelf space limit  $l_i$ . In other words, if Manufacturer  $i$  is the only one to visit the nanostore at time  $t$ , the nanostore will buy a quantity of product  $i$  equal to  $\min\left\{\frac{C(t^-)}{w_i}, l_i - I_i(t^-)\right\}$  and,



for  $i = 1, 2$ , we have:

$$C(t^+) = [C(t^-) - (l_i - I_i(t^-)) w_i]^+ \quad (1)$$

$$I_i(t^+) = \min \left\{ I_i(t^-) + \frac{C(t^-)}{w_i}, l_i \right\} \quad (2)$$

$$I_{3-i}(t^+) = I_{3-i}(t^-). \quad (3)$$

If, at time  $t$ , both manufacturers visit the nanostore, we assume that the nanostore fills both products up to their respective shelf space limits if he has a sufficient cash balance; otherwise, he allocates a share  $s_i \in [0, 1]$  of the available cash to fill up the shelf space of Manufacturer  $i$ . For example the allocation can be proportionally to the cash expenditure necessary to fill up the shelf space of each manufacturer, that is,  $s_i = \frac{(l_i - I_i(t^-)) w_i}{(l_1 - I_1(t^-)) w_1 + (l_2 - I_2(t^-)) w_2}$ . All of our results hold irrespective of the allocation shares  $s_i$ . As a result, we have:

$$C(t^+) = [C(t^-) - (l_1 - I_1(t^-)) w_1 - (l_2 - I_2(t^-)) w_2]^+ \quad (4)$$

$$I_i(t^+) = \begin{cases} l_i & \text{if } C(t^-) \geq \sum_{i=1}^2 (l_i - I_i(t^-)) w_i \\ I_i(t^-) + \frac{s_i C(t^-)}{w_i} & \text{if } C(t^-) < \sum_{i=1}^2 (l_i - I_i(t^-)) w_i. \end{cases} \quad (5)$$

Let  $\tau_{i,n_i}$  denote the time when the sales representative from Manufacturer  $i$  pays her  $n_i$ -th visit to the nanostore (if it takes place). Let  $t_n$  denote the  $n$ -th visit epoch by either of the two manufacturers (if it takes place) where  $n = n_1 + n_2$ .

At the start of the time horizon, we assume that the nanostore has an initial capital of  $C(0^-) = l_1 w_1 + l_2 w_2$  and no inventory of either product, i.e.,  $I_i(0) = 0$  for  $i = 1, 2$ . The two manufacturers' sales representatives simultaneously pay the nanostore a first visit at time zero, i.e.,  $\tau_{1,1} = \tau_{2,1} = t_1 = t_2 = 0$ , so that the nanostore depletes all of his cash balance to purchase  $l_1$  units of product 1 and  $l_2$  units of product 2 at the start of the time horizon, i.e.,  $C(0^+) = 0$  and  $I_i(0^+) = l_i$  for  $i = 1, 2$ . All of our results would continue to hold if the starting point of our model was different; for example, the starting inventory of product  $i = 1, 2$  at the nanostore is less than  $l_i$  units.

In general, for  $n = 1, 2, \dots$ , right before the  $n$ -th visit to the nanostore (if it takes place), the nanostore's inventory level of product  $i$  and the cash balance are given by:

$$I_i(t_n^-) = [I_i(t_{n-1}^+) - (t_n - t_{n-1}) \lambda_i]^+ \quad i = 1, 2 \quad (6)$$

$$C(t_n^-) = \left[ C(t_{n-1}^+) + \sum_{i=1}^2 p_i \min \{ (t_n - t_{n-1})\lambda_i, I_i(t_{n-1}^+) \} - (t_n - t_{n-1})\gamma \right]^+ \quad (7)$$

where  $t_n = t_{n-1}$  if the  $(n-1)$ -th and the  $n$ -th visits occur at the same time.

At time zero, we have  $A(0) = I_1 w_1 + I_2 w_2$ . For  $t_{n-1} \leq t < t_n$  where  $n = 2, 3, \dots$ , let  $j = \arg \min_{i=1,2} \frac{I_i(t_{n-1}^+)}{\lambda_i}$  be the product which would run out first assuming no other visit by either of the sales representatives, then we have:

$$A(t) = \begin{cases} \left[ A(t_{n-1}) + \sum_{i=1}^2 (p_i - w_i)(t - t_{n-1}) - \gamma(t - t_{n-1}) \right]^+ & \text{if } t < t_{n-1} + \frac{I_j(t_{n-1}^+)}{\lambda_j} \\ \left[ A(t_{n-1}) + (p_j - w_j)I_j(t_{n-1}^+) - (p_{3-j} - w_{3-j} - \gamma)(t - t_{n-1}) \right]^+ & \text{if } t_{n-1} + \frac{I_j(t_{n-1}^+)}{\lambda_j} \leq t \leq t_{n-1} + \frac{I_{3-j}(t_{n-1}^+)}{\lambda_{3-j}} \\ \left[ A(t_{n-1}) + \sum_{i=1}^2 (p_i - w_i)I_1(t_{n-1}^+) - \gamma(t - t_{n-1}) \right]^+ & \text{if } t > t_{n-1} + \frac{I_{3-j}(t_{n-1}^+)}{\lambda_{3-j}} \end{cases} \quad (8)$$

where  $A(t_{n-1}) = C(t_{n-1}^+) + w_1 I_1(t_{n-1}^+) + w_2 I_2(t_{n-1}^+)$ . Note that  $A(t) = A(t^+) = A(t^-)$  for all  $t$  and that the nanostore remains to be in the business as long as  $A(t) > 0$ .

Suppose that the two sales representatives have collectively visited the nanostore and sold products to him  $n-1$  times and that the inventory of both products was positive right after this last visit, i.e., at time  $t_{n-1}^+$ . Still let product  $j = \arg \min_{i=1,2} \frac{I_i(t_{n-1}^+)}{\lambda_i}$  be the product which would run out first assuming no other visit by either of the sales representatives. If  $p_{3-j}\lambda_{3-j} > \gamma$ , one of four possible scenarios<sup>1</sup> can occur regarding the  $n$ -th visit:

1. If  $t_n < t_{n-1} + \frac{I_j(t_{n-1}^+)}{\lambda_j}$ , the  $n$ -th store visit happens before the nanostore runs out of stock of either products and his cash balance is equal to  $C(t_n^-) = C(t_{n-1}^+) + (p_1\lambda_1 + p_2\lambda_2 - \gamma)(t_n - t_{n-1})$ , which is positive since we have assumed  $p_1\lambda_1 + p_2\lambda_2 > \gamma$ .
2. If  $t_{n-1} + \frac{I_j(t_{n-1}^+)}{\lambda_j} \leq t_n < t_{n-1} + \frac{I_{3-j}(t_{n-1}^+)}{\lambda_{3-j}}$ , the nanostore has run out of inventory of product  $j$  but still has positive inventory of product  $3-j$  at the time of the  $n$ -th sales representatives' visit. The cash balance is equal to  $C(t_n^-) = C(t_{n-1}^+) + p_j I_j(t_{n-1}^+) + (p_{3-j}\lambda_{3-j} - \gamma)(t_n - t_{n-1})$ , which is positive since  $p_{3-j}\lambda_{3-j} > \gamma$ .
3. If  $t_{n-1} + \frac{I_{3-j}(t_{n-1}^+)}{\lambda_{3-j}} \leq t_n < t_{n-1} + \frac{p_1 I_1(t_{n-1}^+) + p_2 I_2(t_{n-1}^+) + C(t_{n-1}^+)}{\gamma}$ , the nanostore has run out of inventory of both products by the time of the  $n$ -th sales representatives' visit but still has a positive cash balance equal to  $C(t_n^-) = C(t_{n-1}^+) + p_1 I_1(t_{n-1}^+) + p_2 I_2(t_{n-1}^+) - \gamma(t_n - t_{n-1})$ .
4. If  $t_n \geq t_{n-1} + \frac{p_1 I_1(t_{n-1}^+) + p_2 I_2(t_{n-1}^+) + C(t_{n-1}^+)}{\gamma}$ , then the nanostore has run out of both cash and inventory and has therefore closed the business by the time the sales representative reaches the store, therefore, she does not sell any product and this is the last visit she

<sup>1</sup>If  $p_{3-j}\lambda_{3-j} < \gamma$ , then it is possible that the nanostore runs out of cash before running out of inventory of product  $3-j$ ; this case can be handled in a similar way.

makes.

The cost per store visit to the manufacturer is  $K_i$  for  $i = 1, 2$ . The profit Manufacturer  $i$  earns from her  $n_i$ -th visit is:

$$\pi_i(\tau_{i,n_i}) = (w_i - c_i) \left( I_i(\tau_{i,n_i}^+) - I_i(\tau_{i,n_i}^-) \right) - K_i. \quad (9)$$

Let  $\bar{\pi}_{i,n_i} = \frac{(I_i(\tau_{i,n_i-1}^+) - I_i(\tau_{i,n_i}^-))(w_i - c_i) - K_i}{\tau_{i,n_i} - \tau_{i,n_i-1}}$  be the profit rate earned by Manufacturer  $i$  between the  $(n_i - 1)$ -th and  $n_i$ -th visits of her sales representative for  $i = 1, 2$  (provided both take place).

At the start of the time horizon, *simultaneously*, each manufacturer needs to decide on the timing of his visits to the nanostore as well as their wholesale price, in order to maximize their long-run profit rate, while observing the other manufacturer's timing of visits and wholesale price. Let  $N_i$  for  $i = 1, 2$  represent the index of the last visit of Manufacturer  $i$ 's sales representative to the nanostore. If the nanostore closes the business or if Manufacturer  $i$  only sells to the nanostore for a finite period of time, that is,  $N_i$  is finite, Manufacturer  $i$ 's long-run profit rate is zero. Otherwise, each manufacturer solves the following problem:

$$\sup_{w_i, \tau_{i,1}, \tau_{i,2}, \dots} \bar{\pi}_i = \lim_{N_i \rightarrow \infty} \frac{\sum_{n_i=1}^{N_i} \pi_i(\tau_{i,n_i})}{\tau_{i,N_i}} = \lim_{N_i \rightarrow \infty} \frac{1}{\tau_{i,N_i}} \sum_{n_i=2}^{N_i} (\tau_{i,n_i} - \tau_{i,n_i-1}) \bar{\pi}_{i,n_i} \quad i = 1, 2 \quad (10)$$

where we show that the second equality holds in Appendix A.

Finally, we assume that each manufacturer has an alternative business option, which we refer to as the *outside option*, which generates a *reservation* profit rate of  $\beta_i$ . Manufacturer  $i$  therefore enters into business with the nanostore only if the profit rate he earns from selling to the nanostore is greater than or equal to the reservation profit rate  $\beta_i$ .

## 4 Model discussion

Our model is a stylized representation of cash and inventory flows, which assumes a deterministic rate of consumer demand and cash spending by the nanostore owner. This type of fluid analysis is a common way to abstract from the natural randomness of a process (see Harris 1915, Zipkin 2000, Mitra 1988, Ahn and Ramaswami 2003) which allows us to focus on the crucial decisions of the manufacturer's store visit frequency and pricing. We recognize the true stochastic nature of the demand and spending processes of a real-life nanostore but argue that

this fluid simplification generates interesting and valuable insights on a previously unexplored question.

Further, we note that the assumptions we make regarding the nanostore's ordering behavior are consistent with observations by Fransoo et al. (2017) that the amount of cash available is generally the most significant determinant of the size of the order placed by the owner who often lacks any kind of business training and is typically simply driven by the modest goal of providing for his or her family.

## 5 Results

Our first result is to establish bounds on the equilibrium wholesale prices.

**Lemma 1.** *The equilibrium wholesale price for Manufacturer  $i = 1, 2$  is such that  $w_i \in \left[ c_i + \frac{K_i}{l_i} + \frac{\beta_i}{\lambda_i}, p_i - \frac{\gamma - \lambda_{3-i}(p_{3-i} - w_{3-i})}{\lambda_i} \right]$ .*

If  $w_i > p_i - \frac{\gamma - \lambda_{3-i}(p_{3-i} - w_{3-i})}{\lambda_i}$ , which is equivalent to  $\gamma > (p_1 - w_1)\lambda_1 + (p_2 - w_2)\lambda_2$ , then the nanostore will close the business as he does not earn enough to satisfy his family's subsistence needs. We refer to  $(p_1 - w_1)\lambda_1 + (p_2 - w_2)\lambda_2 \geq \gamma$  as the *cash sufficiency condition*. If  $w_i < c_i + \frac{K_i}{l_i} + \frac{\beta_i}{\lambda_i}$ , then Manufacturer  $i$  can make more profit from the outside option even when selling his own entire shelf space  $l_i$  of products at each visit, and therefore prefers not to engage in business with the nanostore. According to the prior assumption that  $w_i < p_i$  for  $i = 1, 2$ , the retail price must be no less than the lower bound of the wholesale price, that is,  $p_i > c_i + \frac{K_i}{l_i} + \frac{\beta_i}{\lambda_i}$ , so that Manufacturer  $i$  can possibly contribute a positive cash inflow to the nanostore after extracting at least its reservation profit. After transformation, we have  $l_i > \underline{l}_i \equiv \frac{K_i}{\left(p_i - c_i - \frac{\beta_i}{\lambda_i}\right)^+}$ , which implies that Manufacturer  $i$  can only join the game if his shelf space limit is above a given threshold which ensures his profitability. Therefore and thereon, we only focus on the parameter space  $\underline{l}_1 < l_1 < L - \underline{l}_2$  where the second inequality of the above condition  $\underline{l}_1 < l_1 < L - \underline{l}_2$  is equivalent to  $l_2 > \underline{l}_2$  since the two manufacturers share the total shelf space. This further implies that we must have  $L > \underline{l}_1 + \underline{l}_2$  otherwise the two manufacturers would not be able to generate enough profit to fund the nanostore's subsistence spending. Lemma 1 also implies another necessary condition:  $c_i + \frac{K_i}{l_i} + \frac{\beta_i}{\lambda_i} \leq p_i - \frac{\gamma - \lambda_{3-i}(p_{3-i} - w_{3-i})}{\lambda_i}$  for  $i = 1, 2$ , that is,  $\left(p_1 - c_1 - \frac{K_1}{l_1}\right)\lambda_1 - \beta_1 + \left(p_2 - c_2 - \frac{K_2}{l_2}\right)\lambda_2 - \beta_2 \geq \gamma$ , which states that the maximal supply chain profit rate (achieved when each manufacturer sells their own entire shelf space  $l_i$  of products at each visit) net the reservation profit rates is able to provide for the nanostore's

subsistence. For ease of representation, let  $M_i(l_i) = p_i - c_i - \frac{K_i}{l_i}$  denote the maximal supply chain profit rate of selling product  $i = 1, 2$ . Then, the necessary condition can be rewritten as:

$$M_1(l_1)\lambda_1 - \beta_1 + M_2(l_2)\lambda_2 - \beta_2 \geq \gamma \quad \text{for } \underline{l}_1 < l_1 < L - l_2.$$

Next, we show that, for fixed wholesale price values satisfying Lemma 1, it is optimal for each manufacturer to schedule store visits by their respective sales representative to match the stock-out epochs of their respective product at the nanostore.

**Theorem 1.** *Assume  $M_1(l_1)\lambda_1 - \beta_1 + M_2(l_2)\lambda_2 - \beta_2 \geq \gamma$  for  $\underline{l}_1 < l_1 < L - l_2$ . For  $w_i \in \left[ c_i + \frac{K_i}{l_i} + \frac{\beta_i}{\lambda_i}, p_i - \frac{\gamma - \lambda_{3-i}(p_{3-i} - w_{3-i})}{\lambda_i} \right]$  where  $i = 1, 2$ , it is optimal for both manufacturers to pay a visit to the nanostore exactly when their own product runs out, i.e., at time  $\tau_{i,n_i} = \frac{(n_i-1)l_i}{\lambda_i}$  for  $n_i = 2, 3, \dots$  Under this sales visit policy, Manufacturer  $i$ 's long-run profit rate is equal to  $\bar{\pi}_i = \left( w_i - c_i - \frac{K_i}{l_i} \right) \lambda_i$ , and the nanostore's total liquid assets at time  $t$  are given by  $A(t) = w_1 l_1 + w_2 l_2 + [(p_1 - w_1)\lambda_1 + (p_2 - w_2)\lambda_2 - \gamma] t$ .*

Visiting the nanostore at stock-out moments avoids lost sales and spreads the fixed visiting cost as much as possible. This not only optimizes the profit for each manufacturer but also injects the highest inflow to the nanostore's cash pool. Therefore, with fixed wholesale prices, such a sales visit policy serves the best interests for both manufacturers and maximizes the likelihood for them to fund the nanostore's subsistence living.

Interestingly, when the cash sufficiency condition is satisfied, i.e.,  $(p_1 - w_1)\lambda_1 + (p_2 - w_2)\lambda_2 \geq \gamma$ , under this policy, the sales visit timing of the two manufacturers decouples. This contradicts the rough intuition that the two manufacturers are supposed to compete fiercely in the timing of their sales visits, given the limited cash availability at the nanostore. However, on the one hand, this conforms to the unspecific nature of cash: the nanostore can use the cash pool (from selling both products) at his discretion to replenish either product. On the other hand, this is due to the cash limitation nature of nanostores: the cash sufficiency condition implies that the two manufacturers have to collectively contribute sufficient cash inflow to provide for the nanostore's subsistence spending. Due to these fundamental and unique characteristics of the nanostore channel, instead of entering into cash competition and free-riding, the two manufacturers display cooperative behavior in supplying to the nanostore.

Given the manufacturers' optimal visit timing, Theorem 2 shows the wholesale pricing equilibrium.

**Theorem 2.** *If  $M_1(l_1)\lambda_1 - \beta_1 + M_2(l_2)\lambda_2 - \beta_2 \geq \gamma$ , then there exists at least one Nash equilibrium where the nanostore enters into business and both manufacturers are willing to sell to the nanostore and set wholesale prices  $(w_1^*, w_2^*)$  such that  $(p_1 - w_1^*)\lambda_1 + (p_2 - w_2^*)\lambda_2 = \gamma$  and  $c_i + \frac{K_i}{l_i} + \frac{\beta_i}{\lambda_i} \leq w_i^* < p_i$  for  $i = 1, 2$ . In equilibrium, Manufacturer  $i$ 's long-run profit rate is  $\bar{\pi}_i^* = \left(w_i^* - c_i - \frac{K_i}{l_i}\right) \lambda_i$ , and both profit rates sum up to  $\bar{\pi}_1^* + \bar{\pi}_2^* = M_1(l_1)\lambda_1 + M_2(l_2)\lambda_2 - \gamma$ . The nanostore's total liquid assets are constant over time and equal to  $A(t) = w_1^*l_1 + w_2^*l_2$ .*

*If  $M_1(l_1)\lambda_1 - \beta_1 + M_2(l_2)\lambda_2 - \beta_2 < \gamma$ , there does not exist an equilibrium where both manufacturers sell to the nanostore.*

Intuitively, the nanostore retail business is profitable only if the supply chain profit rates outweigh the nanostore's subsistence spending rate plus the reservation profit rates that outside options promise for the manufacturers. Since the supply chain operates using take-it-or-leave-it wholesale contracts, the manufacturers simply price the products such that the profit rate earned by the nanostore exactly matches his rate of subsistence spending. This implies that the nanostore's total liquid assets stay constant over time, and the nanostore owner barely makes ends meet, which reflects the subsistence business nature of nanostores.

The existence of a duopoly equilibrium depends on the total shelf space available and the shelf space that each product occupies at the nanostore. The assumption that  $\underline{l}_1 < l_1 < L - \underline{l}_2$  prescribes that each manufacturer is a positive cash contributor to the nanostore. Now, we compare the two minimal wholesale prices that Manufacturer  $i$  can charge, which are  $c_i + \frac{K_i}{l_i} + \frac{\beta_i}{\lambda_i}$  at which the manufacturer earns the reservation profit, and  $p_i - \frac{\gamma}{\lambda_i}$  at which the manufacturer covers the nanostore's subsistence spending solely by himself while the counterpart contributes nothing. This yields the threshold  $l_i^S = \frac{K_i}{\left(p_i - c_i - \frac{\beta_i + \gamma}{\lambda_i}\right)^+}$  for  $i = 1, 2$ . When  $l_i \geq (<)l_i^S$ , Manufacturer  $i$  is (not) able to finance the nanostore's subsistence spending *solely* by himself. In what follows we refer to the manufacturer with the highest value of  $l_i^S$  as the most *cost-efficient* supplier.

In addition, let  $\tilde{L} \equiv \max \left\{ \left(1 + \sqrt{\frac{K_2\lambda_2}{K_1\lambda_1}}\right) l_1, \left(1 + \sqrt{\frac{K_1\lambda_1}{K_2\lambda_2}}\right) l_2 \right\}$  be the value beyond which as long as  $M_1(l_1)\lambda_1 + M_1(l_2)\lambda_2 - \beta_1 - \beta_2 < \gamma$  holds for  $l_1 = \underline{l}_1$  and  $l_1 = L - \underline{l}_2$ , it holds for the entire range  $l_1 \in (\underline{l}_1, L - \underline{l}_2)$ . Also, let  $\underline{L} \equiv \underline{l}_1 + \underline{l}_2$  represent the lower bound of the total shelf space available so that each manufacturer can reach profitability. Let  $\tilde{l}_1^-$  and  $\tilde{l}_1^+$ , such that  $\tilde{l}_1^- \leq \tilde{l}_1^+$ , denote the two roots of the equation  $M_1(l_1)\lambda_1 + M_1(l_2)\lambda_2 - \beta_1 - \beta_2 = \gamma$  which exist for  $L \geq \frac{(\sqrt{K_1\lambda_1} + \sqrt{K_2\lambda_2})^2}{\sum_{i=1}^2 [(p_i - c_i)\lambda_i - \beta_i] - \gamma}$ . In what follows, we use the above-defined thresholds to characterize the existence of a duopoly equilibrium.

**Theorem 3.** For  $l_1 < l_1 < L - l_2$ , there are three cases.

Case 1 When  $L \geq \max \{l_1^S + l_2, l_2^S + l_1\}$ , both manufacturers always enter into business with the nanostore.

Case 2 (a) When  $l_2^S + l_1 \leq L < l_1^S + l_2$ , the manufacturers enter into business with the nanostore only if  $l_1 \leq \tilde{l}_1^+$ ; (b) When  $l_1^S + l_2 \leq L < l_2^S + l_1$ , the manufacturers never enter into business with the nanostore only if  $l_1 \geq \tilde{l}_1^-$ .

Case 3 When  $L < \min \{l_1^S + l_2, l_2^S + l_1\}$ , if  $L \geq \frac{(\sqrt{K_1\lambda_1} + \sqrt{K_2\lambda_2})^2}{\sum_{i=1}^2 [(p_i - c_i)\lambda_i - \beta_i] - \gamma}$  and  $L > \max \{\underline{L}, \tilde{L}\}$ , the two manufacturers enter into business with the nanostore only if  $\tilde{l}_1^- \leq l_1 \leq \tilde{l}_1^+$ ; otherwise, i.e., if  $L < \frac{(\sqrt{K_1\lambda_1} + \sqrt{K_2\lambda_2})^2}{\sum_{i=1}^2 [(p_i - c_i)\lambda_i - \beta_i] - \gamma}$  or  $L \leq \max \{\underline{L}, \tilde{L}\}$ , the two manufacturers do not enter into business with the nanostore.

Figure 1 illustrates Theorem 3 for the case where Manufacturer 1 is the most cost-efficient manufacturer. Intuitively, shifting the shelf space between the two manufacturers while keeping the total equal (to  $L$ ) increases the profitability of one but hurts the other: the one which sees his shelf space increase (decrease) sees his cost of visiting the nanostore go down (up). We say that the split of the total shelf space (i.e., the fraction  $l_1/L$ ) is *balanced* if it is such that the two manufacturers enter into business with the nanostore; otherwise we say it is *imbalanced*.

When the total shelf space is abundant enough that each manufacturer can solely cover the nanostore's subsistence (Case 1), the two of them can enter into business for any possible shelf space split. However, when the total shelf space is just enough for the most cost-efficient manufacturer (i.e., the one with lower  $l_i^S$ ) to cover the nanostore's subsistence needs, the duopoly equilibrium is less likely to occur if the shelf space split shifts away from this manufacturer (Case 2). Finally, when the total shelf space is limited such that no manufacturer can solely provide for the nanostore, the manufacturers only enter into business for balanced shelf space splits so that both contribute to the nanostore's subsistence collectively (Case 3).

For the parameter space such that the duopoly equilibrium exists, the number of equilibrium may be infinite with continuous ranges of equilibrium wholesale prices. Figure 2 illustrates the set of duopoly equilibria<sup>2</sup> by a thick solid segment in three cases. Due to symmetry, we omit the counterpart of the middle graph.

<sup>2</sup>The duopoly equilibrium is only unique in the special case where either (1)  $M_1(l_1)\lambda_1 - \beta_1 + M_2(l_2)\lambda_2 - \beta_2 > \gamma$  and  $l_i = l_i$  for only one Manufacturer  $i = 1, 2$  or (2)  $M_1(l_1)\lambda_1 - \beta_1 + M_2(l_2)\lambda_2 - \beta_2 = \gamma$  and  $l_1 < l_1 < L - l_2$  for  $i = 1, 2$ .

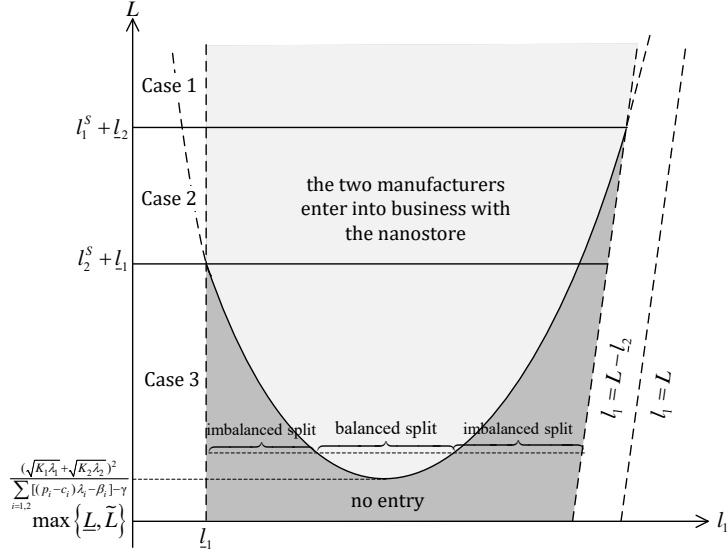


Figure 1: The existence of duopoly equilibrium when  $l_1^S + L_2 > l_2^S + L_1 > \frac{(\sqrt{K_1 \lambda_1} + \sqrt{K_2 \lambda_2})^2}{\sum_{i=1}^2 [(p_i - c_i) \lambda_i - \beta_i] - \gamma} \geq \max \{L_1 + L_2, \underline{L}, \tilde{L}\}$ .

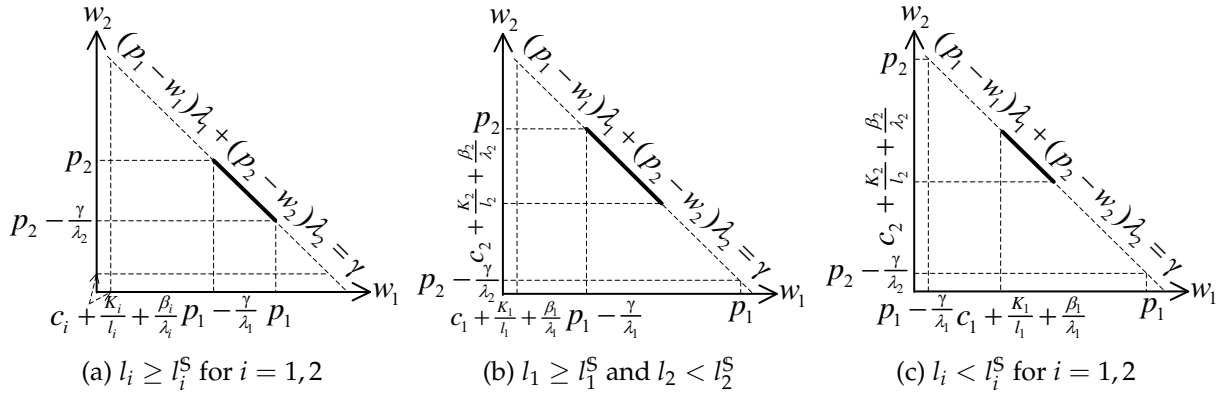


Figure 2: Infinite duopoly equilibria.

When both manufacturers enter into business with the nanostore, they ensure that the profit he earns per unit of time from selling their products exactly offset his subsistence needs. Let  $\phi_i(w_1^*, w_2^*) = \frac{(p_i - w_i^*) \lambda_i}{\gamma}$  be the contribution (fraction) of the nanostore's subsistence needs which are covered from the sale of Manufacturer  $i$ 's product, for  $i = 1, 2$ . From Theorem 2, the sum of the two manufacturers' profit rates is a constant, which does not depend on their wholesale prices. However, their share of this constant total profit rate and their contribution to the nanostore's subsistence needs do depend on their wholesale prices, as shown on Figure 3, where we plot  $\pi_i^*$  and  $\phi_i$  for  $i = 1, 2$  as functions of Manufacturer 1's wholesale price  $w_1^*$  in the three cases defined in Figure 2.

We see that on all three graphs of Figure 3, an increase in Manufacturer 1's wholesale price leads to an increase in her equilibrium profit rate and a decrease in her contribution to the



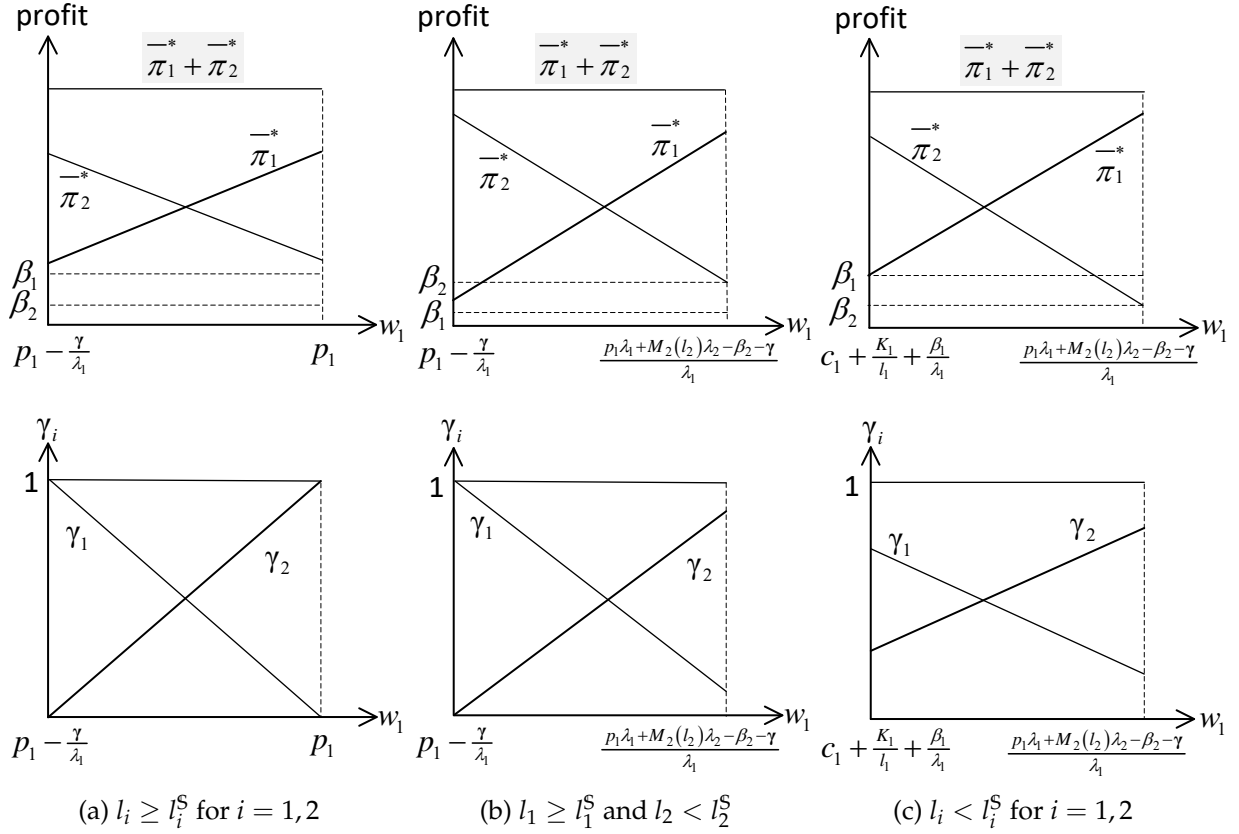


Figure 3: Duopoly equilibrium profit distribution and contribution split to the nanostore's subsistence needs.

nanostore's subsistence needs, while the reverse holds for Manufacturer 2.<sup>3</sup> In Case 1 (leftmost graphs), when both manufacturers could solely cover the nanostore's subsistence needs, Manufacturer 1's equilibrium wholesale price varies between the price he would charge as if he is the only manufacturer selling to the nanostore, i.e.,  $p_1 - \frac{\gamma}{\lambda_1}$ , at which, he provides 100% of the nanostore's subsistence needs, and his product's selling price  $p_1$ , at which she transfers the entire responsibility of providing for the nanostore to Manufacturer 2. In Case 2 (center graphs), the maximum wholesale price value for Manufacturer 1 is such that Manufacturer 2 receives a profit rate equal to his outside option. As a result, both manufacturers always contribute a positive fraction to the nanostore's subsistence needs unless Manufacturer 1 contributes to the nanostore's subsistence needs entirely on his own. Finally, in Case 3 (rightmost graphs), neither manufacturer could fund the nanostore's subsistence individually, so, at equilibrium, both contribute a positive share of the nanostore's subsistence needs.

<sup>3</sup>Note that, on all three graphs of Figure 3, there is value of  $w_1^*$  such that  $\pi_1^* = \pi_2^*$ ; however such a value may not always exist.

### 5.0.1 Fair equilibria

Of all the possible equilibria when both manufacturers enter in business with the nanostore, we single a particular one out in this section, which we refer to as the *fair equilibrium*. Let  $\rho_i$  denote the share of the two manufacturers' total profit rate (in excess of their outside option) which is captured by Manufacturer  $i$ , i.e.,  $\rho_i(w_1^*, w_2^*) = \frac{\bar{\pi}_i^* - \beta_i}{\bar{\pi}_1^* - \beta_1 + \bar{\pi}_2^* - \beta_2}$ . The fair equilibrium wholesales prices  $(w_1^f, w_2^f)$  are such that the share of Manufacturer  $i$ 's total profit rate matches his contribution to the nanostore's subsistence needs, that is,  $\rho_i(w_1^f, w_2^f) = \phi_i(w_1^f, w_2^f)$  for  $i = 1, 2$ . Lemma 2 provides the expression of the fair equilibrium wholesale prices for the cases where the parties enter into business.

**Lemma 2.** *Given  $M_1(l_1)\lambda_1 - \beta_1 + M_2(l_2)\lambda_2 - \beta_2 \geq \gamma$ , the wholesale prices for Manufacturer  $i = 1, 2$  that achieve a fair allocation are given by*

$$w_i^f = p_i - \frac{\gamma}{\lambda_i} \frac{M_i(l_i)\lambda_i - \beta_i}{M_1(l_1)\lambda_1 - \beta_1 + M_2(l_2)\lambda_2 - \beta_2}.$$

Further,  $\rho_i(w_1^f, w_2^f) = \phi_i(w_1^f, w_2^f) \geq \frac{1}{2}$  if and only if  $M_i(l_i)\lambda_i - \beta_i \geq M_{3-i}(l_{3-i})\lambda_{3-i} - \beta_{3-i}$ .

Note that in general, the fair equilibrium does not imply that both manufacturers contribute equally to the nanostore's subsistence needs; the manufacturer with the largest value of the supply chain profit rate (in excess of her outside option) is the one which contributes the bigger share. Figure 4 illustrates the fair equilibrium for the three cases defined in Theorem 6 assuming Manufacturer 1 is the one which contributes the biggest share.<sup>4</sup>

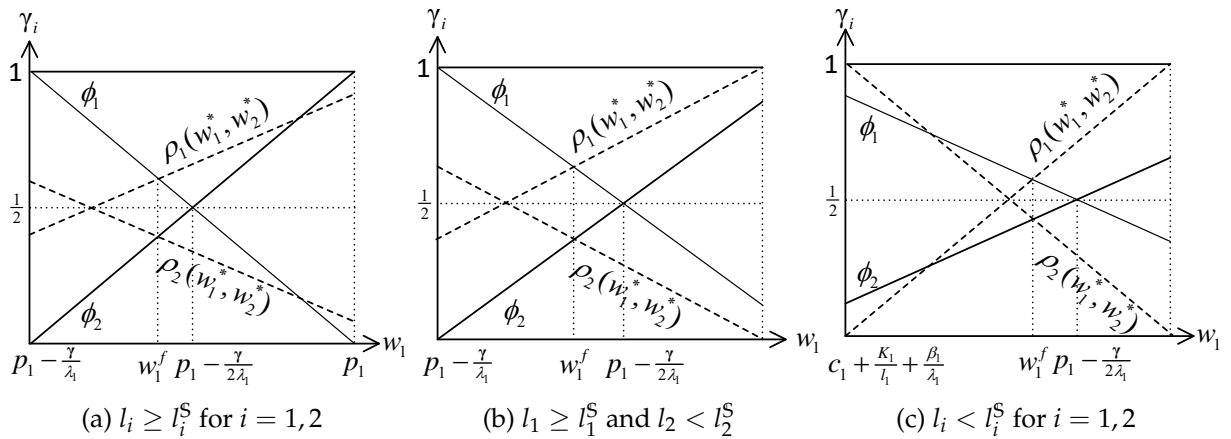


Figure 4: Fair equilibrium where Manufacturer 1 contributes the biggest share.

<sup>4</sup>Note that, on all three graphs of Figure 4, there is value of  $w_1^*$  such that  $\rho_1(w_1^*, w_2^*) = \rho_2(w_1^*, w_2^*)$ ; however such a value may not always exist.

## 5.0.2 Sequential game

In this section, we consider the case of a sequential Stackelberg game, wherein Manufacturer 1 is the leader, and Manufacturer 2 is the follower. Theorem 4 gives the equilibrium Stackelberg wholesale prices.

**Theorem 4.** *Let  $l_2 < l_1 < L - l_2$  and assume  $M_1(l_1)\lambda_1 - \beta_1 + M_2(l_2)\lambda_2 - \beta_2 \geq \gamma$ . In a Stackelberg game, if  $l_2 \geq l_2^S$ , Manufacturer 1 (leader) sets her wholesale price equal to  $w_1^* = p_1$  and Manufacturer 2 (follower) sets her wholesale price equal to  $w_2^* = p_2 - \frac{\gamma}{\lambda_2}$ ; otherwise, Manufacturer 1 sets her wholesale price equal to  $w_1^* = p_1 + \frac{1}{\lambda_1} [M_2(l_2)\lambda_2 - \beta_2 - \gamma]$  and Manufacturer 2 sets her wholesale price equal to  $w_2^* = c_2 + \frac{K_2}{l_2} + \frac{\beta_2}{\lambda_2}$ .*

When  $l_2 \geq l_2^S$ , Manufacturer 2 could provide for the nanostore's subsistence individually and, as a result, Manufacturer 1 transfers the responsibility to provide for the nanostore's subsistence needs entirely to Manufacturer 2, who then earns a profit rate equal to what he would earn on his own. Otherwise, i.e., when  $l_2 < l_2^S$ , Manufacturer 2 could not provide for the nanostore's subsistence individually, so that, in equilibrium, both manufacturers contribute a positive fraction to the nanostore's subsistence needs.

## 6 Two-manufacturer model versus single-manufacturer model

In this section, we compare the (two-manufacturer) model with a single-manufacturer model where the nanostore dedicates all his shelf space  $L$  for a single product from a manufacturer. Please refer to the model details in Appendix B. The nanostore now relies on a single source of revenue to fund its subsistence living, and the nanostore closes the business if he runs out of assets which are the cash balance plus the on-hand inventory of this product evaluated at the wholesale price. The cash balance decreases in rate  $\gamma$  if the inventory is zero and increases otherwise (we assume the revenue rate is higher than the nanostore's subsistence spending rate). We assume that the manufacturer's sales representative pays her first visit to the nanostore at the start of the time horizon when the nanostore has zero inventory and his cash position can exactly afford  $L$  units of product. We assume that the nanostore owner always buys the maximum quantity he can afford given his cash balance up to the shelf space limit  $L$  on the sales visits.

To facilitate the comparison, we assume that the manufacturer as the only supplier to the

nanostore is Manufacturer  $i = 1, 2$  in the two-manufacturer model. The next theorem shows that, in the single-manufacturer model, Manufacturer  $i$  engages in business with the nanostore only if the shelf space  $L$  is above a threshold, which is actually  $l_i^S$ , and the manufacturer prices at the value such that the profit earned by the nanostore exactly matches his rate of subsistence spending. Consistent with the two-manufacturer model, the nanostore's total liquid assets stay constant over time.

**Theorem 5.** *Let Manufacturer  $i = 1, 2$  be the only supplier to the nanostore. If  $L \geq \frac{K_i}{\left(p_i - c_i - \frac{\beta_i + \gamma}{\lambda_i}\right)^+} = l_i^S$ , the optimal wholesale price is  $w_i^* = p_i - \frac{\gamma}{\lambda_i}$  and the optimal long-run profit rate for the manufacturer is  $\bar{\pi}_i^* = \left(p_i - c_i - \frac{K_i}{L}\right) \lambda_i - \gamma$ . Further, the nanostore's total liquid assets are constant over time and equal to  $A(t) = \left(p_i - \frac{\gamma}{\lambda_i}\right) L$ . If  $L < \frac{K_i}{\left(p_i - c_i - \frac{\beta_i + \gamma}{\lambda_i}\right)^+} = l_i^S$ , the manufacturer does not sell to the nanostore.*

Next, we compare the results of the single-manufacturer problem (with shelf space limit  $L$ ) and the two-manufacturer problem (with respective shelf space limits  $l_1$  and  $L - l_1$ ). According to Theorem 5, the sufficient condition for Manufacturer  $i$  as the only supplier to enter into business with the nanostore is  $L \geq l_i^S = \frac{K_i}{\left(p_i - c_i - \frac{\beta_i + \gamma}{\lambda_i}\right)^+}$ . This is because, for  $l_i \geq l_i^S$ , Manufacturer  $i$  can independently finance the nanostore's subsistence needs by setting the wholesale price at  $w_i = p_i - \frac{\gamma}{\lambda_i}$ , regardless of the presence of the other manufacturer. Similarly, the sufficient condition for the two manufacturers to enter into business with the nanostore is  $M_1(l_1)\lambda_1 - \beta_1 + M_2(l_2)\lambda_2 - \beta_2 \geq \gamma$  for  $l_1 < l_1 < L - l_2$ . Note that  $l_i = \frac{K_i}{\left(p_i - c_i - \frac{\gamma}{\lambda_i}\right)^+}$  is the lower bound on Manufacturer  $i$ 's shelf space limit beyond which Manufacturer  $i$  serves as a positive cash contributor to the nanostore in the two-manufacturer setting. Obviously,  $l_i \leq l_i^S$  as being the sole cash contributor requires more shelf space.

Using the above thresholds, we adopt the cases defined in Theorem 3 to compare the solutions to the two models in Theorem 6. For use in the theorem, we define six interesting manufacturer relationships in the following table.

**Theorem 6.** *Table 2 shows how the solutions in the single- and two-manufacturer models compare.*

Figure 5 illustrates Theorem 6. Switching from a single supplier to two suppliers, i.e., adding another product to the nanostore shelves while keeping the total shelf space constant, provides a new cash stream to the nanostore, which also lowers the burden on the first manufacturer to provide for the nanostore's subsistence needs. On the other hand, adding the

	relationships	co-existing	repelling	spilling-over	counteracting	cooperative	unconnected
in business?	individually	✓ ✓	✓ ✓	✓ ×	✓ ×	× ×	× ×
	collectively	✓ ✓	× ×	✓ ✓	× ×	✓ ✓	× ×

Note: ✓ means entry into business while × means quitting.

Table 1: Six interesting manufacturer relationships.

Case 1: $L \geq \max \{l_1^S + l_2, l_2^S + l_1\}$	Case 2: $l_{3-i}^S + l_i \leq L < l_i^S + l_{3-i}$ for $i = 1, 2$		
all splits		balanced splits	imbalanced splits
co-existing	$L \geq l_i^S$	co-existing	repelling
	$L < l_i^S$	spilling-over	counteracting
Case 3: $L < \min \{l_1^S + l_2, l_2^S + l_1\}$			
	$L \geq \frac{(\sqrt{K_1\lambda_1 + \sqrt{K_2\lambda_2}})^2}{\sum_{i=1}^2 [(p_i - c_i)\lambda_i - \beta_i] - \gamma}$ and $L > \max \{L, \tilde{L}\}$		$L < \frac{(\sqrt{K_1\lambda_1 + \sqrt{K_2\lambda_2}})^2}{\sum_{i=1}^2 [(p_i - c_i)\lambda_i - \beta_i] - \gamma}$ or $L \leq \max \{L, \tilde{L}\}$
	balanced splits	imbalanced splits	all splits
$L \geq \max \{l_1^S, l_2^S\}$	co-existing	repelling	repelling
$l_{3-i}^S \leq L < l_i^S$	spilling-over	counteracting	counteracting
$L < \min \{l_1^S, l_2^S\}$	cooperative	unconnected	unconnected

Note: (im)balanced splits are the values of  $l_1$  such that  $M_1(l_1)\lambda_1 + M_2(l_2)\lambda_2 - \beta_1 + \beta_2(<) \geq \gamma$ . Refer to Theorem 3 for the detail.

Table 2: Comparisons between the two models.

second supplier means reducing the shelf space of the first one, which can hurt him. If the supply-chain profit rate gain outweighs the loss, the two manufacturers are more likely to enter into business with the nanostore; otherwise, the opposite happens. This trade-off ramifies into interesting manufacturer relationships.

Interestingly, the total shelf space available and its split between the two manufacturers are among the critical factors which determine the type of relationship characterized in Table 1. Provided with abundant shelf space, the two manufacturers are more likely to enter into business with the nanostore regardless of whether they can do so individually. Given a fixed amount of total shelf space, the two manufacturers (do not) enter into business for (im)balanced splits. This applies even when both manufacturers could profitably enter into business with the nanostore individually: with imbalanced splits, the two manufacturers cannot provide enough subsistence to the nanostore, that is, the two manufacturers end up *repelling* each other rather than *co-existing* under balanced splits. On the other hand, when neither can independently fund the nanostore's subsistence living, given a balanced shelf split,

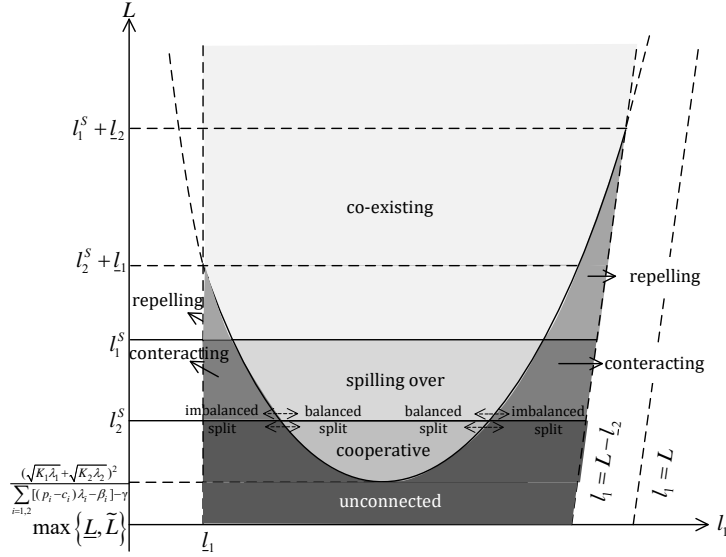


Figure 5: The manufacturer relationships when  $l_1^S + l_2 > l_2^S + l_1 > \frac{(\sqrt{K_1\lambda_1} + \sqrt{K_2\lambda_2})^2}{\sum_{i=1,2} [(p_i - c_i)\lambda_i - \beta_i] - \gamma} \geq \max \{l_1 + l_2, L, \tilde{L}\}$ .

the two manufacturers can work cooperatively to do so, that is, the two manufacturers are in a *cooperative* relationship with each other. Similarly, when only one manufacturer is able to provide for the nanostore's subsistence living on their own, a balanced split generates a healthy *spillover* effect while an imbalanced split makes the two manufacturers enter in a *counteracting* relationship with each other which results in a no-entry outcome.

Finally, we compare the manufacturers' profits in the single- and two-manufacturer models. Obviously, the manufacturers are (weakly) better off if they collectively enter into business with the nanostore but individually do not; on the other hand, the manufacturers are (weakly) worse off if they individually enter into business with the nanostore but collectively do not. The most interesting situation occurs when the two manufacturers enter into business with the nanostore in both models; in this case, the profit split in the two-manufacturer model depends on the equilibrium wholesale prices, as stated in Theorem 7.

**Theorem 7.** Let  $L > \max\{l_1^S, l_2^S\}$  and  $M_1(l_1)\lambda_1 - \beta_1 + M_2(l_2)\lambda_2 - \beta_2 > \gamma$  such that both manufacturers enter into the nanostore both individually and collectively (with an infinite number of duopoly equilibria). Comparing profits obtained collectively versus individually,

- when  $\gamma > K_1\lambda_1 \left(\frac{1}{l_1} - \frac{1}{L}\right) + K_2\lambda_2 \left(\frac{1}{l_2} - \frac{1}{L}\right)$ , if  $w_1^* > p_1 - \frac{K_2\lambda_2}{\lambda_1} \left(\frac{1}{l_2} - \frac{1}{L}\right)$ , Manufacturer 1 (2) is better (worse) off; if  $w_1^* < p_1 - \frac{\gamma}{\lambda_1} + K_1 \left(\frac{1}{l_1} - \frac{1}{L}\right)$ , Manufacturer 1 (2) is worse (better) off; if  $p_1 - \frac{\gamma}{\lambda_1} + K_1 \left(\frac{1}{l_1} - \frac{1}{L}\right) < w_1^* < p_1 - \frac{K_2\lambda_2}{\lambda_1} \left(\frac{1}{l_2} - \frac{1}{L}\right)$ , both manufacturers are better off.

- when  $\max \left\{ K_1 \lambda_1 \left( \frac{1}{l_1} - \frac{1}{L} \right), K_2 \lambda_2 \left( \frac{1}{l_2} - \frac{1}{L} \right) \right\} < \gamma \leq K_1 \lambda_1 \left( \frac{1}{l_1} - \frac{1}{L} \right) + K_2 \lambda_2 \left( \frac{1}{l_2} - \frac{1}{L} \right)$ , if  $w_1^* > p_1 - \frac{\gamma}{\lambda_1} + K_1 \left( \frac{1}{l_1} - \frac{1}{L} \right)$ , Manufacturer 1 (2) is better (worse) off; if  $w_1^* < p_1 - \frac{K_2 \lambda_2}{\lambda_1} \left( \frac{1}{l_2} - \frac{1}{L} \right)$ , Manufacturer 1 (2) is worse (better) off; if  $p_1 - \frac{K_2 \lambda_2}{\lambda_1} \left( \frac{1}{l_2} - \frac{1}{L} \right) < w_1^* < p_1 - \frac{\gamma}{\lambda_1} + K_1 \left( \frac{1}{l_1} - \frac{1}{L} \right)$ , both manufacturers are worse off.
- When  $K_i \lambda_i \left( \frac{1}{l_i} - \frac{1}{L} \right) < \gamma \leq K_{3-i} \lambda_{3-i} \left( \frac{1}{l_{3-i}} - \frac{1}{L} \right)$  for  $i = 1, 2$ , if  $w_1^* > (<) p_1 - \frac{\gamma}{\lambda_1} + K_1 \left( \frac{1}{l_1} - \frac{1}{L} \right)$ , Manufacturer  $i$  is better (worse) off, while the other manufacturer is always worse off.
- when  $\gamma \leq \min \left\{ K_1 \lambda_1 \left( \frac{1}{l_1} - \frac{1}{L} \right), K_2 \lambda_2 \left( \frac{1}{l_2} - \frac{1}{L} \right) \right\}$ , both manufacturers are worse off.

Figure 6 illustrates Theorem 7 where M1 represents Manufacturer 1 while M2 represents Manufacturer 2. When entering into business collectively but not individually, there are two sides to the effect on profits: on the one hand, the two manufacturers provide for the nanostore's subsistence spending collectively; on the other hand, each sells less per store visit due to the shelf space being split between the two of them. We define the *efficiency loss*, equal to  $K_i \left( \frac{1}{l_i} - \frac{1}{L} \right)$ , as the additional amount by which Manufacturer  $i = 1, 2$  needs to mark up his wholesale price in order to earn the same amount in a two-manufacturer setting as in a single manufacturer setting. This efficiency loss may reduce the cash inflow to the nanostore, and therefore jeopardize the whole business even though individually, each manufacturer could enter into business with the nanostore.

Specifically, when the total efficiency loss does not exceed the nanostore's subsistence rate, that is,  $\gamma > \sum_{i=1}^2 K_i \lambda_i \left( \frac{1}{l_i} - \frac{1}{L} \right)$ , both manufacturers earn more profit if they both mark up an additional amount beyond their individual loss amount to recoup the efficiency loss; otherwise, asymmetrically, one manufacturer gains substantially while his counterpart fails to compensate for their efficiency loss. When the total efficiency loss exceeds the nanostore's subsistence rate, that is,  $\gamma < \sum_{i=1}^2 K_i \lambda_i \left( \frac{1}{l_i} - \frac{1}{L} \right)$ , either both manufacturers are hurt or one loses and the other gains.

## 7 Conclusion

We study the sales visit and pricing strategies of CPG manufacturers doing business with a nanostore who aims to make a living. When there is only one manufacturer, his sales representative should visit the nanostore when his product runs out of inventory and set a wholesale

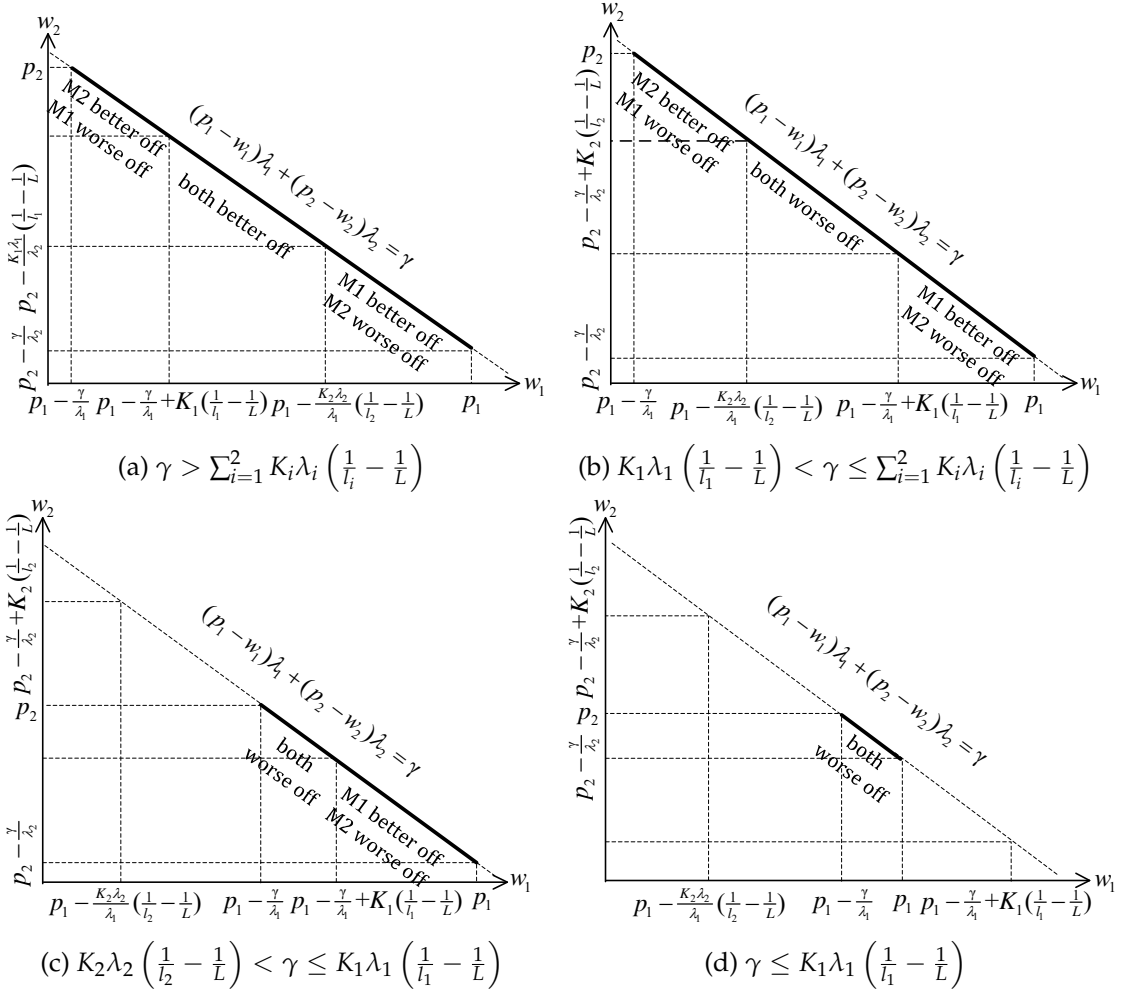


Figure 6: Profit comparison if  $K_1 \lambda_1 \left( \frac{1}{l_1} - \frac{1}{L} \right) > K_2 \lambda_2 \left( \frac{1}{l_2} - \frac{1}{L} \right)$  and  $l_i > l_i^S$  for  $i = 1, 2$ .

price so that the nanostore owner earns exactly enough to cover his family's subsistence needs. When two CPG manufacturers compete for the cash resources of the nanostore, in equilibrium, they should also visit the store when their respective product runs out and set their wholesale prices such that the profit from selling both products is exactly enough to cover the nanostore's subsistence needs.

Our results highlight the mutually beneficial relationship between the nanostore and CPG manufacturers. In some settings, one manufacturer cannot single-handedly sustain the nanostore, but two of them can, and they do so by splitting the responsibility of providing for his subsistence needs. Our analysis reveals that the optimal strategy for a manufacturer highly depends on his cost structure and can be very sensitive to the problem's specific parameters; for example, the shelf space split. This implies that manufacturers may or may not decide to enter in business with the nanostore, and when they do, the outcome of the competition may vary.



In our model, the manufacturers are competing for a joint cash pool, not for shelf space. Further, although it may be argued that manufacturers are subsidizing the nanostore channel, effectively they are doing so only to the extent that the nanostore (barely) survives. This is a marked difference from what is generally known in the modern, organized retail channel where the retailer exploits the competition between manufacturers to reduce their margin. Our model provides an analytical justification for the existence and relative prosperity of multi-brand nanostores in emerging-market mega-cities. One crucial insight, however, shows that such co-existence can only work out if the manufacturer is somehow able to retain efficiency in her supply operations. Hence, economies of scale in logistics remain essential.

Our work provides insights about the practically relevant problem of supplying to a cash-constrained retailer with a simple subsistence goal and, as such, differs from previous work focused on contracting with profit-maximizing retailers. We hope that our work opens up opportunities for further study in this different and important institutional context.

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# Appendix to the paper "Manufacturer Competition in the Nanostore Retail Channel"

## A Alternative formulation of the long-run average profit rate

Let  $\bar{\pi}_{i,n_i} = \frac{(I_i(\tau_{i,n_i}^+) - I_i(\tau_{i,n_i}^-))(w_i - c_i) - K_i}{\tau_{i,n_i} - \tau_{i,n_i-1}}$  be the profit rate earned by Manufacturer  $i$  in  $[\tau_{i,n_i-1}, \tau_{i,n_i}]$ , i.e., the time interval between the  $(n_i - 1)$ -th and  $n_i$ -th visits of her sales representative for  $i = 1, 2$  (provided both take place). Then, Manufacturer  $i$ 's long-run average profit rate can be formulated as a function of  $\bar{\pi}_{i,n_i}$ :

$$\begin{aligned}
 \bar{\pi}_i &= \lim_{N_i \rightarrow \infty} \frac{\sum_{n_i=1}^{N_i} \pi_i(\tau_{i,n_i})}{\tau_{i,N_i}} \\
 &= \lim_{N_i \rightarrow \infty} \left[ \frac{1}{\tau_{i,N_i}} \sum_{n_i=1}^{N_i} \left[ (I_i(\tau_{i,n_i}^+) - I_i(\tau_{i,n_i}^-)) (w_i - c_i) - K_i \right] \right] \\
 &= \lim_{N_i \rightarrow \infty} \left[ \frac{1}{\tau_{i,N_i}} \sum_{n_i=2}^{N_i} \left[ (I_i(\tau_{i,n_i-1}^+) - I_i(\tau_{i,n_i}^-)) (w_i - c_i) - K_i \right] \right] \\
 &= \lim_{N_i \rightarrow \infty} \frac{1}{\tau_{i,N_i}} \sum_{n_i=2}^{N_i} \frac{(I_i(\tau_{i,n_i-1}^+) - I_i(\tau_{i,n_i}^-)) (w_i - c_i) - K_i}{\tau_{i,n_i} - \tau_{i,n_i-1}} (\tau_{i,n_i} - \tau_{i,n_i-1}) \\
 &= \lim_{N_i \rightarrow \infty} \frac{1}{\tau_{i,N_i}} \sum_{n_i=2}^{N_i} (\tau_{i,n_i} - \tau_{i,n_i-1}) \bar{\pi}_{i,n_i}
 \end{aligned}$$

where  $I_i(\tau_{i,1}^-) = I_i(\tau_{i,N_i+1}^-) = 0$ .  $\square$

## B The single-manufacturer model

A manufacturer produces a non-perishable product at a cost of  $c$  and sells it to a nanostore at a wholesale price of  $w$ . The nanostore sells this single product to its customers at a price of  $p$ . The demand from nanostore customers is deterministic with rate  $\lambda$ . Unsatisfied demand is lost. The nanostore has only a limited amount of shelf space in his store: let  $L$  be the maximum amount of inventory of the manufacturer's product, which can be stocked in the store. We assume that the nanostore does not have a backroom so that all inventory units at the nanostore are directly available for purchase by the customers.

Let  $C(t)$  denote the cash balance of the nanostore at time  $t$ . The nanostore owner spends money for his family's subsistence needs (such as food, housing, clothing) from this product's

sales at a rate of  $\gamma$  per time unit as long as the cash balance is positive<sup>5</sup>. We refer to  $\gamma$  as the nanostore's *rate of subsistence spending*. We assume that  $p\lambda > \gamma$ , as a necessary condition for the nanostore owner to be able to satisfy his family's subsistence needs. Let  $I(t)$  denote the inventory of the product at the nanostore at time  $t$ . Let  $A(t)$  be the nanostore's total liquid assets at time  $t$ , which is defined as the cash balance, plus the on-hand inventory evaluated at the wholesale price (as is customary in accounting practices), i.e.,  $A(t) = C(t) + wI(t)$ .

The manufacturer's sales representative visits the nanostore to replenish the inventory at discrete moments in time. If, at time  $t$ , there is no such visit, we have:

$$I'(t) = \begin{cases} -\lambda & \text{if } I(t) > 0 \\ 0 & \text{if } I(t) = 0 \end{cases}$$

$$C'(t) = \begin{cases} p\lambda - \gamma & \text{if } I(t) > 0 \\ -\gamma & \text{if } I(t) = 0 \text{ and } C(t) > 0 \\ 0 & \text{otw} \end{cases}$$

$$A'(t) = \begin{cases} (p - w)\lambda - \gamma & \text{if } I(t) > 0 \\ -\gamma & \text{if } I(t) = 0 \text{ and } C(t) > 0 \\ 0 & \text{otw.} \end{cases}$$

In other words, as long as the inventory of the product is positive, the owner's cash balance increases at a rate of  $p\lambda - \gamma$ . If the nanostore has stocked out, then the cash balance decreases at a rate of  $\gamma$ ; in this case, when the cash depletes to zero, the nanostore runs out of assets and the business closes.

We assume that, when the manufacturer's sales representative visits the nanostore, the nanostore owner always buys the maximum quantity he can afford given his current cash balance, up to the shelf space limit  $L$ . In other words, if the sales representative visits the nanostore at time  $t$ , the quantity purchased by the nanostore is given by  $\min \left\{ \frac{C(t^-)}{w}, L - I(t^-) \right\}$ .

Let  $t_n$  denote the time of the  $n$ -th store visit by the manufacturer's sales representative, if it

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<sup>5</sup>If the nanostore sells more than one product,  $\gamma$  can be seen as the proportion of the nanostore's subsistence spending which is not covered by the revenues from other products sold in the store.

takes place. In general, for  $n = 1, 2, \dots$ , we have:

$$C(t_n^+) = [C(t_n^-) - (L - I(t_n^-))w]^+ \quad (11)$$

$$I(t_n^+) = \min \left\{ I(t_n^-) + \frac{C(t_n^-)}{w}, L \right\}. \quad (12)$$

For ease of analysis, we assume that the manufacturer's sales representative pays her first visit to the nanostore at the start of the time horizon ( $t = 0$ ) when the nanostore's cash balance is exactly equal to  $wL$ , and his inventory is equal to zero, i.e.,  $C(0^-) = wL$  and  $I(0^-) = 0$ . Hence, upon the departure of the sales representative from the nanostore, the store has an inventory level of  $L$  units and a cash balance of zero, i.e.,  $I(0^+) = L$  and  $C(0^+) = 0$ . All of our results would continue to hold if the starting point of our model was different; for example, the starting inventory at the nanostore is less than  $L$  units.

In general, the nanostore's inventory level and cash balance right before the  $n$ -th visit (if it occurs) of the sales representative are given by:

$$I(t_n^-) = [I(t_{n-1}^+) - (t_n - t_{n-1})\lambda]^+ \quad (13)$$

$$C(t_n^-) = [C(t_{n-1}^+) + p \min \{ \lambda(t_n - t_{n-1}), I(t_{n-1}^+) \} - \gamma(t_n - t_{n-1})]^+. \quad (14)$$

At  $t = 0$ , we have  $A(0) = wL$ . For  $t > 0$  such that  $t_{n-1} \leq t < t_n$ , the nanostore's liquid assets are:

$$A(t) = [A(t_{n-1}) + (p - w) \min \{ I(t_{n-1}^+), \lambda(t - t_{n-1}) \} - \gamma(t - t_{n-1})]^+. \quad (15)$$

Note that  $A(t^-) = A(t) = A(t^+)$  holds for all time  $t$  and the nanostore remains in business as long as  $A(t) > 0$ .

The cost per store visit to the manufacturer is  $K$ . Given this, the profit the manufacturer earns from her  $n$ -th visit (if it takes place) is:

$$\pi(t_n) = (w - c) \min \left\{ \frac{C(t_n^-)}{w}, L - I(t_n^-) \right\} - K. \quad (16)$$

Let  $\bar{\pi}_n = \frac{(I(t_{n-1}^+) - I(t_n^-))(w - c) - K}{t_n - t_{n-1}}$  be the profit rate earned by the manufacturer between the  $(n - 1)$ -th and  $n$ -th visits of her sales representative (provided both take place). Let  $N$  represent the index of the last visit to the nanostore. If the nanostore closes the business or if

the manufacturer only sells to the nanostore for a finite period of time, that is,  $N$  is finite, the manufacturer's long-run profit rate is zero. Otherwise, the manufacturer needs to decide on the timing of her visits to the nanostore as well as wholesale price  $w$  in order to maximize her long-run profit rate.

$$\sup_{w, t_1, t_2, \dots} \bar{\pi} = \lim_{N \rightarrow \infty} \frac{\sum_{n=1}^N \pi(t_n)}{t_N} = \lim_{N \rightarrow \infty} \frac{1}{t_N} \sum_{n=2}^N (t_n - t_{n-1}) \bar{\pi}_n, \quad (17)$$

where we can use the approach in Appendix A to show that the second equality holds.

Finally, we assume the manufacturer has an alternative business option, which we refer to as the *outside option*, which generates a profit rate of  $\beta$ . The manufacturer, therefore, enters into business with the nanostore only if the profit rate she earns from selling to the nanostore is greater than or equal to the reservation profit rate  $\beta$ .

## C Proofs

*Proof.* Proof of Lemma 1

If Manufacturer  $i = 1, 2$  prices her product at a value lower than  $c_i + \frac{K_i}{l_i} + \frac{\beta_i}{\lambda_i}$ , she earns less than  $\beta_i$  per unit of time in  $[\tau_{i,n_i-1}, \tau_{i,n_i}]$ . Specifically, if  $I_i(\tau_{i,n_i-1}^+) - I_i(\tau_{i,n_i}^-) = 0$ ,  $\bar{\pi}_{i,n_i}$  is less than zero. Otherwise, we have:

$$\begin{aligned} \bar{\pi}_{i,n_i} &< \frac{\left( I_i(\tau_{i,n_i-1}^+) - I_i(\tau_{i,n_i}^-) \right) \left( \frac{K_i}{l_i} + \frac{\beta_i}{\lambda_i} \right) - K_i}{\tau_{i,n_i} - \tau_{i,n_i-1}} \\ &= \frac{I_i(\tau_{i,n_i-1}^+) - I_i(\tau_{i,n_i}^-)}{(\tau_{i,n_i} - \tau_{i,n_i-1})\lambda_i} \beta_i + \frac{\left( I_i(\tau_{i,n_i-1}^+) - I_i(\tau_{i,n_i}^-) \right) - l_i}{(\tau_{i,n_i} - \tau_{i,n_i-1})l_i} K_i \\ &\leq \beta_i \end{aligned}$$

where the first inequality is because  $w_i < c_i + \frac{K_i}{l_i} + \frac{\beta_i}{\lambda_i}$  and the second inequality is because  $\frac{I_i(\tau_{i,n_i-1}^+) - I_i(\tau_{i,n_i}^-)}{\tau_{i,n_i} - \tau_{i,n_i-1}} \leq \lambda_i$  and  $I_i(\tau_{i,n_i-1}^+) - I_i(\tau_{i,n_i}^-) \leq l_i$ . Then, the Manufacturer  $i$ 's long-run average profit rate is upper bounded by  $\beta_i$ , i.e.,

$$\bar{\pi}_i = \lim_{N_i \rightarrow \infty} \frac{1}{\tau_{i,N_i}} \sum_{n_i=2}^{N_i} (\tau_{i,n_i} - \tau_{i,n_i-1}) \bar{\pi}_{i,n_i} < \beta_i \lim_{N_i \rightarrow \infty} \frac{1}{\tau_{i,N_i}} \sum_{n_i=2}^{N_i} (\tau_{i,n_i} - \tau_{i,n_i-1}) = \beta_i.$$

Therefore, the lowest wholesale price manufacturer  $i = 1, 2$  can charge is  $c_i + \frac{K_i}{l_i} + \frac{\beta_i}{\lambda_i}$  below which she profits less than her outside option.



Next, we show that the nanostore's total liquid assets  $A(t)$  are decreasing in  $t$  until reaching zero if  $w_i > p_i - \frac{\gamma - \lambda_{3-i}(p_{3-i} - w_{3-i})}{\lambda_i}$ , i.e.,  $(p_1 - w_1)\lambda_1 + (p_2 - w_2)\lambda_2 < \gamma$ . Note that the total liquid assets  $A(t)$  comprise of the cash balance plus the inventory value as the inventory positions weighted by the wholesale prices, which remains invariant prior and post inventory purchasing epochs. Therefore, the total liquid assets are exclusively dependent on the profit rate that inflows and the nanostore expense rate that outflows. When the outflow rate  $\gamma$  outweighs the the highest inflow rate, i.e.,  $(p_1 - w_1)\lambda_1 + (p_2 - w_2)\lambda_2$ , which realizes when both products have positive inventory on the shelf space, the total liquid assets continuously decrease until drain. Therefore, if  $w_i > p_i - \frac{\gamma - \lambda_{3-i}(p_{3-i} - w_{3-i})}{\lambda_i}$  for  $i = 1, 2$ , the nanostore eventually closes his business where the manufacturer can only get zero long-run profit rate. This implies the manufacturers should price their product at a value lower than or equal to  $p_i - \frac{\gamma - \lambda_{3-i}(p_{3-i} - w_{3-i})}{\lambda_i}$  for  $i = 1, 2$ .  $\square$

*Proof.* Proof of theorem 1

Let the manufacturers visit the nanostore exactly when their own product runs out, i.e.,  $\tau_{i,n_i} = \frac{(n_i-1)l_i}{\lambda_i}$  for  $n_i = 1, 2, \dots$  and  $i = 1, 2$ . First, we show by induction that (i)  $I_i(\tau_{i,n_i}^+) = l_i$  and  $C(\tau_{i,n_i}^-) \geq w_i l_i$  for  $n_i = 1, 2, \dots$  and (ii)  $A(t) = w_1 l_1 + w_2 l_2 + ((p_1 - w_1)\lambda_1 + (p_2 - w_2)\lambda_2 - \gamma)t$  for all  $t$ .

(i) is true for  $n_i = 1$  by assumption. Since the two manufacturers are symmetrical, we only show (ii) holds until Manufacturer 1's next visit, i.e., for  $t \leq \tau_{1,2}$ . If there is no visit from Manufacturer 2 until 1' next visit at  $\tau_{1,2} = \frac{l_1}{\lambda_1}$ , (ii) holds for  $t \leq \tau_{1,2}$  since no product stocks out meanwhile  $(p_1 - w_1)\lambda_1 + (p_2 - w_2)\lambda_2 \geq \gamma$ . If there is only one visit from Manufacturer 2 at  $\tau_{2,2} = \frac{l_2}{\lambda_2}$  until 1' next visit, the nanostore has enough cash  $C(\tau_{2,2}^-) = (p_1\lambda_1 + p_2\lambda_2 - \gamma)\tau_{2,2}^- \geq (w_1\lambda_1 + w_2\lambda_2)\tau_{2,2}^- \geq w_2 l_2$  to buy a whole shelf of product 2, i.e.  $I_2(\tau_{2,2}^+) = l_2$ ; therefore, (ii) holds for  $t \leq \tau_{1,2}$  due to reasons mentioned above. If there are more than one visits from Manufacturer 2 until 1' next visit, due to the same reasoning, the nanostore always has enough cash to buy a whole shelf of product 2 and (ii) holds for  $t \leq \tau_{1,2}$ .

Now, assume the result is true (induction hypothesis) for  $n_i - 1$  where  $\tau_{i,n_i-1} = \frac{(n_i-2)l_i}{\lambda_i}$  and we only need to show (i) is true for  $n_i$  and (ii) is true for  $t \in (\tau_{i,n_i-1}, \tau_{i,n_i}]$  because of symmetry. If there is no visit from Manufacturer 2 until 1' next visit at  $\tau_{1,n_1}$ , the nanostore has enough cash  $C(\tau_{1,n_1}^-) = C(\tau_{1,n_1-1}^+) + (p_1\lambda_1 + p_2\lambda_2 - \gamma)\frac{l_1}{\lambda_1} \geq (w_1\lambda_1 + w_2\lambda_2)\frac{l_1}{\lambda_1} \geq w_1 l_1$  to buy a whole shelf

of product 1, i.e.  $I_1(\tau_{1,n_1}^+) = l_1$ . Further, since zero stock-out for both products, we have:

$$\begin{aligned} A(t) &= [A(\tau_{1,n_i-1}) + ((p_1 - w_1)\lambda_1 + (p_2 - w_2)\lambda_2 - \gamma)(t - \tau_{1,n_i-1})]^+ \\ &= A(\tau_{1,n_i-1}) + ((p_1 - w_1)\lambda_1 + (p_2 - w_2)\lambda_2 - \gamma)(t - \tau_{1,n_i-1}) \\ &= w_1 l_1 + w_2 l_2 + ((p_1 - w_1)\lambda_1 + (p_2 - w_2)\lambda_2 - \gamma)t. \end{aligned}$$

where the second equality is due to  $A(\tau_{1,n_i-1}) > 0$  by induction hypothesis, zero stock-out of these two products for  $t \in (\tau_{1,n_i-1}, \tau_{1,n_i}]$  and  $(p_1 - w_1)\lambda_1 + (p_2 - w_2)\lambda_2 \geq \gamma$  and the last equality is due to the induction hypothesis.

If there is one Manufacturer 2's visit until  $\tau_{1,n_1}$ , by induction hypothesis, both products never stock out since the Manufacturer 2's previous visit so that the nanostore has abundant cash to replenish a whole shelf of product 2. The same applies if there are multiple visits from Manufacturer 2 until  $\tau_{1,n_1}$ . Again, since zero stock-out for both products, (ii) holds and, at  $\tau_{1,n_1}^-$ , we have  $A(\tau_{1,n_1}^-) = w_1 l_1 + w_2 l_2 + ((p_1 - w_1)\lambda_1 + (p_2 - w_2)\lambda_2 - \gamma)\tau_{1,n_1}^-$ . Since  $(p_1 - w_1)\lambda_1 + (p_2 - w_2)\lambda_2 - \gamma \geq 0$  and there are at most  $l_2$  units of product 2 on the shelf, there is enough cash to buy a whole shelf of product 1, i.e., (i) holds.

Given the results above, the nanostore orders exactly  $l_i$  units from Manufacturer  $i$  at each visit, so that  $\pi_i(\tau_{i,n}) = (w_i - c_i)l_i - K_i$  for  $n_i = 1, 2, \dots$  and  $i = 1, 2$ . Therefore,  $\frac{\sum_{n_i=1}^{N_i} \pi_i(\tau_{i,n_i})}{\tau_{i,N_i}} = \frac{N_i[(w_i - c_i)l_i - K_i]}{N_i \frac{l_i}{\lambda_i}} = (w_i - c_i)\lambda_i - K_i \frac{\lambda_i}{l_i}$  for all  $N_i$ , therefore,  $\bar{\pi}_i = (w_i - c_i)\lambda_i - K_i \frac{\lambda_i}{l_i}$ .

Let  $\bar{\pi}_{i,n_i} = \frac{(I_i(\tau_{i,n_i-1}^+) - I_i(\tau_{i,n_i}^-))(w_i - c_i) - K_i}{\tau_{i,n_i} - \tau_{i,n_i-1}}$  be the time average profit rate in  $[\tau_{i,n_i-1}, \tau_{i,n_i}]$ . We show that  $\bar{\pi}_i < (w_i - c_i)\lambda_i - K_i \frac{\lambda_i}{l_i}$  if  $\tau_{i,n_i} - \tau_{i,n_i-1} \neq \frac{I_i(\tau_{i,n_i-1}^+)}{\lambda_i}$ .

If  $\tau_{i,n_i} - \tau_{i,n_i-1} < \frac{I_i(\tau_{i,n_i-1}^+)}{\lambda_i}$ , we have:

$$\begin{aligned} \bar{\pi}_{i,n_i} &= \frac{(I_i(\tau_{i,n_i-1}^+) - I_i(\tau_{i,n_i}^-))(w_i - c_i) - K_i}{\tau_{i,n_i} - \tau_{i,n_i-1}} \\ &= \frac{(\tau_{i,n_i} - \tau_{i,n_i-1})(w_i - c_i)\lambda_i - K_i}{\tau_{i,n_i} - \tau_{i,n_i-1}} \\ &= (w_i - c_i)\lambda_i - K_i \frac{1}{\tau_{i,n_i} - \tau_{i,n_i-1}} \\ &< (w_i - c_i)\lambda_i - K_i \frac{\lambda_i}{l_i} \end{aligned}$$

where the last inequality is due to  $\tau_{i,n_i} - \tau_{i,n_i-1} < \frac{I_i(\tau_{i,n_i-1}^+)}{\lambda_i} \leq \frac{l_i}{\lambda_i}$ . Otherwise, if  $\tau_{i,n_i} - \tau_{i,n_i-1} >$

$\frac{I_i(\tau_{i,n_i-1}^+)}{\lambda_i}$ , we have:

$$\begin{aligned}
\bar{\pi}_{i,n_i} &= \frac{\left(I_i(\tau_{i,n_i-1}^+) - I_i(\tau_{i,n_i}^-)\right) (w_i - c_i) - K_i}{\tau_{i,n_i} - \tau_{i,n_i-1}} \\
&= \frac{(w_i - c_i)I_i(\tau_{i,n_i-1}^+) - K_i}{\tau_{i,n_i} - \tau_{i,n_i-1}} \\
&\leq \left[ (w_i - c_i) - \frac{K_i}{I_i(\tau_{i,n_i-1}^+)} \right]^+ \frac{I_i(\tau_{i,n_i-1}^+)}{\tau_{i,n_i} - \tau_{i,n_i-1}} \\
&< \left[ (w_i - c_i) - \frac{K_i}{l_i} \right] \lambda_i \\
&= (w_i - c_i)\lambda_i - K_i \frac{\lambda_i}{l_i}
\end{aligned}$$

where the second inequality is because  $\frac{I_i(\tau_{i,n_i-1}^+)}{\tau_{i,n_i} - \tau_{i,n_i-1}} < \lambda_i$ ,  $w_i \geq c_i + \frac{K_i}{l_i} + \frac{\beta_i}{\lambda_i} > c_i + \frac{K_i}{l_i}$  and  $I_i(\tau_{i,n_i-1}^+) \leq l_i$ .

Finally the long-run average profit rate is

$$\begin{aligned}
\bar{\pi}_i &= \lim_{N_i \rightarrow \infty} \frac{1}{\tau_{i,N_i}} \sum_{n_i=2}^{N_i} \bar{\pi}_{i,n_i} (\tau_{i,n_i} - \tau_{i,n_i-1}) \\
&\leq \left( (w_i - c_i)\lambda_i - K_i \frac{\lambda_i}{l_i} \right) \lim_{N_i \rightarrow \infty} \frac{\sum_{n_i=2}^{N_i} (\tau_{i,n_i} - \tau_{i,n_i-1})}{\tau_{i,N_i}} \\
&= (w_i - c_i)\lambda_i - K_i \frac{\lambda_i}{l_i}.
\end{aligned}$$

Since  $\bar{\pi}_i = (w_i - c_i)\lambda_i - K_i \frac{\lambda_i}{l_i}$  can only be realized if  $\tau_{i,n_i} = \frac{(n_i-1)l_i}{\lambda_i}$  for  $n_i = 2, \dots$ , it is optimal for both manufacturer  $i$  to visit the nanostore exactly when his own product runs out.  $\square$

*Proof.* Proof of theorem 2

Both manufacturers' long-run profit rates are non-negative and strictly increasing in their own wholesale price as long as  $(p_1 - w_1)\lambda_1 + (p_2 - w_2)\lambda_2 \geq \gamma$  and  $c_i + \frac{K_i}{l_i} + \frac{\beta_i}{\lambda_i} \leq w_i \leq p_i$  for  $i = 1, 2$ . Therefore, given  $(p_1 - c_1 - \frac{K_1}{l_1})\lambda_1 - \beta_1 + (p_2 - c_2 - \frac{K_2}{l_2})\lambda_2 - \beta_2 \geq \gamma$  and  $c_i + \frac{K_i}{l_i} + \frac{\beta_i}{\lambda_i} \leq p_i$ , i.e.,  $M_1(l_1)\lambda_1 - \beta_1 + M_2(l_2)\lambda_2 - \beta_2 \geq \gamma$  and  $l_1 < l_1 < L - l_2$ , vector  $(w_1^*, w_2^*)$  such that  $(p_1 - w_1^*)\lambda_1 + (p_2 - w_2^*)\lambda_2 = \gamma$  are the Nash equilibrium wholesale prices since unilaterally raising wholesale price results in zero profit rates for both manufacturers, and unilateral lowering wholesale price reduces her own profit rate. Under the equilibrium, the Nash long-run profit rate is  $\bar{\pi}_i^* = (w_i^* - c_i)\lambda_i - K_i \frac{\lambda_i}{l_i}$  and the sum is  $\bar{\pi}_1^* + \bar{\pi}_2^* = M_1(l_1)\lambda_1 + M_2(l_2)\lambda_2 - \gamma$ . Further, theorem 1 implies the nanostore's total liquid assets are constant over

time and equal to  $A(t) = w_1^*l_1 + w_2^*l_2$ .

If  $M_1(l_1)\lambda_1 - \beta_1 + M_2(l_2)\lambda_2 - \beta_2 < \gamma$ , according to Lemma 1, there does not exist an equilibrium where both manufacturers sell to the nanostore.  $\square$

*Proof.* Proof for Theorem 3

Let  $B(l_1) = M_1(l_1)\lambda_1 + M_2(L - l_1)\lambda_2 - \beta_1 - \beta_2 - \gamma$  represent the balance between the supply profit rate and the reservation profit rates plus the nanostore's subsistence spending rate. It's first derivative in  $l_1$  is given by:

$$\frac{\partial B(l_1)}{\partial l_1} = \frac{K_1\lambda_1}{l_1^2} - \frac{K_2\lambda_2}{(L - l_1)^2}$$

where the first term is decreasing while the second term is increasing in  $l_1$ . This implies  $B(l_1)$  is either monotonic or increasing then decreasing in  $l_1 \in (l_1, L - l_2)$ . Next, we will center the discussion on the cases in the theorem.

Case 1 with  $L \geq \max\{l_1^S + l_2, l_2^S + l_1\}$  occurs when  $B(l_1) \geq 0$  and  $B(L - l_2) \geq 0$ . This implies  $B(l_1)$  is positive for any  $l_1 \in (l_1, L - l_2)$ , which means the two manufacturer always enter into business with the nanostore collectively.

Case 2 occurs when one of  $B(l_1)$  and  $B(L - l_2)$  is nonnegative while the other is negative. If  $l_2^S + l_1 \leq L < l_1^S + l_2$ ,  $B(l_1)$  is nonnegative and  $B(L - l_2)$  is negative; so  $B(l_1)$  is nonnegative for  $l_1$  under and equal to the bigger solution to  $M_1(l_1)\lambda_1 + M_2(l_2)\lambda_2 - \beta_1 - \beta_2 = \gamma$  (less than  $L - l_2$ ) such that the two manufacturers enter into business with the nanostore and quit the market for  $l_1$  beyond the threshold. If  $l_1^S + l_2 \leq L < l_2^S + l_1$ , the proof is symmetric.

Case 3 with  $L < \min\{l_1^S + l_2, l_2^S + l_1\}$  happens when  $B(l_1) < 0$  and  $B(L - l_2) < 0$ .  $B(l_1)$  is nonnegative for some value(s) of  $l_1$  only if the solution to the first order differential equation falls in  $(l_1, L - l_2)$ , i.e.,  $L > \tilde{L}$ , and if the peak of the function is nonnegative, i.e.,  $L \geq \frac{(\sqrt{K_1\lambda_1} + \sqrt{K_2\lambda_2})^2}{\sum_{i=1}^2 [(p_i - c_i)\lambda_i - \beta_i] - \gamma}$ . If  $L \geq \frac{(\sqrt{K_1\lambda_1} + \sqrt{K_2\lambda_2})^2}{\sum_{i=1}^2 [(p_i - c_i)\lambda_i - \beta_i] - \gamma} > \max\{\underline{L}, \tilde{L}\}$  where  $L > \underline{L}$  holds such that  $l_1 < L - l_2$ , the two manufacturers enter into business only if  $l_1$  is in between two solutions to  $M_1(l_1)\lambda_1 + M_2(l_2)\lambda_2 - \beta_1 - \beta_2 = \gamma$ , which become equal to  $l_1 = \frac{\sqrt{K_1\lambda_1}(\sqrt{K_1\lambda_1} + \sqrt{K_2\lambda_2})}{\sum_{i=1}^2 [(p_i - c_i)\lambda_i - \beta_i] - \gamma}$  for  $L = \frac{(\sqrt{K_1\lambda_1} + \sqrt{K_2\lambda_2})^2}{\sum_{i=1}^2 [(p_i - c_i)\lambda_i - \beta_i] - \gamma}$ . If  $L > \max\{\underline{L}, \tilde{L}\} \geq \frac{(\sqrt{K_1\lambda_1} + \sqrt{K_2\lambda_2})^2}{\sum_{i=1}^2 [(p_i - c_i)\lambda_i - \beta_i] - \gamma}$ , they enter into business with the nanostore only if  $l_1$  is in between two distinct solutions. Otherwise, the two manufacturers do not enter into business with the nanostore.  $\square$

*Proof.* Proof of Lemma 2

Solving equations  $\frac{\bar{\pi}_i^* - \beta_i}{\bar{\pi}_1^* - \beta_1 + \bar{\pi}_2^* - \beta_2} = \phi_i$  for  $i = 1, 2$  yields the following solution:

$$w_i^f = p_i - \frac{\gamma}{\lambda_i} \frac{M_i(l_i)\lambda_i - \beta_i}{M_1(l_1)\lambda_1 - \beta_1 + M_2(l_2)\lambda_2 - \beta_2}$$

which is greater than  $p_i - \frac{\gamma}{\lambda_i}$  due to  $M_1(l_1)\lambda_1 - \beta_1 + M_2(l_2)\lambda_2 - \beta_2 \geq \gamma$  and  $M_i(l_i)\lambda_i - \beta_i \geq 0$  for  $i = 1, 2$ . It is also greater than or equal to  $c_i + \frac{K_i}{l_i} + \frac{\beta_i}{\lambda_i}$  because

$$w_i^f - \left( c_i + \frac{K_i}{l_i} + \frac{\beta_i}{\lambda_i} \right) = \frac{M_i(l_i)\lambda_i - \beta_i}{\lambda_i} \frac{M_1(l_1)\lambda_1 - \beta_1 + M_2(l_2)\lambda_2 - \beta_2 - \gamma}{M_1(l_1)\lambda_1 - \beta_1 + M_2(l_2)\lambda_2 - \beta_2} \geq 0$$

where the inequality is because  $M_1(l_1)\lambda_1 - \beta_1 + M_2(l_2)\lambda_2 - \beta_2 \geq \gamma$  and  $M_i\lambda_i(l_i) - \beta_i \geq 0$  for  $i = 1, 2$ . Therefore, the fair equilibrium is the unique Nash equilibrium or is always one of the Nash equilibria as stated in Theorem 2. Finally, if and only if  $M_i(l_i)\lambda_i - \beta_i \geq M_{3-i}(l_{3-i})\lambda_{3-i} - \beta_{3-i}$  for  $i = 1, 2$ , we have  $\rho_i(w_1^f, w_2^f) = \phi_i(w_1^f, w_2^f) = \frac{(p_i - w_i^f)\lambda_i}{\gamma} \geq \frac{1}{2}$ .  $\square$

*Proof.* Proof of theorem 4

In the Stackelberg game, both manufacturers' long-run profit rates are non-negative and strictly increasing in their own wholesale price as long as  $(p_1 - w_1)\lambda_1 + (p_2 - w_2)\lambda_2 \geq \gamma$  and  $c_i + \frac{K_i}{l_i} + \frac{\beta_i}{\lambda_i} \leq w_i \leq p_i$  for  $i = 1, 2$ . Given the wholesale price  $w_1$  of Manufacturer 1, i.e., the leader, the best response of Manufacturer 2, i.e., the follower, is such that  $(p_1 - w_1)\lambda_1 + (p_2 - w_2)\lambda_2 = \gamma$ . Therefore, if  $l_2 \geq l_2^S$ , i.e., Manufacturer 2 can provide for the nanostore's subsistence individually, Manufacturer 1 charges at the highest retail price  $w_1^* = p_1$  and Manufacturer 2 charges at the corresponding wholesale price  $w_2^* = p_2 - \frac{\gamma}{\lambda_2}$ . If  $l_2 < l_2^S$ , i.e., Manufacturer 2 cannot provide for the nanostore's subsistence individually, Manufacturer 1 always charges the highest wholesale price  $w_1^* = p_1 + \frac{1}{\lambda_1} [M_2(l_2)\lambda_2 - \beta_2 - \gamma]$  where Manufacturer 2 charges the lowest wholesale price  $w_2^* = c_2 + \frac{K_2}{l_2} + \frac{\beta_2}{\lambda_2}$ .  $\square$

*Proof.* Proof of Theorem 5

Let the manufacturer as the only supplier to the nanostore as Manufacturer  $i$  in the two-manufacturer model. In the following proof, we drop the manufacturer index.

We first show that the optimal wholesale price is such that  $w \in \left[ c + \frac{K}{L} + \frac{\beta}{\lambda}, p - \frac{\gamma}{\lambda} \right]$ . If the manufacturer prices the product at a value lower than  $c + \frac{K}{L} + \frac{\beta}{\lambda}$ , she earns less than  $\beta$  per unit

of time in  $[t_{n-1}, t_n]$ , i.e.,

$$\begin{aligned}\bar{\pi}_n &< \frac{(I(t_{n-1}^+) - I(t_n^-)) \left( \frac{K}{L} + \frac{\beta}{\lambda} \right) - K}{t_n - t_{n-1}} \\ &= \frac{I(t_{n-1}^+) - I(t_n^-)}{(t_n - t_{n-1})\lambda} \beta + \frac{(I(t_{n-1}^+) - I(t_n^-)) - L}{(t_n - t_{n-1})L} K \leq \beta\end{aligned}$$

where the first inequality is because  $w < c + \frac{K}{L} + \frac{\beta}{\lambda}$  and the second inequality is because  $\frac{I(t_{n-1}^+) - I(t_n^-)}{t_n - t_{n-1}} \leq \lambda$  and  $I(t_{n-1}^+) - I(t_n^-) \leq L$ . Then, the manufacturer's long-run average profit rate is upper bounded by  $\beta$ , i.e.,

$$\bar{\pi} = \lim_{N \rightarrow \infty} \frac{1}{t_N} \sum_{n=2}^N (t_n - t_{n-1}) \bar{\pi}_n < \beta \lim_{N \rightarrow \infty} \frac{1}{t_N} \sum_{n=2}^N (t_n - t_{n-1}) = \beta.$$

Therefore, the lowest wholesale price the manufacturer can charge is  $c + \frac{K}{L} + \frac{\beta}{\lambda}$  below which she profits less than the outside option.

We follow to show by induction that the nanostore's total liquid assets  $A(t)$  are decreasing in  $t$  until it reaches zero if  $w > p - \frac{\gamma}{\lambda}$ . At  $t = 0$ , we have  $A(0) = wL$  and for  $t \leq t_2$ , we have:

$$A(t) = \begin{cases} A(0) + [\lambda(p - w) - \gamma]t & \text{if } t \leq t_2 \leq \frac{L}{\lambda} \\ A(0) + ((p - w)\lambda - \gamma) \frac{L}{\lambda} - \gamma \left( t - \frac{L}{\lambda} \right) & \text{if } \frac{L}{\lambda} < t \leq t_2 < \frac{pL}{\gamma} \\ 0 & \text{if } \frac{pL}{\gamma} \leq t \leq t_2 \end{cases}$$

which, provided  $w > p - \frac{\gamma}{\lambda}$ , is decreasing in  $t \leq t_2$  if  $t_2 \leq \frac{pL}{\gamma}$  and is decreasing in  $t$  until it reaches zero before  $t_2 > \frac{pL}{\gamma}$ . Now let us assume (induction hypothesis) that the result is true for  $t \leq t_{n-1}$  and we prove it holds for  $t \in (t_{n-1}, t_n]$ . For  $t \in (t_{n-1}, t_n]$ , we have:

$$A(t) = \begin{cases} A(t_{n-1}) + [\lambda(p - w) - \gamma](t - t_{n-1}) & \text{if } t_{n-1} < t \leq t_n \leq t_{n-1} + \frac{I(t_{n-1}^+)}{\lambda} \\ A(t_{n-1}) + (p - w)I(t_{n-1}^+) - \gamma(t - t_{n-1}) & \text{if } t_{n-1} + \frac{I(t_{n-1}^+)}{\lambda} < t \leq t_n < t_{n-1} + \frac{pI(t_{n-1}^+) + C(t_{n-1}^+)}{\gamma} \\ 0 & \text{if } t_{n-1} + \frac{pI(t_{n-1}^+) + C(t_{n-1}^+)}{\gamma} \leq t \leq t_n \end{cases}$$

which, provided  $w > p - \frac{\gamma}{\lambda}$ , is decreasing in  $t \leq t_n$  if  $t_n \leq \frac{pI(t_{n-1}^+) + C(t_{n-1}^+)}{\gamma}$  and is decreasing in  $t$  until it reaches zero before  $t_n$  otherwise. Therefore, if  $w > p - \frac{\gamma}{\lambda}$ , the nanostore eventually closes his business where the manufacturer can only get zero long-run profit rate.

It follows that a necessary condition for the nanostore to enter into business to sell the manufacturer's product is  $c + \frac{K}{L} + \frac{\beta}{\lambda} \leq p - \frac{\gamma}{\lambda}$ , which is equivalent to  $L \geq \frac{K}{\left( p - c - \frac{\beta + \gamma}{\lambda} \right)^+}$ .

Next, we show that, under the condition  $L \geq \frac{K}{\left( p - c - \frac{\beta + \gamma}{\lambda} \right)^+}$ , for  $w \in \left[ c + \frac{K}{L} + \frac{\beta}{\lambda}, p - \frac{\gamma}{\lambda} \right]$ ,

it is optimal for the manufacturer's sales representative to visit the nanostore exactly when inventory runs out, which implies that  $t_n = \frac{(n-1)L}{\lambda}$  for  $n = 2, 3, \dots$ . Under this policy, the manufacturer's long-run profit rate is equal to  $\bar{\pi} = (w - c - \frac{K}{L}) \lambda$  and the nanostore's total liquid assets at time  $t$  are given by  $A(t) = wL + (p - \frac{\gamma}{\lambda} - w)t\lambda$ .

Let the manufacturer visit the nanostore exactly when the stock runs out, i.e.,  $t_n - t_{n-1} = \frac{L}{\lambda}$  for  $n = 2, 3, \dots$ . First, we show by induction that (i)  $I(t_n^+) = L$  and  $C(t_n^-) \geq wL$  for  $n = 1, 2, \dots$  and (ii)  $A(t) = wL + (p - \frac{\gamma}{\lambda} - w)t\lambda$  for all  $t$ . (i) is true for  $n = 1$  since  $I(t_1^+) = I(0^+) = L$  and  $C(t_1^-) = C(0^-) = wL$  by assumption and (ii) is true for  $t \leq t_2 = \frac{L}{\lambda}$  since  $A(t) = wL + (p - \frac{\gamma}{\lambda} - w)t\lambda$  due to  $w \leq p - \frac{\gamma}{\lambda}$ .

Now, we assume (i) is true for  $n - 1$  and show it is true for  $n$ ; we assume (ii) is true for  $t \leq t_{n-1}$  and show it is true for  $t \in (t_{n-1}, t_n]$ . For (i), using the induction hypothesis, we obtain  $C(t_n^-) = C(t_{n-1}^+) + p \min\{\lambda(t_n - t_{n-1}), L\} - \gamma(t_n - t_{n-1}) \geq (p - \frac{\gamma}{\lambda})L \geq wL$  since  $t_n - t_{n-1} = \frac{L}{\lambda}$  and  $w \leq p - \frac{\gamma}{\lambda}$ . Further,  $I(t_n^+) = \min\{I(t_n^-) + \frac{C(t_n^-)}{w}, L\} = \min\{\frac{C(t_n^-)}{w}, L\} = L$ . As (ii), for  $t \in (t_{n-1}, t_n]$ , we have:

$$\begin{aligned} A(t) &= [A(t_{n-1}) + [\lambda(p - w) - \gamma](t - t_{n-1})]^+ \\ &= A(t_{n-1}) + [\lambda(p - w) - \gamma](t - t_{n-1}) \\ &= wL + (p - \frac{\gamma}{\lambda} - w)t_{n-1}\lambda + [\lambda(p - w) - \gamma](t - t_{n-1}) \\ &= wL + (p - \frac{\gamma}{\lambda} - w)t\lambda \end{aligned}$$

where the the second equality is because  $w \leq p - \frac{\gamma}{\lambda}$  and the third equality is due to induction hypothesis.

Given this, the nanostore orders exactly  $L$  units from the manufacturer at each visit, so that  $\pi(t_n) = (w - c)L - K$  for all  $n = 1, 2, \dots$ . Therefore,  $\frac{\sum_{n=1}^N \pi(t_n)}{t_N} = \frac{N[(w-c)L - K]}{N\frac{L}{\lambda}} = (w - c)\lambda - K\frac{\lambda}{L}$  for all  $N$ , therefore,  $\bar{\pi} = (w - c)\lambda - K\frac{\lambda}{L}$ .

Let  $\bar{\pi}_n = \frac{(I(t_{n-1}^+) - I(t_n^-))(w - c) - K}{t_n - t_{n-1}}$  be the time average profit rate in  $[t_{n-1}, t_n]$ . Following the proof to Theorem 1, we show that  $\bar{\pi}_n < (w - c)\lambda - K\frac{\lambda}{L}$  if  $t_n - t_{n-1} \neq \frac{L}{\lambda}$ . Since  $\bar{\pi} = (w - c)\lambda - K\frac{\lambda}{L}$  can only be realized if  $t_n - t_{n-1} = \frac{L}{\lambda}$ , we have proven that it is optimal for the manufacturer to visit the nanostore exactly when the product stocks out.

Following the previous arguments, if  $L \geq \frac{K}{(p - c - \frac{\beta + \gamma}{\lambda})^+}$ , the manufacturer's problem simpli-

fies to:

$$\max_{w \in [c + \frac{K}{L} + \frac{\beta}{\lambda}, p - \frac{\gamma}{\lambda}]} \bar{\pi}(w) = \left( w - c - \frac{K}{L} \right) \lambda.$$

Since  $\bar{\pi} = (w - c - \frac{K}{L}) \lambda$  is strictly increasing in the wholesale price  $w \in [c + \frac{K}{L} + \frac{\beta}{\lambda}, p - \frac{\gamma}{\lambda}]$ , the value of  $w$  which maximizes  $\bar{\pi}$  is  $p - \frac{\gamma}{\lambda}$ . Therefore, if  $L \geq \frac{K}{(p - c - \frac{\beta + \gamma}{\lambda})^+}$ , the optimal profit rate is  $\bar{\pi}^* = (p - c - \frac{K}{L}) \lambda - \gamma \geq \beta$  and the manufacturer enters into business with the nanostore. Further, according to previous arguments, the nanostore's total liquid assets are constant over time and equal to  $A(t) = (p - \frac{\gamma}{\lambda}) L$ . If  $L < \frac{K}{(p - c - \frac{\beta + \gamma}{\lambda})^+}$ , the manufacturer does not enter into business with the nanostore.  $\square$

*Proof.* Proof for Theorem 6

According to the one-manufacturer model, Manufacturer  $i = 1, 2$  enters into business with the nanostore as long as  $L \geq l_i^S$  where  $l_i^S > L_i$ . Referring to Theorem 3, the results follow directly.  $\square$

*Proof.* Proof for Theorem 7

For Manufacturer  $i = 1, 2$ , the profit of selling individually to the nanostore is  $(p_i - c_i - \frac{K_i}{L}) \lambda_i - \lambda$ , and the profit of selling collectively with the other manufacturer is  $(w_i^* - c_i - \frac{K_i}{L_i}) \lambda_i$ . To earn more profit from selling collectively, it is easy to show Manufacturer  $i$  needs to charge  $w_i^* > p_i - \frac{\gamma}{\lambda_i} + K_i \left( \frac{1}{l_i} - \frac{1}{L} \right)$  where  $p_i - \frac{\gamma}{\lambda_i}$  is the price that Manufacturer  $i$  charges as the only one selling to the nanostore. If the pricing threshold is less than the retail price, i.e.,  $\gamma > K_i \lambda_i \left( \frac{1}{l_i} - \frac{1}{L} \right)$ , Manufacturer  $i$  gets better off when charging above the threshold  $p_i - \frac{\gamma}{\lambda_i} + K_i \left( \frac{1}{l_i} - \frac{1}{L} \right)$ ; otherwise, Manufacturer  $i$  is always worse off. Further, note that the pricing threshold  $p_i - \frac{\gamma}{\lambda_i} + K_i \left( \frac{1}{l_i} - \frac{1}{L} \right)$  is always greater than or equal to the lowest equilibrium wholesale prices  $p_i - \frac{\gamma}{\lambda_i}$  due to  $l_i < L$  and  $c_i + \frac{K_i}{l_i} + \frac{\beta_i}{\lambda_i}$  due to  $L \geq l_i^S$ . Next, we center the discussion on the specific cases in the theorem.

When  $\gamma > K_1 \lambda_1 \left( \frac{1}{l_1} - \frac{1}{L} \right) + K_2 \lambda_2 \left( \frac{1}{l_2} - \frac{1}{L} \right)$ , the supply chain rates under the wholesale pricing thresholds more than covers the nanostore's subsistence spending rate. Then, selling to the nanostore collectively, both manufacturers earn more profit if they both charge higher than their respective wholesale pricing threshold, i.e., if  $p_1 - \frac{\gamma}{\lambda_1} + K_1 \left( \frac{1}{l_1} - \frac{1}{L} \right) < w_1^* < p_1 - \frac{K_2 \lambda_2}{\lambda_1} \left( \frac{1}{l_2} - \frac{1}{L} \right)$ ; one manufacturer gains while the other loses if one charges less than his threshold, i.e., if  $w_1^* > p_1 - \frac{K_2 \lambda_2}{\lambda_1} \left( \frac{1}{l_2} - \frac{1}{L} \right)$  or if  $w_1^* < p_1 - \frac{\gamma}{\lambda_1} + K_1 \left( \frac{1}{l_1} - \frac{1}{L} \right)$ .



When  $\max \left\{ K_1 \lambda_1 \left( \frac{1}{l_1} - \frac{1}{L} \right), K_2 \lambda_2 \left( \frac{1}{l_2} - \frac{1}{L} \right) \right\} < \gamma \leq K_1 \lambda_1 \left( \frac{1}{l_1} - \frac{1}{L} \right) + K_2 \lambda_2 \left( \frac{1}{l_2} - \frac{1}{L} \right)$ , the two manufacturers cannot price higher than their own wholesale pricing threshold simultaneously because under which the supply chain rates are less than the nanostore's subsistence spending rate. Therefore, selling to the nanostore collectively, the two manufacturers are never better off simultaneously. In particular, if  $w_1^* > p_1 - \frac{\gamma}{\lambda_1} + K_1 \left( \frac{1}{l_1} - \frac{1}{L} \right)$ , Manufacturer 1 (2) is better (worse) off; if  $w_1^* < p_1 - \frac{K_2 \lambda_2}{\lambda_1} \left( \frac{1}{l_2} - \frac{1}{L} \right)$ , Manufacturer 1 (2) is worse (better) off; if  $p_1 - \frac{K_2 \lambda_2}{\lambda_1} \left( \frac{1}{l_2} - \frac{1}{L} \right) < w_1^* < p_1 - \frac{\gamma}{\lambda_1} + K_1 \left( \frac{1}{l_1} - \frac{1}{L} \right)$ , both manufacturers are worse off.

When  $K_i \lambda_i \left( \frac{1}{l_i} - \frac{1}{L} \right) < \gamma \leq K_{3-i} \lambda_{3-i} \left( \frac{1}{l_{3-i}} - \frac{1}{L} \right)$  for  $i = 1, 2$ , only Manufacturer  $i$  can price higher than his wholesale pricing threshold to earn more profit in the two-manufacturer setting. In particular, if  $w_1^* > (<) p_1 - \frac{\gamma}{\lambda_1} + K_1 \left( \frac{1}{l_1} - \frac{1}{L} \right)$ , Manufacturer  $i$  is better (worse) off, while the other manufacturer is always worse off.

Finally, when  $\gamma \leq \min \left\{ K_1 \lambda_1 \left( \frac{1}{l_1} - \frac{1}{L} \right), K_2 \lambda_2 \left( \frac{1}{l_2} - \frac{1}{L} \right) \right\}$ , obviously, both manufacturers are worse off selling to the nanostore collectively.  $\square$