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Distributed Price-based Optimal Control of Power Systems

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Abstract—This paper proposes a new control scheme for achieving optimal power balancing in electrical power systems. Optimality is defined via economical criteria related to steady-state operation requirements. Limits on the capacity of the power lines are taken into account via inequality constraints that are added to the optimization problem. Due to these inequality constraints, the resulting control law has a piecewise affine structure. Furthermore, the control action is distributed in the sense that a controller is assigned to each node and each controller communicates only with the controllers associated with the neighboring nodes. We prove that the resulting steady-state solution is optimal. The proposed control structure is a relaxation and an extension of classical “automatic generation control” in power systems, and can be interpreted as a fast acting real-time market for electrical power.

Index Terms—Power systems, Distributed control, Congestion management, Optimality.

I. INTRODUCTION

During the past decade there has been a tremendous amount of research devoted to a market-oriented approach for the electrical power system, see [1] for an overview. Electrical power systems have some unique properties, which make this a challenging task. For example, electrical energy can not be efficiently stored in large amounts, which implies that production has to meet rapidly changing demand in real-time, making electricity a commodity with fast changing prices. Furthermore, unlike other transportation systems, which assume a free choice among alternative paths between source and destination, the flow of power in electrical energy transmission networks is governed by physical laws and, for some fixed pattern of power injections, it can be influenced only to a certain degree. Therefore, physical and security limits on the maximal power flow in the lines of electrical energy transmission networks represent crucial system constraints, which cannot be neglected [2]. Due to the fast changing variable production costs, there is a general tendency in power markets towards increasing the speed with which the market price is updated. The common characteristic of virtually all existing approaches is that the congestion management problem is treated in a *static manner*, while for real-time control of power flow in the lines the AGC (Automatic Generation Control) [3] scheme is utilized.

This paper concerns novel, *explicit, price-based, dynamic* control strategies for real-time power balancing in electrical power systems with network constraints. By *explicit* we mean that the controller is not based on solving on-line

an optimization problem, as it is for instance the case of the model predictive control framework [4]. The proposed explicit controller guarantees that, following any admissible change in the load, the power system will settle in the corresponding economically optimal steady-state point with all line flow constraints satisfied. Due to the inequality constraints representing the line flow limits, the controller has a piecewise affine structure. This explicit control scheme was first presented, in a non-distributed form, in [5]. For completeness of the presentation, some of the results from [5] are recalled. In this paper we focus on the *distributed* implementation of the developed explicit controller. Due to its advantages, e.g. convexity of some synthesis problems and scalability, distributed control has gained a significant attention over the past several years, resulting with a rapidly growing list of papers on the subject. For more details on that subject, interested reader is referred [6], [7] and the references therein. The practical applicability and the efficiency in terms of distributed implementation of the proposed control scheme is illustrated using the IEEE 39-bus New England test system.

A. Nomenclature

The field of real numbers is denoted by \mathbb{R} , while $\mathbb{R}^{m \times n}$ denotes m by n matrices with elements in \mathbb{R} . For a matrix $A \in \mathbb{R}^{m \times n}$, $[A]_{ij}$ denotes the element in the i -th row and j -th column of A . For a vector $x \in \mathbb{R}^n$, $[x]_i$ denotes the i -th element of x . $\ker A$ and $\text{im} A$ denote the kernel and the image space of A , respectively. We use I_n and $\mathbf{1}_n$ to denote an identity matrix of dimension $n \times n$ and a column vector with n elements all being equal to 1, respectively. The operator $\text{col}(\cdot, \dots, \cdot)$ stacks its operands into a column vector, and $\text{diag}(\cdot, \dots, \cdot)$ denotes a square matrix with its operands on the main diagonal and zeros elsewhere. *All inequalities are interpreted elementwise.* With a slight abuse of notation we will often use the same symbol to denote a signal, i.e. a function of time, as well as possible values that the signal may take at any time instant.

II. PROBLEM FORMULATION

Consider a connected undirected graph $G = (V, E, A)$ as an abstraction of an electrical power network. $V = \{v_1, \dots, v_n\}$ is the set of nodes, $E \subseteq V \times V$ is the set of undirected edges, and A is a weighted adjacency matrix. Undirected edges are denoted as $e_{ij} = (v_i, v_j)$, and the adjacency matrix $A \in \mathbb{R}^{n \times n}$ satisfies $[A]_{ij} \neq 0 \Leftrightarrow e_{ij} \in E$ and $[A]_{ij} = 0 \Leftrightarrow e_{ij} \notin E$. No self-connecting edges are allowed, i.e. $e_{ii} \notin E$. We associate the edges with the power lines of the electrical network and, for convenience, we set the weights in the adjacency matrix as follows: $[A]_{ij} = -\frac{1}{z_{ij}} = -b_{ij}$, where z_{ij} is the

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inductive reactance of a line, i.e. the imaginary part of the line impedance, and b_{ij} is the line susceptance, see [2] for details. Note that the matrix A has zeros on its main diagonal and $A = A^\top$. The set of neighbors of a node v_i is defined as $N_i \triangleq \{v_j \in V \mid (v_i, v_j) \in E\}$. Often we will use the index i to refer the node v_i . Define $I(N_i)$ as the set of indices corresponding to the neighbors of node i , i.e. $I(N_i) \triangleq \{j \mid v_j \in N_i\}$. We associate the nodes with the buses in the electrical energy transmission network.

A. Steady-state optimization problem

To define the steady-state optimization problem, with each node v_i we associate a singlet \hat{p}_i and a quadruplet $(p_i, \underline{p}_i, \bar{p}_i, J_i)$, where $p_i, \underline{p}_i, \bar{p}_i, \hat{p}_i \in \mathbb{R}$, $\underline{p}_i < \bar{p}_i$ and $J_i: \mathbb{R} \rightarrow \mathbb{R}$ is a strictly convex, continuously differentiable function. The values p_i and \hat{p}_i denote the reference values for node power injections into the network. Positive values correspond to a flow of power into the network (production), while negative values denote power extracted from the network (consumption). Both p_i and \hat{p}_i can take positive as well as negative values, and the only difference is that, in contrast to \hat{p}_i , the value p_i has an associated objective function J_i and a constraint $\underline{p}_i \leq p_i \leq \bar{p}_i$. In the case of a positive p_i , the function J_i represents the variable costs of production, while for negative values of p_i , it denotes the negated benefit function of a consumer. We will refer to p_i as the power from a price-elastic producer/consumer (or simply, power from a price-elastic unit), and to \hat{p}_i as the power from a price-inelastic producer/consumer (price-inelastic unit). The assumption that one price-elastic unit is associated with each node is made only to simplify the presentation. However, the results of the paper are directly applicable to the case where some nodes in the network have several price-elastic units, or when at some nodes there are no price-elastic units.

We use a “dc power flow” model [2] to determine the power flows in the network for given values of node power injections. With δ_i denoting a voltage phase angle at the node v_i , the power flow in a line $e_{ij} \in E$ is given by $p_{ij} = b_{ij}(\delta_i - \delta_j) = -p_{ji}$. If $p_{ij} > 0$, power in the line e_{ij} flows from node v_i to node v_j . The power balance in a node yields $p_i + \hat{p}_i = \sum_{j \in I(N_i)} p_{ij}$. With the abbreviations $p = \text{col}(p_1, \dots, p_n)$, $\hat{p} = \text{col}(\hat{p}_1, \dots, \hat{p}_n)$, $\delta = \text{col}(\delta_1, \dots, \delta_n)$ the overall network balance condition is $p + \hat{p} = B\delta$, where the matrix B is given by $B = A - \text{diag}(A\mathbf{1}_n)$.

We define the optimal power flow problem as follows.

Problem II.1 Optimal Power Flow (OPF) problem.

For any constant value of \hat{p} ,

$$\min_{p, \delta} J(p) \triangleq \min_{p, \delta} \sum_{i=1}^n J_i(p_i) \quad (1a)$$

subject to

$$p - B\delta + \hat{p} = 0, \quad (1b)$$

$$\underline{p} \leq p \leq \bar{p}, \quad (1c)$$

$$b_{ij}(\delta_i - \delta_j) \leq \bar{p}_{ij}, \quad \forall (i, j \in I(N_i)), \quad (1d)$$

where $\underline{p} = \text{col}(\underline{p}_1, \dots, \underline{p}_n)$, $\bar{p} = \text{col}(\bar{p}_1, \dots, \bar{p}_n)$, and $\bar{p}_{ij} = \bar{p}_{ji}$ is the maximal allowed power flow in the line e_{ij} . \square

We will refer to a vector p that solves the OPF problem as a *vector of optimal power injections*.

For an appropriately defined matrix L and a suitably defined vector of power line limits \bar{p}_L , the set of constraints in (1d) can be written in a more compact form as follows:

$$L\delta \leq \bar{p}_L. \quad (2)$$

Note that in the Problem II.1 we have included δ explicitly as a decision variable. As it will be shown later, this formulation is crucial in defining the controller structure proposed in this paper. Another possibility, common in the literature, is to introduce a “slack bus” with zero voltage phase angle and to solve the equations for the line flows, completely eliminating δ from the problem formulation [2], [8]. However, in that case a specific structure, i.e. sparsity, of the power flow equations is lost.

In traditional power system structure, where the production units are owned by one utility and there is little or no price elastic consumers, adjusting the production according to the solution of the OPF problem is one of the major operational goals of a utility. In such a system, the OPF problem is directly solved at a utility dispatch center, and the optimal reference values p are sent to the production units. In this paper we are concerned with a deregulated, market-based power system, where the OPF problem is important due to its relation to the optimal nodal price problem (which is defined next).

In a liberalized, market-oriented power system, different units are owned by separate parties and each of them acts autonomously to maximize its own benefit. In other words, when a price-elastic unit at node i receives the current price for electrical power, i.e. λ_i , it adjusts its production level p_i to be equal to \bar{p}_i , where $\bar{p}_i = \arg \min_{p_i} \{J_i(p_i) - \lambda_i p_i\}$ subject to $\underline{p}_i \leq p_i \leq \bar{p}_i$. Since J_i is a strictly convex, continuously differentiable function, this relation defines a unique mapping from λ_i to \bar{p}_i for any $\lambda_i \in \mathbb{R}$. For convenience, we denote this mapping with $\Upsilon_i: \lambda_i \rightarrow p_i$, i.e.

$$\bar{p}_i = \Upsilon_i(\lambda_i) \triangleq \arg \min_{p_i \in [\underline{p}_i, \bar{p}_i]} J_i(p_i) - \lambda_i p_i, \quad (3)$$

and define $\Upsilon(\lambda) \triangleq \text{col}(\Upsilon_1(\lambda_1), \dots, \Upsilon_n(\lambda_n))$.

The *operational goal in a liberalized power system* is to determine the nodal price λ_i for each node i in the network, in such a way that the total benefit of the system is maximized, while all constraints are fulfilled. Formally, we define the optimal nodal price problem as follows.

Problem II.2 Optimal Nodal Prices (ONP) problem.

For any constant value of \hat{p} ,

$$\min_{\lambda, \delta} \sum_{i=1}^n J_i(\Upsilon_i(\lambda_i)) \quad \text{subject to} \quad \Upsilon(\lambda) - B\delta + \hat{p} = 0, \quad L\delta \leq \bar{p}_L, \quad (4)$$

where $\lambda = \text{col}(\lambda_1, \dots, \lambda_n)$ is a vector of nodal prices. \square

We will refer to a vector λ that solves the ONP problem with the term *vector of optimal nodal prices*.

The OPF and ONP problems are related through Lagrange duality, as it will be shown later in this section. The ONP problem is employed next to define the control problem considered in this paper

B. Control problem

Consider a power network where each price-elastic unit is a dynamical system, and assign to each such unit an appropriate model G_i of its dynamics. We assume for simplicity that each model G_i is an *LTI system with respect to the input* $p_i = Y_i(\lambda_i)$, which is specified by its state-space realization, i.e.

$$G_i : \begin{cases} \dot{x}_i = A_i x_i + B_i p_i = A_i x_i + B_i Y_i(\lambda_i) \\ p_i^A = C_i x_i \end{cases}, \quad \forall i, \quad (5)$$

where the power reference signal p_i is the input, and the actual node power injection p_i^A is the output. We denote the actual power injection of a price-inelastic unit with \hat{p}_i^A . Note that (1b) is always fulfilled when p and \hat{p} are replaced with $p^A = \text{col}(p_1^A, \dots, p_n^A)$ and $\hat{p}^A = \text{col}(\hat{p}_1^A, \dots, \hat{p}_n^A)$, since in that case (1b) represents the conservation law, i.e. $p^A - B\delta + \hat{p}^A = 0$. Achieving balance in *reference values* (1b), i.e. balance of the *desired* production and consumption, is a control problem. The production/consumption of price-inelastic units is an exogenous signal to the system and $\hat{p}^A = \hat{p}$, which yields:

$$p^A - B\delta + \hat{p} = 0. \quad (6)$$

The desired production/consumption of price-elastic units is a function of current nodal prices. Therefore, nodal prices can be effectively used as a feedback signal for power balance control. Each system G_i receives a price signal and, based on its benefit maximization objective (3), it maps the signal into a reference p_i . We will assume this mapping to be instantaneous, although the model can easily be extended with dynamics, time delays, threshold based rules, etc. *The complete dynamical model of the power system is described with the set of differential algebraic equations (5)-(6)*. For a detailed presentation of power system modeling for real-time power balance control problems we refer to [3], Chapter 11, or [9], Chapter 12.

Note that the mapping $Y_i : \lambda_i \rightarrow p_i$ is linear only if J_i is a quadratic function and $\underline{p}_i = -\infty$, $\bar{p}_i = \infty$. In practice, however, it will always be a nonlinear mapping and therefore, the model (5)-(6) with the nodal prices λ_i as inputs, is nonlinear.

To control the system, a measure of imbalance in (1b) has to be available. The network frequency serves that purpose. The system is in balance, in the sense of equality (1b), if the frequency is equal to its reference value, e.g. 50Hz in Europe. A change in any reference for power value causes the frequency to change. In steady-state, the frequency is equal for all nodes in the network and, if it is above its reference value, the total production in a network exceeds the total consumption. Finally, we are able to define the control problem.

Problem II.3 Optimal steady-state control problem.

Design an explicit feedback controller that has the network frequency and the line power flows as input, and the nodal prices as output, such that the following objective is met: for any constant value of \hat{p} such that the ONP problem is feasible, the state of the closed-loop system converges to a steady-state where the nodal prices are the *optimal nodal prices* as defined in Problem II.2. \square

In the above formulation, *explicit* denotes that the controller is not based on solving on-line an optimization problem, as it is for instance the case of the model predictive control framework [4]. In this paper, the emphasis is on constructing a distributed control scheme such that, if the system is stable, optimal nodal prices are always guaranteed in steady-state. Stability is beyond the scope of this paper and makes the object of future research.

III. CONTROLLER DESIGN

In this section we employ the relation between the solutions of the OPF and the ONP problems to obtain a solution to Problem II.3. Furthermore, we present the distributed implementation of the developed controller.

A. Explicit controller

The following proposition presents a basic result from power system economics.

Proposition III.1 *The optimal dual variable (Lagrange multiplier) associated with the power balance constraint (1b) in the Lagrange dual problem of OPF, is the vector of optimal nodal prices for the corresponding ONP problem.*

For an in-depth analysis of optimal nodal prices, we refer to [10] and [11]. For controller synthesis, based on Proposition III.1, we first analyse the conditions for optimality of the variables in the Lagrange dual problem to OPF.

Consider some constant value \hat{p} such that the OPF and ONP problems are feasible. The OPF problem is a convex problem which satisfies Slater's constraint qualification. This implies that strong duality holds and that first-order Karush-Kuhn-Tucker (KKT) conditions are *necessary and sufficient* conditions for optimality. For the OPF problem, the Lagrangian is given by $\mathcal{L}(p, \delta, v^+, v^-, \lambda, \mu) = J(p) - \lambda^\top (p - B\delta + \hat{p}) + (v^-)^\top (\underline{p} - p) + (v^+)^\top (p - \bar{p}) + \mu^\top (L\delta - \bar{p}_L)$ and the KKT conditions are given by:

$$p - B\delta + \hat{p} = 0, \quad (7a)$$

$$p - \bar{p} \leq 0, \quad v^+ \geq 0, \quad (v^+)^\top (p - \bar{p}) = 0, \quad (7b)$$

$$-p + \underline{p} \leq 0, \quad v^- \geq 0, \quad (v^-)^\top (-p + \underline{p}) = 0, \quad (7c)$$

$$L\delta - \bar{p}_L \leq 0, \quad \mu \geq 0, \quad \mu^\top (L\delta - \bar{p}_L) = 0, \quad (7d)$$

$$\nabla_p J(p) - \lambda + v^+ - v^- = 0, \quad (7e)$$

$$B\lambda + L^\top \mu = 0, \quad (7f)$$

where λ , v^+ , v^- and μ are (*vector*) Lagrange multipliers. The multiplier λ is associated with the equality constraint (7a) and is therefore not sign restricted.

Remark III.2 B is a singular matrix with rank deficiency one and with the kernel space spanned by the vector $\mathbf{1}_n$. This is easy to check due to its specific structure, e.g. by putting B into its Echelon form. It is also apparent that the kernel space of B is spanned by the vector $\mathbf{1}_n$, since $B\mathbf{1}_n = A\mathbf{1}_n - \text{diag}(A\mathbf{1}_n)\mathbf{1}_n = 0$. Physically, this reflects the fact that only the relative voltage phase angles determine the power flow. Note also that $\mathbf{1}_n \notin \text{im} B$, since $B = B^\top$ implies $\mathbf{1}_n^\top B = 0$, i.e. $\mathbf{1}_n$ is orthogonal to each column of B . Finally, analyzing the structure of L (see (1d) and (2)) one can easily observe that $\mathbf{1}_n^\top L^\top = 0$ and therefore, $\mathbf{1}_n \notin \text{im} L^\top$. These properties of B and L will be used later in this section to prove Proposition III.3.

Consider the OPF problem solution for some constant value \hat{p} such that the problem is feasible. We denote the minimizers of OPF with \tilde{p} , $\tilde{\delta}$, and with $\tilde{\lambda}$ the value of the corresponding Lagrange multiplier. Strict convexity of each J_i implies that at the optimum \tilde{p} is unique. On the other hand, due to singularity of B , if $\tilde{\delta}$ is a minimizer so is $\tilde{\delta} + \mathbf{1}_n c$ where $c \in \mathbb{R}$ is an arbitrary constant. However, note that for all minimizers the set of active constraints is uniquely determined. Furthermore, we denote with $\tilde{\mu}$ and $\tilde{\nu}$ the Lagrange multipliers corresponding to inactive and active line power flow constraints, respectively. Analogously, we define \tilde{v}^+ , \tilde{v}^- and \tilde{v}^+ , \tilde{v}^- . For inactive constraints $\text{col}(\tilde{\mu}, \tilde{v}^+, \tilde{v}^-) = 0$. The equality (7f) yields $B\tilde{\lambda} = -L^\top \tilde{\mu}$. This condition implies that $\tilde{\lambda} \in \ker B$ in the case that no lines are congested. This further implies $\tilde{\lambda} = \mathbf{1}_n \lambda^*$, $\lambda^* \in \mathbb{R}$, i.e. at the optimum, there is one price in the network for all nodes. *In case at least one line in the system is congested, it follows that the optimal nodal prices will in general be different for each node in the system.*

Next, we present the explicit dynamic controller that solves Problem II.3. We define $f_i = \frac{\delta_i}{2\pi}$ as the network frequency [Hz] at node i , the abbreviation $f \triangleq \text{col}(f_1, \dots, f_n)$, and we use f_{ref} ($f_{\text{ref}} \in \mathbb{R}$) to denote the frequency reference value, e.g. $f_{\text{ref}} = 50\text{Hz}$. Let K_λ , K_f and K_p be diagonal matrices with positive elements on the diagonal, such that $K_f = \alpha K_\lambda$, $\alpha \in \mathbb{R}$, $\alpha > 0$, and let Γ denote a diagonal matrix of the same size as K_p . Consider the following explicit piecewise affine *dynamic* controller:

$$\begin{pmatrix} \dot{x}_\lambda \\ \dot{x}_\mu \end{pmatrix} = \begin{pmatrix} -K_\lambda B & -K_\lambda L^\top \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \end{pmatrix} + \begin{pmatrix} -K_f & 0 \\ 0 & \Gamma \end{pmatrix} \begin{pmatrix} \Delta f \\ \Delta p_L \end{pmatrix}, \quad (8a)$$

$$\lambda = (I_n \ 0) \begin{pmatrix} x_\lambda \\ x_\mu \end{pmatrix}, \quad (8b)$$

$$\begin{cases} [\Gamma]_{ii} = [K_p]_{ii} & \text{if } [x_\mu]_i \geq 0 \ \& \ [\Delta p_L]_i \geq 0 \\ [\Gamma]_{ii} = [K_p]_{ii} & \text{if } [x_\mu]_i > 0 \ \& \ [\Delta p_L]_i < 0 \\ [\Gamma]_{ii} = 0 & \text{if } [x_\mu]_i = 0 \ \& \ [\Delta p_L]_i < 0, \end{cases} \quad (8c)$$

$$x_\mu(0) \geq 0, \quad (8d)$$

where x_λ and x_μ denote the controller states and the matrices K_λ , K_f and K_p represent the controller gains. In (8), the

inputs $\Delta p_L = L\delta - \bar{p}_L$ and $\Delta f = f - \mathbf{1}_n f_{\text{ref}}$ denote the line power overflow and the frequency deviation, respectively, while the output λ denotes the vector of nodal prices. Note that, due to the initialization constraint (8d), it holds that $x_\mu(t) \geq 0$ for all $t \geq 0$.

Assumption 1 The closed-loop system resulting from the interconnection of the explicit dynamic controller (8) with the overall network model given by the differential algebraic equations (5)-(6) is globally asymptotically stable for any constant value of \hat{p} (i.e. with respect to the corresponding steady-state) such that the ONP problem is feasible. \square

Proposition III.3 *Suppose that Assumption 1 holds. Then the explicit dynamic controller (8) solves the optimal steady-state control problem, as defined in Problem II.3.*

Proof: To prove Proposition III.3, it suffices to show that in steady-state, the vector of nodal prices λ in (8) coincides with the Lagrange multiplier λ in (7), and therefore, by Proposition III.1, is a vector of optimal nodal prices. In steady-state, the frequency is equal for all nodes, i.e. $\Delta f = \mathbf{1}_n \Delta f^*$, $\Delta f^* \in \mathbb{R}$. Since in the power system dynamics (5)-(6) there is no direct feedthrough from the input λ to the outputs Δf and Δp_L , in *steady-state* the following conditions are satisfied: (i) for $\mu := x_\mu$ and from (8a) and (8b) it follows that $B\lambda + L^\top \mu + \mathbf{1}_n \alpha \Delta f^* = 0$, which, together with $\mathbf{1}_n \notin \text{im}(B \ L^\top)$ (see Remark III.2), implies that $\Delta f^* = 0$ and $B\lambda + L^\top \mu = 0$, i.e. that the power balance constraint (7a) and the optimality condition (7f) are satisfied; (ii) from (8a) and (8c) it follows that (7d) is satisfied for $\mu := x_\mu$; (iii) (7b), (7c), (7e) are satisfied since they correspond to the KKT conditions for optimization problem in (3), which concludes the proof. \blacksquare

Remark III.4 Due to the steady-state related complementarity conditions in (7d), there does not exist an LTI controller that solves Problem II.3, and a hybrid controller, such as the piecewise affine controller (8), is a necessity. For fixed values of the gain matrices K_λ , K_f and K_p in (8), one can a posteriori check asymptotic stability of the resulting closed-loop system by searching for quadratic Lyapunov functions, and therefore, validate Assumption 1.

B. Distributed implementation of the explicit controller

Matrices B and L in (8) are highly structured and related in such a way that this structure can be effectively utilized for *distributed* implementation of the proposed explicit controller. First, we illustrate this with the following simple example. In the next section the efficiency of developed methodology will be demonstrated on the IEEE 39-bus New England test network.

Example. Consider a simple network depicted in Figure 1 and assume that at the optimum the lines e_{12} and e_{13} are congested so that $p_{12} = \bar{p}_{12}$ and $p_{13} = \bar{p}_{13}$. With μ_{12} and μ_{13} denoting the corresponding Lagrange multipliers from (7d), the optimality condition (7f) relates the optimal nodal

prices with the following equality:

$$\begin{pmatrix} b_{12,13} & -b_{12} & -b_{13} & 0 & b_{12} & b_{13} \\ -b_{12} & b_{12,23} & -b_{23} & 0 & -b_{12} & 0 \\ -b_{13} & -b_{23} & b_{13,23,34} & -b_{34} & 0 & -b_{13} \\ 0 & 0 & -b_{34} & b_{34} & 0 & 0 \end{pmatrix} \times \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \bar{\mu}_{12} \\ \mu_{13} \end{pmatrix} = 0, \quad (9)$$

where $b_{12,13} = b_{12} + b_{13}$ and so on. Each row in (9) represents an equality related to the corresponding node in the network, i.e. the first row is related to the first node etc. Note that the i -th row directly relates the nodal price λ_i only with the nodal prices of its neighboring nodes, i.e. with λ_j , $j \in I(N_i)$. Similarly, only the nodal prices in the nodes corresponding to the congested line e_{ij} are directly related to the corresponding Lagrange multiplier μ_{ij} . Since in practice B is usually sparse, the number of neighbors for most of the nodes is small, e.g. two to four. These highly structured relations from the optimality conditions (7) are as well present in the proposed controller (8), allowing for its *distributed* implementation. This means that the control law (8) can be implemented through a set of “nodal controllers”, where one nodal controller (NC) is assigned to each node in the network, and each NC communicates only with the NC 's of the neighboring nodes. From (8) and (9) it is easy to derive that the NC corresponding to node 1 in the network depicted in Figure 1 is given by:

$$\begin{pmatrix} \dot{x}_{\lambda_1} \\ \dot{x}_{\mu_{12}} \\ \dot{x}_{\mu_{13}} \end{pmatrix} = \begin{pmatrix} -k_{\lambda_1} b_{12,13} & k_{\lambda_1} b_{12} & k_{\lambda_1} b_{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_{\lambda_1} \\ x_{\mu_{12}} \\ x_{\mu_{13}} \end{pmatrix} + \begin{pmatrix} k_{\lambda_1} b_{12} & k_{\lambda_1} b_{13} & -k_{f_1} & 0 & 0 \\ 0 & 0 & 0 & \Gamma_{12} & 0 \\ 0 & 0 & 0 & 0 & \Gamma_{13} \end{pmatrix} \begin{pmatrix} x_{\lambda_2} \\ x_{\lambda_3} \\ \Delta f_1 \\ \Delta p_{12} \\ \Delta p_{13} \end{pmatrix}, \quad (10a)$$

$$\lambda_1 = (1 \ 0 \ 0) \text{col}(x_{\lambda_1}, x_{\mu_{12}}, x_{\mu_{13}}) \quad (10b)$$

where $k_{\lambda_1} = [K_{\lambda}]_{11}$, $k_{f_1} = [K_f]_{11}$, and $\Gamma_{12} = [\Gamma]_{11}$, $\Gamma_{13} = [\Gamma]_{22}$ are defined according to (8c).

Note that the state $x_{\mu_{ij}}$ is present only in one of the adjacent nodal controllers, i.e. in node i or in node j , and is communicated to the NC in the other node.

The distributed implementation of developed explicit controller is graphically illustrated in Figure 2.

IV. APPLICATION CASE STUDY

To illustrate the potential of the developed methodology for practical application we consider the widely used IEEE 39-bus New England test network. The network topology, generators and loads are depicted in Figure 3. The complete network data, including reactance of each line and load values can be found in [12]. All generators in the system are

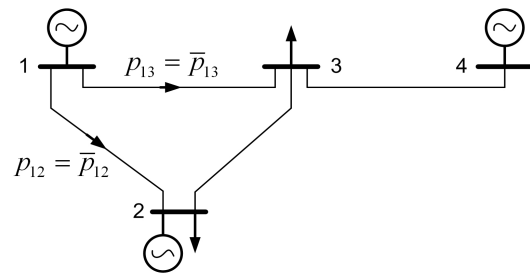


Fig. 1. An example of a simple congested network.

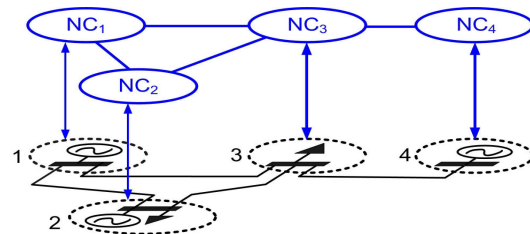


Fig. 2. Distributed control scheme for power balance and congestion control.

modeled using a third order model consisting of governor, turbine and rotor dynamics. This is a standard model used in “automatic generation control” studies [3]. The parameter values, in per units, are taken to be in the $\pm 20\%$ interval from the values given in [9], pp. 545. Each generator is taken to be equipped with a proportional feedback controller for frequency control with the gain in the interval [18, 24]. We have used quadratic functions to represent the variable production costs, i.e. $J_i(p_i) = \frac{1}{2} c_{g,i} p_i^2 + b_{g,i} p_i$, with the values of parameters $c_{g,i}$, $b_{g,i}$, for $i = 1, \dots, 10$ as listed in Table 5 in [13]. The lower saturation limit and the upper saturation limit for each generator was set to 0 and 10, respectively. All loads are taken to be price-inelastic, with the values from [12].

The proposed distributed controller was implemented with the following values of gain matrices: $K_{\lambda} = 3I_{39}$, $K_f = 8I_{39}$. For simplicity of exposition, the line power flow limit was assigned only for the line connecting nodes 25 and 26, and the corresponding gain K_p in the controller was set equal to 1.

The simulation results are presented in Figure 4 and Figure 5. In the beginning of the simulation, the line flow limit $\bar{p}_{25,26}$ was set to infinity, and the corresponding steady-state operating point is characterized by the unique price of 39.28 for all nodes. At time instant 5s, the line limit constraint $\bar{p}_{25,26} = 1.5$ was imposed. The solid lines in Figure 4 are simulated trajectories of nodal prices for the generator buses, i.e. for buses 30 to 39, which is where the generators are connected. In the same figure, dashed lines indicate the off-line calculated values of corresponding steady-state optimal nodal prices. For clarity, the trajectories of the remaining 29 nodal prices were not plotted. In the simulation, all these trajectories converge to the corresponding optimal values of nodal prices as well. The optimal nodal prices for all buses are presented in Figure 6. In this figure, the nodal prices corresponding to generator buses 30-39 are emphasized with

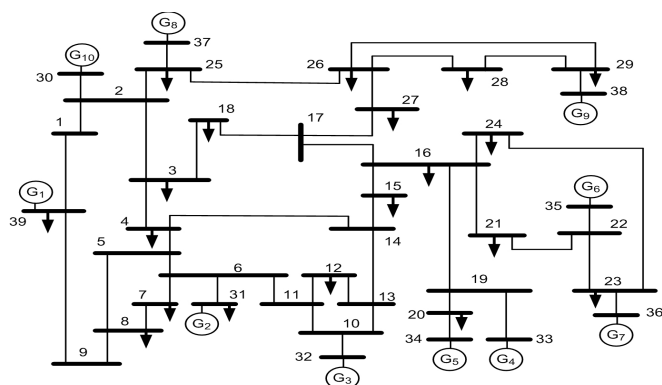


Fig. 3. IEEE 39-bus New England test system.

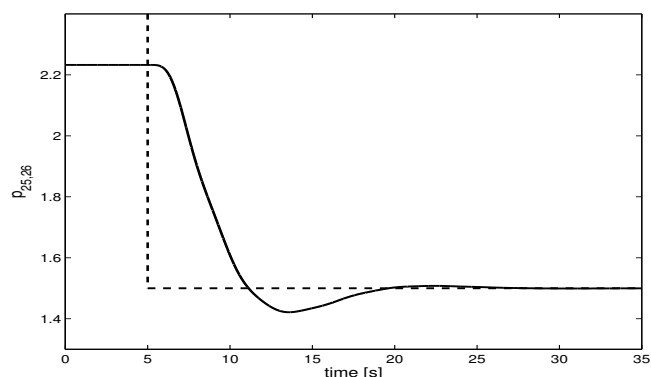


Fig. 5. Power flow in the line connecting buses 25 and 26.

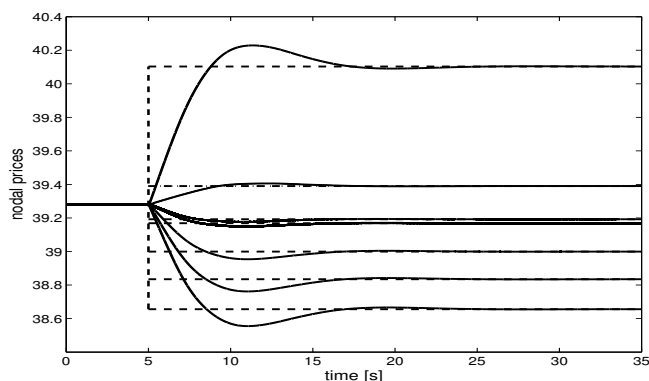


Fig. 4. Trajectories of nodal prices for generator buses 30-39.

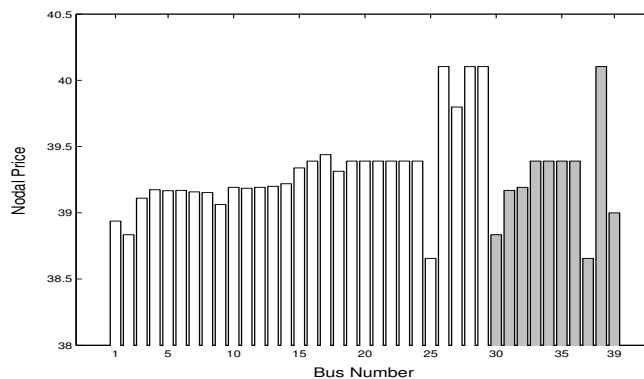


Fig. 6. Nodal prices in the case of congestion.

the gray shaded bars. The solid line in Figure 5 represents the simulated trajectory of the line power flow $p_{25,26}$. In the same figure, the dashed line indicates the limits on the power flow $\bar{p}_{25,26}$. The obtained simulation results clearly illustrate the efficiency of the proposed distributed control scheme.

V. CONCLUSIONS

This paper proposed a new distributed control structure for optimal power balancing in electrical power systems. Optimality is defined by economic criteria for steady-state operation and includes inequality constraints induced by limited capacity of power lines. Due to the inequality constraints, the controller has a piecewise affine structure. The proposed controller has several desired properties such as simplicity, computational efficiency (e.g. no on-line optimization is required) and distributed structure. Its main advantage is that, besides network parameters, such as line impedances, it requires no knowledge of cost/benefit functions of producers/consumers to converge to the economically optimal working point, while respecting all network constraints. A realistic case study illustrated the effectiveness of the proposed control scheme.

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