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A healthcare facility location problem for a multi-disease, multi-service environment under risk aversion

S. Taymaz a, b, C. Iyigun b, *, Z.P. Bayindir b, N.P. Dellaert c

a The United Nations High Commissioner for Refugees, Ankara, Turkey
b Department of Industrial Engineering, Middle East Technical University, Ankara, Turkey
c Department of Industrial Engineering and Innovation Sciences, Eindhoven University of Technology, Eindhoven, the Netherlands

ABSTRACT

This paper presents a stochastic optimisation model for locating walk-in clinics for mobile populations in a network. The walk-in clinics ensure a continuum of care for the mobile population across the network by offering a perpetuation of services along the transportation lines, and also establishing referral systems to local healthcare facilities. The continuum of care requirements for different diseases is modelled using coverage definitions that are designed specifically to reflect the adherence protocols for services for different diseases. The risk of not providing the required care under different realisations of health service demand is considered. In this paper, for a multi-disease, multi-service environment, we propose a model to determine the location of roadside walk-in clinics and their assigned services. The objective is to maximise the total expected weighted coverage of the network subject to a Conditional-Value-at-Risk (CVaR) measure. This paper presents developed coverage definitions, the optimisation model and the computational study carried out on a real-life case in Africa.

1. Introduction

According to the global assessment of the World Health Organisation [1], more than one-fifth of global deaths are attributable to modifiable environmental factors, defined as all the physical, chemical and biological factors external to a person, and all related behaviors. This relation between environmental factors and disease burden demonstrates that with healthier environments, 12.6 million deaths worldwide could be prevented with improved quality of life and well-being. Even though the health risks are preventable or reducible, there are substantial obstacles that must first be surmounted, such as unsubstantiated healthcare policies and interventions, poor healthcare systems, inefficient resource planning, limited access to healthcare services, and poverty. With an increased knowledge of environment-health interactions, practicable strategies and interventions can be developed to decrease the morbidity and mortality associated with diseases.

One part of health related problems is concerned with mobile populations, groups of people migrating across borders and changing location. They are subject to further vulnerabilities compared to the rest of the population. Some of the reasons for this are limited access to healthcare facilities, the inadequate dissemination of information, and poor working conditions [2]. As the population moves along the network, they act as vectors of transmission, carrying diseases along with themselves and playing a major role in the spread of diseases. The improvement of their health environment, based on the “social and contextual realities faced by the mobile populations”, is a must to reduce their vulnerability and curtail the chain of transmission [3].

Our study particularly focuses on transport workers and commercial sex workers in Africa (also referred to as mobile workers) as a part of the mobile population, although the findings can be extended to cover other groups. The large number of hubs, harbours and airports in Africa results in goods flowing continuously, requiring an extensive number of transport workers working in hard conditions. The risks of infection and transmission are increased by factors such as being away from home for long durations, delays caused by border crossings, the increased possibility of sexual behaviours, lack of knowledge and limited access to health care facilities. More than one-third of truck drivers have reported that they have been repeatedly stopping at a “hot-spot”, referred to as a delay location, for interaction with sex-workers, and 30% of interactions were unprotected [3]. As our motivation arises from the African case, looking at the prevalence rates, it is seen that 60% of truck drivers reported having contracted Sexually Transmitted Infections (STIs) in the previous six-month period, and in total, truck drivers and sex-workers have a 56% HIV prevalence rate in the South African region [4].

* Corresponding author.
E-mail address: iyigun@metu.edu.tr (C. Iyigun).

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In a study in 2004, 15.2% of truck drivers and their assistants in East Africa reported experiencing STI symptoms in the previous twelve-month period, and 84.7% reported that the cases occurred while travelling [5]. The prevalence rate of HIV for truck drivers and their assistants was reported as 34% which is higher than the one for the general population in East Africa and Kenya [5]. In a more recent study, out of 261 truck drivers in Uganda, 57.8% reported experiencing STI symptoms in the previous twelve-month period, and it was reported that 17% of the cases did not complete their treatment [6]. The prevalence rate of HIV for truck drivers was 26%, that is almost three times more than HIV of the cases did not complete their treatment [6]. The prevalence rate of general population of the same age group men in South Africa [7]. These prevalence rates demonstrate the connection between healthcare problems and mobile population, and highlight the urgent need for interventions for specific conditions and needs of mobile population.

Access to health care services for both transport workers and sex workers is unfortunately not straightforward. For transport workers, having to work to strict time frames and so being unable to deviate from defined routes makes challenging to access to healthcare facilities located away from transportation lines. Obstacles such as unsuitable road conditions and inadequate parking near healthcare facilities further complicate problem. A possible solution is the provision of small walk-in clinics located along transportation lines and at hot-spots to provide health services for mobile populations. North Star Alliance, a non-governmental organisation, developed the concept of Roadside Wellness Centers (RWC): a walk-in clinic, which can be either mobile or not, located at critical locations where the mobile population congregates. The aim of establishing RWCs is to extend the local healthcare infrastructure and the coverage of the transportation lines by offering continuous access to and primary help in basic healthcare facilities. Each RWC also acts like a “honeypot” by attracting mobile populations through offering what they need, but at the same time their awareness can be increased and they can be referred to better equipped healthcare facilities if needed.

The idea of a “continuum of care” is embodied by the provision of sustainable access to health and safety services for the mobile workers who require healthcare not only at a single point but continuously as they travel. The continuum of care, a critical term for our study and also a part of North Star’s strategy, can be categorised in terms of “vertical” and “horizontal” continuity. The “horizontal” continuity is concerned with the mobility along the transportation lines, where a high quality health service is ensured by the provision of follow-up of activities for patients along the transportation lines even if the services are required in different districts, regions or countries. In terms of “vertical” continuity, the type of care is ensured by aligning the RWCs with local district health systems. It is provided by establishing referral systems to other preferred local healthcare facilities when the services in the RWCs are insufficient.

This study proposes a novel stochastic programming model for a network design problem of walk-in clinics (hereinafter referred to as clinics), designed to specify the optimal locations of limited number of clinics, as well as the allocation of services offered for different diseases in each clinic, under the assumption that the flow of mobile workers with different diseases is random. To reach this decision, a coverage definition is formulated and employed, based on the continuum of care description provided above. Coverage can be defined as the extent of the healthcare services offered within each clinic along the network to meet the demand. This demand in this problem is the mobile workers’ requirement to have access to healthcare services. This demand occurs between the origin and destination pairs, the transportation lines, also referred as the traffic flows, and it is specifically defined for different services and diseases. The traffic flows are represented via random variables and different realisations (scenarios) of the flows that occur daily on the transportation lines. The average values of flows for different transportation lines vary, as some can be more congested than the others. In addition, the flow on certain transportation lines might also have greater fluctuations from the average value. Due to these fluctuations, a determined set of clinic locations and an allocation of healthcare services may not be able to provide an effective coverage of the demand under some cases (scenarios), and this poses a risk for the mobile workers. Hence, the optimal decision should acknowledge the different scenarios of flow and the associated risk. This is achieved with the introduction of a loss function to the model. Here, the loss can be defined as the deviation of the provided coverage from the maximum possible coverage that can be attained over the network. Based on the loss definition, for each disease under consideration, the risk is measured by calculating the expected flow that is not covered under the worst-case scenarios of the flow. This measure is referred to as Conditional-Value-at-Risk (CVaR), which is a coherent risk measure in the literature and it is adapted in the model to control the risk associated with the uncertain flows [8]. The aim is to keep the risk below a pre-determined threshold, which is set by the decision maker. Here, the decision maker is considered as the person/entity who is responsible for finalising the decision on the optimal location of clinics, as well as allocation of services, by providing the required inputs to the model to reflect the problem environment and characteristics. The threshold set by the decision maker should reflect his/her risk perception as it indicates how much of loss of coverage can be tolerated.

One of the main contributions of this study comes with the definition and formulation of the coverage. The definition of coverage differentiates by considering the nature of different diseases and we propose a coverage definition that can handle the particular requirements of each disease and service pair. This differentiation is adapted into our model by employing a multi-disease, multi-service approach. Furthermore, the model utilises the risk measure CVaR, which takes into consideration the uncertain nature of demand flow along the transportation lines. The integration of risk measure into the model enables the decision maker to account for possible losses that may arise due to a lack of coverage under different realisations of the demand. Furthermore, the control over the risk is disease specific, meaning under limited resources (i.e., having an upper bound on the number of clinics that can be opened), the decision maker can also make judgements on the coverage trade-off between different diseases and services. Computational study results also show that reflecting different disease and service requirements in the coverage definition, as well as risk measure definitions, results in specialised networks that are targeted towards real-life concerns.

The reminder of this paper is organized as follows. The literature review is given in Section 2 with a focus on location problems and the application of risk measures. Section 3 gives the problem definition and the problem environment considered. Section 4 and 5 further elaborate on the formulation details presented in the given problem definition. The results of the formulations are analyzed in the computational studies given in Section 6 and the study concludes in Section 7.

2. Literature review

Providing sustainable access to healthcare services for mobile populations with a clinic network is the core of our study. For this purpose, a coverage definition is employed in the study to reflect the “continuum of care” requirements for different disease and service pairs. In the literature on facility location problems, similar problems are studied, while a facility’s coverage is defined with respect to the characteristics of the demand. As discussed by Sterle et al. [9], the demand can be flow based, i.e., the demand travels between an origin and destination pair (similar to the problem in this study) or it can be point based, i.e., the demand does not change locations but occurs at a given point. A collection of research that is discussed under facility location problems with flow-based demand deals with flow capturing location models (FCLM). The objective is to maximise the captured flow by the facility, and a flow can be categorised as captured only if a facility is located on the flow between the origin and destination points. One of the earliest studies about FCLM is by Berman et al. [10], where customers
move along pre-defined routes and are captured by a facility that is located on the pre-defined route. The problem is later extended by relaxing the assumption of a pre-defined route and customers are allowed to deviate from the routes [11]. Dandan et al. [12] study a multi-type flow considering multi-purpose customer flows which are mutually influencing each other. Sterle et al. [9] extends the FCLM to multi-period, and determines the optimal location of facilities on a network, where the flow patterns change over time. The dynamic environment of the problem is tackled with the formulation of two objectives, one is to maximise the flow intercepted and one is to minimise the cost of relocation over a time horizon. An extension to FCLM is the flow refuelling location model (FRLM). FRLM challenges the assumption of FCLM and it studies the case where more than once facility can be required for coverage on a path [13]. In this study, it is a refuelling problem, where the drivers are required to have access to refuelling stations (facilities) throughout the trip between the origin and destination. Upchurch et al. [14] extend the FRLM problem with capacitated fuelling stations, where each fuelling station can accommodate up to a certain level of customers. Capar et al. [15] extend the previous formulation with a more efficient one, which is referred to as the arc-cover-path cover formulation. Riemann et al. [16] also extend the arc-cover-path cover formulation by taking into account traffic congestion and the availability of fuelling stations, both of which affect the driver's routing choice. FCLM and FRLM problems are particularly relevant to our problem because the demand occurs as traffic flows between origin and destination pairs, and the demand requires continuous access to facilities throughout the trip in order to be categorised as covered. In our problem, the flow is not deterministic and a unique formulation is developed that meets the problem’s needs.

Another type of facility location model that is relevant to our study is the maximal covering location problem (MCLP), which focuses on the facility location problem with point-based demand. One of the earliest models for MCLP, developed by Church and ReVelle [17], optimises the location of a pre-determined number of facilities to maximise the demand covered. The demand is assumed to be covered or not, depending on the distance from the facilities and the required coverage radius. Many extensions to the MCLP problem have been studied and the first relevant extension is relaxing the binary coverage assumption of the MCLP, as studied by Berman and Krass [18], Berman et al. [19] and Drezner et al. [20]. The authors model the coverage not in a binary manner, but as a value that declines with respect to increased distance from the facility. Another type of extension incorporates the necessity of completing a tour by visiting a certain number of facilities in order for the demand to be referred to as covered [21]. Ruby and Lim [22] develop a model for the case where multiple facilities are required on the paths in the network. A demand point is considered to be covered if sufficient numbers of stops are available. Another extension is studied by O’Hanley and Church [23], where the worst-case scenario of the ratio of critical facilities is examined. These facilities are considered as the ones which together cover the largest group of demand. In this problem, in addition to maximising the initial coverage, the minimum coverage offered by the remaining facilities after the loss of critical facilities is also considered as objective function.

The most recent survey focusing on the MCLP problem can be found in Farahani et al. [24] and Berman et al. [25]. Even though in our problem, the demand is not point based, our proposed coverage formulation focuses on the distance between the demand and the nearest critical head, at a given point in time. This approach relates to the facility location problem with point-based demand, and it is relevant to the developed coverage definitions.

Facility location problems and coverage definitions are also widely applied in the field of humanitarian operations and emergency planning. Within these problems, coverage definitions are adopted according to the characteristics of the problem and needs of the problem nature. Drezner [26] developed a model to locate a pre-determined number of casualty collection points by testing the problem on five different objective functions. Among the optimal solutions, they suggest that the best performing objective is the one that maximises coverage, as it aims to cover as much as possible within a given distance. Lee et al. [27] developed an emergency-response tool to determine the locations for dispensing medication, as well as making other relevant decisions. Initially, the number of facilities to be opened is determined by the model, and based on this number, the locations of the facilities are optimised by minimising the travel time between the demand and the facility. Abounacer et al. [28] extend the MCLP problem to a multi-objective version with three conflicting objectives: minimising the travel time between the facilities and the demand, minimising number of facilities and minimising the demand without cover. Jia et al. [29] address the issue of large-scale emergencies in which the MCLP is extended by incorporating quality of coverage requirements. Li et al. [30] extend the classical coverage problem to a co-operative coverage problem where the demand is covered by multiple facilities, and operated by different actors. Budget considerations are included in the model to guide the relief organisations in terms of budget allocations as well as the trade-off between costs and benefits. Balcik and Beamon [31] develop a MCLP model for the locating of facilities for relief operations, where the coverage is defined in an hierarchical way, meaning demand located further than a given threshold is considered to be partially covered. A review of an MCLP problem applied to emergency response can be found in Li et al. [32], as well as Caunhye et al. [33]. Another extended review of facility location problems applied to humanitarian logistics is in Celik et al. [34].

Unlike with deterministic models, in our problem, the traffic flow between the origin and destination is stochastic and so the developed coverage model needs to incorporate this uncertainty. Therefore, facility location problems for an uncertain problem environment need to be considered. For the uncappeditated plant location problem, Jucker and Carlson [35] describe the objective function with a mean-variance formulation, unlike mean-outcome models where only the expected performance of the system is considered and the variance is abandoned. The application of portfolio approaches to location analysis is studied by Hanink [36] and Hodder [37] with the integration of contemporary portfolio theory. For the robust capacitated international sourcing problem, Velarde and Laguna [38] deal with the uncertain parameters by including risk measures in the objective function, which minimises the expected cost and penalises positive deviations from the expected value. The penalisations of deviations are multiplied with a coefficient that is adjusted by the decision-maker according to the significance of the risk. Another approach is α-reliable minimax problems, where rather than minimising the maximum regret, the maximum regret over a possible subset of scenarios with a total probability of at least α is minimised [39]. The set of scenarios where maximum regret is minimised is referred to as a reliable set and is endogenously determined. Chen et al. [40] extend the model that minimises the mean expected regret and this extension also accounts for the magnitude of regrets for the scenarios not included in the reliability set. The review paper by Snyder [41] presents different models developed in the literature for facility location problems under uncertain environments.

The application of stochastic facility location models in the field of humanitarian logistics is also extensive. Murali et al. [42] develop a model to determine the location of medicine distribution facilities in the case of bio-terror attack. The demand uncertainty is handled in the model with chance constraints. The constraints deal with the uncovered demand that they accommodate. The distance between the demand and the facility. Rawls and Turnquist [43] model the optimal pre-positioning of emergency supplies in terms of the location of the centers and the quantity at each center. It is modelled as a two-stage stochastic program and the uncertainties in the demand values and availability of the transportation roads are addressed. Rawls and Turnquist [44] also study pre-positioning and dynamic delivery planning for natural disasters, integrating the concept of reliability. The model guarantees that with a pre-defined certainty level, the demand is covered.
In our study, in order to decide on the optimal location of clinics in an uncertain problem environment, the risk associated with the lack of coverage based on the location of clinics is measured and integrated into the decision-making process with a risk averse approach. Out of the risk measures proposed by Rockafellar [8], Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) can be considered for our problem. In the problem context, VaR refers to the minimum amount of uncovered flow value which will not be exceeded by a given confidence level. CVaR, on the other hand, refers to the “expected” flow that is not covered in the worst-case scenarios, defined by a given confidence level. Unlike VaR, CVaR accounts for the magnitude of extreme losses and therefore, a CVaR measure is employed in our problem instead of VaR.

In this study, novel coverage definitions were developed in the light of the problem nature. The availability of a single clinic between the origin and the destination does not necessarily indicate that the demand is covered throughout the transportation line. The demand should ideally be met by having access to a clinic for the required treatment of a disease at any point throughout an instance of travel. Therefore, at any point along the transportation line, the coverage of the demand is calculated and the total coverage for the travel between the origin and destination is an accumulation of these values. Additionally, the definition of coverage at any point in time is dependent on the specific requirements of the disease and service pairs. Given these requirements, a unique formulation for the problem is developed. This problem has been studied by Taymaz [45] in her thesis by incorporating the properties of both coverage and flow interception problems and developing formulations that account for the continuum of care requirements. De Vries et al. [46] consider the problem of locating healthcare facilities that offer a single type of service in order to maximise the patient volume visiting them under a deterministic demand setting. This current study extends the problem, since we consider the stochastic nature of the flow on the network and examine the risk of not satisfying the continuum of care requirements of a disease with the open clinics that arises from the stochastic nature of the flow. Moreover, our study considers a multi-disease, multi-service environment where the continuum of care requirements differ for each specific disease and service. Besides the horizontal dimension, the vertical dimension of a continuum of care that accounts for referrals to the local healthcare services is also included in our model.

3. Environment considered and problem definition

In the current problem, a clinic is the foundation for sustainable and quality health services for a mobile worker. They are situated along the transportation lines on hot-spots (i.e., intersections of transportation lines, checkpoints and weighbridges) where mobile workers congregate and wait for long periods of time (called “delay durations”). These hot-spots are visited by a large number of mobile workers every day, and having large numbers in one place often eventuates to sexual interaction between the mobile workers and high-risk groups, such as sex workers. Due to the many vulnerabilities explained in Section 1, mobile workers suffer from communicable diseases and become a transmission vector of these diseases as they travel along the transportation lines. Clinics, commonly the only convenient place to receive healthcare services when travelling, are approached by the mobile workers to alleviate their conditions whenever needed. Within the spectrum of services offered (such as diagnosis and treatment), many of the diseases with high prevalence rates are addressed. Clinics are adapted to enable the continuation of the service at any clinic along the transportation lines with electronic patient data being available in each of the clinics. Furthermore, with collaboration from other healthcare providers, any condition that cannot be resolved within the clinic is referred to better equipped facilities. This viewpoint on the delivery of services is structured under the “continuum of care” definition, which can be elaborated as:

The horizontal continuum of care, as the name implies, refers to the availability of clinics along the transportation line that are equipped with a certain service for a disease. To clarify, an example case related with the diagnosis of malaria can be given. For effective malaria treatment, a timely and correct diagnosis of malaria is an essential step [47]. Since diagnosing malaria within 24–48 h of onset of the illness is critical, a malaria diagnosis service should be accessible as the mobile worker travels along the transportation line within those limits [48]. Therefore, when a network of diagnosis of malaria is designed in a region with a high risk level of malaria, such protocols should be regarded.

The vertical continuum of care is concerned with the availability of local healthcare facilities that the clinics can refer to. As clinics are only able to provide a limited extent of service, they should be aligned with the local health care provider for further treatment.

The continuum of care requirement for each service for a disease is distinct, and the network design should be able to capture all these requirements effectively to provide the imperative healthcare services for mobile workers. Given limited funds, there is an upper-bound on the number of clinics that can be opened. This limitation together with the distinctions in the continuum of care requirements for services and disease pairs result in trade-offs in the location of clinics and their allocated services for different diseases (as it may be in the benefit of one disease and not for another). The uncertainties in the number of mobile workers transporting along the transportation lines further complicate the problem.

In the model developed, the aim is to determine the location of clinics on the network together with the information on which services for diseases are to be provided. The network consists of a set of arcs, representing the transportation lines for major transportation corridors and a set of nodes of the network, which are the potential clinic locations. The flow of mobile workers travelling along the transportation lines is called as the traffic flow and it is specifically defined for different services and diseases. The traffic flow is uncertain and it is assumed to be stationary with a known probability mass function.

The continuum of care requirements are integrated into the model with coverage definitions that are able to reflect the characteristics of the requirements for distinct service-disease pairs. The objective function maximises the attained service level with the clinics, which is defined as the total weighted expected coverage for different diseases. Coverage values of different diseases are aggregated with respect to the weight given by the decision-maker. The assignment of the weight is within the scope of the decision-makers responsibilities, as they are able to rely on facts to understand the requirements of the network in focus and can make informed decisions about the relative importance of different diseases. Furthermore, due to additional constraints, such as stakeholder preferences, the weight might also require adjustment. Stakeholders can include government authorities, non-governmental organisations and donors that financially support the clinic establishment. This external inference enables interaction with the decision-makers to make solid decisions applicable in the field.

To deal with the uncertainty in demand and to design a network where a lack of coverage does not lead to high values of loss, a cap is put on the expected lack of coverage for a certain portion of the worst-case scenarios. This is achieved with the CVaR measure to enable control over the risk for the decision-maker in accordance with their risk perception. The measure is included in the model separately for each disease. Another limitation is imposed by the upper-bound on the number of clinics to be opened, as discussed. Given the definition for the problem, the notation for the mathematical formulation is summarised in Table 1.

The associated network design problem, P, can be expressed in a closed form as,
The scores network translating the risk

\[ \phi \] constraints

\[ \gamma_{qd} \]

\[ \beta \]

\[ U \]

Upper limit for the number of clinics that can be opened

Decision Variables:

\( x_k \)

Binary decision variables representing whether there is a clinic at location \( k \), \( k \in K \)

\( y_{qd} \)

Binary decision variables representing offered services at an open clinic, \( k \in K \), \( d \in D \), \( s \in S \)

\( \psi_{qs} \)

Risk function defined as \( \beta \)-CVaR risk measure for the locations \( q, d \in D \)

\( \xi_{qds} \)

Random variable denoting the flow of mobile workers’ demand on path \( q, d \), \( q \in Q \), \( d \in D \)


\[ \Phi(\{x_k\}, \{y_{qd}\}) = \sum_{x \in x \in X} \sum_{q \in Q} \sum_{d \in D} \sum_{s \in S} w_{qds} \Gamma_{qdh} \psi_{qs} \]

s.t.

\[ \phi(\{x_k\}, \{y_{qd}\}) = \omega_{qs} \sum_{q \in Q} \sum_{d \in D} \sum_{s \in S} \Gamma_{qdh} \psi_{qs} / |S| \quad \forall d \in D \]

\[ \gamma_{qd} x_k \quad \forall k \in K, \forall s \in S, \forall d \in D \]

\[ \sum_{k \in K} U \]

\[ x_k \in \{0, 1\} \quad \forall k \in K \]

\[ \gamma_{qd} \in \{0, 1\} \quad \forall k \in K, \forall s \in S, \forall d \in D \]

The objective function (1) represents the weighted expected coverage provided by the clinic network. Every path \( q \) of the network has a coverage score for service \( s \) of disease \( d \), calculated through the function \( \Gamma_{qdh} \) with respect to the location of open clinics. The coverage scores are weighted by the parameter \( w_{qds} \), determined by the decision-maker. The expected coverage value for disease \( d \) is attained by calculating the expected value of the mobile demand covered, the product of coverage score for disease \( d \) along the path, and the expected mobile demand values of the corresponding disease. The total coverage of the network is the weighted sum of coverage of different diseases, each having a weight of \( r_d \) to represent the preference information of the decision-maker. The details of the calculation of coverage scores, \( \Gamma_{qdh} \), are presented in Section 4.

The risk-averse measure, CVaR, is integrated into the model with constraints (2), where the minimum service level for disease \( d \) is defined. \( \psi_{qs} \) is the risk function defined as the \( \beta \)-CVaR risk measure for disease \( d \). It denotes the expected lack of coverage in a certain portion of the worst-case scenarios. The right-hand side of the constraints denote the value of coverage that is allowed for the risk exposure. This is based on the maximum coverage that can be attained on the network given the user-defined risk tolerance level \( \omega_q \) and confidence level \( \beta_q \). With these constraints, the risk caused by the random demand and the lack of coverage of the network is limited up to a degree specified by the decision-maker. The motives for the risk function \( \psi_{qs} \) and its details for the calculation with the underlying loss function are presented in Section 5.

With the constraints (3), if a clinic is offering any service for any disease \( \text{(i.e., } \gamma_{qd} = 1) \), \( x_q \) is assigned a value of 1, showing it is open. Constraint (4) puts an upper bound on the number of clinics that can be opened. Constraints (5) and (6) are for the integrality of decision variables.

### 4. Coverage for mobile populations

In this section, coverage definitions and formulations are elaborated in terms of integrating the continuum of care definitions as an expression of the medical requirements of disease and service pairs. As these requirements differ for disease and service pairs, several definitions for coverage are developed. Following the definitions, the formulations are illustrated with an example.

#### 4.1. Coverage definitions

Coverage is formulated under four definitions to acknowledge the different continuum of care requirements of disease and service pairs. For the formulations, it is assumed that the mobile workers make circular trips over the paths and cannot change their route, so once they start from the origin, they arrive at the destination and return back to the origin. It is further assumed that travel times in the network are symmetrical.

**Definition 1. Binary coverage.** This definition is designated for disease and service pairs that have a strict maximum intervention time, within which the demand for the service must be satisfied. The intervention times refer to the time requirement that the service should be offered within, for a given disease. When the travelling time between the location of a mobile worker (on a path) that requires the health service and the clinic providing a specific service for a disease is less than the intervention time, the mobile worker is “covered” for this particular disease and service. Otherwise, it is “not covered”. Using the notation in Table 2, the coverage score \( \Gamma_{qdh} \) along path \( q \) for disease \( d \) of service \( s \) is:

\[ \Gamma_{qdh} = \int_{0}^{t_{d}^{i}} c_{t}^{i} dt \]

where \( c_{t}^{i} \) is the coverage at point \( t \) (in terms of time) and,

\[ c_{t}^{i} = \begin{cases} 1, &\text{if there exists an open clinic within the intervention time of the current position, } 0 \text{ to } t_{d}^{i} \text{ including delay duration} \\ 0, &\text{otherwise.} \end{cases} \]

An example is supervised tuberculosis treatment. As a recommended method for supervision, directly observed therapy aims to ensure the observation of every drug intake of the patient by a healthcare provider and the recording of such on the treatment card [49]. This ensures the right doses of the right anti-tuberculosis drug are taken at the right time (often daily) and avoids worsening the patient’s symptoms. Given such conditions and requirements for supervision, adherence to the protocol

<table>
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<th>Table 1</th>
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<tbody>
<tr>
<td><strong>Notation used.</strong></td>
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<tr>
<td><strong>Sets and Parameters:</strong></td>
</tr>
<tr>
<td>( Q ) &amp; Set of paths, ( q \in {1, 2, \ldots</td>
</tr>
<tr>
<td>( K ) &amp; Set of locations, ( k \in {1, 2, \ldots</td>
</tr>
<tr>
<td>( S ) &amp; Set of services that are offered within the clinics, ( s \in {1, 2, \ldots</td>
</tr>
<tr>
<td>( D ) &amp; Set of diseases that are covered within the clinics, ( d \in {1, 2, \ldots</td>
</tr>
<tr>
<td>( r_d ) &amp; Weight given to disease ( d ), ( d \in D )</td>
</tr>
<tr>
<td>( w_{sds} ) &amp; Weight given to service ( s ) of disease ( d ), ( d \in D ), ( s \in S )</td>
</tr>
<tr>
<td>( \omega_q ) &amp; Risk tolerance level for disease ( d ), ( d \in D )</td>
</tr>
<tr>
<td>( \beta_q ) &amp; Confidence level for disease ( d ), ( d \in D )</td>
</tr>
<tr>
<td>( U ) &amp; Upper limit for the number of clinics that can be opened</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Notation used for binary coverage.</strong></td>
</tr>
<tr>
<td>( t_q ) &amp; Total travel time on path ( q ), from the origin to the destination including delay durations</td>
</tr>
<tr>
<td>( t_{qk}^{i} ) &amp; Total travel time on path ( q ), from the origin to the location ( k ) including delay durations</td>
</tr>
<tr>
<td>( t_{sd}^{il} ) &amp; Intervention time for service ( s ) and disease ( d )</td>
</tr>
</tbody>
</table>
can only be achieved if a clinic is available within at most the limits specified for the patient. Relaxation of the drug intake intervals is improbable.

**Definition 2. Partial coverage.** This definition can be considered as the relaxed version of the binary coverage, where two intervention times for every service and disease combination are used. The mobile worker is not only categorised as either “covered” or “not covered”. There is also a partial value that is assigned regarding the travelling time to the next clinic and the two intervention times. The two additional parameters, $t_1^{sd}$, which is the lower intervention time for service $s$ and disease $d$, and $t_2^{sd}$, which is the upper intervention time for service $s$ and disease $d$, are used for the calculation of the coverage score along the path $q$ for service $s$ and disease $d$:

$$
\Gamma_{\text{par}} = \int_{0}^{\infty} \frac{c}{t_{q}} \frac{t}{t_2^{sd} - t_1^{sd}} \, dt,
$$

where

$$
c = \begin{cases} 
1, & \text{if there exists an open clinic within the intervention time of the current position} \, \text{or} \, 0 < t_k^{sd} < t_1^{sd} \, \text{or} \, t_2^{sd} < t_k^{sd} \\
0, & \text{otherwise.}
\end{cases}
$$

An example case can be given for malaria diagnosis. Malaria should be diagnosed within 24–48 h of the onset of the symptoms [47]. Early diagnosis is favourable as it would hinder the progress of malaria, so rapid access to a clinic is attributed with larger coverage. However, the distance can be increased to a certain extent at the expense of decreased coverage.

**Definition 3. Coverage based on expected travelling time.** For certain disease and service pairs, unlike with binary and partial coverage, the application of intervention times is irrelevant. The demand for a health service does not require an immediate service within the pre-defined time limits; however, it is preferable to satisfy the demand within the least time possible. Such a definition minimises the expected travel time to the next clinic along the transportation line. The parameter $E_{q, k} \cdot t$ refers to the expected travel time to the next clinic, which includes service $s$ for disease $d$ at position $t$ on path $q$. The following equation is used for the calculation of the coverage score along the path $q$ for service $s$ and disease $d$:

$$
\Gamma_{\text{exp}} = \int_{0}^{\infty} \frac{E_{q, k} \cdot t}{t_{q}} \, dt.
$$

HIV care falls under this definition. For patients living with HIV, who are either eligible or ineligible for Antiretroviral Therapy (ART), the provision of care services is essential. Care services are particularly important due to the benefits of reducing failures to show up for follow-up treatments, encouraging adherence to treatment regimes, supporting ART and access to medication, or ensuring the timely initiation of ART [50]. HIV care service does not need to be provided within strict time frames, yet a broad availability of such services is favourable. For such services, coverage based on expected travelling time is appropriate as higher coverage on a path is attributed to smaller values in expected travelling time.

**Definition 4. Referral coverage.** In order to ensure the clinic network is aligned with the local health care infrastructure, a local healthcare facility that is providing the services that a clinic lacks should be accessible within a pre-defined distance to the clinic. The information on the referral coverage can be obtained by observing the referral availability of the clinics along the transportation line, which is the percentage of clinics having referral availabilities on path $q$. The referral coverage can be expressed as (6), where $y'_{q, k}$ refers to clinics that are available to offer a referral service for a given disease $d$, and $K_q$ is the set of all locations on the path $q$.

$$
\Gamma_{\text{ref}} = \sum_{q \in K_q} y'_{q, k} \sum_{k \in K_q} y_{q, k}
$$

4.2. Coverage formulations

The covering area embodied by a clinic can increase the coverage score along a path in two possible ways: (i) the coverage provided for the mobile workers while travelling towards the clinic, (ii) the coverage provided for the mobile workers in the delay duration that is spent at the location where the clinic is located. To assess this and the coverage definitions presented in Section 4.1, it is required to examine the neighbourhood relation between the locations where clinics are open. Also, based on the time spent travelling between two consecutive open clinics, coverage scores should be determined in the mathematical model.

Two clinic locations are defined as neighbours if and only if while travelling from a clinic, the other clinic location is reached without passing any other open clinic. Every clinic is a neighbour of another open clinic on the path. However, due to the assumption of circular trips, an exception to this occurs when a clinic is not open at the origin or the destination of the path. In the case where there is not a clinic at the origin or destination of the path, the next open clinic before the origin or destination will be a neighbour to itself since the first clinic that will be visited by the mobile worker will be the same clinic that the worker most recently departed from. The neighbourhood relations can be explained by an example from a sample path illustrated in Fig. 1, with origin $o$ and

![Fig. 1. Representation of neighbourhood of clinics on a path.](image-url)
destination \(d\) and \(x, y, z\) being possible clinic locations. The clinics are open at \(x, z\) and \(d\) (denoted by a shaded area). Accordingly, \(x\) is neighbour to \(z\) and \(z\) is neighbour to \(d\), and \(d\) is neighbour to \(z\) and \(z\) is neighbour to \(x\). Furthermore, \(x\) is neighbour to itself as there is no open clinic for a mobile worker on their round trip from \(x\) to \(o\) and \(o\) to \(x\).

To represent the neighbourhood relation, the notation given in Table 3 is employed. For the sake of brevity in notation, the index \(d\) for disease and \(s\) for service are dropped in the sequel. The following set of logical expression has to be satisfied in order to calculate the neighbourhood defining binary variables.

\[
\Gamma_q \ y \quad \sum_{k \in KL_q} n_{klq} \min \ t_{kq} \ y \quad \lambda_k \quad \sum_{k \in KL_q} m_{klq} \left[ \min \left\{ t_{kq} \ y \right\} \right] \quad \lambda_k \quad \sum_{k \in KL_q} m_{klq} \left[ \min \left\{ t_{kq} \ y \right\} \right] \quad \lambda_k
\]

where

- \(n_{klq}\) is the total time spent (all travel and delay times excluding delays at \(k\) and \(l\) in travelling from location \(k\) to location \(l\),
- \(t_{kq}\) is the total time spent (all travel and delay times excluding delay at \(k\)) travelling from location \(k\) to location \(k\) again, passing through the origin,
- \(t_{kq}^0\) is the total time spent (all travel and delay times excluding delay at \(k\)) travelling from location \(k\) to location \(k\) again, passing through the destination,
- \(\lambda_k\) is the total delay time at location \(k\),
- \(T_q\) is the completion time for entire total tour along the path.

With the formulation for \(\Gamma_q \ y\), the travel time between each neighbouring clinic is compared with the pre-defined critical intervention time over all existing neighbourhood pairs to accumulate the time spent within the coverage area, and correspondingly, to calculate the coverage score.

Other coverage definitions. The formulations for Partial Coverage and Coverage Based on Expected Travel Time are similar to that of Binary Coverage. The major difference emerges from the method of expressing the coverage between each neighbouring clinic as it is based on the requirements imposed by the coverage definitions. For Referral Coverage, further linearization is also required to integrate the formulation into the developed linear model. The details for each of the remaining definitions are provided in A.

5. Decision under risk

As described in Sections 1 and 3, the risk associated with the uncertain demand values and morbidity/mortality can be controlled with the input facilitated by the decision-maker by adopting a coherent risk

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Additional notation for neighbourhood definition.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(K_q)</td>
<td>Set of all locations on path (q)</td>
</tr>
<tr>
<td>(KE_{kq})</td>
<td>Set of all locations that are visited prior to location (k) as the mobile worker travels from origin to destination on path (q).</td>
</tr>
<tr>
<td>(KL_{kq})</td>
<td>Set of all locations that are visited after location (k) as the mobile worker travels from origin to destination on path (q).</td>
</tr>
<tr>
<td>(n_{klq})</td>
<td>Binary decision variable equals to 1 if (k) and (l) are bordering clinics along the path (q), 0 otherwise.</td>
</tr>
<tr>
<td>(m_{klq}^0)</td>
<td>Binary decision variable equals to 1 if (k) is bordering to itself and passes through origin in between as the mobile worker travels on path (q), 0 otherwise.</td>
</tr>
<tr>
<td>(m_{klq}^0)</td>
<td>Binary decision variable equals to 1 if (k) is bordering to itself and passes through destination in between as the mobile worker travels on path (q), 0 otherwise.</td>
</tr>
</tbody>
</table>
measure to the problem formulation. As a class of risk measures, both VaR and CVaR values can be used in optimisation models where risk is involved. VaR risk measure (which is referred as \( \beta \)-quantile minimax regret) is considered to be not very powerful as it focuses only on the \( \beta \)-quantile and ignores the magnitude of losses at the tail [40]. This can result in overwhelmingly high loss values at the tail. On the other hand, CVaR considers the conditional expectation of the loss values larger than the \( \beta \)-quantile when the loss function is atomless. Regarding these discussions, the CVaR risk measure is more advantageous than VaR.

For the integration of CVaR measure, a loss function should be defined based on the set of decision variables and set of random parameters. As the aim is to design a network by maximising the attained service level, which is defined through expected weighted coverage, the loss function should target the coverage scores. Consequently, the loss function is defined as the \textit{deviation of the provided coverage from the maximum possible value over the whole network as formulated in Equation (9)}. The maximum possible coverage can be achieved by opening a clinic that provides every service for every disease at all locations. Loss is given as the sum of the deviation of coverage values in the network from the maximum coverage values, multiplied with the mobile worker flow \( (z_{qd}) \), defined with a known probability mass function.

\[
\ell_{ad} = \sum_{q \in D} \sum_{i \in J} \frac{z_{qdi} \Gamma_{qdi} T}{S} - \sum_{i \in J} w_{adi} \Gamma_{adi} Y, \quad \forall d \in D. \tag{8}
\]

While the network optimisation problem for a distinct set of diseases and services determines the location of the open clinics, it considers the measures on risk for each disease. This is achieved by the integration of risk measures as a constraint in the model. Referring to Theorem 14 by Krokhmal et al. [51], which proposes the use of CVaR as constraint in the linear programs, the following constraint is added:

\[
\phi_{\beta d} = \alpha_{d} \sum_{q \in D} E\left[ z_{qdi} \Gamma_{qdi} T \right] / \sum_{i \in J} w_{adi} \Gamma_{adi} Y, \quad \forall d \in D \tag{9}
\]

where \( \phi_{\beta d} \) is the risk function defined as the \( \beta \)-CVaR risk measure for disease \( d \) for the loss function given in (9). Here, \( \alpha_{d} \) is a percentage of the maximum number of people that can be covered with the network for disease \( d \) and it denotes the ratio of people that is allowed for risk exposure.

Based on Theorem 14 by Krokhmal et al. [51] and the approximation for \( \beta \)-CVaR function by Rockafellar and Uryasev [52], which does not require the explicit calculation of VaR value, the constraint (10) is re-arranged. A scenario-based approach is proposed, where the random variable for mobile demand is described by discrete scenarios as the probability information is assumed to be known. The following set of constraints are included in the network design problem, \( P \), using the notation given in Table 4.

\[
\ell_{ai} = \sum_{q \in D} \sum_{i \in J} \frac{z_{qai} \Gamma_{qai} T}{|S|} - \sum_{i \in J} w_{aidi} \Gamma_{aidi} Y, \quad \forall d \in D \text{ and } \forall i \in I, \tag{10}
\]

### Table 4

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I )</td>
<td>Set of scenarios defined for the distribution of demand, ( i = 1, 2, \ldots,</td>
</tr>
<tr>
<td>( \ell_{ai} )</td>
<td>The value of loss for disease ( d ) in scenario ( i )</td>
</tr>
<tr>
<td>( u_{ad} )</td>
<td>The positive difference between loss function ( \ell_{ai} ) and ( a_{d} )</td>
</tr>
<tr>
<td>( a_{d} )</td>
<td>The ( \beta )-VaR value given the confidence level ( \beta_{d} )</td>
</tr>
<tr>
<td>( \epsilon_{qdi} )</td>
<td>The mobile flow for disease ( d ) on path ( q ) in scenario ( i )</td>
</tr>
<tr>
<td>( \epsilon_{i} )</td>
<td>Probability of scenario ( i )</td>
</tr>
</tbody>
</table>

### Table 5

<table>
<thead>
<tr>
<th>Disease</th>
<th>Services</th>
<th>Coverage</th>
<th>Time</th>
<th>Care</th>
<th>Referral</th>
</tr>
</thead>
<tbody>
<tr>
<td>HIV</td>
<td>Diagnosis</td>
<td>Expected travelling time</td>
<td>Partial</td>
<td>Partial</td>
<td>Expected travelling time</td>
</tr>
<tr>
<td>Malaria</td>
<td>Partial</td>
<td>Partial coverage</td>
<td>Partial</td>
<td>Partial</td>
<td>Referral coverage</td>
</tr>
<tr>
<td>STIs</td>
<td>Partial</td>
<td>Partial coverage</td>
<td>Partial</td>
<td>Partial</td>
<td>Referral coverage</td>
</tr>
<tr>
<td>Tuberculosis</td>
<td>Partial</td>
<td>Partial coverage</td>
<td>Binary</td>
<td>Binary</td>
<td>Referral coverage</td>
</tr>
</tbody>
</table>

6. Computational study

One of the main objectives of the computational study is the validation of the proposed model. The other objectives are to investigate (i) the effects of different coverage definitions based on disease and service pair requirements on the objective function, (ii) the effects of the risk measure on the objective function as well as expected coverage values for different diseases and clinic locations, and (iii) the improvements achieved with the risk measure by observing the changes in the number of people at risk calculated through CVaR.

6.1. Experimental setting

Today, the non-governmental organisation North Star establishes walk-in clinics along transportation lines and works to address the problem of mobile populations for both communicable and non-communicable diseases. Our computational study is limited to the communicable diseases and among them, the high impact diseases such as HIV and tuberculosis, malaria and STIs are considered. Furthermore, the health service package that North Star offers involves five services: (1) screening, (2) diagnosis, (3) treatment, (4) care and (5) referral. As the screening service mainly includes symptom recording and physical examination, it can be offered in any clinic, regardless of the equipment available there. Hence, the services that will be in the scope of the study are limited to diagnosis, treatment, care and referral.

For all communicable diseases, a different set of actions is required under different services. Therefore, distinct continuum of care requirements emerge for the specific services of the specific diseases. The match between the disease and service pairs and coverage definitions are summarised in Table 5. This match has been conducted with the consultation from North Star professionals.

The computational study is based on a data set provided by North Star, which includes the coordinates of 32 potential clinic locations and coordinates for the 18 origin and destination locations for the transportation lines. All these locations are mainly in South-East Africa and they are determined upon the real-life studies of North Star. The average
traffic flow values between the origin-destination pairs are also provided by North Star. For the stochastic optimisation model, we consider different realisations of the flows on the transportation lines and each realisation is called as a flow scenario. It is assumed that each scenario is equally likely to occur, so a Uniform Distribution is considered for scenario generation with using the provided average flow values as mean values. Additionally, North Star provided an extensive set of coordinates for the existing health care facilities in Africa. These facilities are considered for the referral services offered in the clinics. This data also includes the services offered for different diseases in each existing health care facility.

In addition to the real-life data provided by North Star, the transportation lines are generated based on the origin and destination coordinates. First, each origin-destination pair is assumed to be connected with a direct line and any potential clinic that falls within a certain distance from this line is considered to be part of the path between the given origin and destination points. Then each transportation line is presented as a combination of the piecewise lines connecting the potential clinic locations and the origin-destination points. Some of these lines may be short, referring to include a single location for a potential clinic or can be longer with eight potential locations, excluding the origin and destination points.

Even though assuming the roads are direct paths may not reflect the actual transportation lines in real life, this approach allows the quick generation of paths for different origin and destination pairs that can be utilised for computational purposes and it enables flexibility.

As explained above, the average traffic flows are fit to a Uniform Distribution to create flow scenarios to understand the behaviour of the model under different flow realisations. In order to define upper and lower limits for defining a Uniform Distribution, coefficient of variation ($C_v$) measure is used. The $C_v$ measure can be interpreted as the dispersion of the variable around the mean value. In the problem context, $C_v$ is the volatility of the traffic flow in comparison to the amount of demand that is expected to occur along the paths. Hence, it represents the variance associated with the mobile demand.

The set of paths are categorised into three, depending upon the regions of the paths and clinics that are situated on them. Within each category, the high and low volatility paths are divided equally with respect to the average traffic flow. Therefore, in the end, the set of data is composed of equally assigned high and low volatility, all around South East Africa. Based on the assigned degree of volatility, the upper and lower bounds of the Uniform Distribution are calculated. Given the calculated ranges, the scenarios are generated with the random sampling method. For each flow of disease $d$ along path $q$, 100 random instances are generated and the flow data is created for each of the four diseases, on each of the 18 paths.

The proposed model is solved using CPLEX solver running with a time limit of 2000 s on a Windows 8 PC with 4 GBs of RAM, and a 256 GB Solid-State Disk.

### 6.2. Results and discussion

The objective function of the developed model calculates the weighted expected coverage of the demand (in terms of the number of mobile workers) for different diseases on the established network clinic. In this section, in addition to the objective function values, the performance of the model is presented in terms of (i) the coverage scores for disease $d$ along a particular path, which varies from 0 to 1, and (ii) the expected number of mobile workers covered in the network for a particular disease. As the aim of the model is to establish different networks, by adjusting the weights given to each disease, it is important to observe the changes in the coverage scores and the number of mobile workers covered for different paths and for different diseases. These different networks will respond to the needs of the decision-maker. Even though the objective function may increase or decrease with different weights, it is also important to observe the change in values of mobile

**Table 6**: Coverage score along the paths for HIV- and malaria-focused care for different numbers of clinics ($U$).

<table>
<thead>
<tr>
<th>U</th>
<th>Paths</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HIV Coverage</td>
<td>7</td>
<td>1.000</td>
<td>0.967</td>
<td>0.768</td>
<td>0.061</td>
<td>0.009</td>
<td>0.009</td>
<td>0.009</td>
<td>0.009</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>Malaria Coverage</td>
<td>8</td>
<td>1.000</td>
<td>0.967</td>
<td>0.768</td>
<td>0.061</td>
<td>0.009</td>
<td>0.009</td>
<td>0.009</td>
<td>0.009</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>HIV Focus</td>
<td>9</td>
<td>1.000</td>
<td>0.967</td>
<td>0.768</td>
<td>0.061</td>
<td>0.009</td>
<td>0.009</td>
<td>0.009</td>
<td>0.009</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>Malaria Focus</td>
<td>10</td>
<td>1.000</td>
<td>0.967</td>
<td>0.768</td>
<td>0.061</td>
<td>0.009</td>
<td>0.009</td>
<td>0.009</td>
<td>0.009</td>
<td>1.000</td>
</tr>
</tbody>
</table>
workers covered for specific diseases, as well as specific paths. The results are provided for the risk-neutral model first, followed by the risk-averse model.

Initially, for a risk-neutral decision-maker, the effects of the weights given to the different diseases \((r_d)\) are analyzed. Four cases are compared, where a single disease is given a higher weight relative to the others (i.e., \(r_{HIV} \gg r_{malaria} \gg r_{tuberculosis}\), which is a HIV-focused case). The results are analyzed by setting the upper limit for the number of clinics from one to ten.

For the two diseases with different coverage requirements and high demand values, HIV-focused and malaria-focused models are compared. With fewer than three clinics, the clinics are opened at the same locations, namely Beitbridge and Dar es Salaam, as they are located on more than a single path and contribute to the coverage score greatly. However, with more than three clinics, the network obtained from each of the models starts to differ. HIV coverage can be easily attained by locating the clinics at distant locations, as it would span a greater area. However for malaria, locating clinics closer to each other contributes to higher coverage values compared to distant clinics as the coverage requirements for malaria are stricter. For example, in the HIV-focus model, 12 out of 18 paths are located with clinics serving for malaria and this leads to a 58.6% malaria coverage on average. However, in the malaria-focused model, the number of paths that are located with clinics serving for malaria increases to 14, with an average of 79.7% malaria coverage.

A further observation can be made by looking at the changes in the network for seven to ten clinics. Table 6 shows the expected coverage scores along each path for HIV- and malaria-focused models for a different number of clinics (represented by \(U\) in the Table). In the malaria-focused case, with the increase in the number of clinics, clinics are located along the paths with high malaria flow to increase the malaria coverage in return of sacrificing from the HIV coverage. This can be observed in the case of eight clinics in malaria-focused model. Coverage for the paths 10, 11 and 15 for malaria are increased compared to the HIV-focused case, whereas for the paths 10 and 15, HIV coverage is decreased. The change in the optimal location decisions for clinics under HIV-focused and malaria-focused models for 7 to 10 clinics can be seen in Fig. 2, where the blue dots denote the potential locations and the red dots denote the open clinics.

Another comparison is made between the HIV-and tuberculosis-focused cases. Tuberculosis is a disease with very strict intervention requirements and low demand, so the coverage scores are increased slowly with the opening of additional clinics. Table 7 shows the expected number of covered mobile workers with tuberculosis, in HIV- and tuberculosis-focused cases (the percentages in parentheses show the proportion of covered tuberculosis demand over the total expected tuberculosis demand in the network). When the number of clinics is less than 5, the expected number of covered mobile workers with tuberculosis coverage does not change. However, when the number of clinics is more than five, the gap in tuberculosis coverage provided by the two models is increased and the captured tuberculosis demand is improved. This implies that before reaching a certain number of clinics, the focus is on interrupting the flow as much as possible rather than focusing on tuberculosis coverage. However, increasing the number of clinics allows for the strategic positioning of the clinics, leading to greater coverage for tuberculosis along the paths. For the diseases that have strict coverage requirements, the number of clinics targeted in medium and long term investment periods is also important since the overall impact achieved by the network is not only dependent on the contribution of the added clinic, but also on the composition of them. Additionally, as seen from the table, the expected coverage of mobile workers with tuberculosis in the tuberculosis-focused case is increased steadily with the addition of each clinic; however, for the HIV-focused case, there is no steady increase, and in the nine-clinic case there is even a decrease. In the HIV-focused case, the model focuses on HIV coverage and that may lead to a decrease in tuberculosis coverage, even if number of clinics is increased. This indicates that once the focus on a disease is emphasised, coverage of the demand for other diseases might drop.

The numerical analysis is further conducted for a risk-averse decision-maker by including the constraints associated with the risk measures in the model. It is important to determine the problem parameters related to the risk-averse measures accurately to interpret the results of the study. Two parameters are set by the decision-maker in the proposed model: \(B_d\) shows the confidence level for disease \(d\) which means that the scenarios falling into \(1 - B_d\) upper tail of the loss distribution are in

Fig. 2. The optimal location decisions for clinics under HIV-focused and malaria-focused cases.
consideration. Those scenarios are called as the worst-case scenarios for the disease $d$. The other parameter $\alpha_d$ denotes the risk tolerance level for disease $d$ which defines the allowed risk and it is represented as a percentage of maximum number of people that can be covered on the network. To exemplify, for disease $d = \text{HIV}$ setting $\alpha_{\text{HIV}} = 0.10$ and $\beta_{\text{HIV}} = 0.95$ asserts that, the mean of the 5% upper tail (worst case) distribution of the loss must not exceed the 10% of the maximum number of people with HIV that can be covered. Thereupon, for the case $\alpha_{\text{HIV}} = 1$, all members of the HIV population would be under risk and risk-averse constraints in the model would not have any influence. It refers that the coverage is not guaranteed and possible fluctuations from the average HIV traffic flow will not be considered. On the contrary, a value of 0 for $\alpha_{\text{HIV}}$ would mean no risk is allowed for HIV. This means that considering all possible traffic flow realisations, HIV demand in the network should be covered.

Keeping the weight $r_d$ for each disease equal, risk constraints are introduced to the model by varying $\beta_d$ and $\alpha_d$ levels to observe the impact on the objective function (the weighted expected coverage over the clinic network) and the coverage scores on the individual paths for a given disease $d$. The numerical analysis for the risk-averse case is conducted for HIV, malaria and tuberculosis with different risk tolerance levels, $\alpha_d$, and by changing the number of clinics from four to ten. When the number of clinics is less then four, infeasibilities are observed as the desired control over the risk is not possible, meaning that no possible combination of open clinics would be able to satisfy the desired control over the risk. For seven or more clinics, there is no risk of not covering the demand considering the flow realisations (flow scenarios) and therefore, the risk constraint is not binding in the model. Therefore, in Fig. 3, the resulting objective function values for different $\beta_d (0.95, 0.9$ and 0.8) and $\alpha_d$ are depicted for the cases of opening four to six clinics only.

For each of the HIV, malaria and tuberculosis risk-averse cases, it is observed that when increasing the risk tolerance level beyond a certain limit (for example, 4 clinics, $\alpha_{\text{HIV}} = 0.35$, $\beta_{\text{HIV}} = 0.9$), the objective function values do not show any changes and remain the same throughout different risk tolerance and confidence levels. This indicates that the risk constraints are not binding. Any risk tolerance level beyond this limit is satisfied and the model gives equal results with the risk-neutral model. This can also be observed from the objective function values among different risk-averse cases. Below a given number of clinics, the

<table>
<thead>
<tr>
<th>Number of Clinics Open</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
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<td>28.97</td>
<td>32.73</td>
<td>35.55</td>
<td>35.99</td>
<td>40.19</td>
<td>38.19</td>
<td>41.67</td>
<td>37.42</td>
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<tr>
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<td>23.73</td>
<td>28.97</td>
<td>32.73</td>
<td>35.55</td>
<td>35.99</td>
<td>40.19</td>
<td>38.19</td>
<td>41.67</td>
<td>42.97</td>
</tr>
</tbody>
</table>

Table 7

Expected number of mobile workers with demand for tuberculosis who are covered under HIV- and Tuberculosis-focused cases.

![Fig. 3. Objective function values of the model with a CVaR measure.](image-url)
Table 8
Expected number of people covered per disease on the paths with different risk tolerance levels.

<table>
<thead>
<tr>
<th>Paths</th>
<th>$\omega_{HV}$</th>
<th>Expected Number of People Covered</th>
<th>$\omega_{HV}$</th>
<th>Expected Number of People Covered</th>
<th>$\omega_{HV}$</th>
<th>Expected Number of People Covered</th>
</tr>
</thead>
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<tr>
<td></td>
<td>$\Gamma_{1}$</td>
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<td>$\Gamma_{1}$</td>
<td>HIV to Malaria to STI to TB</td>
<td>$\Gamma_{1}$</td>
<td>HIV to Malaria to STI to TB</td>
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<td>1</td>
<td>1</td>
<td>60.00</td>
<td>1</td>
<td>60.00</td>
<td>1</td>
<td>60.00</td>
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<td>2</td>
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<td>0.86</td>
<td>0.86</td>
<td>0.86</td>
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<tr>
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<td>1.69</td>
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<tr>
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<tr>
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</table>
This confirms that with tightening the CVaR constraint, the average loss in 10% of the worst cases is also decreased, and the associated risk can be controlled. In the case where only a limit is defined on the threshold value that the loss function should not exceed, the magnitude of the loss is not captured. Similar results can also be examined for the CVaR measure on malaria and tuberculosis in Table 11, second and third blocks. With these results, it is assured that the model with CVaR constraints is capable of capturing the risk-averse behaviour and preference of the decision-maker through the $\alpha_d$ and $\omega_{d}$ parameters.

7. Conclusions

Over the years, the health problems faced by mobile workers have drawn the attention and interest of many groups, including the private
companies that employ the mobile workers, and the NGOs that assist the mobile workers and researchers. In this paper, the issue is addressed from both a mitigation and response point of view by formulating a model that would optimise the locations of healthcare clinics that specifically serve the group in focus. Unlike the rest of the population, the mobile workers travel along routes for long durations under poor conditions, which make them more vulnerable to many diseases. Their working conditions together with their lack of access to healthcare services only exacerbate their medical problems. Furthermore, as they are continuously travelling, they become vectors of transmission and create a greater burden for the whole community. In this paper, a healthcare facility location model, which has the characteristics of both the MCP and FCLM, is developed so that the medical needs of the mobile workers can be addressed specifically. This is achieved by developing different coverage definitions that match with the requirements of the different services for different diseases, which is also referred to as the continuum of care requirements. Additionally, due to both the unpredictability of human behaviour and possibility of sudden onset of emergencies such as epidemics that would significantly impact the demand, variation in the demand of mobile workers is an important part of the problem that requires attention. Therefore, a risk-averse approach is integrated into the model to reduce the risks associated with the lack of coverage by adopting the risk measure, CVaR.

As discussed in the computational study, the developed model is able to capture the different coverage requirements needed to provide a service for distinct diseases, which enables health policy makers to generate and design specialised networks. The study also confirms that a CVaR measure is capable of re-designing the network so as to locate clinics on the high flow volatility paths and control the number of mobile workers at risk. Although CVaR is commonly applied in financial practice, it is also well suited for humanitarian applications as it imposes rigid limitations on the loss of human lives.

Even though the external parameters that are decided on by the decision-maker bring a flexibility to the modelling of the network for different conditions (i.e., limits imposed by stakeholders, change in the conditions in the region, or the outbreak of an epidemic), the interpretation of the parameters might become hard. To fully benefit from the model, the parameters should first be fully comprehended by the decision-maker.

The developed coverage definitions can also be further extended for applications in other fields, which include mobility. Furthermore, the model can be extended by imposing capacity or reliability limits on the clinics in order to analyze the effects of coverage over the network when a clinic becomes unavailable for service.

Acknowledgements

First author was a student of TU/e & METU Industrial Engineering’s double degree program. She began this study while working as a graduate intern in North Star, and later continued her research during her MS studies, see Taymaz (2013) for further information. The authors would like extend their gratitude to H. de Vries for discussions during the project, and to North Star, ORTEC and healthcare professionals working in collaboration with North Star for providing healthcare insight and input data.

Appendix A. Coverage Formulations

Definition 2. Partial coverage. Based on the partial coverage definition, the coverage for the mobile worker depends on two intervention times, \( \tau_1 \) and \( \tau_2 \), which are defined specifically for disease and service pairs. Accordingly, with the following equation, the coverage score for a given solution can be expressed as:

\[
\Gamma_{\eta, \tau} = \frac{1}{\tau_0} \sum_{i \in K_0} \left( \sum_{i \in K_0} \min \left( \lambda_i \tau, \lambda_i \tau_i \right) \right) - \left( \sum_{i \in K_0} \min \left( \lambda_i \tau, \lambda_i \tau_i \right) \right)
\]

where

\[
\begin{align*}
\tau_1 & = \begin{cases} 
\frac{t_{\text{thr}}}{2 \tau_1} & \text{if } \lambda_i \tau \geq \lambda_i \tau_i \\
\frac{t_{\text{thr}}}{2 \tau_1} & \text{if } \lambda_i \tau_i \geq \lambda_i \tau \\
0.5 \tau_1 & \text{otherwise}
\end{cases} \\
\tau_2 & = \begin{cases} 
\frac{t_{\text{thr}}}{2 \tau_2} & \text{if } \lambda_i \tau \geq \lambda_i \tau_i \\
\frac{t_{\text{thr}}}{2 \tau_2} & \text{if } \lambda_i \tau_i \geq \lambda_i \tau \\
0.5 \tau_2 & \text{otherwise}
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\tau_1 & = \begin{cases} 
\lambda_i \tau_i & \text{if } \lambda_i \tau \leq \lambda_i \tau_i \\
\lambda_i \tau_i & \text{if } \lambda_i \tau_i \leq \lambda_i \tau \\
0.5 \lambda_i \tau_i & \text{otherwise}
\end{cases}
\end{align*}
\]

Equation (A.1) is developed in a similar manner to the coverage score calculation for Binary Coverage. Based on the partial coverage definition, the coverage provided by the clinic decreases with the distance at which the mobile worker is located. These differences are described in Equations A.2, A.3 and A.4 by linearly decreasing coverage as the mobile worker’s distance to the next clinic increases and taking into account the intervention times \( \tau_1 \) and \( \tau_2 \).
Definition 3. Coverage based on the expected travel time. For the disease and service pairs for which coverage does not have to be provided urgently, the aim is to minimise the expected travelling time between clinics. This requires the formulation of the expected travel time between two clinics \( k \) and \( l \) on path \( q \), and it is defined to be equal to \( \frac{q}{T_{k,l}} \) [53]. Accordingly, the expected travel time for solution \( \alpha, ET_q \) \( \alpha \), along the path \( q \) by integrating the neighbourhood relations can be expressed as follows:

\[
ET_q \alpha = \frac{\sum_{k,l,k,l} n_{klq} q_{kl}^2}{T_{k,l}},
\]

The expected travel time along path \( q \) is transformed into the coverage score (which is on a scale from 0 to 1) with the help of a linear function based on the threshold time values represented by \( \theta_1, \theta_2, \theta_3, \) and \( \theta_4 \). The threshold time values are defined specifically for the requirements of the service and disease pair that is of concern. If the expected travel time is between \( \theta_1 \) and \( \theta_2 \), a coverage value of one is assigned. If it is between the critical times \( \theta_2 \) and \( \theta_3 \), coverage is a linearly decreasing function having a value between 0 and 1. Finally, for the expected travel time between \( \theta_3 \) and \( \theta_4 \), a coverage value of 0 is assigned. The following equations and inequalities are used for formulation:

\[
ET_q \alpha \theta_1, m_{q\alpha} = \frac{\sum_{k,l,k,l} n_{klq} q_{kl}^2}{T_{k,l}}, \quad \forall q \in Q,
\]

\[
\mu_{q\alpha} \mu_{q\alpha} = \mu_{q\alpha} \mu_{q\alpha} = 1, \quad \forall q \in Q,
\]

\[
\sum_{k,l} y_{k,l} \alpha \mu_{q\alpha} = \sum_{k,l} y_{k,l} \alpha \mu_{q\alpha} = 0, \quad \forall q \in Q.
\]

where \( \mu_{q\alpha} \) for \( n = 1, \ldots, 4 \) is the non-negative weight for path \( q \). With this formulation, \( \mu_{q\alpha} \) values are calculated with respect to the expected travel time values. In (A.6), if no clinic is located along the path \( q \) (which is defined through \( z_q \)), the expected travel time is increased and a coverage score is assigned with the value of 0. In addition to these constraints, no more than two adjacent \( \mu_{q\alpha} \) can be greater than 0.

Definition 4. Referral coverage. The referral coverage in Section 4.1 is defined as the percentage of clinics over path \( q \) having referral availability for disease \( qd \). \( \gamma_{qd\alpha} \) shows the clinics on a path that have a referral service ability for disease \( d \). Those clinics need to be in the vicinity of a local healthcare facility. The vicinity of a local healthcare facility is defined by a pre-set threshold distance. Consequently, Equation (A.11) can be formulated as,

\[
\Gamma_{qd\alpha} \alpha = \frac{\sum_{q,l} y_{qd\alpha} q_{kl}}{\sum_{q,l} y_{qd\alpha} q_{kl}}, \quad \forall q \in Q \text{ and } d \in D.
\]

Equation (A.11), is the ratio of two functions defined by decision variables, which implies non-linearity in the optimisation model and so requires to be adjust. Equation (A.11) is linearised with the help of auxiliary binary variables, and corresponding linear constraints are integrated in model P.

References
