Broadening the attenuation range of acoustic metafoams through graded microstructures

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Low frequency sound attenuation is a challenging task, because of the severe mass, stiffness and volume constraints on the absorbing and/or reflecting barriers. Recently, significant improvements in low frequency sound attenuation has been achieved by introducing the acoustic metafoam concept, which combines the mechanism of conventional acoustic foams - high viscothermal dissipation - with the working principle of locally resonant acoustic metamaterials - wave attenuation at low frequencies. However, the attenuation improvement provided by periodic materials containing identical resonators is confined to a narrow frequency range. To overcome this limitation, graded acoustic metafoams are proposed and studied here, where a distribution of local resonators with varying properties (mass and stiffness) is introduced. It is demonstrated that, through a suitable design of mass and stiffness distribution of the resonators, the broadening of the frequency attenuation ranges can be effectively achieved. Graded acoustic metafoams are, therefore, a natural development direction for achieving broad frequency attenuation zones.

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1. Introduction

Low frequency noise is a significant problem, especially in urban environments that are saturated with sources of unwanted sounds [1]. Acoustic waves of long wavelengths propagate easily in space, which makes their attenuation challenging [2]. The available material solutions for low-frequency sound insulation and absorption are typically demanding in terms of space [3,4]. A potential solution to this noise problem could be the use of acoustic metamaterials, i.e. man-made materials targeting this functionality with properties beyond natural ones [5]. However, most of the metamaterials are still not suitable for mass production [6].

Along this line, the recently proposed acoustic metafoams [7], which consist of a poro-elastic material endowed with local resonators, might be a promising approach, since this foam based microstructure can be potentially manufactured via well-controlled foaming processes. Such a material can show advantages both for manufacturability and weight reduction, which are important bottlenecks for the usage of LRAMs in real applications. As demonstrated in Lewińska et al. [7], acoustic metafoams combine the benefits of standard acoustic foams and locally resonant acoustic metamaterials (LRAMs), thus exhibiting improved attenuation at low frequencies while preserving the visco-thermal dissipation effects typical of standard foams. Moreover, as shown therein, the fluid-solid coupling associated with visco-thermal fluid is indispensable for observing the attenuation effect. However, this improved attenuation performance at low frequencies is primarily related to the local resonance mechanism,
which is confined to a narrow frequency range. In order to overcome this limitation, in this paper, functionally graded acoustic metafoams are introduced and explored for the purpose of broadening the attenuation zone.

Functionally graded microstructures, with a spatial variation in the composition and/or microstructure, have been proposed in the literature as an advanced alternative to uniform or random microstructures. It has been demonstrated that such microstructural arrangements often result in improved material performance for multiple applications, such as in Zhang et al. [8], where a superior load-bearing capacity of elastically graded ceramics has been shown, or in Li et al. [9], where a higher strength has been reported for a gradient cellular structure compared to its uniform metallic equivalent. In some cases, using an appropriate graded design can also trigger exotic material behaviour. For instance, in wave propagation problems, asymmetric transmission has been achieved in Chen et al. [10] using a graded grating of phononic crystal slabs, while in Deng et al. [11] a graded phononic crystal microstructure enabled wave focusing.

Recently, functionally graded microstructures have been considered as a potential direction for improving acoustic/elastic wave insulation performance of materials. For example, for the class of porous materials that are widely used for sound absorption purposes, it has been demonstrated that adopting adequate material gradients can result in a broadening of the absorption regions and an increase of the absorption levels. Such an effect has been observed by Doutres and Atalla [12], who numerically analysed polyurethane foams with graded reticulation rates. They reported a considerable increase of the absorption values at high frequencies along with a smoothing of the absorption spectra due to the impedance mismatch reduction. A poly lactide open cell foam and an Al alloy open cell foam with a gradual pore size variation have been investigated, respectively, in Mosanenzadeh et al. [13] and Ke et al. [14] by means of numerical and experimental analyses. In these studies, the graded microstructures outperformed samples with uniform cell sizes as long as the large cells were facing the incoming wave.

Furthermore, in the field of locally resonant acoustic metamaterials (LRAMs), which are a promising solution for low frequency sound attenuation problems [5], it has been demonstrated that the use of functionally graded microstructures can address the main application limit of these materials, i.e. the narrowness of the band gaps. In Banerjee et al. [15], a microstructure with a graded arrangement of resonating units has been used in order to extend the upper border of the attenuation bandwidth. By means of a 1D theoretical analysis, it has been shown that gradually varying the resonator stiffness is, in general, more effective compared to the variation of the resonator mass. An extended attenuation bandwidth has also been reached by Kröde et al. [16], who proposed graded microstructures for seismic applications and investigated their performance experimentally, exploiting the so-called rainbow trapping effect. Finally, in Jiménez et al. [17] a rainbow metamaterial has been proposed consisting of graded Helmholtz resonators. The excellent absorption behaviour has been demonstrated for this metastructure in the low frequency range.

The impact of microstructural grading on the performance of acoustic metafoams has not been explored yet in the literature, which is the focus of this paper. It is shown that by using graded microstructures, it is possible to broaden the attenuation zone while preserving the unique attenuation features of the acoustic metafoams. The analysis is performed by means of computational homogenisation, which enables the identification of macroscopic effective parameters that control the acoustic response of the material. The dependence of these parameters on the stiffness and mass variation of the resonators is computed and used to design functionally graded metafoam models. The broadening of the attenuation regime occurring in graded microstructures is rationalized by means of simpler computational models containing two masses with distinct resonance frequencies. The simplified two-mass analysis enables the identification of an improved spatial grading to be used in order to achieve attenuation range broadening. Additionally, for non-smooth mass gradients, bridging of attenuation zones is achieved by means of material damping in the solid skeleton. The attenuation band in the graded microstructures approximates to the envelope of the attenuation bands of the single mass/stiffness resonators, with a non-trivial interplay between wave energy absorption and reflection. This computational strategy proves to be a versatile tool for the design of new graded metafoams with remarkably broad attenuation ranges. Finally, the performance of two graded microstructures with continuously varying mass and stiffness of the resonators is demonstrated.

The paper is organised as follows. In Section 2, the computational homogenisation scheme for acoustic foams [18,19], is briefly recapitulated. In Section 3, the configurations analysed in this work are presented. The results obtained for several graded microstructural configurations at the micro- and the macro-scales are shown in Section 4. The paper ends with concluding remarks.

2. Computational homogenisation

This section summarises the main aspects of the multi-scale computational homogenisation framework for porous acoustic materials initially proposed by Gao et al. [18,20], and recently applied to the case of acoustic metafoams in Lewińska et al. [19]. In this framework, two coupled problems are considered at two scales. At the micro-scale, a multi-phase representative volume element (RVE) is modelled, including both solid (polymer or metal) and fluid (air) domains. At the macro-scale, the heterogeneous poro-elastic material is homogenised towards an equivalent homogeneous material. The framework assumes complete scale separation, implying that the characteristic length of the external excitation (i.e. the wavelength) is much larger than the microscopic characteristic size. At both macro- and micro-scales, the study is conducted in the frequency domain by adopting the “$+\, i\omega$” convention with $i$ denoting the imaginary unit and $\omega$ being the angular frequency.
2.1. Micro-scale

The RVE of the poro-elastic microstructure consists of fluid (denoted by the superscript “f”) and solid (denoted by superscript “s”) domains. In order to simplify the notation, no subscripts are used in the following for variables associated with the microscale. The solid is assumed linear elastic, obeying the classical momentum balance equation:

\[-\rho_0^s \omega^2 \mathbf{u}^s = \nabla \cdot \mathbf{\sigma}^s,\]  

(1)

where \( \mathbf{u}^s \) is the solid displacement vector, \( \rho_0^s \) the solid density and \( \mathbf{\sigma}^s \) is the stress tensor given by \( \mathbf{\sigma}^s = 4 \mathbf{C}^s \mathbf{:\nabla u}^s \), with \( \mathbf{C}^s \) the fourth-order elasticity tensor, which is assumed isotropic. When skeleton damping is also considered, the elasticity tensor becomes complex-valued: \( 4 \mathbf{C}^s = (1 + i \eta)^4 \mathbf{C} \), with \( \eta \) being an isotropic loss factor. The symbol \( \nabla \) denotes the gradient operator at the microscopic scale.

The governing equations in the fluid domain are the linearised Navier-Stokes-Fourier equations:

\[

\begin{align*}
\text{i}\omega \rho_0^f \mathbf{v}^f &= \nabla \cdot \left[ -p^f \mathbf{I} + \mu^f \left( \nabla \mathbf{v}^f + (\nabla \mathbf{v}^f)^T \right) - \frac{2}{3} \mu^f (\nabla \cdot \mathbf{v}^f) \mathbf{I} \right], \\
i\omega \rho_0^f c_i^f \theta^f &= i\omega p^f - \nabla \cdot \left( -\kappa^f \nabla \theta^f \right), \\
i\omega \frac{p^f}{\rho_0^f} &= i\omega \frac{\theta^f}{\theta_0} - \nabla \cdot \mathbf{v}^f.
\end{align*}
\]

(2) \hspace{1cm} (3) \hspace{1cm} (4)

These equations represent the balance of momentum, the energy balance and the mass conservation (considering the ideal gas law), respectively, and they already include the constitutive equations for the stress in the fluid \( \mathbf{\sigma}^f \) and the heat flux \( \mathbf{q}^f \) (Fourier’s law). In these equations, \( \mathbf{v}^f \) denotes the fluid velocity vector (note that \( \mathbf{v}^f = i\omega \mathbf{u}^f \)), \( p^f \) indicates the pressure, \( \theta^f \) the temperature change, \( \rho_0^f \) the equilibrium density, \( c_i^f \) the heat capacity at constant pressure, \( P_0 \) and \( \theta_0 \) are the ambient pressure and temperature, respectively, \( \mu^f \) the fluid viscosity, \( \kappa^f \) the thermal conductivity and \( \mathbf{I} \) the second order identity tensor.

The domain coupling at the fluid–solid interface \( S_i \) is achieved by enforcing continuity of the velocities and tractions:

\[

\text{i}\omega \mathbf{u}^s = \mathbf{v}^f \quad \text{at} \quad S_i,
\]

(5)

\[

\mathbf{\sigma}^s \cdot \mathbf{n}_i = \mathbf{\sigma}^f \cdot \mathbf{n}_i \quad \text{at} \quad S_i,
\]

(6)

with \( \mathbf{n}_i \) the normal vector pointing from the fluid to the solid domain. An isothermal boundary condition is imposed at the fluid–solid interface, due to the large difference between thermal conductivity of the solid and fluid phases:

\[

\theta^f = 0 \quad \text{at} \quad S_i.
\]

(7)

Periodic boundary conditions are applied for the solid displacements on the outer RVE boundaries (thus on the solid external RVE surface \( S_e^s \)):

\[

\mathbf{u}^i = \mathbf{u}_M^i + (\nabla_m \mathbf{u}_M^i)^T \cdot (\mathbf{x}^i - \mathbf{x}_0^i) + \mathbf{w}_M^i, \quad \mathbf{w}_M^i = \mathbf{w}_M^i^+ \quad \mathbf{w}_M^i \in S_e^s
\]

(8)

while on the fluid external surface \( S_e^f \) tractions are prescribed according to:

\[

\mathbf{n}_e \cdot \mathbf{\sigma}^i = -p^f \mathbf{n}_e^i \quad \text{with} \quad p^f = p_M^f + \mathbf{v}_M^f \cdot (\mathbf{x}^f - \mathbf{x}_0^f), \quad \mathbf{x}^f \in S_e^f
\]

(9)

where \( \mathbf{x}_0^i \) and \( \mathbf{x}_0^f \) are the position vectors in the micro-scale solid \( V^s \) and fluid \( V^f \) domains, respectively. \( \mathbf{x}_0^i \) is a reference position vector, \( \mathbf{n}_e^i \) is the outward normal vector on the fluid surface and \( \mathbf{w}_M^i \) is the displacement microfluctuation field in the solid (with + and − denoting opposite corresponding boundaries).

2.2. Macro-scale

The problem at the macroscale is defined in terms of the solid displacement \( \mathbf{u}_M^i \) and the fluid pressure \( p_M^f \). Sound propagation is considered as an adiabatic process, while the viscous stress of the macroscopic fluid is assumed to be negligible in comparison with the macroscopic pressure. The porosity \( \phi \), which is defined as the volume fraction of the fluid domain, is assumed to remain constant. Thus, the governing equations, i.e. momentum conservation for the solid and mass conservation for the fluid, are given by:

\[

\mathbf{f}_M^s = \nabla M \cdot \mathbf{\sigma}_M^s, \quad \mathbf{f}_M^f = \nabla M \cdot \mathbf{u}_M^f
\]

(10) \hspace{1cm} (11)
where $\sigma_M^s$ is the macroscopic stress tensor in the solid, $f_M$ is the inertial force exerted on the solid, $c_M^f$ is the macroscopic volumetric change of the fluid, $u_M^f$ is the fluid displacement vector and $\nabla_M$ is the spatial gradient at the macroscopic scale. The system of equations (10) and (11) is closed by the macroscopic constitutive equations and boundary conditions.

Following Gao et al. [18,20], the macroscopic constitutive equations take the following form:

$$ (1 - \phi)\sigma_M^s = 4D : \nabla_M u_M^s - \Psi_M^f p_M^f $$  \hspace{1cm} (12)  

$$ \phi c_M^f = -\Psi_M^f : \nabla_M u_M^s - S_f^f p_M^f $$ \hspace{1cm} (13)  

$$ (1 - \phi)\mathbf{f}_M^e = -\omega^2 \mathbf{M}^f \cdot \mathbf{u}_M^s + \mathbf{K}^f \cdot \nabla_M p_M^f $$ \hspace{1cm} (14)  

$$ \phi \mathbf{u}_M^f = (\mathbf{K}^f)^T \cdot \mathbf{u}_M^s + (\omega^2 \rho_0^M)^{-1} \mathbf{K}^f \cdot \nabla_M p_M^f $$ \hspace{1cm} (15)  

where the fourth order tensor $4D$ is the homogenised stiffness tensor, which is dominated by the stiffness of the solid skeleton, while the scalar $S_f^f$ is dominated by the compliance of the fluid. The symmetric second–order tensor $\Psi_M^f$ is the coupling parameter and $\mathbf{M}^f$, $\mathbf{K}^f$ and $\mathbf{K}^f$ are also second–order tensors associated with the effective solid density, the effective fluid permeability and the coupling term, respectively. The effective constitutive properties in equations (12)–(15) are obtained from the microscopic RVE through a number of microscopic simulations as detailed in Refs. [19].

2.3. Micro-to-macro relations

In computational homogenisation, the Hill–Mandel condition typically establishes the relationship between the micro- and macro-scales. This condition states that the energy variation (increment) per unit volume in a macroscopic point should equal the volume average of the microscopic energy variation (increment) of the associated unit cell:

$$ \delta E_M = \frac{1}{V} \int_V \delta E_m \, dV. $$ \hspace{1cm} (16)  

For the considered poro-elastic constituents, the macroscopic virtual energy rate can be written as:

$$ \delta E_M = (1 - \phi)\mathbf{f}_M^e \cdot \delta \mathbf{u}_M^s + (1 - \phi)\sigma_M^s \cdot \delta \mathbf{u}_M^s + \delta (\nabla_M \mathbf{u}_M^s) - \phi c_M^f \delta p_M^f - \phi \mathbf{u}_M^f \cdot \delta (\nabla_M p_M^f) $$ \hspace{1cm} (17)  

and the volume average of the microscopic virtual energy rate can be computed from the virtual power at the RVE boundary as:

$$ \int_V \delta E_m \, dV = \int_{S_e^s} (\mathbf{n}_e^s \cdot \sigma^s) \, dA \cdot \delta \mathbf{u}_M^s + \int_{S_e^s} (\mathbf{n}_e^s \cdot \sigma^s)(\mathbf{x}^s - \mathbf{x}^s) \, dA : \delta (\nabla_M \mathbf{u}_M^s) $$ \hspace{1cm} (18)  

$$ - \partial p_M^f \int_{S_e^f} \mathbf{n}_e^f \cdot \mathbf{u}_M^f \, dA - \delta (\nabla_M p_M^f) \cdot \int_{S_e^f} (\mathbf{x}^f - \mathbf{x}^f)(\mathbf{n}_e^f \cdot \mathbf{u}_M^f) \, dA, $$

where relations (8) and (9) have been used; $\mathbf{n}_e^s$ and $\mathbf{n}_e^f$ are the outward normal vectors on the solid and fluid surfaces, respectively. The energy consistency principle (16) then leads to the following micro-to-macro relations:

$$ (1 - \phi)\mathbf{f}_M^e = \frac{1}{V} \int_{S_e^s} \mathbf{n}_e^s \cdot \sigma^s \, dA, $$ \hspace{1cm} (19)  

$$ (1 - \phi)\sigma_M^s = \frac{1}{V} \int_{S_e^s} (\mathbf{n}_e^s \cdot \sigma^s)(\mathbf{x}^s - \mathbf{x}^s) \, dA, $$ \hspace{1cm} (20)  

$$ \phi c_M^f = \frac{1}{V} \int_{S_e^f} \mathbf{n}_e^f \cdot \mathbf{u}_M^f \, dA, $$ \hspace{1cm} (21)  

$$ \phi \mathbf{u}_M^f = \frac{1}{V} \int_{S_e^f} (\mathbf{n}_e^f \cdot \mathbf{u}_M^f)(\mathbf{x}^f - \mathbf{x}^f) \, dA. $$ \hspace{1cm} (22)  

Equations (19)–(22) are used to compute the macroscopic quantities $\mathbf{f}_M^e$, $c_M^f$, $\mathbf{u}_M^f$ and $\mathbf{u}_M^f$ from a set of microscopic simulations as described in detail in section 3.2. The resulting effective properties can be then obtained using Equations (12)–(15).
Fig. 1. (a) The reference unit cell without added mass (left), and the unit cell with a resonator with a heavy mass at its tip (right), (b) a double cell consisting of two unit cells with different resonator masses. Blue colour represents polyurethane (PU) solid skeleton, dark grey and dark orange are the different heavy masses at the tip of the resonator, light grey is the air domain. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

Table 1
Material properties of air (\(\rho_f^0\) denotes fluid equilibrium density, \(\kappa_f^0\) thermal conductivity, \(c_p^0\) heat capacity at constant pressure, \(\mu_f^0\) dynamic viscosity, \(\theta_0\) and \(P_0\) equilibrium temperature and pressure, respectively) [21].

<table>
<thead>
<tr>
<th>(\rho_f^0) (kg/m(^3))</th>
<th>(\kappa_f^0) (W/(m·K))</th>
<th>(c_p^0) (J/(kg·K))</th>
<th>(\mu_f^0) (Pa·s)</th>
<th>(\theta_0) (K)</th>
<th>(P_0) (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2</td>
<td>0.0257</td>
<td>1005</td>
<td>1.84·10(^{-3})</td>
<td>293</td>
<td>1.01·10(^{5})</td>
</tr>
</tbody>
</table>

3. Simulation set-up

3.1. Model configurations

In this work, the acoustic metafoam material is parametrised using the geometry of the unit cell shown in Fig. 1a and its variations. This unit cell has been introduced and used in Lewińska et al. [7,19]. This simplified representation of a single foam pore consists of a solid polyurethane (PU) skeleton filled with air. The skeleton is made out of thicker solid struts connected by thin membranes (among which four out of six are partially open, the top and bottom membranes are closed). A local resonator is attached to one of the membranes in the form of a cantilever beam with an added mass at the tip. As a reference case, the same geometry of the unit cell is used without the added mass (but with the cantilever) referred to as the ‘reference case’.

The following dimensions for the pore are assumed. The size of the unit cell is \(a = 100\) μm, with a strut thickness of 25 μm and a membrane thickness of 1 μm. The membrane opening is assumed to be 1% of the membrane surface. The default unit cell has a resonator cantilever with dimensions 45 μm × 10 μm × 10 μm, with a mass of 10\(^{-10}\) kg and dimensions 10 μm × 10 μm × 10 μm attached at its tip. The mass and cantilever length/stiffness variations considered in this paper will be normalised with respect to this default unit cell.

Material properties for the solid and fluid domains used in the analysis are listed in Tables 1 and 2.

In order to identify the combinations of the resonator mass and stiffness that may induce a broad attenuation zone, Bloch analyses have been performed, following a standard lossless approach as described in Lewińska et al. [7]. The resonator’s mass is varied through its density and the resonator’s stiffness is changed by varying the length of the cantilever to which the resonating mass is attached. The band gap frequency ranges for solid-born waves are shown in Fig. 2 with respect to the varied parameters. It can be observed that the lower the mass, the higher and narrower the frequency band gaps are. On the other hand, an increase of the resonator stiffness (by shortening the cantilever length) results in broader band gaps but higher frequency ranges. The same dependency has been observed for standard LRAMs, for instance, in Krushynska et al. [23].

Based on the intrinsic dependency of the band gap frequencies on the resonator tip mass (Fig. 2a), several double cells have been constructed and investigated. These consist of two single unit cells with different masses. For case A, the mass difference is small, i.e. mass ratios are 1.0 and 0.9. In case B, the mass difference is larger, i.e. density ratios are 1.0 and 0.6. In case A, the band gap frequencies formed due to the two masses are close to each other (445–560 Hz and 465–575 Hz), whereas in case B, the

Table 2
Material properties of the polyurethane (PU) solid phase (\(\rho^s\) denotes solid density, \(E\) Young’s modulus, \(\nu\) Poisson’s ratio) [22].

<table>
<thead>
<tr>
<th>(\rho^s) (kg/m(^3))</th>
<th>(E) (MPa)</th>
<th>(\nu) (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>1</td>
<td>0.4</td>
</tr>
</tbody>
</table>
Fig. 2. Band gap frequency ranges at the variation of the cantilever resonator properties (a) mass and (b) cantilever length, both normalised with respect to the default values. Blue lines depict the opening and closing frequencies of the band gap regions, black dashed lines mark the total width of the attenuation zone targeted in Section 4.2 for graded microstructures. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

Fig. 3. Sketch of the functionally graded microstructures: variation of mass (top), variation of cantilever length, i.e. stiffness (bottom).

band gap opening frequency of the smaller mass corresponds approximately to the closing band gap frequency of the heavier mass (570–665 Hz and 445–560 Hz).

Finally, fully graded microstructures (Fig. 3) have been studied, where the mass and stiffness vary linearly in space in the ranges (0.3–1.0) and (0.6–1.0) in terms of the normalised mass density and normalised cantilever length, respectively. For such parameter variation ranges, an expected total attenuation zone may be estimated from Fig. 2 lying between 445 and 800 Hz indicated in Fig. 2 by dashed black lines.

3.2. Computation of effective properties

In order to obtain the effective properties for the macroscopic constitutive equations (12)-(15) for each of the considered unit cells, five simulations with different loading conditions are conducted, which are sufficient for investigating wave propagation in the $x$ direction (Fig. 1). The specific loading conditions for each simulation set are listed in Tables 3 and 4.

<table>
<thead>
<tr>
<th>Set</th>
<th>$u_s^0$ (m)</th>
<th>$V_{u_s}p_{\alpha}^0$ (Pa/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$10^{-4}e_x$</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>$10^4e_x$</td>
</tr>
</tbody>
</table>
Table 4

Loading conditions used to determine parameters $C, \Psi_f, S_f$, with vectors $e_x, e_y$ denoting unit vectors in $x$ and $y$ directions, respectively (see Fig. 1). The solid displacement $u_s$ and the fluid pressure gradient $\nabla p_f$ are both prescribed zero.

<table>
<thead>
<tr>
<th>Set</th>
<th>$\nabla M u_s$ (−)</th>
<th>$p_f'$ (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$10^{-6} e_x e_x$</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>$10^{-6} e_y e_y$</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Fig. 4. Transmission set-up for homogenised sample. The green faces depict the excitation planes, the red faces are pressure probing points. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

3.3. Transmission analysis

A transmission analysis is performed in order to assess the acoustic attenuation performance of the homogenised poro-elastic medium. To this end, a transmission impedance tube set-up is reconstructed numerically (Fig. 4). A sample with a thickness of 10 unit cells is embedded in a homogeneous acoustic domain, which is described by the Helmholtz equation and by using the properties of the air: the wave speed is taken as $c = 343$ m/s, while the density is $\rho = 1.2$ kg/m$^3$. The sample thickness is consistent with the numerical simulations shown in Ref. [19] for better comparison. On the left boundary, a plane wave excitation with a unit pressure amplitude is applied. On the right boundary, a perfectly matched layer is used in order to reduce spurious reflections. On the lateral planes, periodic boundary conditions are assumed and interface conditions between the acoustic domain and the homogenised metafoam layer are applied, following [24]. A three point method [7,25,26] is adopted to obtain the transmission, reflection and absorption spectra. According to this method, the averaged pressure values are evaluated at three positions, namely two planes before and one plane after the sample. The numerical model is solved using COMSOL Multiphysics. The adopted quadratic mesh has a size not exceeding 1/6 of the wavelength and for the graded case, it is refined to 100 elements per sample length.

4. Results and discussion

In this section, the graded acoustic metafoams are analysed. To this aim, first the effective properties for each configuration are obtained through the computational homogenisation procedure. Next, the macroscopic acoustic response of the material samples is investigated.

4.1. Configuration with two masses

The configurations A and B with a double cell with two different masses described in Section 3 are compared with the single cell performance. The comparison of the configurations with two different stiffnesses leads to analogical conclusions. Therefore only the double cell with different masses is presented in this paper.

4.1.1. Effective properties

As stated in Lewińska et al. [19], the tensors $M^s$ and $K^f$ (where $M_1^s, M_2^s, M_3^s$ and $K_1^f, K_2^f, K_3^f$ denote the diagonal components with respect to the unit cell basis, see Fig. 1) contain the effective properties that are significantly influenced by the presence of the resonators with an added mass. Moreover, these are also the parameters that determine the acoustic response of the sample. Since the difference in $K^f$ for the analysed cases is insignificant and the behaviour of both tensors $M^s$ and $K^f$ has a
similar character, in Fig. 5 only the real and imaginary parts of $M_1^c$ are shown. In Fig. 5a and b, the values obtained for the cases A and B (described in section 3) are compared with the single unit cell cases, with the resonator mass ratios 0.9 or 1.0 and 0.6 or 1.0 are considered, respectively. Here, the main characteristics of $M_1^c$ as a function of frequency that govern the transmission performance are described, as discussed in the next subsection. Multiple peaks and dips can be observed for the double cell cases that coincide with the locations of the peaks and dips in the relevant single unit cell configurations. However, the maxima and minima for the double cell do not reach the same values as in the single cells, which alters the magnitude of the attenuation as will be demonstrated in the transmission analysis. Moreover, compared with the single unit cell cases, the real part of $M_1^c$ is zero at four distinct frequencies instead of two.

The zeros occurring for the sign change for this parameter from negative to positive can be correlated with the reduction of the reflection, which in the two-mass cell will, in general, occur at two frequencies, in contrast with the single mass cell, where this phenomenon occurs at a single frequency. The extent of the reduction in reflection and specifically the accompanying bandwidth, further correlates with the slope of the real part of $M_1^c$ at the zero crossing. This slope is steep in case A while it changes gradually in case B, implying a negligible reduction of the reflection bandwidth for case A and a large bandwidth for case B. Furthermore, in case A, the imaginary part of $M_1^c$ does not vanish at the zeros of the real part, which implies the persistence of absorption.

One of the solutions proposed in the literature for bridging attenuation zones formed by multiple resonators of different eigenfrequencies, is by using material damping [27,28]. Therefore, case B is also considered with a material damping coefficient equal to 0.1 for the PU skeleton. In Fig. 6, the smoothing effect of the damping can be observed for both the real and imaginary parts of $M_1^c$; moreover, the real part of $M_1^c$ stays positive and the values of the imaginary part of $M_1^c$ decrease in between the dips.
4.1.2. Transmission simulations

In Fig. 7, transmission, reflection and absorption spectra are shown for the double cell case A and the two corresponding single mass unit cell cases. The attenuation provided by the mixture of two resonating masses is slightly broader compared to each single case. It is however not a true envelope of the single resonator cases due to the lower mass content in the double cell (lower mass content results in narrowing the band gap regions). At the frequency of 455 Hz, corresponding to the zero of Re($M_s$) in Fig. 5a, a narrow dip in the reflection spectrum and a sharp peak in the absorption can be observed. This result correlates with the behaviour of Re($M_s$) as discussed previously.

In Fig. 8, transmission, reflection and absorption spectra are shown for case B and for the two corresponding single mass unit cell cases. The attenuation zone in case B is covering a broader frequency range than for case A since the resonance frequencies are more separated, but the transmission peak in between these frequencies has a larger width with higher values. Again, the location and width of this peak can be associated with the behaviour of Re($M_s$) shown in Fig. 5b. The increase of transmission is a result of the significant decrease of reflection at this frequency. Two higher peaks can in turn be observed in the absorption spectrum. In between these peaks the absorption values are instead decreasing, and this result correlates with the vanishing of Im($M_s$).

In Fig. 9, the influence of the skeleton damping on the macroscopic attenuation response of case B is reported. Bridging of the attenuation zones emerges as a result of the smoothing of the reflection spectrum (which is a consequence of the modification of the effective parameters due to material damping). In particular, the reflection improves in the region between the reflection peaks. Moreover, the absorption level increases in the entire attenuation zone, as expected. It should be noted that polymeric materials such as PU can exhibit typically larger losses in shear deformation than in volumetric deformation. By accounting for larger loss moduli in shear than in bulk, the qualitative results shown here are not expected to change since the resonators exhibit mostly shear deformation.
To summarise, this study shows the microstructural origin of the improved transmission response of microstructures with multiple masses. The importance of using fine mass gradings (% relative steps 0.1) in order to broaden attenuation regimes without introducing intermediate transmission peaks is highlighted. It has been also demonstrated that in the case of coarser mass gradings, the attenuation zones can be smoothened by skeleton material damping. Note, that all considered metafoam configurations can be assumed to have approximately the same impedance mismatch with the outside air. Therefore, the differences in reflection/transmission performance of different metafoam structures can be almost entirely attributed to the internal resonance mechanism and its dependence on the particular internal structure configuration.

4.2. Graded metafoam

In this section, the performance of functionally graded acoustic metafoams with varying mass and stiffness of the resonators is analysed. The acoustic response of graded metafoams is compared with metafoams endowed with uniform resonators, as well as with the reference foam without added mass.

4.2.1. Effective properties

The effective properties of the unit cells studied until now vary with frequency, and this dependence changes as a function of the resonator mass and the cantilever length (the stiffness). In order to obtain the effective properties of the graded structures where mass or stiffness varies along the sample length, several single unit cell computations at the micro-scale level have been performed for a discrete number of density ratios and cantilever lengths (eight sets of simulations per graded structure, denoted by dot-markers in Fig. 2). It is thereby assumed that either the mass or the cantilever length change linearly in space (i.e. along the sample length). The local effective properties of the graded structure have then been determined by linear interpolation between the computed effective properties, yielding a continuous dependency of each effective property on both frequency and space.

In Fig. 10, both real and imaginary parts of the parameter $M_s$ normalised with respect to the PU density $\rho_s$ are shown for the graded mass case with normalised mass ratios varying between 1.0 and 0.3 along the sample length. With decreasing mass (i.e. increasing position along the sample length) the characteristic peak and dip for this parameter are gradually mitigated and shifted to higher frequencies in a non-linear manner.

Similarly, when the cantilever length is decreased from 1.0 to 0.6 (with increased normalised position), the local resonance frequency increases, shifting the peak and dip of the parameter $M_s$ (normalised with respect to PU density $\rho_s$, Fig. 11). The higher the frequency, the higher the peak and dip values. This is a consequence of keeping the resonator mass constant throughout the sample, since the magnitude of the peaks and the dips is proportional to the unit cell mass content.

4.2.2. Transmission simulations

The acoustic responses of the considered graded metafoams are shown in the transmission spectra plotted in Figs. 12 and 13. Although the maximum attenuation level is locally lower than for the sample with the uniform resonators, in both cases (graded mass and stiffness) the attenuation zones are significantly broadened for the graded structure, spanning the entire targeted frequency width (denoted by black dashed lines). Moreover, compared with the uniform case with the same total mass (Fig. 12), the graded mass case is characterised by the attenuation reaching lower frequencies. Similarly to the two-mass problem discussed in the previous section, the incorporation of a mass gradient results in a transmission dip with a varying level of attenuation. The smaller the mass, the higher the number of cells that are needed in order to provide the same level
of attenuation. In turn, stiffness variation (Fig. 13) results in a flat attenuation zone, meaning that each wave frequency in the range 440–800 Hz is almost equally attenuated. This is a consequence of the fact that, although the stiffness is varied, the mass content is constant.

In both cases, with the introduction of a graded microstructure, the peak reflection level drops. It can also be observed that the amount of reflection varies in the attenuation zone. The absorption instead is improved in the entire frequency span for both cases. Note, that for acoustic metafoams, the dissipation enhancement occurs due to the local resonance mechanism, and therefore the overall broader absorption spectrum originates from the contribution of the graded resonators with varying eigenfrequencies.

Note, that the small oscillations in the transmission, reflection and absorption spectra within the attenuation zones are a consequence of the linear interpolation of the effective properties between the set of discrete points (Figs. 10 and 11). Indeed, the number of wiggles is equal to the number of simulations sets for which the effective properties have been computed and between which the interpolation has been performed. These wiggles can be eliminated either by using a larger number of discrete simulations sets, or by adopting a nonlinear interpolation scheme. However, this does not alter the main result of the broadened attenuation zone.

Within the scope of this study, also the opposite grading direction (positive) has been considered for variation of both stiffness and mass of the resonators. Transmission, reflection and absorption spectra (not shown in this paper) overlap with the
Fig. 12. Transmission, reflection and absorption spectra for a metafoam sample with a graded mass distribution, a metafoam sample with uniform resonators of normalised mass 1.0 and 0.65 (for which the total mass equals the total mass of the graded material) and the reference sample without added mass.

Fig. 13. Transmission, reflection and absorption spectra for a metafoam sample with graded stiffness, a metafoam with uniform resonators and the reference foam sample without added mass.

presented results (negative gradient), which demonstrates that there is no influence of the grading direction on the acoustic performance of the sample. Since the order of the resonators in the grading seems to have a negligible impact, it is suggested to investigate further a random polydisperse distribution of resonators which might be favourable from a manufacturing point of view.

5. Conclusions

In this paper, the acoustic performance of functionally graded metafoams with a broadened attenuation range has been demonstrated. First, the concept of extending the frequency attenuation region by combining metafoam unit cells with two differently tuned resonators has been investigated, resulting in two transmission dips corresponding to the performance of each individual unit cell. Next, microstructures with spatially graded resonating masses or resonator stiffnesses have been considered, resulting in a single, broad attenuation zone delimited by the minimum and maximum local resonant frequencies of the corresponding unit cells.

The analyses, based on a computational homogenisation technique, have shown that for the design of functionally graded acoustic metafoams, the continuous variation of the resonator properties is crucial in order to avoid the occurrence of transmission peaks inside the attenuation zone. However, the presence of such undesired disturbances can be mitigated by means of solid skeleton (viscous) damping. Such damping generally occurs in real applications, for example when a polymeric foam material is adopted. Finally, it has been shown that for the graded microstructures investigated in this paper, the resulting attenuation levels are higher and more uniform in frequency when using a graded resonator stiffness, compared to resonator mass grading. Moreover, the lack of sensitivity of the transmission simulations to the gradient direction suggests that a random distribution of
different resonators could be efficient as well. The analysis of acoustic metafoams with a polydisperse distribution of resonators is therefore recommended. The findings presented in this paper can be used as a guidance for design and manufacturing of acoustic metafoams with an improved low frequency attenuation response compared to conventional acoustic foams. Future work will include the analysis of more realistic unit cells for acoustic foams, i.e. based on a Kelvin cell. The benefit of using computational homogenisation will be fully exploited while considering such complex geometries and wave propagation in different directions.

CRediT authorship contribution statement

**M.A. Lewińska:** Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Software, Validation, Visualization, Writing - original draft. **J.A.W. van Dommelen:** Conceptualization, Methodology, Supervision, Writing - review & editing. **V.G. Kouznetsova:** Conceptualization, Methodology, Supervision, Writing - review & editing. **M.G.D. Geers:** Conceptualization, Funding acquisition, Methodology, Resources, Supervision, Writing - review & editing.

Declaration of competing interest

This study and the manuscript present no conflict of interest.

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