Direct-quadrature Sequence Models for Energy-function based Transient Stability Analysis of Unbalanced Inverter-based Microgrids

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Direct-quadrature Sequence Models for 
Energy-function based Transient Stability Analysis 
of Unbalanced Inverter-based Microgrids

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and J.G. Slootweg, Senior Member, IEEE

Abstract—The reliability of supply in distribution networks can be improved by islanding (parts of) the network as autonomous microgrids in case of a contingency. Microgrids are commonly fed by inverter-based distributed energy resources, which causes transient stability challenges due to the lack of inertia and strongly coupled dynamics. The transient stability of inverter-based microgrids is conventionally analyzed by time-domain analysis. However, energy-function based transient stability analysis has advantages over time-domain analysis, as it provides a stability result for all initial conditions and allows uncertainty to be taken into account with the domain of attraction. Microgrids are generally unbalanced, however models for unbalanced energy-function based transient stability analysis are lacking in the literature. In this paper, coupled sequence components models of distributed energy resources and load devices in the dq reference frame are proposed, and experimentally validated with unbalanced voltage transients. The validated models are used for energy-function based transient stability analysis of an unbalanced inverter-based case study microgrid. Finally, the impact of unbalanced connection of devices on transient stability is analyzed. The results indicate that the proposed models accurately represent the behavior of physical devices and that the unbalanced connection of devices significantly impacts the transient stability of inverter-based microgrids.

Index Terms—Inverter-based microgrid, unbalanced conditions, dynamic models, Lyapunov methods, power system stability.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0, 1, 2</td>
<td>Zero, positive, negative sequence superscript.</td>
</tr>
<tr>
<td>c, s, k</td>
<td>Controller reference value superscript.</td>
</tr>
<tr>
<td>Re, Im</td>
<td>Real, imaginary part.</td>
</tr>
<tr>
<td>ωn</td>
<td>Nominal frequency (rad/s).</td>
</tr>
<tr>
<td>mp, nq</td>
<td>Droop control parameters.</td>
</tr>
<tr>
<td>Kpc, Kic</td>
<td>Current controller P,I gains.</td>
</tr>
<tr>
<td>Kpu, Kiv</td>
<td>Voltage controller P,I gains.</td>
</tr>
<tr>
<td>Kppt, Kiv</td>
<td>PLL controller P,I gains.</td>
</tr>
<tr>
<td>VDC</td>
<td>Primary DC source voltage.</td>
</tr>
<tr>
<td>Rdc, Ldc, Cdc</td>
<td>DC-link resistance, inductance, capacitance.</td>
</tr>
<tr>
<td>r_f, L_f</td>
<td>Inverter-side LCL filter resistance, inductance.</td>
</tr>
<tr>
<td>r_c, L_c</td>
<td>Grid-side LCL filter resistance, inductance.</td>
</tr>
<tr>
<td>C_f</td>
<td>LCL filter capacitance.</td>
</tr>
</tbody>
</table>

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MICROGRIDS can improve the reliability of supply of local demand in case of a contingency the distribution network, by operating as autonomous islands [1], [2]. Microgrids stability characteristics are different from conventional power systems due to their small size, generation from intermittent inverter-based distributed energy resources (DERs) and highly unbalanced connection of DERs and loads. The (in)stability phenomena in microgrids can be classified into two main categories: (i) control system (in)stability (ii) power supply and balance (in)stability, which can be triggered by transient events such as faults, large load changes and loss of generation units [3]. In this paper, the transient stability of microgrids is defined as: the ability to reach a stable equilibrium point which lies within the acceptable microgrid operating range after a transient occurs [3], [4]. Inverter-based microgrids (IBMs) are especially susceptible to transient instability due to low inertia, nonlinear dynamics, strongly coupled dynamics, and a lack of reference voltage and frequency. Therefore, the system-wide, nonlinear transient stability of islanded IBMs has to be critically evaluated.

The transient stability of IBMs can be evaluated with either time-domain analysis or energy-function based transient stability analysis (EBTSA) [3], [5]. With time-domain analysis, the stability of an initial condition is evaluated by numerical integration of the nonlinear IBM model equations. This allows different types of modeling with continuous and discrete variables, and produces exact stability results for a single initial condition. In contrast, EBTSA allows derivation of transient stability directly from the IBM model equations. EBTSA allows conservative estimation of the domain of attraction (DOA), which quantifies the stability margin. This provides a stability result for all initial conditions, allows uncertainty to be taken into account, and allows IBM design and control actions to be optimized [6]–[8]. However, EBTSA generally requires the IBM model to be of a methodology-specific form.

Y_R, Y_L, Y_C | RLC load resistor, inductor, capacitor admittance. |
| K_pf | Flux controller P gain. |
| K_pt, K_it | Torque controller P,I gains. |
| K_pc, K_ic | Speed controller P,I gains. |
| p | Number of induction machine pole pairs. |
| Ilim, Tlim | Current and torque limit values. |
| R012, ρ012 | Line sequence resistance, inductance. |

I. INTRODUCTION
Due to the capability of time-domain analysis to perform exact transient stability analysis and accept different types of models, time-domain analysis is commonly used to validate the transient stability of IBMs as performed by e.g. [2], [7], [9]–[11]. In contrast, the estimation of the DOA allows EBTSA to be used for optimization of the design of an IBM based on the size of the DOA by [7], and design an energy storage controller and a nonlinear droop controller to maximize the DOA of DC microgrids with constant power loads by [6] and [8] respectively. However, due to the unavailability of suitable unbalanced models, EBTSA has only been performed on DC microgrids and balanced IBMs until now, while IBMs are commonly unbalanced.

To allow EBTSA of unbalanced IBMs, the IBM model has to have a constant equilibrium point [12], [13], and describe the nonlinear dynamics under balanced and unbalanced conditions. Several IBM models in the dq reference frame with constant equilibrium points have been proposed in literature e.g. [7], [14], [15]. However, these models only include the positive sequence components and are therefore unsuitable for transient stability analysis of unbalanced IBMs. Models of IBMs which can describe unbalance have been proposed by [10], [11], [16]. The IBM dynamic phasor model in the abc reference frame proposed by [16] is able to accurately describe the IBM dynamics with single-phase DERs and induction machines. An IBM dynamic phasor model in the abc reference frame which can describe the IBM dynamics with a single-phase DER has been proposed by [10]. An IBM model in a mixed abc and dq reference frame is proposed by [11] which is able to describe the IBM dynamics with unbalanced loads. However, the components in the abc reference frames of the aforementioned models have to be linearized to obtain a constant equilibrium point, which neglects the nonlinear dynamics of the IBMs. In addition, the part of the model by [11] in the dq reference frame is based on decoupled sequence components and thus assumes balanced connection of devices. An unbalanced IBM model which preserves the nonlinear dynamics and has constant equilibrium points has not yet been proposed in the literature.

Unbalance can be represented by positive, negative and zero sequence components. Sequence components are commonly used to simplify the analysis of unbalanced faults by decoupling unbalanced networks into three balanced components. However, this assumes a balanced connection of devices i.e. symmetrical components [17], while devices in IBMs such as single-phase DER and loads are usually unbalanced. Unbalanced devices cause the positive, negative and zero sequence components to be coupled [18]. In this case, the state variables in each sequence component model impacts the state variables in the other sequence component models. Dynamic coupled sequence components (CSC) models of IBMs are not yet described or experimentally validated in the literature.

This paper analyzes the coupling between the sequence components and proposes CSC models of three-phase DER, single-phase DER, parallel RLC load and variable frequency drive (VFD) load devices in the dq reference frame. The description of the CSC allow the models to describe device unbalance, while providing a constant equilibrium point. The proposed models are validated with large balanced and unbalanced voltage transients, and are used for EBTSA of an unbalanced case study IBM.

The main contributions of this paper are:
1) Proposition of dynamic CSC models of DER and load devices in the dq reference frame.
2) Experimental validation of the CSC models in the dq reference frame with balanced and unbalanced voltage transients.
3) EBTSA of an unbalanced case-study IBM using the CSC models in the dq reference frame.
4) Analysis of the impact of generation and load unbalance on IBM transient stability.

The CSC models of DER and load devices in the dq reference frame are described in the next section. The models are experimentally validated in section III. In section IV, the transient stability of an unbalanced case study IBM is analyzed with EBTSA. The results are discussed in section VI and conclusions are given in section VII.

II. SEQUENCE MODELS IN THE dq REFERENCE FRAME

A. Coupled sequence components

This subsection analyzes the contribution of the devices to different sequence components and the coupling between the sequence components, before developing the dq CSC models in sections II-C and II-D.

1) Three-phase DERs: In general, four types of DER control modes are distinguished: constant current, constant PQ (grid-feeding), constant V/f (grid-forming) and droop (grid-supporting) control [19]. Three-phase DERs with constant current control only inject positive sequence current, while grid-feeding, grid-forming and grid-supporting controlled three-phase DERs can inject negative sequence current when the negative sequence component is not properly filtered in the controller [20], [21].

This paper considers three-phase DERs operating in either grid-feeding or grid-supporting control mode in the dq reference frame, using either a phase-locked loop (PLL) or droop control for grid synchronization [19]. The controllers are equipped with negative sequence bandstop (notch) filters as discussed by [22]. With proper filtering, the inverters of three-phase DERs only inject positive sequence current, while (relatively small) negative and zero sequence currents are only allowed to flow through the LCL filter capacitor as shown in Figs. 13, 14 and 24, and discussed by [20].

2) Single-phase DERs: Single-phase DERs are usually synchronized to the grid by using a PLL and individually control the power injected into the connected phases [23]. Single-phase DERs can therefore contribute to positive, negative and zero sequence current in the network. The output current of three single-phase DERs $I^{abc}$ is proportional to their power references $S^{abc*}$ divided by the phase voltages $V^{abc}$ as described in (1).

$$I^{abc} = S^{abc*}V^{abc} = \begin{bmatrix} S^a* & 0 & 0 \\ 0 & S^b* & 0 \\ 0 & 0 & S^c* \end{bmatrix} \begin{bmatrix} \frac{1}{V^a} \\ \frac{1}{V^b} \\ \frac{1}{V^c} \end{bmatrix}$$ (1)
The expression of the CSC current injected by single-phase DERs \( I^{012} \) is determined by using (1) with identities \( V^{012} = T^{-1} V^{abc} \), \( I^{012} = T^{-1} I^{abc} \) and \( S^{012} = T^{-1} S^{abc} + T \). The sequence components of the single-phase DER are coupled via expression \( V^{012} \), as the voltage in a sequence component impacts the current in all of the sequence components.

\[
I^{012} = S^{012} V^{012} = S^{012} \begin{bmatrix} \left( \frac{V^0}{V^1} \right)^2 + \frac{1}{V^2} \end{bmatrix} \begin{bmatrix} \frac{V^0}{V^1} + \frac{1}{V^2} \end{bmatrix} \begin{bmatrix} \left( \frac{V^0}{V^1} \right)^2 + \frac{1}{V^2} \end{bmatrix}
\]

Where: \( T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}, a = \cos \left( \frac{2\pi}{3} \right) + j \sin \left( \frac{2\pi}{3} \right) \) (2)

3) Constant impedance loads and branches: The impact of constant impedance load and branches on the positive, negative and zero sequence currents in the network can be determined by transforming the impedance matrix \( Z^{abc} \) to the CSC admittance matrix \( Y^{012} \) using (3). When the off-diagonal elements of the CSC admittance matrix are nonzero, the sequence voltages across the constant impedance loads and branches impact all of the absorbed current sequence components, which are therefore coupled.

\[
Y^{012} = T^{-1} (Z^{abc})^{-1} T = \begin{bmatrix} y^{00} & y^{10} & y^{20} \\ y^{01} & y^{11} & y^{21} \\ y^{02} & y^{12} & y^{22} \end{bmatrix} \]

(3)

4) Three-phase uncontrolled rectifier load devices: An extensive analysis of the characteristics of three-phase uncontrolled rectifiers under unbalanced voltage is given by [24]. During normal six-pulse operation, the positive sequence current \( I^1 \) absorbed by this type of devices mainly depends on the load current, while the negative sequence current \( I^2 \) depends mainly on the voltage unbalance and DC-link capacitor impedance \( \left( \frac{\pi X_c}{3} \right) \). The absorbed sequence currents during six-pulse operation are described by (4), where \( I_{L,dc} \) is the current absorbed by the DC load.

\[
I^{012} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sqrt{\frac{2}{3}} e^{-j\frac{\pi}{3}} & 0 \\ 0 & 0 & \frac{1}{3} X_c \end{bmatrix} \begin{bmatrix} \frac{I_{L,dc}}{V^2} \end{bmatrix} \]

(4)

B. DERs current limiting

The current limiting behavior of DERs should be accurately modeled as it can significantly impact the transient behavior of the DERs [20], [21]. Instantaneous current limiting is often implemented with a non-smooth function (5), where \( I^* \) is the current reference and \( \tilde{I}^* \) is the limited current reference. Sigmoid functions are used by [25] and [26] to model current controller saturation and sliding mode controller saturation with smooth functions respectively. To enable EBTSA, the current limiting behavior of DERs is described by smooth sigmoid function (6) in this paper.

\[
\tilde{I}^* = \begin{cases} I_{lim} & \text{if } |I^*| > I_{lim} \\ I^* & \text{if } |I^*| \leq I_{lim} \end{cases}
\]

(5)

\[
\tilde{I}^* = \frac{I_{lim} I^*}{\sqrt{I_{lim}^2 + (I^*)^2}}
\]

(6)

1) Three-phase DERs: As discussed in section II-A, only the positive sequence components are present in the controllers of three-phase DERs. Therefore, current limiting is only applied to the positive sequence component as described by (13).

2) Single-phase DERs: In case of three single-phase DERs, current limiting of individual DERs on the separate phases should be considered, which can be described in the abc reference frame by (6) for each phase. The expression of the limited CSC current injected by single-phase DERs (7) is then determined by using identity \( I^{012} = T^{-1} I^{abc} \).

\[
\tilde{I}^0 = \frac{I_{lim}}{3} \left( (\phi^a + \phi^b + \phi^c) I^0 + (\phi^a + a^2 \phi^b + a \phi^c) I^1 + (\phi^a + a \phi^b + a^2 \phi^c) I^2 \right)
\]

\[
\tilde{I}^1 = \frac{I_{lim}}{3} \left( (\phi^a + \phi^b + \phi^c) I^0 + (\phi^a + \phi^b + \phi^c) I^1 + (\phi^a + a \phi^b + \phi^c) I^2 \right)
\]

\[
\tilde{I}^2 = \frac{I_{lim}}{3} \left( (\phi^a + \phi^b + \phi^c) I^0 + (\phi^a + \phi^b + \phi^c) I^1 + (\phi^a + \phi^b + \phi^c) I^2 \right)
\]

Where:

\[
\phi^a = \frac{1}{\sqrt{I_{lim}^2 + (I^0 + I^1 + I^2)^2}} \]

\[
\phi^b = \frac{1}{\sqrt{I_{lim}^2 + (a I^0 + I^1 + a I^2)^2}} \]

\[
\phi^c = \frac{1}{\sqrt{I_{lim}^2 + (a^2 I^0 + a I^1 + a^2 I^2)^2}}
\]

(7)

C. DER models

In this section, detailed sequence models of grid-supporting three-phase, grid-feeding three-phase and grid-feeding single-
The DC-link dynamics are described by (8). The models are based on the theory presented in section II-A, phase DER in the \(dq\) current controller.

The positive sequence LCL filter dynamics are described by (10), (11) and (13). The positive sequence current limiting factor \(\phi^1 = \sqrt{I_{\text{lim}}^2 + (I_{\text{lim}}^*)^2} \).

\[
\begin{bmatrix}
  \dot{I}_{dc} \\
  \dot{V}_{Cdc}
\end{bmatrix} = \begin{bmatrix}
  -\frac{R_{dc}}{C_{dc}} & \frac{1}{C_{dc}} \\
  -\frac{1}{C_{dc}} & \frac{1}{C_{dc} L_{dc}}
\end{bmatrix} \begin{bmatrix}
  I_{dc} \\
  V_{Cdc}
\end{bmatrix} + \begin{bmatrix}
  \frac{V_{\text{dc}}}{L_{dc}}
\end{bmatrix}
\]  
\tag{8}

\[
\begin{bmatrix}
  \dot{V}_{\text{iq}}^1 \\
  \dot{V}_{\text{iq}}^2
\end{bmatrix} = \begin{bmatrix}
  K_v \psi_{\text{iq}}^1 + K_p (I_{\text{iq}}^* - I_{\text{iq}}^1) \\
  K_v \psi_{\text{iq}}^2 + K_p (I_{\text{iq}}^* - I_{\text{iq}}^2)
\end{bmatrix}
\]  
\tag{9}

\[
\begin{bmatrix}
  \dot{\theta} \\
  \dot{P}_1 \\
  \dot{Q}_1
\end{bmatrix} = \begin{bmatrix}
  0 & 0 & nq \\
  0 & -\omega_c & 0 \\
  0 & 0 & -\omega_c
\end{bmatrix} \begin{bmatrix}
  \theta \\
  P_1 \\
  Q_1
\end{bmatrix} + \begin{bmatrix}
  \frac{\omega^* - \omega}{2 \omega_c (V_{\text{iq}}^1 R_{\text{iq}}^1 + V_{\text{iq}}^2 R_{\text{iq}}^2)} \\
  \frac{3 \omega_c (V_{\text{iq}}^1 F_{\text{iq}}^1 + V_{\text{iq}}^2 F_{\text{iq}}^2)}{2 \omega_c (V_{\text{iq}}^1 R_{\text{iq}}^1 - V_{\text{iq}}^2 R_{\text{iq}}^2)}
\end{bmatrix}
\]  
\tag{10}

\[
\begin{bmatrix}
  \dot{T}_{ld} \\
  \dot{T}_{lq}
\end{bmatrix} = \begin{bmatrix}
  K_{iv} \psi_{ld} + K_{pv} (-m P_1 + V^*) - K_{pv} V_{\text{iq}}^1 \\
  K_{iv} \psi_{lq} - K_{pv} V_{\text{iq}}^2
\end{bmatrix}
\]  
\tag{12}

\[
\begin{bmatrix}
  \ddot{I}_{ld}^1 \\
  \ddot{I}_{lq}^1
\end{bmatrix} = \begin{bmatrix}
  \frac{V_{\text{ld}}}{L_{\text{ld}}} \\
  \frac{V_{\text{lq}}}{L_{\text{lq}}}
\end{bmatrix}
\]  
\tag{14}

Fig. 3: Proposed \(dq\) sequence model of a three-phase DER with grid-feeding control.

Fig. 4: Grid-feeding DER control in the \(dq\) reference frame. CC: current controller.

As opposed to the droop control, grid-feeding DERs rely on different grid synchronization and controller as shown in Figs. 1 and 2. The DC-link dynamics are described by (8). The three-phase inverter is modeled by a power source \(P\) and voltage source \(V_{\text{edq}}\) described by the inverse droop controller and (9) respectively, where the droop control low-pass filter bandwidth \(\omega_c = 31.41\text{rad/s}\) [14], [27]. The inverse droop, voltage and current controllers are described by (10), (11) and (12), and (13). The positive sequence LCL filter dynamics are described by (14), while the negative and zero sequence LCL filter dynamics are described by (15). The positive sequence current limiting factor \(\phi^1 = \sqrt{I_{\text{lim}}^2 + (I_{\text{lim}}^*)^2} \).

1) Grid-supporting three-phase DER: The grid-supporting DER sequence model proposed in this paper is shown in Figs. 3 and 4. The three-phase inverter is modeled by a power source \(P\) and voltage source \(V_{\text{edq}}\) described by the inverse droop controller and (9) respectively, where the droop control low-pass filter bandwidth \(\omega_c = 31.41\text{rad/s}\) [14], [27]. The inverse droop, voltage and current controllers are described by (10), (11) and (12), and (13). The positive sequence LCL filter dynamics are described by (14), while the negative and zero sequence LCL filter dynamics are described by (15). The positive sequence current limiting factor \(\phi^1 = \sqrt{I_{\text{lim}}^2 + (I_{\text{lim}}^*)^2} \).

2) Grid-feeding three-phase DER: The grid-feeding three-phase DER model is similar to the grid-supporting three-phase DER model as they are the same electrical device with different grid synchronization and controller as shown in Figs. 3 and 4. The grid-feeding three-phase DER model consists of (9), (13) and (14) from the grid-supporting DER model. As the output power of grid-feeding DER is constant, the DC-link voltage is well described by an algebraic equation (16). As opposed to the droop control, grid-feeding DERs rely on a PLL to determine the phase angle as described by (17).
The output power reference determines the output current as described by (18).

\[
V_{C_{dc}} = \frac{V_{DC} + \sqrt{V_{DC}^2 - 4P_{1s}R_{dc}}}{2} \tag{16}
\]

\[
\left[ \gamma \right] = \left[ \begin{array}{c}
\begin{array}{c}
\gamma \\
\theta \\
\end{array}
\end{array} \right] = \left[ \begin{array}{c}
\begin{array}{c}
K_{ip} V_{c_{dq}} + \omega_s \\
K_{ip} V_{c_{dq}} + \omega_s \\
\end{array}
\end{array} \right] \tag{17}
\]

\[
\begin{bmatrix}
I_{id}^* \\
I_{iq}^*
\end{bmatrix} = \begin{bmatrix}
2P_{1s} & 0 \\
0 & 2Q_{1s}
\end{bmatrix} \begin{bmatrix}
\omega_s \\
\omega_s
\end{bmatrix} \tag{18}
\]

3) Grid-feeding single-phase DER: The grid-feeding single-phase DER model is similar to the grid-feeding three-phase DER model. However, as discussed in section II-A, single-phase DERs contribute to all sequence components, which can be represented by the same model for each sequence component with different output power references as shown in Figs. 5 and 6. The sequence current references (19) are determined from the output power references \(S_{101s} = P_{010s} + Q_{010s}\) and \(d\)-axis LCL filter voltage \(V_{cd}^0\) based on the CSC transformation (2). The positive, negative and zero sequence current controllers are described by (20). The inverter output voltage is described by (21), while the positive, negative and zero sequence LCL filter dynamics are described by (14) as opposed to the only positive sequence dynamics in the three-phase DER model. The phase angle is determined from the positive sequence \(q\)-axis voltage as described by (17).

\[
I_{id}^{012s} = P_{010s} V_{cd}^0 \quad I_{iq}^{012s} = Q_{010s} V_{cd}^0 \tag{19}
\]

\[
\begin{bmatrix}
I_{id}^{012s} \\
I_{iq}^{012s}
\end{bmatrix} = \begin{bmatrix}
I_{id}^{012s} - I_{id}^{012s} \\
I_{iq}^{012s} - I_{iq}^{012s}
\end{bmatrix} \tag{20}
\]

\[
\begin{bmatrix}
V_{id}^{012s} \\
V_{iq}^{012s}
\end{bmatrix} = \begin{bmatrix}
K_{ic} s_{id} + K_{pc} (I_{id}^{012s} - I_{id}^{012s}) \\
K_{ic} s_{iq} + K_{pc} (I_{iq}^{012s} - I_{iq}^{012s})
\end{bmatrix} \tag{21}
\]

D. Load and branch sequence models

1) Parallel RLC load: The dynamics of the parallel RLC load are described by (22), (23) and (24) in terms of capacitor voltage, inductor current and resistor current respectively.

Where, \(I_{012s}\) are the \(dq\) sequence currents absorbed by the VFD load, and \(V_{C_{cd}}^0\) and \(V_{L_{R}}^0\) and \(V_{L_{L}}^0\) are determined by (3).

\[
\begin{aligned}
V_{C_{cd}}^0 &= - \text{Re}(Y_{C_{cd}}^0(I_{bdq}^0 - I_{bdq}^0 - I_{bdq}^0)) \\
&- \text{Im}(Y_{C_{cd}}^0(I_{bdq}^0 - I_{bdq}^0 - I_{bdq}^0)) + \omega V_{C_{cd}}^0 \\
V_{C_{dq}}^0 &= - \text{Re}(Y_{C_{dq}}^0(I_{bdq}^0 - I_{bdq}^0 - I_{bdq}^0)) \\
&- \text{Im}(Y_{C_{dq}}^0(I_{bdq}^0 - I_{bdq}^0 - I_{bdq}^0)) + \omega V_{C_{dq}}^0 \\
\end{aligned} \tag{22}
\]

\[
\begin{aligned}
I_{L_{d}}^0 &= - \text{Re}(Y_{L_{d}}^0 V_{C_{cd}}^0) - \text{Im}(Y_{L_{d}}^0 V_{C_{cd}}^0) \\
&+ \text{Im}(Y_{L_{d}}^0 V_{C_{cd}}^0) \omega V_{C_{cd}}^0 \\
I_{L_{q}}^0 &= - \text{Re}(Y_{L_{q}}^0 V_{C_{cd}}^0) - \text{Im}(Y_{L_{q}}^0 V_{C_{cd}}^0) \\
&- \text{Im}(Y_{L_{q}}^0 V_{C_{cd}}^0) \omega V_{C_{cd}}^0 \\
\end{aligned} \tag{23}
\]

2) Variable frequency drive load: The VFD sequence model proposed in this paper is shown in Fig. 7. The three-phase rectifier of the original model is replaced by a transformer, where \(S_{d} = \frac{3}{2} \frac{2}{\sqrt{3}}\) as described by [28]. The DC-link dynamics are then described by (25). The three-phase inverter is replaced by voltage source \(V_{sd} = \frac{3}{2} V_{sd, V_{sq}}\) and a power source \(P_{e} = \frac{3}{2} (V_{sd} I_{sd} + V_{sq} I_{sq})\). The speed and direct torque controllers are described by (26) and (27), where \(T_{lim} = 75 \text{Nm} \) and \(\psi^*\) is the nominal flux. The synchronous reference frame induction motor model is described by (29), where \(V_{sd}, V_{sq}, I_{sd}, I_{sq}\) and \(\psi_{sd}, \psi_{sq}\) are the \(dq\) stator voltages, currents and fluxes, and \(\omega_m\) is the rotational speed. As described by (4), the positive and negative sequence currents absorbed by the VFD load \(I_{id}^1, I_{iq}^1\) and \(I_{id}^2, I_{iq}^2\) are modeled by (28).

\[
\begin{bmatrix}
\dot{V}_{C_{dc}} \\
\dot{I}_{dc}
\end{bmatrix} = \begin{bmatrix}
-\frac{R_{dc}}{L_{dc}} & -\frac{1}{C_{dc} L_{dc}} \\
\frac{-1}{C_{dc} L_{dc}} & \frac{1}{C_{dc} L_{dc}}
\end{bmatrix} \begin{bmatrix}
I_{dc} \\
V_{C_{dc}}
\end{bmatrix} + \begin{bmatrix}
\sqrt{3} \frac{2}{\sqrt{3} L_{dc}} V_{bdq}^1 \\
0
\end{bmatrix} \tag{25}
\]

\[
\begin{bmatrix}
\dot{V}_{sd} \\
\dot{V}_{sq}
\end{bmatrix} = \begin{bmatrix}
K_{pf} \psi^* - K_{pf} \sqrt{\psi_{sd}^2 + \psi_{sq}^2} \\
K_{pf} \psi^* + K_{pf} \frac{3}{2} \psi_{sq} + \frac{3}{2} K_{pf} I_{sd} \psi_{sd}
\end{bmatrix} \tag{26}
\]

\[
\begin{bmatrix}
\dot{\phi}_q \\
\dot{\phi}_q
\end{bmatrix} = \begin{bmatrix}
0 \\
\frac{3}{2} I_{sd} \psi_{sq} - \frac{3}{2} I_{sd} \psi_{sd}
\end{bmatrix} \tag{27}
\]

\[
\begin{bmatrix}
I_{id}^1 \\
I_{iq}^1 \\
I_{id}^2 \\
I_{iq}^2
\end{bmatrix} = \begin{bmatrix}
\sqrt{\frac{2}{3}} S_d & 0 & 0 \\
0 & \sqrt{\frac{2}{3}} S_d & 0 \\
\frac{3}{\pi N_{C_{dc}}} \frac{1}{V_{C_{dc}}^2} & 0 & \frac{3}{\pi N_{C_{dc}}^2}
\end{bmatrix} \begin{bmatrix}
I_{dc} \end{bmatrix} \tag{28}
\]
3) Branch and grounding transformer: The branch model proposed in this paper is based on the four-wire series RL model, which is commonly used to model unbalanced distribution networks. The branch impedance is considered to be symmetrical and can therefore be modeled by decoupled sequence components. The sequence currents through the branch model are described by (29), where \( V_{bi} \) and \( V_{bj} \) are the voltages at the sending and receiving end of the branch.

\[
\begin{bmatrix}
I_{\text{line}012} \\
I_{\text{line}012}
\end{bmatrix} =
\begin{bmatrix}
-R_{\text{line}012} + R_{\text{line}} & \omega \\
-\omega & -R_{\text{line}012} + R_{\text{line}}
\end{bmatrix}
\begin{bmatrix}
V_{\text{line}012} \\
V_{\text{line}012}
\end{bmatrix}
\]

(29)

Grounding transformers are designed to have a very high positive and negative sequence impedance, and a low zero sequence impedance [17]. Grounding transformers can therefore be modeled as branches in the zero sequence component, and as an open circuit in the positive and negative sequence components.

E. Alignment of the \( dq \) reference frame in different operation modes

The alignment of the \( dq \) reference frame is chosen depending on the operational mode of the IBM. During grid-connected operation, the \( dq \) reference frame is aligned with the external grid to which the IBM is connected, therefore the frequency of the \( dq \) reference frame is equal to the external grid frequency (\( \omega = \omega_n = 100\pi \)). During islanded operation the \( dq \) reference frame is aligned with the phase angle of a DER in the IBM as proposed by [14]. The frequency is determined by the PLL (\( \omega = K_{\text{ipd}}\gamma + K_{\text{pfd}}V_{\text{eq}} + \omega_n \)) or by the droop controller (\( \omega = nq\omega_1 + \omega_n \)) of the chosen DER.

III. Experimental Validation

To validate the accuracy of the CSC models proposed in the last section, experimental results are compared to simulation results as shown in Fig. 8. The determination of the model parameters is described in [30]. During the experiments, balanced and unbalanced voltage transients \( V_m(t) \) with a duration of 200ms are generated by a California Instruments MX45-3PI programmable voltage source (PVS) and provided to a three phase DER, single phase DERs, VFD load and constant impedance load devices in balanced and unbalanced connections. The current absorbed or injected by the devices \( I_m(t) \) is measured by a Hioki PW6001 power analyzer with Hioki CT6841-05 and Hioki CT6843-05 current clamps. The devices used for experimental validation shown in Figs. 11, 9 and 10 are:

- KEB F5 inverter (17kVA, Fig. 11)
- 3x Soladin Web 3000/2200 (3x 3/2.2kVA, Fig. 9)
- Cressall AC30 (30kVA, Fig. 10)
- ABB ACS550 with ABB M2QA160M4A (11kVA, Fig. 10)

A DC source is used as primary source of the three-phase inverter, while the primary sources of the single-phase inverters are emulated by an Ametek TerraSAS photovoltaic simulator ETS (10kVA) as shown in Fig. 11.

As shown in Fig. 8, the voltage signals produced by the PVS are recorded, transformed to sequence components and provided to the models described in the last section. To determine the error between the measured and simulated current, the measured current signals are transformed to sequence components and compared to the simulated sequence current. The normalized mean absolute errors (NMAEs) are determined with (30), where \( I_m \) is the magnitude of the current absorbed or injected by the device under nominal and balanced voltage. To validate the pre-, on- and post-transient behavior of the proposed models, the NMAEs are determined 200ms before, during and after the transient for a total period of \( T_{er} = 600\text{ms} \).

\[
e^{012} = \frac{1}{T_{er}I_m} \int_{t=0}^{T_{er}} |I_m(t) - I_m^{012}(t)| dt
\]

(30)
The measurements are performed with an unbalanced steady-state voltage $V^{abc} = [0.6; 0.8; 1.0]$ p.u. (S), a balanced voltage transient to $V^{abc} = [0.6; 0.6; 0.6]$ p.u. (T1) and four unbalanced voltage transients to $V^{abc} = [0.5; 0.6; 0.7]$ p.u. (T2), $V^{abc} = [0.4; 0.6; 0.8]$ p.u. (T3), $V^{abc} = [0.3; 0.6; 0.9]$ p.u. (T4) and $V^{abc} = [0.2; 0.6; 1.0]$ p.u. (T5). The three-phase DER is tested with a power output of $P^{11*} = 3$ kW and a current limit of $I_{lim} = 41.5$ A (TP1), and a current limit of $I_{lim} = 8.2$ A (TP2). The measurements of the single-phase DERs are performed with three different power outputs of $\text{diag}(S^{abc}) = [1.8; 1.8; 1.8]$ kW (SP1), $\text{diag}(S^{abc}) = [1.8; 0.9; 0.9]$ kW (SP2), $\text{diag}(S^{abc}) = [1.8; 0.0; 0.0]$ kW (SP3) and $\text{diag}(S^{abc}) = [3.0; 0.9; 0.9]$ kW (SP4). The VFD load has a constant torque load of 35 Nm (VFD). The measurements of the constant impedance load are performed with a resistance of $\text{diag}(R^{abc}) = [16.0; 16.0; 16.0]$ Ω (R1), $\text{diag}(R^{abc}) = [16.0; 31.1; 31.1]$ Ω (R2) and $\text{diag}(R^{abc}) = [16.0; \infty; \infty]$ Ω (R3).

IV. CASE STUDY

A. Description

The islanded case study IBM considered in this paper is shown in Fig. 12. DER1 has a power rating of 10 kVA and is in grid-supporting control mode with active and reactive droop control parameters of 0.025 p.u. and 0.05 p.u. respectively. DER2 consists of three single-phase grid-feeding DERs with individual power references and a total power rating of 10 kVA. The parallel RLC load consists of three single-phase loads, while the VFD load is connected to all three phases. The parameters of the DER and VFD load devices are equal to those determined in [30]. The branches in the network consist of four-wire 50 mm$^2$ XLPE underground power cables with a wire resistance and reactance of 0.71 Ω/km and 0.075 Ω/km respectively, while the grounding transformer allows zero-sequence current to flow.

The impact of unbalanced operation on the transient stability of the IBM is demonstrated by determining the domain of attraction (DOA) with EBTSA. The results of EBTSA are compared to a large set of time-domain simulations around the equilibrium point with different initial conditions. The size of the DOA is determined in three different scenarios. In scenario 1, the single-phase RLC loads have a balanced impedance of $R = 11.1$ Ω, $X_L = X_C = 33.9$ Ω and the single-phase DER2 have balanced power references of $S^{abc*} = 3.33 + 0$ jkVA. In scenario 2, the power reference of DER2 $S^{a*} = 8.33 + 0$ jkVA, while $S^{b*} = S^{c*} = 0.83 + 0$ jkVA. In scenario 3 the RLC load resistance $R^c = 4.45$ Ω, while $R^a = R^b = 44.5$ Ω. The VFD load torque is equal to 20 Nm in all scenarios.

To analyze the impact of current limiting on the DOA of the case study IBM, scenario 2 is also analyzed with where the current limit of DER1 and DER2 is reduced from 2.0 p.u. to 1.2 p.u.

B. Energy-Function Based Transient Stability Analysis

The energy-function based stability analysis performed in this paper is based on the methodology proposed by [13]. In order to perform the analysis, the system nonlinear state-space model should be autonomous. An IBM model is generated in the form $\dot{x} = A(x)x + b$ by using the model proposed in section II. Subsequently, the autonomous state-space model $\dot{x} = A(x)x$ is generated by performing change of variables $\ddot{x} = x - x_0$, where $x_0$ are the system states in the equilibrium point [31].

As proposed by [13], a fuzzy model of $r = 2^q$ matrices are generated from the system matrix $A(x)$ by replacing the each unique nonlinear term in the system matrix by all possible combinations of $q$ functions $f_{\min}$ and $f_{\max}$, where $q$ is the number of unique nonlinear terms. Initially the value of $f_{\min}$ and $f_{\max}$ are set to zero. The DOA can be found by iteratively
To improve conditioning of the fuzzy model matrices $A_i$ and enable the EB TSA to be performed within reasonable time, some model simplifications are performed. To prevent an empty system matrix column caused by the droop controller and PLL phase angle states (10) and (17), the trigonometric variables are replaced by their 7th order Taylor expansions. To reduce the number of linear matrix inequalities to be solved, the VFD torque limit is removed, the VFD input voltage is equal to the $d$-axis voltage and the single-phase DER current reference calculation is based on the $d$-axis voltage.

\[
A_i^T(f_{\text{min}}, f_{\text{max}})P + PA_i(f_{\text{min}}, f_{\text{max}}) < 0, \forall i \in \{1, ..., r\} 
\]  

(31)

C. Modeling inaccuracy sensitivity analysis

As discussed in section V-A, the main model inaccuracies are present in the single-phase DER positive, negative and zero sequence current, and the VFD negative sequence current. To analyze the impact of the model inaccuracies on the estimated DOA and especially whether these model inaccuracies can lead to overestimation the DOA, a modeling inaccuracy sensitivity analysis is performed.

As shown in Figs. 15 and 17, the peak error of the sequence currents injected by the single-phase DER and the negative sequence current absorbed by the VFD load are approximately 0.25p.u. and 0.5p.u. of the positive sequence current in steady-state for a duration of 200ms. The sensitivity of overestimation of the DOA of the case study IBM due to the model inaccuracies is therefore analyzed by performing time-domain simulations, while the current sources indicated in red in Fig. 12 absorb currents $I_{\text{DER}}^{012}$ and $I_{\text{VFD}}^{012}$ for 200ms according to table I.

V. RESULTS

A. Experimental validation

The positive, negative and zero sequence NMAE of the proposed CSC models during steady-state and voltage transients are shown in table II. The NMAE in both steady-state and transient situations is low, which indicates that the proposed CSC models accurately represent the physical devices. The measured ($I_{m}^{012}$) and simulated ($I_{s}^{012}$) sequence current of the three-phase DER, single phase DERs, VFD load and constant impedance load under voltage transient T3 is shown in Figs. 13, 14, 15, 16, 17 and 18.

1) Three-phase DER: The current of the three-phase DER model closely matches the measured current before, during and after the transient as shown in Figs. 13 and 14. The physical device outputs a small negative sequence current which is not present in the model, causing a minor inaccuracy. The current limiter reduces the positive sequence current during the transient and is accurately represented in the proposed model as shown in Fig. 14.

2) Single-phase DER: The current of the single-phase DERs model with unbalanced power references SP2 and SP4 closely match the measured current before and after the voltage transient as shown in Figs. 15 and 16. Due to the low voltage of phase $a$ during the transient, the single-phase DER device connected to this phase briefly injects a reduced current between $t = 0.34s$ and $t = 0.39s$. This causes the simulated current to be smoother than the measured current during the transient, which causes some inaccuracy. The current limiter reduces the positive, negative and zero sequence currents and is accurately represented in the proposed model as shown in Fig. 16.

3) Variable frequency drive load: As shown in Fig. 17, the positive and zero sequence currents of the VFD load model closely matches the measured currents. Due to the highly unbalanced voltage during the transient, the VFD load device briefly enters two-pulse operation between $t = 0.26$ and $t = 0.35$. Since the negative sequence current absorbed by the proposed model is valid for six-pulse operation as described in section II-A, there is some inaccuracy in the negative sequence current. As described by [24] and shown in Fig. 17, the negative sequence current during the two-pulse operation period is accurately described by $I_0^2 = I_1^1$. 

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{Var1} & \text{Var2} & \text{Var3} & \text{Var4} \\
\hline
I_{\Delta \text{DER}}^{012} & 0.25 & 0.5 & 0.75 & 1.0 \\
I_{\Delta \text{VFD}}^{012} & 0.25 & 0.5 & 0.75 & 1.0 \\
I_{\Delta \text{DER}}^{012} & 0.5 & 1.0 & 1.5 & 2.0 \\
\hline
\end{array}
\]
TABLE II: Positive, negative and zero sequence NMAE of the proposed models during steady-state and voltage transients.

<table>
<thead>
<tr>
<th></th>
<th>T1</th>
<th>T2</th>
<th>T3</th>
<th>T4</th>
<th>T5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S</td>
<td>z</td>
<td>p</td>
<td>n</td>
<td>z</td>
</tr>
<tr>
<td>TP1</td>
<td>0.005</td>
<td>0.011</td>
<td>0.001</td>
<td>0.014</td>
<td>0.013</td>
</tr>
<tr>
<td>TP2</td>
<td>0.005</td>
<td>0.011</td>
<td>0.001</td>
<td>0.006</td>
<td>0.004</td>
</tr>
<tr>
<td>SP1</td>
<td>0.003</td>
<td>0.012</td>
<td>0.009</td>
<td>0.035</td>
<td>0.002</td>
</tr>
<tr>
<td>SP2</td>
<td>0.003</td>
<td>0.012</td>
<td>0.006</td>
<td>0.027</td>
<td>0.011</td>
</tr>
<tr>
<td>SP3</td>
<td>0.005</td>
<td>0.018</td>
<td>0.015</td>
<td>0.041</td>
<td>0.044</td>
</tr>
<tr>
<td>VFD</td>
<td>0.006</td>
<td>0.010</td>
<td>0.000</td>
<td>0.030</td>
<td>0.024</td>
</tr>
<tr>
<td>R1</td>
<td>0.000</td>
<td>0.002</td>
<td>0.007</td>
<td>0.003</td>
<td>0.003</td>
</tr>
<tr>
<td>R2</td>
<td>0.000</td>
<td>0.002</td>
<td>0.001</td>
<td>0.003</td>
<td>0.001</td>
</tr>
<tr>
<td>R3</td>
<td>0.000</td>
<td>0.002</td>
<td>0.001</td>
<td>0.002</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Fig. 15: Measured $I_{12}^{0}$ and simulated $I_{12}^{0}$ sequence currents injected by the single phase DERs with power references SP2 during voltage transient T3 from $t = 0.22s$ to $t = 0.42s$.

Fig. 16: Measured $I_{12}^{0}$ and simulated $I_{12}^{0}$ sequence currents injected by the single phase DERs with power references SP4 during voltage transient T3 from $t = 0.22s$ to $t = 0.42s$.

4) Constant impedance load: The current of the constant impedance load model with unbalanced impedance R2 closely matches the measured current before, during and after the transient as shown in Fig. 18.

B. Case study

1) Domain of attraction: The case study IBM model has a total of 75 states. To allow a concise analysis, this subsection focuses on the DER1 DC-link voltage ($V_{C_{dc,DER1}}$), VFD DC-link voltage ($V_{C_{dc,VFD}}$) and phase angle between DER2 and DER1 ($\theta_{DER2} - \theta_{DER1}$) of the case study IBM model. These states are chosen as they (i) reflect both power supply and balance stability, and control system stability as defined by [3], and (ii) occur in the nonlinear terms of the system matrix $A(x)$. The maximum deviations of $V_{C_{dc,DER1}}$, $V_{C_{dc,VFD}}$ and $\theta_{DER2} - \theta_{DER1}$ from the equilibrium point determined by the EBTSA and time-domain analysis in the three scenarios are shown in table III. Additionally, the DOAs estimated by EBTSA using the proposed CSC models (DOAu) and using only positive sequence models (DOAb) are compared to time-domain analysis results in the $V_{C_{dc,DER1}}, V_{C_{dc,VFD}}$ plane in Figs. 19, 20 and 21.

The results of the EBTSA are more conservative than time-domain analysis in all scenarios. When $V_{C_{dc,DER1}}$ becomes too low, the output power of DER1 is reduced. If the output power of DER1 becomes too low, instability occurs due to a lack of positive sequence power generation. A low $V_{C_{dc,VFD}}$ increases the power absorbed by the VFD and reduces the rotational speed of the VFD, which in turn causes the VFD to absorb more power to increase the rotational speed back to nominal. When the absorbed power becomes too high for the available generation capacity, instability occurs. A large $\theta_{DER2} - \theta_{DER1}$ causes circulating current to occur, reducing the power injected into the loads. When $\theta_{DER2} - \theta_{DER1}$ becomes too large, the system does not converge to the stable equilibrium. Since the IBM is balanced in scenario 1, the DOAs estimated by EBTSA with the proposed CSC models
TABLE III: Maximum state deviations from the equilibrium point determined by time-domain/energy-function based analysis in different scenarios.

| Scenario | $V_{Cdc,DER1}$ (V) | $V_{Cdc,VFD}$ (V) | $|\theta_{DER2} - \theta_{DER1}|$ (rad) |
|----------|-------------------|------------------|---------------------------------|
| Scenario 1 | 600/480          | 395/263          | 2.15/1.57                       |
| Scenario 2 | 400/280          | 198/154          | 2.15/1.26                       |
| Scenario 3 | 720/520          | 417/285          | 2.15/1.73                       |

(DoAu) and only positive sequence models (DoAb) are equal as shown in Fig. 19.

The maximum $V_{Cdc,DER1}$ and $V_{Cdc,VFD}$ deviation in scenario 2 is smaller than in scenario 1, as the reduced positive sequence power injection by DER2 requires DER1 to inject more positive sequence power. When the proposed CSC models are used, the DOA estimated by EBTSA (DoAu) is also smaller than in scenario 1 as shown in Fig. 20. However, when only positive sequence models are used, the estimated DOA (DoAb) is larger than the actual DOA.

The maximum $V_{Cdc,DER1}$ deviation in scenario 3 is larger than in scenario 1, as the total positive sequence load in the network is reduced. When the proposed CSC models are used, the DOA estimated by EBTSA (DoAu) is also larger than in scenario 1 as shown in Fig. 21. The estimated DOA with only positive sequence models (DoAb) is equal to scenario 1.

2) Impact of current limiting: The stability boundary determined with time-domain analysis and the DOA estimated with EBTSA of the case study IBM in scenario 2 with DERs current limits of 2.0 p.u. and 1.2 p.u. are shown in Fig. 22. The results show that stricter current limits reduce the size of the DOA and therefore the transient stability of the case study IBM.

The load voltage, and output current of DER1 and DER2 for scenario 2 and initial conditions $\Delta x_{TDA} = [-200, -95]$ V (also indicated in Fig. 22) with the original and reduced DERs current limits are shown in Figs. 23, 24 and 25. The dq sequence component values of the proposed models are transformed to the abc reference frame with matrix $T$ in (2). Both the proposed CSC models and switched models [30] show that the case study IBM with original current limits converges to a stable equilibrium point within the acceptable operating region, while the reduced current limits prevent the IBM from converging to this equilibrium point, resulting in a very low network voltage.

3) Modeling inaccuracy sensitivity analysis: The stability boundary of the case study IBM with different currents $I_{\Delta DER}^{12}$ and $I_{\Delta VFD}^{2}$ according to Table I are compared to the original stability boundary and estimated DOA for scenarios 2 and 3 in Figs. 26 and 27 respectively. As shown in the figures, $I_{\Delta DER}^{12}$ and $I_{\Delta VFD}^{2}$ reduce the size of the DOA as they can cause a temporary generation deficiency, which in turn can lead to instability.

As discussed in section IV-C, Var1 gives an upper boundary of the impact of the inaccuracies of the proposed models, which does not cause overestimation of the DOA as shown in Figs. 26 and 27. Slight overestimation of the DOA with EBTSA occurs in scenario 2 when $I_{\Delta DER}^{12} >= 0.5$ p.u. and $I_{\Delta VFD}^{2} >= 1.0$ p.u., and in scenario 3 when $I_{\Delta DER}^{12} >= 0.75$ p.u. and $I_{\Delta VFD}^{2} >= 1.5$ p.u.

4) Computational burden: The computations are performed on a desktop PC with an Intel Xeon E5 CPU and 24 GB of RAM. The case study IBM model is developed in Matlab using the CSC models described in section II and in Matlab/Simulink using the component-based models described in [30]. To determine the 2-dimensional DOAs shown in Figs. 19, 20, 21, 26 and 27, time-domain simulations with $C = c' = 1681$ initial conditions are performed for each DOA using the Matlab ode15s solver, where $c = 41$ is the number of tested values for each system state and $v$ is the number of dimensions. To perform the EBTSA, the LMIs described in (31) are constructed using the YALMIP toolbox.
and solved using the MOSEK solver. The average computation time to determine a 2-dimensional DOA with time-domain analysis is 2128 seconds, while the average computation time of estimating a DOA with EBTSA is 49.3 seconds.

To determine the DOA for all system states/dimensions with time-domain analysis, time-domain simulations of $C^75$ initial conditions have to be performed, which is unfeasible within reasonable time. In contrast, the average computation time of EBTSA to determine a DOA for all 75 system states is 13871 seconds.

VI. DISCUSSION

The proposed CSC models in the $dq$ reference frame described in section II accurately represent the physical devices used for the experimental validation in section III. The main inaccuracies are present in the positive, negative and zero sequence current of the single-phase DER model, and the negative sequence current of the VFD load. The inaccuracy of the single-phase DER is due to a brief reduction of output power by the physical device in case of a voltage transient to less than 0.5p.u.. The inaccuracy of the VFD is due to the switch from (normal) six-pulse to two-pulse operation in case of large voltage unbalance. This can be taken into account by changing the description of the negative sequence current to $I^2 = I^1$ in heavily unbalanced networks as described by [24]. As analyzed with the sensitivity analysis, the model inaccuracies are within the error margin granted by the conservativeness of EBTSA.

The results of the case study IBM show that unbalanced connection of DER and load devices can significantly impact the transient stability of islanded IBMs. The reduction of the DOA is visible with the proposed CSC models, but not with commonly used positive sequence models. This indicates that it is critical to use the proposed CSC models for EBTSA of unbalanced IBMs. The case study time-domain analysis and EBTSA results also show that current limiting reduces the size of the DOA, and that the current limiting behavior of DERs is accurately described by the proposed CSC models.

Although EBTSA of IBMs is much faster in estimating the DOA than exhaustive time-domain simulations with different initial conditions as discussed in section V-B4, the computational burden of EBTSA of highly nonlinear IBMs is relatively high. To allow EBTSA of IBMs within reasonable time, the IBM model is often slightly simplified as described by [7] and in section IV-B of this paper. To allow EBTSA of IBM models without simplifications, the computational burden of EBTSA of IBMs will be reduced in future research by analyzing the system as interconnected subsystems, as proposed for LTI systems by [32].
Fig. 26: Stability boundaries of the original and sensitivity analysis IBM models determined with time-domain analysis, and the original DOA estimated with EBTSA in scenario 2.

Fig. 27: Stability boundaries of the original and sensitivity analysis IBM models determined with time-domain analysis, and the original DOA estimated with EBTSA in scenario 3.

VII. CONCLUSION

The proposed CSC DER and load models in the dq reference frame can describe both voltage unbalance and device unbalance, and allow EBTSA of unbalanced IBMs. Extensive experimental validation shows that the proposed models accurately represent the steady-state and transient behavior of physical devices. The proposed models are used for EBTSA of an islanded case study IBM.

The results show that EBTSA accurately determines the DOA when the proposed CSC models are used, while EBTSA with commonly used positive sequence models can over- or underestimate the DOA of unbalanced IBMs. The DOA becomes smaller when DERs are more unbalanced and the DOA becomes larger when the constant impedance load is more unbalanced. Future research should focus on reducing the computational burden of EBTSA.

REFERENCES


as predictive and corrective grid control functions. Deep learning, real-time system awareness using (IoT) data integrity, as well communication networks. His research of interests includes data analytics with domains of mathematical programming, stochastics, data mining, and composite networks and 2013. Dr. Nguyen has committed his research effort to realize synergies of advanced monitoring and control functions for the distribution networks. This distinctive combination of competences allows him to develop a research pathway crossing over various domains of mathematical programming, stochastics, data mining, and communication networks. His research of interests includes data analytics with deep learning, real-time system awareness using (IoT) data integrity, as well as predictive and corrective grid control functions.


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