# A closed-form solution to estimate space-dependent parameters in heat and mass transport

# A closed-form solution to estimate space-dependent parameters in heat and mass transport

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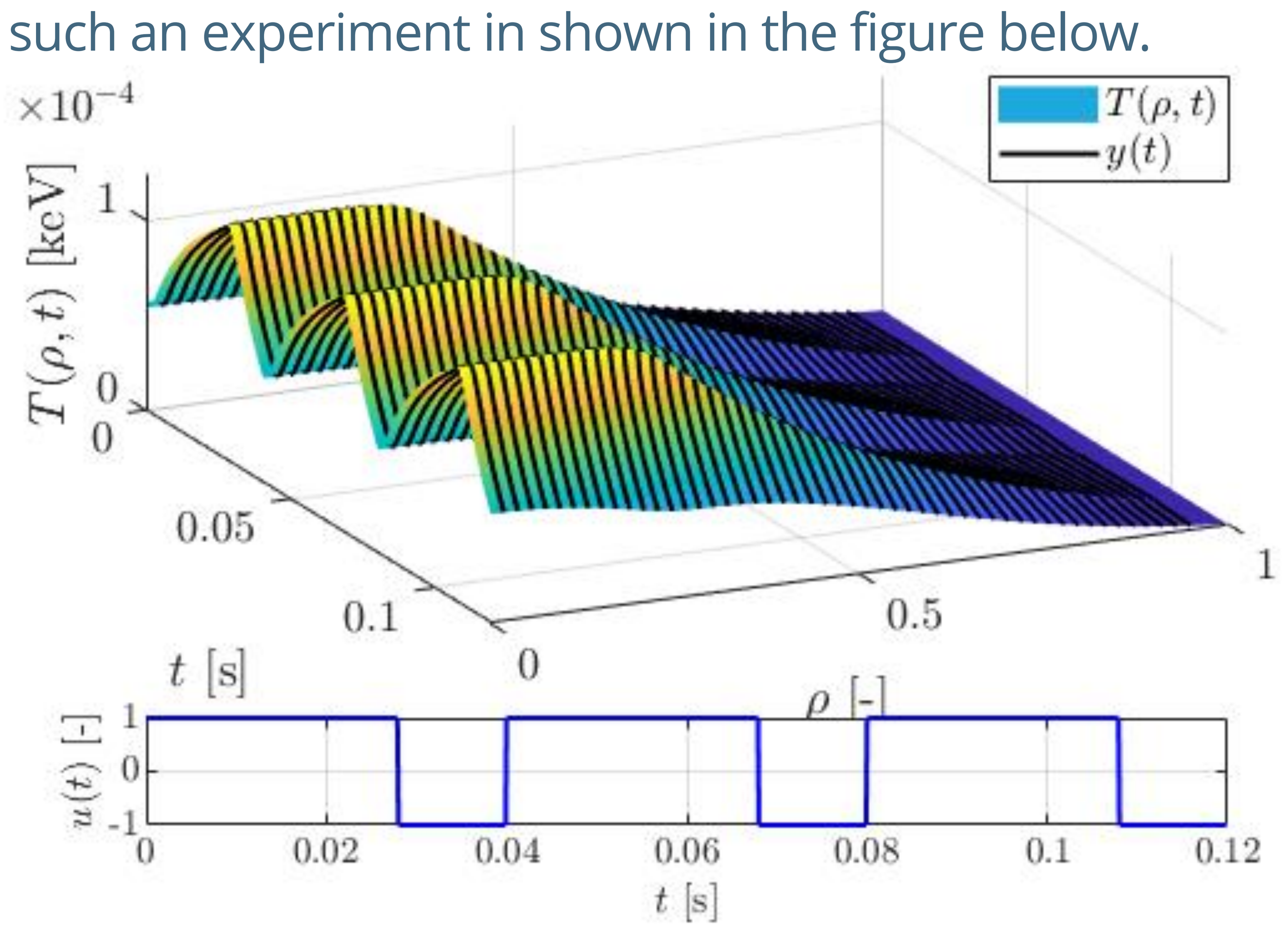
## Introduction

Heat and mass transport are important for various problems [1-3]. We consider the partial differential equation (PDE) that governs heat and mass transport in a cylindrical geometry

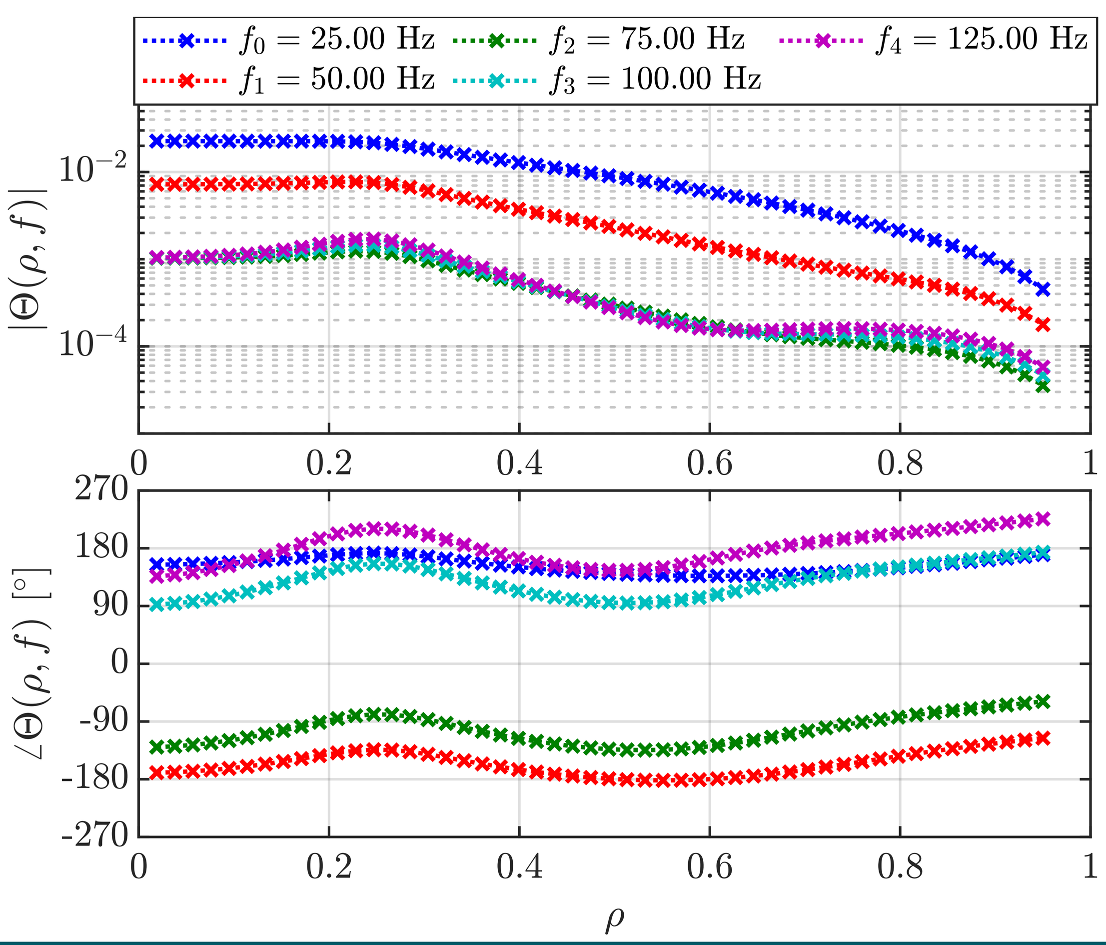
$$\frac{\partial T(\rho,t)}{\partial t} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho D(\rho)\frac{\partial T(\rho,t)}{\partial \rho} + \rho V(\rho) T(\rho,t)\right) + P(\rho)u(t)$$

with the state (e.g. temperature) $T$, that depends on the time and spatial coordinate $t$ and $\rho$, respectively. The unknown space-dependent parameters, are the diffusion $D$, convection $V$ and source profile $P$. To estimate $\{D, V, P\}$, data is generated by (periodically) perturbing the system via the input $u(t)$ and measuring the state at $M$ spatial locations $\check{\rho}_m$, $m = 1, \ldots, M$, resulting in $y(t) = \text{col}(y_1, \ldots, y_M)$, with $y_m(t) = T(\check{\rho}_m, t)$. An example of data generated by such an experiment in shown in the figure below.




## Filtering and Frequency domain

Only the forced response is considered by applying advanced filtering techniques that remove noise, drifts and transients (i.e. dependency on the initial condition) [4]. Furthermore, the PDE is considered in the frequency domain such that it is sufficient to only consider the base frequency and a few of its harmonics that are above the noise floor. Such data is shown in the figure below.

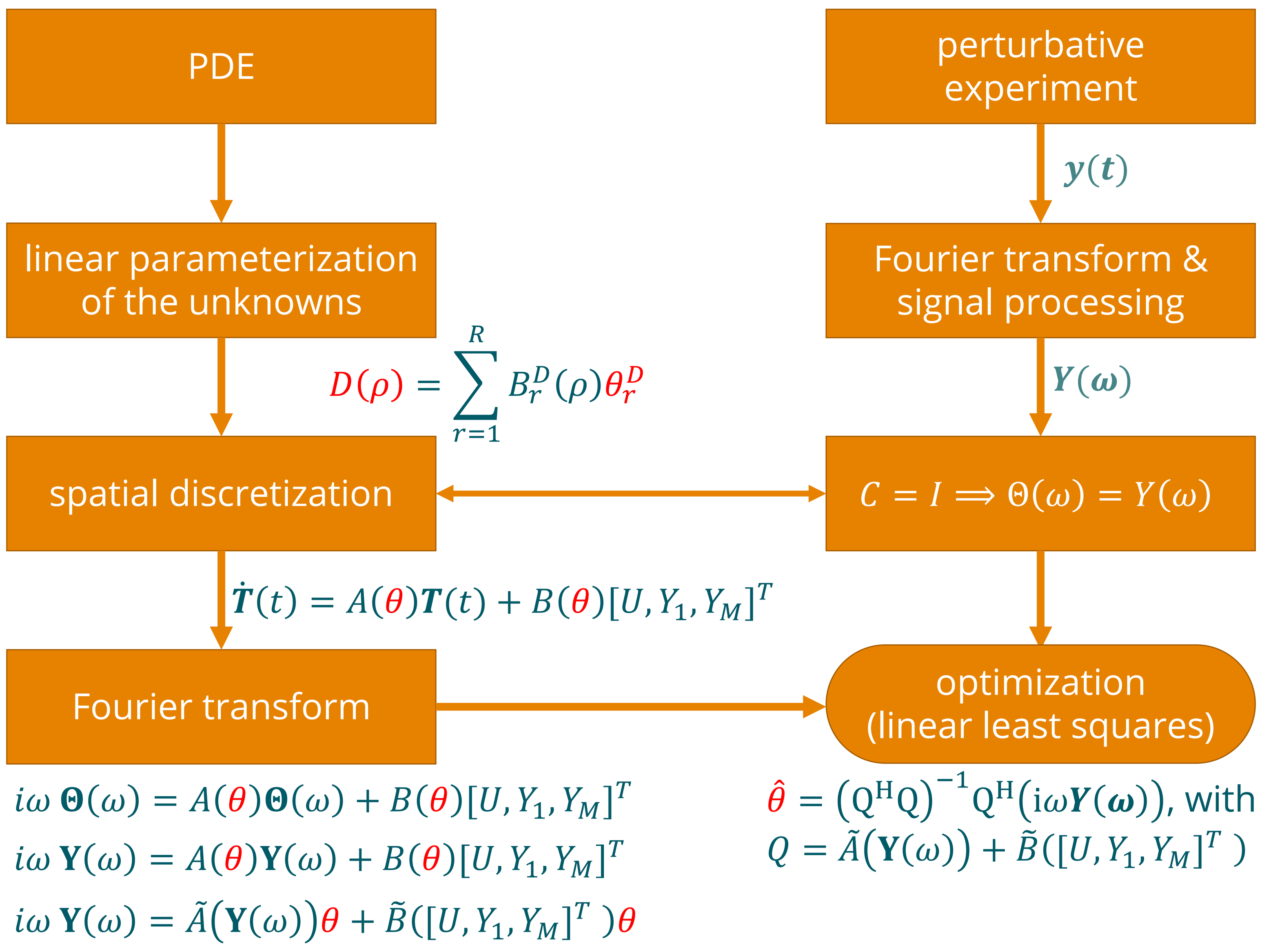



## Methodology

We consider the PDE in the frequency domain

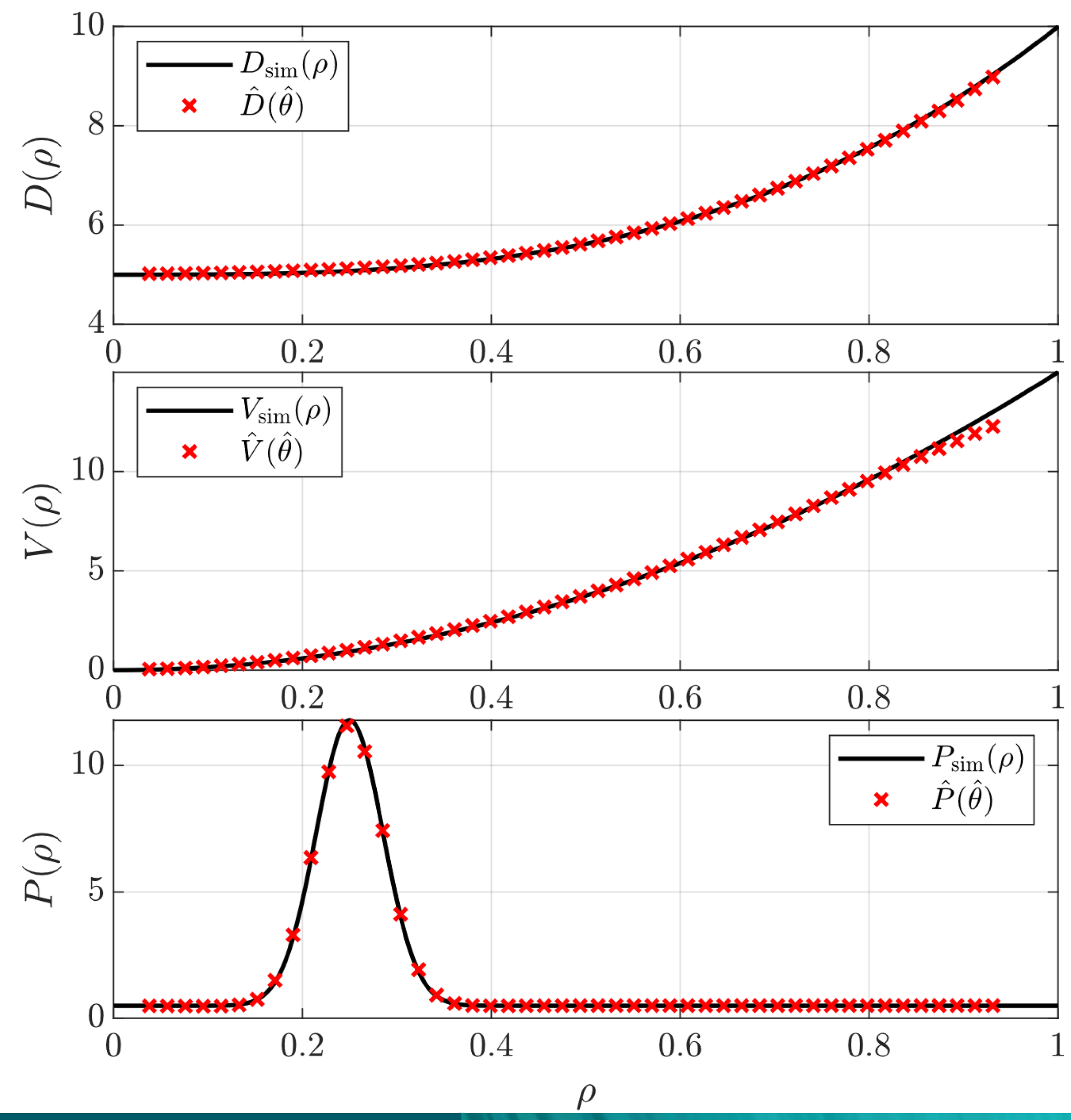
$$i\omega\Theta(\rho,\omega) = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho D(\rho)\frac{\partial \Theta(\rho,t)}{\partial \rho} + \rho V(\rho)\Theta(\rho,t)\right) + P(\rho)u(t)$$

and follow the methodology from [5] such that the inverse problem is rewritten as a linear least square problem which has an analytic solution for the optimum. This method is visualized below.




## Results

Due to the availability of the analytic solution, it guarantees the global minimum and does not require computational expensive optimization methods! The simulated and the estimated parameters for shown data set are shown below.




## References

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[2] U. Schneidewind et al., *Water Resour. Res.*, vol. 52, no. 8, pp. 6596-6610, 2016
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## Acknowledgements

DIFFER is part of the institutes organisation of NWO.
This work has been carried out within the framework of the EUROfusion Consortium and has received funding from the Euratom research and training programme 2014-2018 and 2019-2020 under grant agreement No 633053. The views and opinions expressed herein do not necessarily reflect those of the European Commission.