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A closed-form solution to estimate space-dependent parameters in heat and mass transport

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Introduction
Heat and mass transport are important for various problems [1-3]. We consider the partial differential equation (PDE) that governs heat and mass transport in a cylindrical geometry

$$\frac{\partial T(\rho, t)}{\partial t} = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho D(\rho) \frac{\partial T(\rho, t)}{\partial \rho} \right) + \rho V(\rho) T(\rho, t) + P(\rho) u(t)$$

with the state (e.g. temperature) $T$, that depends on the time and spatial coordinate $t$ and $\rho$, respectively. The unknown space-dependent parameters, are the diffusion $D$, convection $V$ and source profile $P$. To estimate $D(V, P)$, data is generated by (periodically) perturbing the system via the input $u(t)$ and measuring the state at $M$ spatial locations $\rho_m$, $m = 1, ..., M$, resulting in $y(t) = \text{col}(y_1, ..., y_M)$, with $y_m(t) = T(\rho_m, t)$. An example of data generated by such an experiment is shown in the figure below.

Filtering and Frequency domain
Only the forced response is considered by applying advanced filtering techniques that remove noise, drifts and transients (i.e. dependency on the initial condition) [4]. Furthermore, the PDE is considered in the frequency domain such that it is sufficient to only consider the base frequency and a few of its harmonics that are above the noise floor. Such data is shown in the figure below.

Methodology
We consider the PDE in the frequency domain

$$\omega \Theta(\omega) = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho D(\rho) \frac{\partial \Theta(\omega)}{\partial \rho} \right) + \rho V(\omega) \Theta(\omega) + P(\omega) u(t)$$

and follow the methodology from [5] such that the inverse problem is rewritten as a linear least square problem which has an analytic solution for the optimum. This method is visualized below.

Results
Due to the availability of the analytic solution, it guarantees the global minimum and does not require computational expensive optimization methods! The simulated and the estimated parameters for shown data set are shown below.

References

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