Joint ordering in multiple news-vendor problems : a game-theoretical approach

Document status and date:
Published: 01/01/2001

Document Version:
Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

• A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
• The final author version and the galley proof are versions of the publication after peer review.
• The final published version features the final layout of the paper including the volume, issue and page numbers.

Link to publication

Download date: 23. Sep. 2021

Joint ordering in multiple news-vendor problems: a game-theoretical approach[*]

Marco Slikker[†]   Jan Fransoo[†]   Marc Wouters[‡]

September 3, 2001

Abstract

We study a situation with n retailers, each of them facing a news-vendor problem, i.e., selling to customers over a finite period of time (product with a short life cycle, such as fashion). Groups of retailers might improve their expected joint profit by cooperating. We analyze these situations by defining a cooperative game, called a general news-vendor game, for such a situation with n retailers. We concentrate on whether it makes sense to cooperate by studying properties of general news-vendor games. Besides some results on convexity we prove that general news-vendor games have non-empty cores, which answers an open question of Hartman et al. (2000).

1 Introduction

Consider a group of retailers who buy from the same supplier and who have to place their purchase orders well in advance of receiving customer orders. For example, imagine a supplier located in Asia and retailers located in Europe who have to place orders for a seasonal product. They have to place their orders in advance of knowing actual demand to cover the manufacturing and transportation lead time. After all orders have been received, the supplier has to make a release decision on how much to produce in total (i.e., for all retailers). However, it is still possible to postpone the allocation decision (i.e., which portion of the quantity manufactured to allocate to each of the retailers). This raises the question of whether in such a setting it is always beneficial for companies to cooperate and to order jointly. In this paper we use cooperative game theory to analyze this question. We look at the benefits for the total supply chain and at benefits for the individual companies, assuming that under cooperation demand information is shared and orders are based on combined demand.

Companies may benefit even more from cooperation by not sharing all information. For example, they may have private information about actual demand as it materializes after placing the order and they may use this information to their own benefit. In case of shortages, they order more than they really need while expecting to actually receive what they anticipate to need (shortage gaming, see Lee et al. (1997)). See, for example, Cachon and Lariviere (1999) for a model in a news-vendor setting where retailers have private information about demand and retailers can influence their allocation through their orders. Cachon and Lariviere (1999) investigate allocation rules that maximize expected total profit and are attractive for the individual companies. They show that supply chains may not benefit from allocation rules that lead retailers to tell the truth (i.e., order exactly their needs). We refer to Cachon (1999) for a review and analysis of non-cooperative game theory in supply chain settings. Further, single period supply chain ordering (usually referred to as contracts), possibly consisting of two consecutive decisions (initial order - reallocation) is reviewed by Tsay et al. (1999).

Strategic behavior with non-cooperative gaming is outside the scope of this paper. We consider a general news-vendor situation in a supply chain consisting of a single supplier (wholesaler) and n retailers. The retailers order the same product from a single supplier and resell the product to consumers. Each retailer i orders a quantity q_i at the supplier, who in its turn orders a quantity q at the manufacturer of the goods. Every retailer experiences stochastic demand and realization of demand is not known at the moment of ordering. We consider a single-period demand, i.e., items that are delivered and are not sold are being disposed of, and inventory is not carried. We show

that expected profits would be higher if the retailers would order jointly. Besides price-effects, this is because if some companies have ordered more and others have ordered less than they can sell, products are transferred between these companies. This builds on the traditional news-vendor problems (see, e.g., Silver et al. (1998) and many other textbooks; see also Khouja (1999) for a review). Although ordering jointly is collectively rational, the feasibility of such an arrangement depends on whether the expected profits of individual companies increase as a result of cooperation. We use cooperative game theory to investigate if it makes sense for companies to cooperate.

Modeling inventory situations as newsboy problems does not only provide us with some nice properties of the model, it also exemplifies the increasing importance of style goods in industry as discussed by Fisher and Raman (1996). Newsboy models are single period models, which means that inventory is not carried over to another period. Furthermore, any remaining products at the end of the period can be disposed of at a certain expense, or can be sold at a lower price than the market price. Initially, this type of modeling was applied to products with very high perishability, such as newspapers. Later, especially in the fashion industry, newsboy models were proven to be of use (see Fisher and Raman (1996) who study the single period setting in the fashion industry), and following the decrease of product life cycles in high-tech, such as personal computers and mobile phones, newsboy models are now well-accepted to model ordering decisions in these environments (see, e.g., Tayur et al. (1999), for a series of papers using this setting). This means that the newsboy setting studied in this paper has been widely accepted as one of practical relevance.

We formulate a coalitional game associated with a general news-vendor situation that we call a general news-vendor game. This coalitional game corresponds essentially to the inventory centralization game introduced by Hartman et al. (2000). The main difference being that they concentrate on costs of a coalition, whereas we concentrate on profits. Furthermore, extending the model of Hartman et al. (2000), we allow that different retailers may experience different wholesale and/or customer prices. We refer to the games corresponding to the situations that were studied by Hartman et al. (2000) as news-vendor games (as opposed to general news-vendor games). Hartman et al. (2000) remark that Hartman and Dror (1997) show by means of a simple 3-person example that news-vendor games are not necessarily convex. Furthermore, they prove that news-vendor games have a non-empty core for specific demand distributions of the retailers. A non-empty core means that no group of players has an incentive not to cooperate, because with cooperation there exists a division of the joint profit that gives each group of players at least as much as they can obtain for themselves. Hartman et al. (2000) end with an open question, which can be reformulated as follows: 'Do news-vendor games

have a non-empty core?'. In this paper we answer this question affirmatively for the class of general news-vendor games. Furthermore, we show that a news-vendor game is convex if the companies face independent normally distributed stochastic demands. Convexity does not necessarily hold if the demand distribution is not normal or if demand is not independent.

News-vendor games have been studied in the literature before. Gerchak and Gupta (1991) compare four simple allocation rules in a continuous review single period inventory model with complete back-ordering. They show that individual stores may be unhappy. Robinson (1993) reexamines their results in terms of the core and subsequently studies the Shapley value (cf. Shapley (1953)) for these games and an alternative allocation rule for games with a large number of retailers. Hartman and Dror (1996) formulate three criteria for allocation rules in this setting: non-emptiness of the core, computational ease, and justifiability. This last criterion demands the existence of an appealing dual allocation rule in case the situation is analyzed in terms of profits rather than in terms of costs. There results mainly deal with the existence of justifiable pairs, i.e., the analysis of an allocation rule and its dual. Finally, we mention Meca-Martínez et al. (1999) who consider cooperation within the field of inventory management. They analyze cooperative games that depend on the information that is revealed by the companies and then focus on proportional cost-allocation mechanisms. Borm et al. (2001) provide a recent survey of cooperative games associated with operations research games, in which five types of underlying OR-problems are distinguished, one of them being 'inventory'.

The plan of this paper is as follows. We start in section 2 with some preliminaries on cooperative game theory to make this paper self-contained. Then, in section 3, we introduce general news-vendor situations and associated cooperative games with these situations. These so-called general news-vendor games are studied in sections 4 and 5, which concentrate on convexity and balancedness, respectively. We conclude in section 6 with some final remarks and discussion.

2 Preliminaries cooperative game theory

For reasons of self-containedness, we here give a brief introduction to cooperative game theory, based on the work by Slikker and van den Nouweland (2001). Cooperative game theory covers the problem setting where different parties cooperate to reach a common goal. Let us assume we have n different companies pursuing similar objectives. In the setting studied in this paper, these companies are retailers possibly ordering jointly in order to reach a better overall service level to serve a set of independent markets. In game theory, these companies are referred to as *players*, with N the set of players. For

convenience, we number the players such that the player set is $N = \{1, 2, \ldots, n\}$. A subset of N is called a *coalition* and is denoted by S. We are now interested in various coalitions S, and particularly to what extent a specific coalition can reach the common objective without the players who are not part of the coalition. We assume that the benefits of the cooperation within the coalition S are transferable between the players of S, and denote the benefits of the cooperation as $v(S)$. In the example of the retailers described above, this can be defined as the reduced cost of ordering jointly, or as the increased profits of reaching a higher service level. The function v that assigns to every coalition $S \subseteq N$ its value or worth $v(S)$, with $v(\emptyset) = 0$, is commonly referred to as the *characteristic function*. A pair (N, v) consisting of a player set N and a characteristic function v constitutes a *cooperative game* or *coalitional game*.

In our analysis, we sometimes focus on only a few of the players involved in a coalitional game (N, v). For a coalition $S \subseteq N$, $v_{|S}$ denotes the restriction of the characteristic function v to the player set S, i.e., $v_{|S}(T) = v(T)$ for each coalition $T \subseteq S$. The pair $(S, v_{|S})$ is a cooperative game with player set S that is then closely related to the game (N, v).

An interesting property of a characteristic function is that for any two disjoint coalitions S and T of players it holds that $v(S) + v(T) \leq v(S \cup T)$. Such a characteristic function, or a game with this characteristic function, is called *superadditive*. An important consequence of a superadditive characteristic function, is that it is always attractive for these two disjoint coalitions to form one big coalition $S \cup T$ rather than operating separately. In general, if there are no negative effects from having larger coalitions, then the addition of more players will increase the value obtainable. Such a characteristic function is called *monotonic*, i.e., $v(S) \leq v(T)$ and $S \subseteq T$.

In line with any reality involving individual companies and increasingly with papers in the Operations Management literature (e.g., Cachon (1999), Fransoo et al. (2001)), players are not primarily interested in the benefits of a coalition, but in the individual benefits they are getting out of the coalition. The *allocation* is represented as a *payoff vector* $x = (x_i)_{i \in N} \in \mathbb{R}^N$, specifying for each player $i \in N$ the benefit (e.g., (extra) profit) x_i that this player can expect if he cooperates with the other players. An allocation is called *efficient* if the payoffs to the various players add up to exactly $v(N)$. The set consisting of all efficient allocations is the set $\{x \in \mathbb{R}^N \mid \sum_{i \in N} x_i = v(N)\}$. Note that not all these allocations will be acceptable to the players, as each player will require that he gets at least as much as what he can obtain when staying alone. An allocation $x \in \mathbb{R}^N$ with the property that $x_i \geq v(i)$ for all $i \in N$ is called *individually rational*.

The set of all individually rational and efficient allocations is the *imputation set* $I(N, v) = \{x \in \mathbb{R}^N \mid \sum_{i \in N} x_i = v(N) \text{ and } x_i \geq v(i) \text{ for each } i \in N\}$. This type of

rationality requirement can be extended to all coalitions, not just individual players, to obtain the *core* $C(N,v) = \{x \in \mathbb{R}^N \mid \sum_{i \in N} x_i = v(N) \text{ and } \sum_{i \in S} x_i \geq v(S) \text{ for all } S \subseteq N\}$.[1] The interpretation of the core is that it consists of all imputations that are such that no group of players has an incentive to split off from the grand coalition N and form a smaller coalition S because they collectively receive at least as much as what they can obtain for themselves as a coalition. A second interpretation of the core is that no group of players gets more than what they collectively add to the value obtainable by the grand coalition N. This follows since for each $x \in C(N,v)$ and $S \subseteq N$ it holds that $\sum_{i \in S} x_i = \sum_{i \in N} x_i - \sum_{i \in N \setminus S} x_i \leq v(N) - v(N \setminus S)$.

Bondareva (1963) and Shapley (1967) independently identified the class of games that have non-empty cores as the class of balanced games. To describe this class, we define for all $S \subseteq N$ the vector e^S by $e_i^S = 1$ for all $i \in S$ and $e_i^S = 0$ for all $i \in N \setminus S$. A map $\kappa : 2^N \setminus \{\emptyset\} \to [0,1]$ is called a *balanced map* if $\sum_{S \in 2^N \setminus \{\emptyset\}} \kappa(S) e^S = e^N$. Further, a game (N,v) is called *balanced* if for every balanced map $\kappa : 2^N \setminus \{\emptyset\} \to [0,1]$ it holds that $\sum_{S \in 2^N \setminus \{\emptyset\}} \kappa(S) v(S) \leq v(N)$. The following theorem is due to Bondareva (1963) and Shapley (1967).

Theorem 2.1 Let (N,v) be a coalitional game. Then $C(N,v) \neq \emptyset$ if and only if (N,v) is balanced.

A coalitional game (N,v) is called totally balanced if it balanced and each of its subgames is balanced as well.

The last notion we wish to introduce is the notion of convexity. A coalitional game is *convex* if a player's marginal contribution increases if he joins a larger coalition. Formally, coalitional game (N,v) is convex if for each $i \in N$ and for all $S \subseteq T \subseteq N \setminus \{i\}$ it holds that $v(S \cup i) - v(S) \leq v(T \cup i) - v(T)$. We remark that a convex coalitional game has a non-empty core.

3 The model

In this section we introduce general news-vendor situations. These are situations where several companies order the same good from a producer. This good can be bought by company i at (wholesale) price c_i and subsequently sold to customers at customer price p_i. Every company experiences a stochastic demand. As in the standard news-vendor problem it is assumed that the realization of this stochastic demand is not known at the moment of ordering.

---

[1]We define the empty sum to be equal to 0.

A *general news-vendor situation* is a tuple $(N, (X_i)_{i \in N}, (c_i)_{i \in N}, (p_i)_{i \in N})$, where N denotes a set of companies and X_i the stochastic demand for the good at company $i \in N$. Furthermore, c_i and p_i denote the prices that companies pay to the producer and the customers pay to the companies, respectively. Throughout this work we assume that $p_i > c_i > 0$ for all $i \in N$ and that each company has a finite expected demand. A general news-vendor situation with one company only corresponds to the standard news-vendor problem.[2]

This model extends the model of Hartman et al. (2000), who restrict themselves to anonymous wholesale prices ($c_i = c$ for all $i \in N$) and anonymous customer prices ($p_i = p$ for all $i \in N$). Furthermore, they have a slightly different point of view by focusing on costs rather than on profits. They start from the point of view that all demand has to be satisfied. Oversupplying comes at a cost of h per unit and undersupplying at a cost of p' per unit. With $h = c$ and $p' = p - c$ (or $c = h$ and $p = p' + h$) it follows straightforwardly that an optimal order quantity in their model corresponds to an optimal order quantity in our model and vice versa.

Consider a general news-vendor situation and a collection of retailers S who will jointly determine an order size. Together they place an order by the company in S that faces the lowest wholesale price. So, coalition S faces wholesale price $c_S = \min_{i \in S} c_i$. Suppose coalition S has ordered quantity q_S and subsequently, they face demand vector $x_S \in \mathbb{R}^N$ with $x_i = 0$ for all $i \in N \backslash S$. Obviously, the profit of the coalition is optimized by letting the retailers with the highest customer prices sell as much as possible. Assume for notational convenience that $S = \{1, \ldots, s\}$ and that $p_1 \geq p_2 \geq \ldots \geq p_s$. Then the optimal profit of coalition S with order q_S and demand realization x_S is described by

$$\pi^S(q_S, x_S) = - c_S q_S + p_1 \min\{q_S, x_1\} + p_2 \min\{\max\{q_S - x_1, 0\}, x_2\}$$
$$+ \ldots + p_s \min\{\max\{q_S - \sum_{i \in S \backslash \{s\}} x_i, 0\}, x_s\}.$$

The expected profit of coalition S depends on the size of their order q_S and the demand faced by each retailer. We introduce for any vector of demands $(X_i)_{i \in N}$ and any $S \subseteq N$ the vector of demands X_S, where $(X_S)_i = X_i$ for all $i \in S$ and $(X_S)_i = 0$ for all $i \in N \backslash S$. The expected profit of a coalition S that ordered q_S and faces stochastic demand vector X_S is then given by

$$\bar{\pi}^S(q_S, X_S) = E_{X_S}[\pi^S(q_S, .)] \tag{1}$$

[2]The model can easily be extended by including disposal costs for each item that is ordered but not sold and lost-sales costs for each item that was not ordered but could have been sold. By some straightforward manipulation one can reduce this extended model to the original model with adapted prices.

A coalition may achieve higher expected profits than the sum of the expected profits of the individual companies. Hence, retailers may have an incentive to cooperate. We will construct a coalitional game with each general news-vendor situation. In this game, the value of a coalition is the maximum expected profit this coalition can obtain if they order jointly. Formally, let $\Gamma = (N, (X_i)_{i \in N}, (c_i)_{i \in N}, (p_i)_{i \in N})$ be a general news-vendor situation. The associated *general news-vendor game* (N, v^Γ) is defined by

$$v^\Gamma(S) = \max_{q \geq 0} \bar{\pi}^S(q, X_S) \text{ for all } S \subseteq N, \tag{2}$$

where X_S and $\bar{\pi}^S$ are as defined above. In the following theorem we will show that each coalition has an optimal order size, implying that the general news-vendor game is well-defined.

Theorem 3.1 Let $\Gamma = (N, (X_i)_{i \in N}, (c_i)_{i \in N}, (p_i)_{i \in N})$ be a general news-vendor situation and let $S \subseteq N$. Then there exists an order size q_S^* that maximizes the expected profit of coalition S.

Proof: We will show that the optimal profit $\bar{\pi}^S(., X_S)$ is a continuous function of the order size. Additionally we will show that any order outside a specific compact set results in lower expected profits than an order equal to zero. Hence, coalition S optimizes a continuous function over a compact set, which implies that an optimal order size exists.

First, we prove that the optimal profit is a continuous function of the order size. Therefore, let $q \geq 0$ denote an order size and let $\epsilon > 0$. Define $\delta = \frac{\epsilon}{c_S + \max_{i \in S} p_i}$. Let $q' \in (q - \delta, q + \delta)$ with $q' \geq 0$. The difference in costs of purchasing these orders equals the purchasing price times the difference in order size, whereas for any realization x_S of stochastic demand X_S the revenue differs at most the maximum customer price times the difference in order size. Using the triangle inequality we have for any realization x_S

$$|\pi^S(q, x_S) - \pi^S(q', x_S)| \leq |q - q'|(c_S + \max_{i \in S} p_i). \tag{3}$$

Hence, for the expected profit functions we have

$$|\bar{\pi}^S(q, X_S) - \bar{\pi}^S(q', X_S)| \leq |q - q'|(c_S + \max_{i \in S} p_i) < \delta(c_S + \max_{i \in S} p_i) = \epsilon. \tag{4}$$

Hence, $\bar{\pi}$ is continuous function of the order size.

It remains to show that any order outside a specific compact set results in lower expected profits than an order equal to zero. Let

$$q_S^m = \frac{\max_{i \in S} p_i}{c_S} \left[\sum_{i \in S} E[X_i] + \sum_{i \in S} E[X_i^-] \right], \tag{5}$$

8

where $X_i^- = \max\{-X_i, 0\}$.[3] Then for all $q > q_S^m$ we have

$$\bar{\pi}^S(q, X_S) \leq -c_S q_S^m + \max_{i \in S}(p_i) \times \sum_{i \in S} E[X_i] < -\max_{i \in S}(p_i) \times \sum_{i \in S} E[X_i^-] \leq \bar{\pi}^S(0, X_S).$$

This completes the proof. □

If S consists of a single retailer or if all retailers in S experience the same customer price then the determination of an optimal order size for this coalition comes down to a standard news vendor problem with stochastic demand $\sum_{i \in S} X_i$, wholesale price c_S and customer price p $(= p_i$ for all $i \in S)$. It is well known that an optimal order size is then given by the $(1 - \frac{c}{p})$-quantile, which need not be a unique order size. In case of a completely continuous stochastic variable with inverse cumulative demand function F^{-1} this quantity equals $F^{-1}(1 - \frac{c}{p})$. For a normal distribution $\text{Norm}(\mu, \sigma)$ this implies that the optimal order quantity is given by $\mu + \sigma z_{c/p}$, where z_α is defined as the unique real number such that $P(X \geq z_\alpha) = \alpha$ if X is a stochastic variable with the standard normal distribution $\text{Norm}(0, 1)$. A general news-vendor situation with anonymous wholesale and customer prices $(c_i = c$ and $p_i = p$ for all $i \in N)$ is called a news-vendor situation with associated news-vendor game.

Consider the following example, which illustrates some of the concepts introduced above.

Example 3.1 Consider the 2-person news-vendor situation $(N, (X_i)_{i \in N}, (c_i)_{i \in N}, (p_i)_{i \in N})$ with $N = \{1, 2\}$, $c_1 = c_2 = 1$, $p_1 = p_2 = 16$, and a stochastic demand X_1 for company 1 described by probability mass function

$$p_1(x) = \begin{cases} 0 & \text{if } x \notin \{0, 1\}; \\ \frac{7}{8} & \text{if } x = 0; \\ \frac{1}{8} & \text{if } x = 1, \end{cases} \tag{6}$$

while company 2 faces a similar demand that is independent of the demand of company 1. It is easily verified that $(1 - \frac{c_1}{p_1})$-quantile of the distribution of X_1 equals 1, implying that the optimal order size of coalition $\{1\}$ equals 1, resulting in expected profit

$$\bar{\pi}^{\{1\}}(1, X_1) = -1 * 1 + \frac{1}{8} * 1 * 16 = -1 + 2 = 1.$$

Obviously, similar calculations result in the optimal expected profit for coalition $\{2\}$. The optimal order for coalition $\{1, 2\}$ equals 1 as well, resulting in an optimal profit for the grand coalition equal to

$$\bar{\pi}^{\{1,2\}}(1, X_{12}) = -1 * 1 + \frac{15}{64} * 1 * 16 = 2\frac{3}{4}.$$

[3]The introduction of X_i^- is superfluous if only non-negative demands are possible.

By cooperating, the companies can increase their joint profit from 2 to $2\frac{3}{4}$. This is due to the fact that they can reduce the total amount they order significantly, while this results in a small increase in lost sales only.

The associated news-vendor game (N, v^{Γ}) is described by

$$v^{\Gamma}(S) = \begin{cases} 0 & \text{if } S = \emptyset; \\ 1 & \text{if } |S| = 1; \\ 2\frac{3}{4} & \text{if } S = N. \end{cases}$$

Note that this news-vendor game is balanced, i.e., it has a non-empty core. Moreover, it is convex. $\diamond$

In the following sections we will study properties of general news-vendor games. We remark that if we restrict ourselves to anonymous wholesale prices and anonymous customer prices, i.e., to news-vendor games, then balancedness (convexity) of the news-vendor game corresponds to balancedness (concavity) of the associated inventory centralization game of Hartman et al. (2000) since the characteristic function of our news-vendor game can be rewritten as the characteristic function of an additive game minus the characteristic function of the associated inventory centralization (cost) game. The additive game represents for each coalition the profits that could be obtained in case demand would be perfectly predictable, i.e., the expected demand times the margin per unit.

4 Convexity

In this section we will study convexity of general news-vendor games. We will first show that not every (general) news-vendor game is convex. The main result of this section, however, states that if the companies face independent normally distributed stochastic demands, anonymous wholesale prices, and anonymous customer prices then the news-vendor game is convex.

First, we will show that news-vendor games need not be convex, even if the stochastic demands of the companies are independent.

Example 4.1 Consider the 3-person news-vendor situation $\Gamma = (N, (X_i)_{i \in N}, (c_i)_{i \in N}, (p_i)_{i \in N})$ with $N = \{1, 2, 3\}$, $c_1 = c_2 = c_3 = 1$, $p_1 = p_2 = p_3 = 16$, and a stochastic demand for company 1 as described by (6), while companies 2 and 3 face a similar demand. Demands are assumed to be independent.

The optimal order size of the grand coalition can be shown to be equal to 1, implying an optimal expected profit equal to $4\frac{9}{32}$. Combining this with the results of example 3.1 implies that the associated news-vendor game is described by

$$
v^{\Gamma}(S) = \begin{cases} 0 & \text{if } S = \emptyset; \\ 1 & \text{if } |S| = 1; \\ 2\frac{3}{4} & \text{if } |S| = 2; \\ 4\frac{9}{32} & \text{if } S = N. \end{cases}
$$

Using this description of the news-vendor game (N, v^{Γ}), we find that

$$
v^{\Gamma}(\{1,2\}) - v^{\Gamma}(\{2\}) = 1\frac{3}{4} > 1\frac{17}{32} = v^{\Gamma}(N) - v^{\Gamma}(\{2,3\}).
$$

We conclude that the news-vendor game associated with news-vendor situation Γ in this example is not convex. $\diamond$

Secondly, we focus on situations in which the demands of the companies are not only independent, but additionally we assume that the demand of any company is normally distributed. Furthermore, we assume that wholesale prices and customer prices are anonymous. The following theorem states that the associated news-vendor games are convex. The proof is included in the appendix.

Theorem 4.1 Let $\Gamma = (N, (X_i)_{i \in N}, (c_i)_{i \in N}, (p_i)_{i \in N})$ be a news-vendor situation with independent stochastic demands $X_i \sim \text{Norm}(\mu_i, \sigma_i)$ for all $i \in N$, $c_i = c$ for all $i \in N$, and $p_i = p$ for all $i \in N$. The associated news-vendor game (N, v^{Γ}) is convex.

Proof: See appendix $\square$

This theorem is related to a result of Hartman et al. (2000) who showed that if retailers face a joint demand distribution that is multinormal then the associated news-vendor game is balanced. For their result, it is not required that the demand of different firms is independent. The following example shows that we cannot extend our convexity result to a setting with dependent stochastic demand. In this example we let $X = (X_i)_{i \in N}$ be a multinormal distribution. Formally, we assume $X \sim \text{Norm}(\mu, \Sigma)$, where $\mu \in \mathbb{R}^N$ denotes a vector with as elements the expected demand of the different companies and $\Sigma \in \mathbb{R}^{N \times N}$ denotes the covariance-matrix.

Example 4.2 Consider news-vendor situation $\Gamma = (N, (X_i)_{i \in N}, (c_i)_{i \in N}, (p_i)_{i \in N})$ with $N = \{1,2,3\}$, $X = (X_i)_{i \in N} \sim \text{Norm}(\mu, \Sigma)$ with $\mu = (10, 10, 10)$ and $\Sigma =$

$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ c_1 = c_2 = c_3 = 1,\ \text{and}\ p_1 = p_2 = p_3 = 3.\ \text{For the standard devia-}$$

tion of the joint demand distributions we find $\sigma_{\{1,2\}} = 0$, $\sigma_{\{1,3\}} = \sqrt{2}$, $\sigma_{\{2,3\}} = \sqrt{2}$, and $\sigma_N = 1$. For notational convenience, let $l(c, p) = cz_{c/p} + p \int_{z_{c/p}}^{\infty} (y - z_{c/p}) f_Y(y) dy$, where $Y \sim \mathrm{Norm}(0, 1)$. It is easily (numerically) checked that $l(1, 3) > 0$.

Using lemma A.2 in the appendix we find that

$$v^\Gamma(S) = \begin{cases} 0 & \text{if } S = \emptyset; \\ 20 - l(1, 3) & \text{if } |S| = 1; \\ 40 & \text{if } S = \{1, 2\}; \\ 40 - \sqrt{2}l(1, 3) & \text{if } S = \{1, 3\} \text{ or } S = \{2, 3\}; \\ 60 - l(1, 3) & \text{if } S = N. \end{cases}$$

Hence, with $i = 1$, $S = \{2\}$, and $T = \{2, 3\}$ we derive

$$v^\Gamma(S \cup \{i\}) - v^\Gamma(S) = 20 + l(1, 3) > 20 + (\sqrt{2} - 1)l(1, 3) = v^\Gamma(T \cup \{i\}) - v^\Gamma(T),$$

where the inequality holds since $l(1, 3) > 0$. We conclude that (N, v^Γ) is not convex. $\diamondsuit$

The following example shows that by dropping anonymity of the wholesale price, convexity might be lost as well.

Example 4.3 Let $\Gamma = (N, (X_i)_{i \in N}, (c_i)_{i \in N}, (p_i)_{i \in N})$ be a general news-vendor situation with $N = \{1, 2, 3\}$, independent stochastic demands $X_i \sim \mathrm{Norm}(100, 0)$ for all $i \in N$, $c_1 = c_2 = 3$, $c_3 = 5$, and $p_i = 6$ for all $i \in N$. Note that the demand is in fact deterministic. It is a straightforward exercise to determine the associated general news-vendor game, which is described by

$$v^\Gamma(S) = \begin{cases} 0 & \text{if } S = \emptyset; \\ 300 & \text{if } S \in \{\{1\}, \{2\}\}; \\ 100 & \text{if } S = \{3\}; \\ 600 & \text{if } |S| = 2; \\ 900 & \text{if } S = N. \end{cases}$$

Hence, with $i = 1$, $S = \{3\}$, and $T = \{2, 3\}$ we derive

$$v^\Gamma(S \cup \{i\}) - v^\Gamma(S) = 500 > 300 = v^\Gamma(T \cup \{i\}) - v^\Gamma(T).$$

We conclude that (N, v^Γ) is not convex. $\diamondsuit$

5 Balancedness

In the previous section we showed that general news-vendor games need not be convex. Though normal distributed demands combined with anonymous wholesale and customer prices result in convex games, dropping independence of the demand distributions or anonymity of the wholesale price implies that convexity might be lost. A game that is not convex, however, may still have stable allocations, i.e., it might still have a non-empty core. In this section we focus on this issue and show that any general news-vendor game has a non-empty core.

We start with an example.

Example 5.1 Consider the 3-person (general) news-vendor situations in examples 4.1, 4.2, and 4.3. Though none of the associated (general) news-vendor games are convex it follows that all three of them are balanced since payoff vectors $(1\frac{41}{96}, 1\frac{41}{96}, 1\frac{41}{96})$, $(20, 20, 20 - l(1, 3))$, and $(300, 300, 300)$ belong to the respective cores. $\diamond$

This example shows that non-convex general news-vendor games may still have a non-empty core. In the process of proving that this is not a coincidence, we will use the following lemma.

Lemma 5.1 Let $(N, (X_i)_{i \in N}, (c_i)_{i \in N}, (p_i)_{i \in N})$ be a general news-vendor situation. Let κ be an associated balanced map. Denote an optimal order size of coalition $S \subseteq N$ by q_S^*. Then

$$\bar{\pi}^N \Big(\sum_{S \subseteq N : S \neq \emptyset} \kappa(S) q_S^*, \sum_{S \subseteq N : S \neq \emptyset} \kappa(S) X_S \Big) \geq \sum_{S \subseteq N : S \neq \emptyset} \bar{\pi}^S \Big(\kappa(S) q_S^*, \kappa(S) X_S \Big)$$

Proof: Since $c_N \leq c_S$ for all $S \subseteq N$ we have that

$$c_N \sum_{S \subseteq N} \kappa(S) q_S^* \leq \sum_{S \subseteq N} c_S \kappa(S) q_S^*. \tag{7}$$

For convenience denote for all $S \subseteq N$, order q_S, and realization of demand x_S, $R^S(q_S, x_S) = \pi^S(q_S, x_S) + c_S q_S$, i.e., the revenue part of the profit function. Let x_N be a realization of demand for the grand coalition and let x_S be the associated demand of any coalition $S \subset N$. A selling profile of coalition $S \subseteq N$ with order size q_S^* is a vector $y^S \in \mathbb{R}_+^N$ such that $y_i^S = 0$ for all $i \in N \backslash S$, $y_i^S \leq x_i$ for all $i \in S$, and $\sum_{i \in S} y_i^S \leq q_S^*$. Denote a selling profile that maximizes the profit of coalition S by $y^{S,*}$.

Then $z^N = \sum_{S \subseteq N} \kappa(S) y^{S,*}$ denote a possible selling profile of the grand coalition associated with order size $\sum_{S \subseteq N: S \neq \emptyset} \kappa(S) q_S^*$. Denote the optimal selling profile of N associated with this order size by w^N. Then

$$R^N \Big(\sum_{S \subseteq N: S \neq \emptyset} \kappa(S) q_S^*, \sum_{S \subseteq N: S \neq \emptyset} \kappa(S) x_S \Big) = \sum_{i \in N} p_i w_i^N \geq \sum_{i \in N} p_i z_i^N$$

$$= \sum_{i \in N} \Big[p_i \sum_{S \subseteq N} \kappa(S) y_i^{S,*} \Big] = \sum_{S \subseteq N} \Big[\sum_{i \in S} p_i \kappa(S) y_i^{S,*} \Big] = \sum_{S \subseteq N} R^S(\kappa(S) q_S^*, \kappa(S) x_S), \quad (8)$$

where the last equality holds since $\kappa(S) y^{S,*}$ is obviously an optimal selling profile for coalition S and demand realization $\kappa(S) x_S$.

Combining (7) with (8) gives

$$\pi^N \Big(\sum_{S \subseteq N: S \neq \emptyset} \kappa(S) q_S^*, \sum_{S \subseteq N: S \neq \emptyset} \kappa(S) x_S \Big) \geq \sum_{S \subseteq N: S \neq \emptyset} \pi^S \Big(\kappa(S) q_S^*, \kappa(S) x_S \Big)$$

Since this inequality holds for any realization x_N of X_N, taking expectations over the distribution of X_N completes the proof. □

The following theorem states that general news-vendor games are balanced. This answers the open question by Hartman et al. (2000), which dealt with balancedness of news-vendor games with anonymous wholesale and retail prices. Moreover, it extends the affirmative answer to the class of news-vendor situations in which wholesale and/or retail prices are not anonymous.[4]

Theorem 5.1 Let $(N, (X_i)_{i \in N}, (c_i)_{i \in N}, (p_i)_{i \in N})$ be a general news-vendor situation. Then the associated general news-vendor game has a non-empty core.

Proof: Let $\kappa : 2^N \backslash \{\emptyset\} \to [0, 1]$ be a balanced map. Note that $X_N = \sum_{i \in N} X_{\{i\}}$ and also $\sum_{S \subseteq N: S \neq \emptyset} \kappa(S) X_S = \sum_{S \subseteq N: S \neq \emptyset} \kappa(S) \sum_{i \in S} X_{\{i\}} = \sum_{i \in N} X_{\{i\}} \sum_{S \subseteq N: i \in S} \kappa(S) = \sum_{i \in N} X_{\{i\}}$, where the last equality follows since κ is a balanced map.

Let $(q_S^*)_{S \subseteq N: S \neq \emptyset}$ be optimal orders for the different coalitions. Then $t_N = \sum_{S \subseteq N: S \neq \emptyset} \kappa(S) q_S^*$ denotes a possible order for coalition N, while q_N^* is the optimal order. Hence, we know that $\bar{\pi}^N(q_N^*, X_N) \geq \bar{\pi}^N(t_N, X_N)$. Furthermore, we know that for all $S \subseteq N$ with $S \neq \emptyset$ it holds that $\kappa(S) q_S^*$ maximizes profit $\bar{\pi}^S(q, \kappa(S) X_S)$ since $\bar{\pi}^S(\lambda q, \lambda X) = \lambda \bar{\pi}^S(q, X)$ for any stochastic demand X and any order quantity q. This implies that $\kappa(S) q_S^*$ is an optimal order size for coalition S and stochastic demand $\kappa(S) X_S$.

---

[4]During the revision of this paper it appeared that independent from the current work Müller et al. (2001) provided an answer to the open question or Hartman et al. (2000) by a proof built on similar ideas as our proof. However, they restrict themselves to anonymous wholesale and customer prices.

Furthermore, it implies that if the stochastic demand changes by some factor then the optimal expected profit changes by the same factor. Using this, we find

$$v(N) = \bar{\pi}^N(q_N^*, X_N) \geq \bar{\pi}^N(t_N, X_N) = \bar{\pi}^N\Big(\sum_{S \subseteq N: S \neq \emptyset} \kappa(S)q_S^*, \sum_{S \subseteq N: S \neq \emptyset} \kappa(S)X_S\Big)$$

$$\geq \sum_{S \subseteq N: S \neq \emptyset} \bar{\pi}^S(\kappa(S)q_S^*, \kappa(S)X_S) = \sum_{S \subseteq N: S \neq \emptyset} \kappa(S)\bar{\pi}^S(q_S^*, X_S) = \sum_{S \subseteq N: S \neq \emptyset} \kappa(S)v(S). \quad (9)$$

The second inequality follows by lemma 5.1.

Hence, the general news-vendor game is balanced, i.e, it has a non-empty core. □

The result in theorem 5.1 can be strengthened by noting that every subgame of a general news-vendor game is a general news-vendor game itself. Hence, the following corollary follows immediately from theorem 5.1.

Corollary 5.1 Let $(N, (X_i)_{i \in N}, (c_i)_{i \in N}, (p_i)_{i \in N})$ be a general news-vendor situation. Then the associated news-vendor game is totally balanced.

6   Remarks and discussion

This work provides a further impulse to a game-theoretical analysis of situations in which retailers can combine their orders. The next step for research in this area can be taken in at least three directions. First, one could drop the assumption that retailers are risk-neutral. This demands an application of stochastic cooperative game theory. For a survey of research in this area we refer to Suijs (1999). Secondly, strategic behavior of the players could be taken into account. In our model, at two moments information is needed to make optimal decisions: At the point in time where the quantity of the joint order is determined and at the moment that retailers have to reveal their actual demand, i.e., the point in time at which their actual demand is known. The events at the second point in time can be modeled as a process of allocation or reallocation. Here, the work of Klaus (1998) can be a guidance. Thirdly, we mention the possibility to drop the assumption that actual demand is known with certainty at the point in time at which the joint order has to be allocated among the retailers. For each retailer, demand might at that point in time better be modeled by a stochastic variable with less uncertainty (e.g., lower standard deviation) than at the first point in time.

A   Appendix: Proof of theorem 4.1

This appendix contains the proof of theorem 4.1.

Recall from section 3 that the optimal order for company i is given by $q_i^* = \mu_i + \sigma_i z_{c_i/p_i}$. What happens if several companies cooperate and join their orders. The first step that needs to be taken, is to determine the joint demand. Since $c_i = c$ and $p_i = p$ for all $i \in N$ the optimal order size and optimal expected profit of coalition S depends on the sum of the demand of the retailers in S only and not on the distribution of this sum over the different retailers. For convenience denote this sum by $\hat{X}_S = \sum_{i \in S} X_i$.

Let $S \subseteq N$ be a set of retailers. Since the stochastic demands X_i are independent, the sum of the orders has demand distribution $\hat{X}_S \sim N(\mu_S, \sigma_S)$, where $\mu_S = \sum_{i \in S} \mu_i$ and $\sigma_S = \sqrt{\sum_{i \in S} (\sigma_i)^2}$. Using these relations between characteristics of the distributions X_i and $\hat{X}_S$ we derive that the optimal order of coalition S is given by

$$q_S^* = \mu_S + \sigma_S z_{c/p} = \sum_{i \in S} \mu_i + \sqrt{\sum_{i \in S} (\sigma_i)^2} \; z_{c/p}. \tag{10}$$

In order to analyze the associated news-vendor game, we will derive an expression for the value of a coalition. In deriving this expression we will use the following lemma.

Lemma A.1 Let $X \sim \text{Norm}(\mu, \sigma)$ and $Y \sim \text{Norm}(0, 1)$. Then for all $\alpha \in (0, 1)$ we have

$$\int_{\mu + \sigma z_\alpha}^{\infty} (x - (\mu + \sigma z_\alpha)) f_X(x) dx = \sigma \int_{z_\alpha}^{\infty} (y - z_\alpha) f_Y(y) dy. \tag{11}$$

Proof: Let $\alpha \in (0, 1)$. Then

$$\int_{\mu + \sigma z_\alpha}^{\infty} (x - (\mu + \sigma z_\alpha)) f_X(x) dx = \int_{\mu + \sigma z_\alpha}^{\infty} (x - (\mu + \sigma z_\alpha)) \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \int_0^{\infty} x \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{(x+\mu+\sigma z_\alpha)-\mu}{\sigma}\right)^2} dx$$

$$= \int_0^{\infty} x \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x+\sigma z_\alpha}{\sigma}\right)^2} dx$$

$$= \int_0^{\infty} \sigma y \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\sigma y+\sigma z_\alpha}{\sigma}\right)^2} d\sigma y$$

$$= \sigma \int_0^{\infty} y \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y+z_\alpha}{1}\right)^2} dy$$

$$= \sigma \int_{z_\alpha}^{\infty} (y - z_\alpha) \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(y)^2} dy$$

$$= \sigma \int_{z_\alpha}^{\infty} (y - z_\alpha) f_Y(y) dy.$$

$\square$

Using this lemma we can derive an expression for the value of a coalition in the news-vendor game.

Lemma A.2 Let $\Gamma = (N, (X_i)_{i \in N}, (c_i)_{i \in N}, (p_i)_{i \in N})$ be a news-vendor situation with independent stochastic demands $X_i \sim \text{Norm}(\mu_i, \sigma_i)$ for all $i \in N$, $c_i = c$ for all $i \in N$, and $p_i = p$ for all $i \in N$. For all $S \subseteq N$ it holds that

$$v^\Gamma(S) = \mu_S(p - c) - \sigma_S \left(cz_{c/p} + p \int_{z_{c/p}}^{\infty} (y - z_{c/p}) \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2} dy \right).$$

Proof: Consider a coalition $S \subseteq N$. By definition of $v^\Gamma(S)$ and q_S^* we have

$$v^\Gamma(S) = \max_q \bar{\pi}(q, X_S) = \bar{\pi}(q_S^*, X_S).$$

Using this and equations (1) and (10) we then find that

$$
\begin{aligned}
v^\Gamma(S) &= -cq_S^* + \int_{-\infty}^{q_S^*} px f_{X_S}(x) dx + \int_{q_S^*}^{\infty} pq_S^* f_{X_S}(x) dx \\
&= -cq_S^* + p\mu_S - p \int_{q_S^*}^{\infty} (x - q_S^*) f_{X_S}(x) dx \\
&= -c(\mu_S + \sigma_S z_{c/p}) + p\mu_S - p \int_{\mu_S + \sigma_S z_{c/p}}^{\infty} (x - (\mu_S + \sigma_S z_{c/p})) f_{X_S}(x) dx \\
&= -c(\mu_S + \sigma_S z_{c/p}) + p\mu_S - p\sigma_S \int_{z_{c/p}}^{\infty} (y - z_{c/p}) \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2} dy \\
&= \mu_S(p - c) - \sigma_S \left(cz_{c/p} + p \int_{z_{c/p}}^{\infty} (y - z_{c/p}) \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2} dy \right),
\end{aligned}
$$

where the fourth equality follows from lemma A.1. $\qquad \square$

The following lemma derives a relation between standard deviations of demands of several coalitions.

Lemma A.3 Let N be a finite set and let $X_i \sim \text{Norm}(\mu_i, \sigma_i)$ for all $i \in N$ be independent stochastic demands. For all $S, T \subseteq N$ with $S \subseteq T$ and all $i \in S$ it holds that

$$\sigma_S - \sigma_{S \setminus \{i\}} \geq \sigma_T - \sigma_{T \setminus \{i\}}.$$

Proof: Let $S, T \subseteq N$ with $S \subseteq T$ and let $i \in S$. Then

$$
\begin{aligned}
\sigma_S - \sigma_{S \setminus \{i\}} &= \sqrt{\sum_{j \in S} (\sigma_j)^2} - \sqrt{\sum_{j \in S \setminus \{i\}} (\sigma_j)^2} = \frac{(\sigma_i)^2}{\sqrt{\sum_{j \in S} (\sigma_j)^2} + \sqrt{\sum_{j \in S \setminus \{i\}} (\sigma_j)^2}} \\
&\geq \frac{(\sigma_i)^2}{\sqrt{\sum_{j \in T} (\sigma_j)^2} + \sqrt{\sum_{j \in T \setminus \{i\}} (\sigma_j)^2}} = \sigma_T - \sigma_{T \setminus \{i\}},
\end{aligned}
$$

where the last equality follows similar to the first two. $\qquad \square$

The next lemma will be used in the proof of the theorem.

Lemma A.4 Let $Y \sim \text{Norm}(0,1)$ be a stochastic variable with a standard normal distribution and let $p > c > 0$. Then

$$cz_{c/p} + p \int_{z_{c/p}}^{\infty} (y - z_{c/p}) f_Y(y) dy \geq 0. \tag{12}$$

Proof: We will distinguish between two cases, $z_{c/p} \geq 0$ and $z_{c/p} < 0$. First, if $z_{c/p} \geq 0$ we find directly, using that $y - z_{c/p} \geq 0$ for all $y \in (z_{c/p}, \infty)$, that (12) holds.

Secondly, suppose $z_{c/p} < 0$. Then

$$cz_{c/p} + p \int_{z_{c/p}}^{\infty} (y - z_{c/p}) f_Y(y) dy = cz_{c/p} + p(E[Y] - z_{c/p}) - p \int_{-\infty}^{z_{c/p}} (y - z_{c/p}) f_Y(y) dy$$

$$= (c - p)z_{c/p} - p \int_{-\infty}^{z_{c/p}} (y - z_{c/p}) f_Y(y) dy,$$

where the second equality holds since $E[Y] = 0$. Using that $(c - p)z_{c/p} \geq 0$ and $(y - z_{c/p}) \leq 0$ for all $y \in (-\infty, z_{c/p})$, we derive that (12) holds. $\qquad\square$

We can now prove theorem 4.1.

Proof of theorem 4.1: Let $Y \sim \text{Norm}(0,1)$ be a stochastic variable with a standard normal distribution. Let $S, T \subseteq N$ with $S \subseteq T$ and let $i \in S$. Then

$$v^{\Gamma}(S) - v^{\Gamma}(S \backslash \{i\}) = \mu_S(p - c) - \sigma_S \left(cz_{c/p} + p \int_{z_{c/p}}^{\infty} (y - z_{c/p}) f_Y(y) dy \right)$$

$$- \mu_{S \backslash \{i\}}(p - c) - \sigma_{S \backslash \{i\}} \left(cz_{c/p} + p \int_{z_{c/p}}^{\infty} (y - z_{c/p}) f_Y(y) dy \right)$$

$$= (\mu_S - \mu_{S \backslash \{i\}})(p - c)) - (\sigma_S - \sigma_{S \backslash \{i\}}) \left(cz_{c/p} + p \int_{z_{c/p}}^{\infty} (y - z_{c/p}) f_Y(y) dy \right)$$

$$= \mu_i(p - c) - (\sigma_S - \sigma_{S \backslash \{i\}}) \left(cz_{c/p} + p \int_{z_{c/p}}^{\infty} (y - z_{c/p}) f_Y(y) dy \right)$$

$$\leq \mu_i(p - c) - (\sigma_T - \sigma_{T \backslash \{i\}}) \left(cz_{c/p} + p \int_{z_{c/p}}^{\infty} (y - z_{c/p}) f_Y(y) dy \right)$$

$$= v^{\Gamma}(T) - v^{\Gamma}(T \backslash \{i\}),$$

where the first equality follows by lemma A.2, the inequality by combining lemmas A.3 and A.4, and the last equality follows similar to the first three. $\qquad\square$

References

Bondareva, O. (1963). Certain applications of the methods of linear programming to the theory of cooperative games (In Russian). *Problemy Kibernetiki*, 10:119–139.

Borm, P., Hamers, H., and Hendrickx, R. (2001). Operations research games: A survey. CentER Discussion Paper 2001-45, Tilburg University, Tilburg, The Netherlands.

Cachon, G. (1999). Competitive supply chain inventory management. In Tayur, S., Ganeshan, R., and Magazine, M., editors, *Quantitative Models for Supply Chain Management*. Kluwer Academic Publishers, Boston.

Cachon, G. and Lariviere, M. (1999). Capacity choice and allocation: Strategic behavior and supply chain performance. *Management Science*, 45:1091–1108.

Fisher, M. and Raman, A. (1996). Reducing the cost of demand uncertainty through accurate response to early sales. *Operations Research*, 44:87–99.

Fransoo, J., Wouters, M., and de Kok, T. (2001). Multi-echelon multi-company inventory planning with limited information exchange. *Journal of the Operational Research Society*, 130:830–838.

Gerchak, Y. and Gupta, D. (1991). Technical note: on apportioning costs to customers in centralized continuous review inventory systems. *Journal of Operations Management*, 10:546–551.

Hartman, B. and Dror, M. (1996). Cost allocation in continuous review inventory models. *Naval Research Logistics Quarterly*, 43:549–561.

Hartman, B. and Dror, M. (1997). Inventory centralization costs with varying cost parameters. Working paper, MIS Department, University of Arizona.

Hartman, B., Dror, M., and Shaked, M. (2000). Cores of inventory centralization games. *Games and Economic Behavior*, 31:26–49.

Khouja, M. (1999). The single-period (news-vendor) problem: literature review and suggestions for future research. *Omega*, 27:537–553.

Klaus, B. (1998). *Fair Allocation and Reallocation: An Axiomatic Study*. PhD thesis, Maastricht University, Maastricht, The Netherlands.

Lee, H., Padmanabhan, P., and Wang, S. (1997). Information distortion in a supply chain: the bullwhip effect. *Management Science*, 43:546–558.

Meca-Martínez, A., Timmer, J., García-Jurado, I., and Borm, P. (1999). Inventory games. CentER Discussion Paper 9953, Tilburg University, Tilburg, The Netherlands.

Müller, A., Scarsini, M., and Shaked, M. (2001). The newsvendor game has a non-empty core. *Games and Economic Behavior* (to appear).

Robinson, L. (1993). A comment on Gerchak and Gupta's: On apportioning costs to customers in centralized continuous review inventory systems. *Journal of Operations Management*, 11:99–102.

Shapley, L. (1953). A value for n-person games. In Tucker, A. and Kuhn, H., editors, *Contributions to the Theory of Games II*, pages 307–317. Princeton University Press, Princeton.

Shapley, L. (1967). On balanced sets and cores. *Naval Research Logistics Quarterly*, 14:453–460.

Silver, E., Pyke, D., and Peterson, R. (1998). *Inventory Management and Production Planning and Scheduling*. Wiley, New York.

Slikker, M. and van den Nouweland, A. (2001). *Social and Economic Networks in Cooperative Game Theory*. Kluwer Academic Publishers, Boston.

Suijs, J. (1999). *Cooperative Decision-Making under Risk*. Kluwer Academic Publishers, Boston.

Tayur, S., Ganeshan, R., and Magazine, M., editors (1999). *Quantitative Models for Supply Chain Management*. Kluwer Academic Publishers, Boston.

Tsay, A., Nahmias, S., and Agrawal, N. (1999). Modeling supply chain contracts: A review. In Tayur, S., Ganeshan, R., and Magazine, M., editors, *Quantitative Models for Supply Chain Management*. Kluwer Academic Publishers, Boston.