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Iterative Pole-Zero Model Updating Using Generic Parameters

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Introduction

A crucial step in the control of a high-tech system is having a very accurate dynamic model of the system from actuators to sensors and to the unmeasured performance variables. A (reduced) Finite Element (FE) dynamic model may be a good candidate apart from the fact that its accuracy is often limited, since it does not fully match with the real structure. Using a FE model updating algorithm the accuracy of FE model can be significantly improved. An Iterative Pole-Zero (IPZ) model updating procedure is proposed that updates the eigenvalues of the stiffness and/or damping matrix. These are considered as the generic parameters needed to update the FE dynamic model. Updating the poles and zeros is done using estimates from measured Frequency Response Functions (FRFs).

Case Study

Consider the mass-spring-damper system in Fig. 1. The original model consists of six point masses $m_i = 1$ kg, six springs $k_i = 10^6$ N/m, and six viscous dampers $c_i = 10$ Ns/m ($i = 1, \dots, 6$). This results in a system mass M_n, damping C_n, and stiffness matrix K_n, respectively.

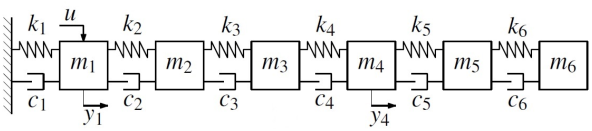

Figure 1: Six DOFs mass-spring-damper system.

In general, the original model differs from the real structure. Assume a similar structure which differs in terms of spring stiffnesses $k_2 = k_6 = 1.2 \times 10^6$ N/m and viscous damping constants $c_1 = c_5 = 5$ Ns/m. This structure is called the experimental model. It is assumed that FRF $H_{11} = Y_1/U$ can be measured and FRF $H_{41} = Y_4/U$ corresponds to the unmeasured performance variable. The poles of the original H_{11} can be calculated via the eigenvalue problem

$$(\lambda_p^2 M_n + \lambda_p C_n + K_n) u_p = 0, \qquad (1)$$

while the zeros are calculated via

$$(\lambda_z^2 M_s + \lambda_z C_s + K_s) u_z = 0, \qquad (2)$$

where M_s, C_s, K_s are the substructure matrices. These matrices can be derived from M_n, C_n, K_n by deleting the column and row corresponding to the actuator and sensor DOFs.

Generic Parameter Assignment

If no information is available on possible locations of errors in the stiffness and damping matrix, it is proposed to use only a limited number of eigenvalues of the stiffness and damping matrix as generic parameters to be updated. For instance, eigenvalue decomposition of the stiffness matrix is given by

$$K_n = V \Sigma V^{-1} = V \Sigma V^T, \qquad (3)$$

where V is a square matrix containing column-wise the real static stiffness mode shapes, and $\Sigma = \mathrm{diag}(\sigma_1, \dots, \sigma_n)$ is a diagonal matrix containing the corresponding real eigenvalues of the stiffness matrix.

Iterative Pole-Zero Model Updating

IPZ model updating is a procedure which tries to minimize the following pole and zero error functions iteratively by updating the generic parameters

$$\varepsilon_p^i = (\lambda_p^e - \lambda_p^{n,i})^H W_p (\lambda_p^e - \lambda_p^{n,i}) \qquad (4)$$

$$\varepsilon_z^i = (\lambda_z^e - \lambda_z^{n,i})^H W_z (\lambda_z^e - \lambda_z^{n,i}), \qquad (5)$$

where $W_p > 0$ and $W_z > 0$ are weighting matrices.

Simulation Results and Conclusions

The IPZ model updating is performed on the original H_{11} using the measured poles and zeros from the experimental H_{11}. In Fig. 2, the original, experimental, and updated FRFs H_{11} and H_{41} are shown. 0 dB is 1 m/N. It is clear that, in contrast to the original H_{11}, the updated H_{11} matches very well with the experimental H_{11}. Furthermore, as a result of IPZ model updating using H_{11}, the updated unmeasured variable reflected by H_{41} also matches well with the experimental H_{41} in contrast to the original H_{41}.

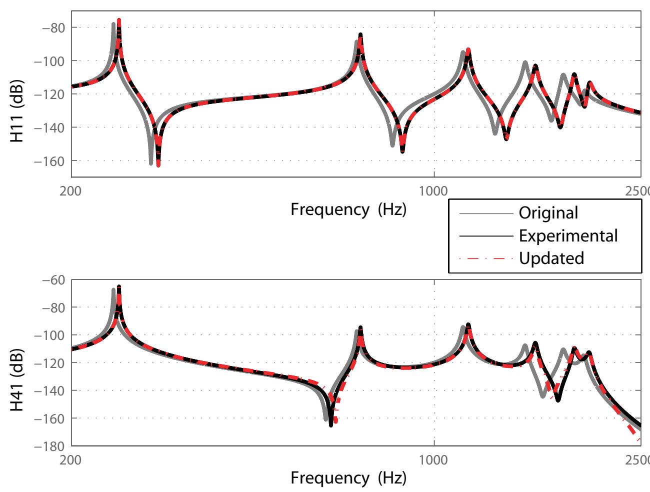

Figure 2: Original, experimental, and updated H_{11} (top) and H_{41} (bottom) after IPZ model updating.

References

[1] J. E. Mottershead et al, The sensitivity method in finite element model updating: A tutorial, Journal of Mechanical Systems and Signal Processing, 2011.