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Multivariable Repetitive Control: Decentralized Designs With Application to Continuous Media Flow Printing

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Abstract—The print productivity and medium dimensions of wide-format roll-to-roll printing systems are limited by position errors induced by step-wise medium transportation. The aim of this article is to develop a design framework for multivariable repetitive control (RC), that enables improved productivity and accuracy through continuous media flow printing. The developed technical framework for RC explicitly addresses the tradeoff between performance and modeling requirements through decentralized designs. In particular, systematic design approaches are developed that explicitly address unmodeled interaction as uncertainty, i.e., through robustness. The result is a range of solutions, including 1) independent single-input single-output (SISO) designs, and 2) sequential SISO design. Experimental results confirm that the developed RC framework in conjunction with continuous media flow printing outperforms existing approaches.

Index Terms—Mechatronics, motion control, multivariable systems, repetitive control.

I. INTRODUCTION

PRINT productivity, print quality, medium size, and medium versatility in wide-format inkjet printing systems are restricted by medium positioning errors. Positioning errors are caused by step-wise medium transportation in between print-passes, and predominantly originate from internal medium deformations due to, e.g., friction and hysteresis [1], and medium-dependent dynamics, which can be uncertain and nonlinear. Positioning errors deteriorate the alignment of print-passes, distorting the printed product. To conceal this adverse effect, typically ink jetting is not started after a medium step before the medium has settled, and multiple overlapping print-passes are performed, both at the cost of production speed [2].

The reproducibility of positioning errors in step-wise transportation is exploited in [3] through the use of iterative learning control (ILC). Batches of medium position measurements are collected using a scanner, which are used to iteratively improve medium positioning in subsequent print passes. However, the achievable accuracy is limited by the uncertain and nonlinear nature of the medium dynamics.

Despite the substantial developments that have been made to reduce positioning errors, these approaches are inherently limited by the pursued approach to model and control the uncertain, nonlinear and medium-dependent dynamics. The aim of this article is to explore a radically different approach, called continuous media flow printing, in which the medium is transported with constant velocity throughout the process, such that uncertain and nonlinear medium dynamics can be represented as a constant force disturbance.

Continuous media flow printing requires the carriage to accurately perform a multidimensional and repetitive trajectory over the print surface, see Figs. 1 and 2. This requires simultaneous actuation of multiple axes in the horizontal plane, which is enabled by flatbed printing systems. The motion degree of freedom in transport direction is offered by a gantry beam, which is not present in typical roll-to-roll printers [3].

To achieve the tracking accuracy required for printing, a repetitive control (RC) [4], [5] framework for multi-input multi-output (MIMO) systems is required, which enables to learn from reproducible errors as in [3], yet in continuous instead of batch-wise operation. RC enables to track or reject disturbances that...
repeat continuously in time, see, e.g., [6], [7] for applications, whereas the ILC in [3] aims to track repeating trajectories of finite-length that are unrelated in time.

Robust stability of RC algorithms is crucial to deal with modeling errors. Most robust RC design approaches for MIMO systems are based on optimization techniques, such as $\mathcal{H}_\infty/\mu$-synthesis, see, e.g., [8]–[11]. MIMO parametric descriptions of a nominal model and its uncertainty in a certain prespecified form are required to guarantee robust stability. Despite being systematic, this may hamper industrial implementation, since a large burden is inflicted on modeling requirements [12]. Particularly for lightly damped motion systems, such as that in Fig. 1, these MIMO parametric models can be cumbersome and expensive to construct because of its complex dynamics [13], [14], and numerical issues [15].

In view of multivariable continuous media flow printing, as is developed in the present article, a fundamentally different RC framework is essential to meet the performance and modeling cost requirements. In many applications, see, e.g., [5], [16], [17], repetitive controllers are designed using manual loop-shaping in the frequency domain, which is often preferred by control engineers. Robust stability can be verified by using inexpensive frequency response function (FRF) measurements to model the uncertainty [18], [19]. However, such manual design approaches are typically limited to single-input single-output (SISO) systems, and their application to MIMO systems is largely undeveloped. Indeed, robust stability issues can arise in case multivariable interaction is ignored.

The contributions of this article are twofold.

C1) A systematic framework is developed for robust decentralized RC design for general MIMO systems, that explicitly addresses the tradeoffs between modeling requirements and performance. C2) The potential of the proposed RC framework is experimentally demonstrated for continuous media flow printing, enabling increased productivity and medium sizes.

The proposed design techniques, constituting the framework in C1, rely on manual SISO loop-shaping design tools and SISO parametric models, hence considerably simplify the design, yet guarantee robust stability of the MIMO system through nonparametric FRF measurements. In particular, interaction is addressed as structured uncertainty, i.e., through robust stability. The design approaches include i) independent robust SISO designs, and ii) sequential SISO design, building upon results in [20]–[22], including the use of the structured singular value (SSV) [23, Section 11.2]. The framework relates to [24] for MIMO ILC, yet in addition allows sequential design, whereas this is not directly relevant for ILC. The present article generalizes preliminary results in [25] with new theoretical and application results, and detailed proofs.

Notation: All systems are discrete-time, MIMO, and linear time-invariant. Let $\mathcal{R}(z)$ denote the set of rational discrete-time transfer matrices. The space of real rational functions bounded on the unit circle and analytic in $|z| > 1$ is denoted $\mathcal{RH}_\infty$. The imaginary unit is denoted $i$, i.e., $i^2 = -1$. The $ij$-th element of a matrix $A$ is denoted $a_{ij}$.

II. PROBLEM DESCRIPTION

The printing operation for continuous media flow is introduced, and the repetitive control design problem is formulated.

A. Printing Operation With Continuous Media Flow

Fig. 1 depicts an industrial flatbed printer with roll-based media supply, which enables to print on flexible media. The printheads, which contain many closely spaced nozzles that deposit ink onto the medium, are located underneath the carriage. In traditional operation, the carriage performs lateral passes over the stationary medium, see Fig. 2(a). In between passes, the medium is transported over the roll-to-roll print surface, which induces positioning errors. The key idea in continuous media flow printing is to transport the medium with constant velocity over the print surface. This requires that the carriage performs a multivariable trajectory, see Fig. 2(b), synchronized with the medium to maintain the relative positioning in transport direction. Note that the trajectory in Fig. 2(b) leads to single-pass printing, and can be modified for multipass printing, see [2]. To achieve the required printing accuracy while tracking the repeating motion, a multivariable RC framework is developed.

B. Repetitive Control Setup

The control scheme is depicted in Fig. 3, consisting of plant $G \in \mathcal{R}^{n_y \times n_x}(z)$, stabilizing feedback controller $C \in \mathcal{R}^{n_u \times n_y}(z)$, and repetitive controller $R \in \mathcal{R}^{n_y \times n_u}(z)$, connected as an add-on controller, see, e.g., [26]. The presented
results can be appropriately modified for other RC implementations, e.g., serial configurations. The control objective is to reject a periodic exogenous disturbance with period $N \in \mathbb{N}$, e.g., $r(k) \in \mathbb{R}^{n_u}$ with $r(k + N) = r(k)$. That is, minimization of tracking error $e = r - y$, given by

$$e = (I + GC)(I + R)^{-1}(r - d)$$

$$= ((I + GC)(I + TR))^{-1} (r - d) = S_R (r - d), \quad (1)$$

with sensitivity $S = (I + GC)^{-1} \in \mathcal{R}^{n_y \times n_y}$ the transfer function matrix from $r$ to $e$ with $R = 0$, complementary sensitivity $T = (I + GC)^{-1} GC = I - S$, and modifying sensitivity $S_R = (I + TR)^{-1}$.

According to the internal model principle [27], perfect asymptotic rejection of an exogenous disturbance is achieved if a model of the disturbance generating system is included in a stable feedback loop. For general $N$-periodic disturbances, this corresponds to a memory loop with $N$ samples delay. Typically, the repetitive controller is designed as, see Fig. 4.

$$R = L z^{-N} Q \left( I - z^{-N} Q \right)^{-1}, \quad (2)$$

with learning filter $L \in \mathcal{R}^{n_y \times n_y}$ and robustness filter $Q \in \mathcal{R}^{n_y \times n_y}$ such that $z^{-N} L(z) Q(z) \in \mathcal{R}^{n_y \times n_y}$. That is, $L$ and $Q$ can have finite preview, i.e., be noncausal, provided that $R(z)$ is causal, by embedding their preview in the delay $z^{-N}$. Although in this work a single memory loop is used, multiple memory loops can be included for uncertain $N$, see, e.g., [16], [28], [29].

C. Background: RC Design for SISO Systems

For the SISO case, i.e., $n_u = n_y = 1$, design techniques for $L$ and $Q$ are well developed. Fundamental to these techniques is the following well-known result for stability, e.g., [5], which is recovered as a special case of the multivariable case in Section III. Assuming stable $S, T, L, Q$, the closed-loop system (1) is stable for all $N \in \mathbb{N}$ if

$$|(1 - T(e^{j \omega}) L(e^{j \omega}) Q(e^{j \omega}))| < 1 \quad \forall \omega \in [0, \pi] \quad (3)$$

Based on this result, typically the following two-step design procedure is considered [4], [26], [28].

**Procedure 1:** Frequency-Domain SISO RC Design.

1. Given a parametric model of $T(z)$, construct $L(z)$ as an approximate stable inverse, i.e., $L(z) \approx \frac{1}{T(z)}$, with learning gain $\alpha \in (0, 1]$.
2. Using a nonparametric FRF model of $T(e^{j \omega})$, design $Q(z)$ such that (3) is satisfied.

Procedure 1 enables a systematic and inexpensive robust design, particularly since no accurate parametric model of the system is required. In step 1, the inverse approximation can be based on, often deliberately, coarse parametric model. In case $T$ is nonminimum phase, algorithms for inversion include ZPETC [26], FIR models [30], and $\mathcal{H}_\infty$-synthesis with [31] and without finite preview [9]. Then, in step 2, robustness to modeling errors and uncertainty, originating from step 1, can effectively be dealt with through the use of nonparametric FRF models, which are for mechatronic systems often inexpensive, accurate and fast to obtain [18]. In addition, in view of robust stability with respect to plant uncertainty, the use of FRF models straightforwardly allows to include confidence intervals, which are typically readily available for FRF estimates that result from experimental data [18].

For the MIMO case however, practical robust designs such as in Procedure 1 do not straightforwardly apply. Crucial yet largely unexplored aspects herein are modeling requirements, and dealing with (deliberate) modeling errors and uncertainty.

D. Problem Formulation and Contributions

Despite the availability of Procedure 1 for SISO RC design, a systematic RC design for MIMO systems, which addresses model quality and model requirements, is not yet available. The aim of the present article is to bridge this gap by extending and generalizing Procedure 1 to the multivariable case.

The first contribution (C1) is the development of a range of robust decentralized design techniques. The designs require only SISO parametric models, since robustness to modeling errors and uncertainty, including multivariable interaction, can be dealt with through the use of FRF models. The second contribution (C2) is an experimental demonstration of the proposed RC framework for continuous media flow printing.

In the next section, the multivariable design problem is analyzed, forming the basis for the decentralized designs in Sections IV and V. In Section VI, a design procedure is presented that connects all developed approaches. The experimental validation on a flatbed printer with continuous media flow is presented in Section VII.

III. ANALYSIS OF MULTIVARIABLE RC DESIGN

Next, the multivariable RC problem is analyzed. A stability theory is developed that is fundamental to forthcoming developments, and the implications of decentralized design on modeling requirements are investigated.
and there are no unstable pole-zero cancellations between

Suppose all poles of

A. Stability Analysis

Stability is investigated of the system (1). Observe that

\[
S_R = (I + TR)^{-1} = \left( I + TLz^{-N}Q(I - z^{-N}Q)^{-1} \right)^{-1} \\
= \left( I - z^{-N}Q + TLz^{-N}Q(I - z^{-N}Q)^{-1} \right)^{-1} \\
= (I - z^{-N}Q)(I - (I - TL)z^{-N}Q)^{-1}. \tag{4}
\]

The following results, see, e.g., [5], enable subsequent derivations, including decentralized RC designs.

**Theorem 1** (A Nyquist stability theorem for MIMO RC): Consider the control configuration of Fig. 3 with \( R \) as in (2). Suppose all poles of \( S, T, L, \) and \( Q \) are in the open unit disk, and there are no unstable pole-zero cancellations between \( S, (I - z^{-N}Q) \), and \( (I - (I - TL)z^{-N}Q)^{-1} \). Then, the closed-loop multivariable system (1) is stable if and only if the image of \( \text{det}(I - (I - TL)z^{-N}Q) \)

1) does not encircle the origin,
2) and does not pass through the origin,
as \( z \) traverses the Nyquist contour \( \Gamma \), depicted in Fig. 5.

**Theorem 2** (Stability independent of \( N \)): Let the assumptions of Theorem 1 hold. If \( (I - TL)z^{-N}Q \) is strictly proper, then the closed-loop system (1) with \( R \) as in (2) is stable for all \( N \in \mathbb{N} \) if

\[
\rho \left( (I - T(e^{i\omega})L(e^{i\omega}))Q(e^{i\omega}) \right) < 1 \quad \forall \omega \in [0, \pi]. \tag{5}
\]

Proofs of Theorem 1 and Theorem 2 are provided in the Appendix. The results are crucial for forthcoming developments. It is emphasized at this point that Theorem 1 gives a necessary condition for stability for a fixed value of \( N \), and Theorem 2 guarantees stability for all \( N \in \mathbb{N} \).

**Remark 1:** Theorem 1 reveals the importance of factorization (4) for RC design. Note that in Theorem 1, no encirclements of the origin are required for stability. In sharp contrast, directly applying Nyquist’s theorem to \( S_R = (I + TR)^{-1} \) may severely complicate RC design, since the loop-gain \( TR \) may contain unstable poles. For example, selecting \( Q = I \) in (2) yields \( N \) open-loop unstable poles on the unit circle. In the multivariable case, the Nyquist contour in Fig. 5 should be adapted to include indentations into the unit disk, around the poles on the unit circle [32]. Hence for stability, the image of \( \text{det}(I + TR) \) should encircle the origin \( N \) times in a counterclockwise direction, which may be cumbersome to verify.

**Remark 2:** Theorem 2 is of crucial importance from a practical perspective, since the conservatism introduced by the underlying small-gain argument is negligibly small, and it enables independent and robust design with respect to \( N \). This is explained by the term \( z^{-N} \), which has very fast decaying phase for large values of \( N \): The phase decays linearly with frequency up to \( \pi N \) radians at the Nyquist frequency. Suppose now that \( \rho((I - TL)e^{-i\omega N}Q) < 1 \) is violated for some frequency \( \omega \neq 0 \) and fixed \( N \), i.e., (5) does not hold. Then, counting all (clockwise and counterclockwise) encirclements of the origin, as is required by Theorem 1, can be a very cumbersome task. Moreover, varying the value of \( N \) requires a reevaluation of counting all encirclements, whereas (5) guarantees stability independent of \( N \).

Motivated by Theorem 2 and the internal model principle [27], typical design aims for multivariable RC are \( TL \approx \alpha I, \alpha \in (0, 1] \), and \( Q \) as close to \( I \) as possible, while satisfying (5). This requires a multivariable parametric model of \( T \). To avoid this requirement, a decentralized approach can be taken.

B. Decentralized RC Design: Stability Considerations

The aim is to design the decentralized controllers

\[
L(z) = \text{diag}\{l_1(z), l_2(z), \ldots, l_n(z)\}, \tag{6}
\]

\[
Q(z) = \text{diag}\{q_1(z), q_2(z), \ldots, q_n(z)\} \tag{7}
\]
such that the multivariable closed-loop system (1) with \( R \) as in (2), \( L \) in (6) and \( Q(z) = q_d(z)I \) with SISO filter \( q_d(z) \) leads to the next result.

**Corollary 1:** Let the assumptions of Theorem 2 hold. Then, the closed-loop multivariable system (1) with \( R \) as in (2), \( L \) in (6) and \( Q(z) = q_d(z)I \) is stable for all \( N \in \mathbb{N} \) if

\[
|q_d(e^{i\omega})\rho(I - T(e^{i\omega})L(e^{i\omega}))| < 1 \quad \forall \omega \in [0, \pi]. \tag{8}
\]

The bound on \( q_d(z) \) in (8) is nonconservative for the case \( Q(z) = q_d(z)I \). However, \( q_d \) accounts for the worst-case modeling errors and interaction over all loops. Consequently, e.g., large modeling errors in a single loop affect the \( Q \)-filter applied to all other loops, which may deteriorate performance. This motivates the development of systematic procedures for independent \( Q \)-filter design, where each loop \( i \) is robustified through the corresponding \( q_i \) separately.

IV. Decentralized RC: Independent Designs With Robustness to Interaction

In this section, the MIMO design problem of Theorem 2 is reformulated as sets of independent SISO design problems, that account for interaction through robustness in \( Q \). The developed techniques are closely related to results for decentralized feedback interconnections, see, e.g., [20], [33], yet differ fundamentally regarding multivariable interaction. This is analyzed in Section IV-A. Based on this, sets of independent robust SISO
design conditions are developed in Sections IV-B and IV-C. These result in a design procedure for independent decentralized RC design in Section IV-D.

A. Factorization of Interaction

A factorization is performed to analyze the role of interaction and enable robust designs. Let $M = I - TL$. Then

$$MQ = (I + E)M_dQ$$

where $E = (M - M_d)M_d^{-1}$ represents normalized interaction in $M$, and $M_d = \text{diag}\{M_{11}, M_{22}, \ldots\}$. Hence, reformulating Theorem 2, stability is achieved for all $N \in \mathbb{N}$ if

$$\rho((I + E)M_dQ) < 1 \quad \forall \omega \in [0, \pi]$$

(10)

The diagonal term $M_dQ$ is to be designed, and the interaction term $I + E$ can be used to analyze robust stability.

Remark 3: The pursued factorization-based approach for decentralized RC is closely related to decentralized feedback control, see, e.g., [20] and [33, Section 10.6], yet fundamentally differs regarding the employed factorization. In decentralized feedback design, i.e., $K = \text{diag}\{k_i\}$ with loop gain $GK$, typically the return difference is factored as

$$I + GK = (I + ET_d)(I + G_dK)$$

(11)

with $E = (G - G_d)G_d^{-1}$ and $T_d = \text{diag}\{\frac{q_i}{\sigma_i}, \frac{\sigma_i}{q_i}, \frac{k_i}{\sigma_i}, \frac{\sigma_i}{k_i}\}$. Assuming stable $(I + G_dK)^{-1}$ and $T_d$, the closed-loop system is stable if $\rho(ET_d) < 1 \ \forall \omega \in [0, \pi]$, see [20, Thm. 2], [33, Sec. 10.6]. This is in sharp contrast with decentralized RC. The difference can be observed from (4). Essentially, the term $(I - (I - TL)z^{-N})^{-1}$ constitutes a positive feedback interconnection with loop gain $(I - TL)z^{-N}Q$. In (9), this loop gain is factored. As a result, $E$ appears affinely in (10). Alternative to this approach, also in RC the return difference can be factored (i.e., dual to decentralized feedback design):

$$I - (I - TL)z^{-N}Q = I - (I + E)M_dz^{-N}Q$$

$$= (I - E \underbrace{M_dz^{-N}Q(I - M_dz^{-N}Q)^{-1}}_{T_RC})(I - M_dz^{-N}Q).$$

(12)

Similar to feedback design on the basis of (11), conditions on $\rho(ET_{RC})$ can be developed for stability. It is emphasized that since $T_{RC}$ is a rational function of $z^{-N}$, such conditions in general guarantee stability only for a specific value of $N$. Factorizations (9) and (12) are both suitable alternatives. In the present article, the former is pursued, i.e., factoring the loop gain, since it enables design independent of $N$. The presented results can be appropriately modified for decentralized design based on (12), i.e., by factoring the return difference.

Next, sufficient conditions for robust stability are developed based on the use of 1) Gershgorin’s theorem, and 2) the structured singular value (SSV).

B. Independent Design Based on Gershgorin Bounds

Application of Gershgorin’s theorem, see, e.g., [34, Theorem 8.2], to the factorization (9) leads to the following result.

Theorem 3 (Independent RC design based on Gershgorin’s theorem): Let the assumptions of Theorem 2 hold. Then, the closed-loop system (1) with $R$ in (2), $L$ in (6), and $Q$ in (7) is stable for all $N \in \mathbb{N}$ if either

$$\left|(1 - t_{ii}(e^{i\omega})t_{ii}(e^{i\omega}))q_i(e^{i\omega})\right| < \frac{1}{\mu_i(I + E)} \quad \forall i, \omega \in [0, \pi]$$

(13)

$$\left|(1 - t_{ii}(e^{i\omega})t_{ii}(e^{i\omega}))q_i(e^{i\omega})\right| < \frac{1}{\mu_i(I + E)} \quad \forall i, \omega \in [0, \pi]$$

(14)

A proof is provided in the Appendix. Theorem 3 provides individual bounds for each loop. Yet, these bounds impose no restriction on the structure of $Q$, which is potentially conservative for decentralized design. Next, alternative conditions are investigated that exploit the diagonal structure of $Q$.

C. Independent Design Based on the Structured Singular Value

Robust stability conditions are developed using the SSV, see, e.g., [20], [23]. The key idea is to exploit the structured form (9) in Theorem 2.

Definition 1: For $A \in \mathbb{C}^{n \times n}$, the SSV $\mu(A)$ is defined

$$\mu(A) = \frac{1}{\min\{\sigma(D) : D \in \Delta, \det(I - AD) = 0\}},$$

with $\Delta = \{A_j, \ldots, A_m\} : A_j \in \mathbb{C}^{m_j \times m_j}, \sum_{j=1}^m m_j = n\}$ a prescribed set of block diagonal matrices, unless no $\Delta$ in $\Delta$ makes $I - AD$ singular, in which case $\mu(A) = 0$.

Theorem 4 (Independent RC design based on SSV): Let the assumptions of Theorem 2 hold. Then, the closed-loop system (1) with $R$ in (2), $L$ in (6), and $Q$ in (7) is stable for all $N \in \mathbb{N}$ if

$$\left|(1 - t_{ii}(e^{i\omega})t_{ii}(e^{i\omega}))q_i(e^{i\omega})\right| < \frac{1}{\mu_i(I + E)} \quad \forall i, \omega \in [0, \pi]$$

(15)

where $\mu_i(\cdot)$ is taken with respect to a diagonal structure.

A proof is provided in the Appendix. In Theorem 4, the SSV is employed in a fundamentally different way than in traditional stability analyses of feedback interconnections. In robust control approaches, see, e.g., [23, Ch. 9, 11], [33, Ch. 8], typically $\mu(M)$ is taken with respect to a structured uncertainty $\Delta$, and $M$ denotes a nominal model. In sharp contrast, here $I + E$ has the role of nominal model, and $M_dQ$ is the structured uncertainty, i.e., is yet to be designed.

D. Design Considerations and Procedure

Theorems 3 and 4 enable systematic robust decentralized design using only SISO parametric models. Interaction does not have to be included in parametric models, since the right-hand-sides of (13), (14), (15) can be computed based on FRF models. This gives rise to the following design procedure.

1) Given SISO parametric models of \( t_{ii}(z) \), \( i = 1, \ldots, n_y \), construct \( \hat{l}_i(z) \) as approximate stable inverses, i.e., \( \hat{l}_i(z) \approx \frac{a}{l_i(z)} \).
2) Given a MIMO nonparametric FRF model of \( T(e^{j\omega}) \), design \( q_i(z) \) according to joint evaluation of Theorems 3 and 4, such that for each separate frequency \( \omega \in [0, \pi] \), at least one of (13), (14), (15) is satisfied.

Remark 4: In the SISO case, Theorems 3 and 4 recover the SISO condition for stability (3), since in this case \( E = 0 \).

Remark 5: The sufficient conditions for stability (13), (14), (15) complement each other in the sense that the ordering of their tightness may vary as a function of frequency, see [24]. Hence, they should be considered jointly during design, see Procedure 2. Note that they can not be combined over loops \( i \): Stability is guaranteed only if, for each \( \omega \in [0, \pi] \), at least one condition is satisfied for all loops \( i \) simultaneously. Further results on their tightness can be found in literature, e.g., [20], [23], [35], and [34, Ch. 8].

In Procedure 2, interaction is dealt with through independent robust design of each SISO filter \( q_i \). In the next section, a sequential robust design is presented, in which only the interaction present in previously designed loops needs to be accounted for, at the cost of a more involved procedure.

V. DECENTRALIZED RC: SEQUENTIAL DESIGN FOR INTERACTION

The MIMO design problem of Theorem 1 is reformulated as a set of sequential SISO designs. Crucially, interaction is explicitly accounted for, since \( \hat{t}_{ii} \) contains all interactions from all preceding loops \( 1, \ldots, i-1 \).

Remark 6: Note that since \( \hat{t}_{ii} \) contains all repetitive controllers from preceding loops, it is a function of \( z^{-N} \). Hence, Theorem 5 guarantees stability for a specific value of \( N \) only.

B. Design Considerations and Procedure

Theorem 5 is particularly useful for design of \( Q \), while sequential design of \( L \) is significantly more involved. Although inversion-based design of \( l_i \) based on \( \hat{t}_{ii} \) may seem appealing, constructing parametric models of \( \hat{t}_{ii} \) may be a cumbersome task, since 1) the delays \( z^{-N} \) from preceding loops may lead to very a high order of \( \hat{t}_{ii} \), and 2) iterative redesign of \( \tilde{t}_{i[1,i-1]} \) and remodeling of \( \hat{t}_{ii} \) may be required, as is motivated next.

The loop-closing order in Theorem 5 must be selected with care, since the resulting closed-loop system may depend on the loop-closing order. Important considerations include, see, e.g., [22], 1) performance deterioration of previously designed loops due to successive designs, and 2) influences of previously designed loops of subsequent designs. This implies that iterative redesign may be needed. Guidelines that have been proven in practice are 1) to close the loops in decreasing order of required robustness, and 2) to end with loops for which performance guarantees are desired, see [22], [36], [37] for results in sequential feedback control design. Without loss of generality, the loop-closing order can be altered using a permutation matrix \( P \), and replacing \( T \) with \( TP \). This leads to the following design procedure.

Procedure 3: Sequential Decentralized RC Design.

Given a nonparametric MIMO model of \( T(e^{j\omega}) \) and decentralized filter \( \hat{L}(z) \), perform the following sequence of steps.
1) Choose the order in which the loops are designed.
2) Set the index \( i = 1 \), and perform the following steps.
   a) Construct \( \hat{t}_{ii}(e^{j\omega}) \) according to (17).
   b) Design \( q_i \) according to (18) in Theorem 5, based on the nonparametric model of \( T \).
   c) Until \( i = n_y \), set \( i \rightarrow i + 1 \) and return to step 2a.
   d) If the resulting closed-loop system is unsatisfactory, reset \( i = 1 \), return to 2a, and redesign \( q_i \).
3) If the resulting closed-loop system after iterations is unsatisfactory, return to step 1 and change the loop closing order.
In the previous sections, decentralized design procedures are presented for robust multivariable RC. Next, a design procedure is provided that connects all approaches.

VI. OVERVIEW OF APPROACHES AND PROCEDURE

In this article, a range of decentralized design techniques is developed that guarantee robust stability of the closed-loop system, i.e., Theorems 3, 4, and 5. The decentralized techniques are complementary in the sense that, in general, neither of these results implies satisfaction of the others, see, e.g., Remark 5. Implications regarding stability guarantees between the results are illustrated in Fig. 6. Indeed, the ordering of conservatism in the developed criteria for closed-loop stability depends on the particular system at hand. Hence, all are valid alternative approaches.

This leads to the following procedure, which connects all results that are presented throughout the article.

Procedure 4: Multivariable RC Design.

Given a multivariable system, possibly after decoupling transformations, perform the following steps.

1) Decentralized RC design:
   a) If period $N$ is fixed, use sequential designs (Procedure 3, Theorem 5), as this in general yields the best performance.
   b) If RC is to be implemented with various $N$, use independent design (Procedure 2, Theorems 3, 4) for stability $\forall N \in \mathbb{N}$.
   c) Only if the user effort in terms of algorithmic complexity is severely limited, use robust SISO design (Corollary 1).

Only if performance is unsatisfactory, and this justifies the increased modeling requirements, proceed to next step.

2) Centralized RC design: Full MIMO design according to Theorems 1 or 2, see, e.g., [8]–[11] for possible approaches.

VII. EXPERIMENTAL VALIDATION: CONTINUOUS MEDIA FLOW ON A FLATBED PRINTER

In this section, the decentralized RC design techniques are experimentally validated and compared on a flatbed printer. It is demonstrated that the proposed RC framework enables continuous media flow operation with the accuracy required for printing, constituting contribution C2 of the present article. The experimental system is introduced next. In Section VII-B, the developed RC design techniques are applied to this system, and the corresponding results are provided in Section VII-C.

A. Experimental Setup

A multivariable Océ Arizona 550GT flatbed printer is considered, see Figs. 1 and 7. The gantry translates in $x$-direction and rotates in $\varphi$, and the carriage translates along the gantry in $y$-direction. The inputs are the currents to the brushless electrical motors, located on the left and right side of the gantry, denoted $U_l[A]$ and $U_r[A]$, and along the carriage, $U_y[A]$. Static input transformations are applied that approximately decouple the rigid-body dynamics into the carriage translation, gantry translation, and rotation, yielding a system $G(z)$ with inputs $U_y, U_x, U_\varphi[A]$ and outputs $y[m], x[m], \varphi[rad]$. The encoder resolution is $10^{-6} m$. The system is controller in discrete time with sampling time 1 ms. A stabilizing feedback controller is implemented, yielding closed-loop bandwidths of 6, 3, 4 Hz in $y, x, \varphi$-directions, respectively (lowest frequencies where $|g_{ii}(e^{i\omega})c_{ii}(e^{i\omega})| = 1$). An FRF measurement of $T$ is depicted in Fig. 8, together with parametric models $f_{ii}$ for RC design. Trajectory $r = [r_y, r_x, r_\varphi]^\top$ for continuous media flow printing is shown in Fig. 9, where $r_\varphi = 0$ and $N = 4734$.

B. Decentralized RC Designs

Four repetitive controllers are designed, according to Corollary 1, and Procedures 1, 2, and 3. Each $l_i(z)$ is obtained through inversion of the minimum phase SISO models $f_{ii}$, see Fig. 8. The robustness filters $q_i(z)$ are designed as 50th order zero-phase low-pass FIR filters, with associated cutoff frequencies $f_{c,i}$ specified in Table I. The loop-closing order of the sequential approach is 2, 1, 3, i.e., in decreasing order of required robustness.
**C. Experimental Results**

The results are presented in Figs. 10, 11, and 12. In Fig. 10, the matrix norm \( \|e_j\|_F = \sqrt{\sum_{i,k} |e_{ij}(i,k)|^2} \) is depicted as a function of periods, where \( e_j = [e_{j,y}, e_{j,x}, e_{j,\varphi}]^\top \in \mathbb{R}^{3 \times N} \) and \( e_{j,k} = e(k + jN) \). The following observations are made:

- Application of multiloop SISO RC designs after the performed rigid-body decoupling transformations, i.e., ignoring remaining interaction, leads to an unstable system. This stresses the importance of accounting for interaction in multivariable RC, including decentralized designs.
- Using robustly stable repetitive control, the tracking error is reduced by 99% in terms of \( ||e||_F \), compared to the case without repetitive control, i.e., \( \frac{||e_0||_F}{||e_1||_F} \approx 0.01 \).

- The sequential and independent design approaches outperform the robust SISO design by 38% and 25%, respectively, in terms of \( ||e_{10}||_F \). Note that each approach uses exactly the same (non)parametric models; performance is increased only through more sophisticated design of \( Q \).

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**TABLE I**

Comparison of Decentralized RC Designs

| Design | \( f_{c,1} \) [Hz] | \( f_{c,2} \) [Hz] | \( f_{c,3} \) [Hz] | \( \frac{||e_{10}||_F}{\sqrt{N}} \) |
|--------|-----------------|----------------|----------------|------------------|
| Multiloop SISO (Procedure 1) | 40 | 30 | 60 | N/A |
| Robust SISO (Corollary 1) | 25 | 25 | 25 | \( 3.64 \times 10^{-6} \) |
| Independent (Procedure 2) | 32 | 25 | 35 | \( 2.71 \times 10^{-6} \) |
| Sequential (Procedure 3) | 37 | 30 | 42 | \( 2.24 \times 10^{-6} \) |

**Fig. 8.** Bode magnitude diagram of identified FRF of \( T \) (–), and SISO parametric models \( \hat{t}_{ii}(\cdot) \) used for decentralized RC design.

**Fig. 9.** Periodic references for continuous media flow printing, see also Fig. 2. The start/end position of each period is depicted by \( \bullet \), and the time windows for printing are indicated in grey. Note that \( r_y = 0 \).

**Fig. 10.** Multiloop SISO RC design (+ Procedure 1), i.e., ignoring interaction, leads to unstable behavior. By systematically addressing interaction, the sequential design approach (○, Procedure 3) outperforms the independent design approach (□, Procedure 2) and the robust SISO design (x, Theorem 2). The error obtained without RC is shown as (φ).

**Fig. 11.** Tracking errors with sequentially designed RC (—) and without RC (—) during the 10th period of \( r \). During printing (grey areas), the errors \( e_y \) and \( e_x \) are inside \( \pm 5 \mu m \), approaching the encoder resolution of \( 1 \mu m \).
increased productivity, print quality, and medium versatility in industrial printing.

APPENDIX

PROOFS OF THEOREMS

Proof of Theorem 1: With $S$ and $(I - z^{-N}Q)$ are stable, the closed-loop $SRs$ is stable iff $(I - H)^{-1}$ is stable with $H = (I - TL)z^{-N}Q$, see (4), assuming no unstable pole-zero cancellations. $(I - H)^{-1}$ essentially constitutes a positive feedback loop with loop gain $H$. The determinant of return difference $1 - H$ can be expressed as [33, Sec. 4.9]

$$\det(I - H(z)) = c(\phi_{cl}(z) - \phi_{ol}(z))$$

where $\phi_{ol}(z)$ is the characteristic polynomial of $H(z)$, $\phi_{cl}(z)$ is the characteristic polynomial of $(I - H)^{-1}$, and $c$ is a nonzero constant if $(I - H)^{-1}$ is well posed. The Cauchy argument principle states, assuming that

1) $\det(I - H(z))$ is analytic along $\Gamma$, i.e., $\phi_{ol}$ has no roots on $\Gamma$,
2) $\det(I - H(z))$ has $P$ poles inside $\Gamma$, i.e., $\phi_{ol}$ has $P$ roots inside $\Gamma$,
3) and $\det(I - H(z))$ has $Z$ zeros inside $\Gamma$, i.e., $\phi_{cl}$ has $Z$ roots inside $\Gamma$,

then the image of $\det(I - H(z))$ as $z$ traverses $\Gamma$ encircles the origin $Z - P$ times in a clockwise direction, see, e.g., [33, Lemma 4.10]. Hence, $(I - H)^{-1}$ is stable, i.e., has $Z = 0$ poles inside $\Gamma$, iff the image of $\det(I - H(z))$ encircles the origin $P$ times in a counterclockwise direction as $z$ traverses $\Gamma$. Since all poles of $H$ are inside the unit disk, i.e., $P = 0$, zero encirclements of the origin are needed. Note that as $H$ has no poles on $\Gamma$, Assumption 1 is automatically satisfied.

Proof of Theorem 2: Let $H = (I - TL)z^{-N}Q$, and consider Theorem 1 with Nyquist contour $\Gamma$ in Fig. 5. Note that if $H$ is strictly proper, which is the case for sufficiently large $N$, only the part of $\Gamma$ around the unit circle needs to be evaluated. This follows since $\lim_{|z| \to \infty} \det(I - H(z)) = 1$, i.e., the part of $\Gamma$ at infinity maps to $+1$, and the parallel branches along the real axis of $\Gamma$ do not influence the stability test [38, p. 86]. Next, consider $\det(I - (I - TL)e^{-iw}Q) = \prod_{i=1}^{N}(1 - \lambda_i((I - TL)e^{-iw}Q))$. Hence, if all $\lambda_i((I - TL)e^{-iw}Q)$ are smaller than 1 for all frequencies, then $\det(I - (I - TL)e^{-iw}Q)$ does not encircle the origin. Finally, since $|z^{-N}| = 1$ for $z$ on the unit circle, (5) implies stability for all $N \in \mathbb{N}$.

Proof of Theorem 3: First, since the products of two square matrices have identical spectra, note that $\rho((I + E)M_0Q) = \rho(M_0Q(I + E))$. Gershgorin’s theorem states that the eigenvalues of a $n \times n$ matrix $A$ lie in the union of the set of disks defined by $|z - a_{ii}| \leq \sum_{j \neq i} |a_{ij}|$, and also in the union of the set of disks defined by $|z - a_{ii}| \leq \sum_{j} |a_{ij}|$. Hence, if $\sum_{i,j} |a_{ij}| < 1\forall i$, or $\sum_{i} |a_{ij}| < 1\forall i$, then $\rho(A(e^{iw})) < 1$. Combining these results with Theorem 2, closed-loop stability hence follows if either

$$\sum_{j} |((I + E)M_0Q)_{ji}| = \sum_{j} |(I + E)_{ji}||M_0Q_{ij}| < 1 \forall i, w \in [0, \pi]$$

VIII. CONCLUSION

A design framework is developed for multivariable repetitive control that enables continuous media flow printing with enhanced positioning accuracy. The developed framework explicitly addresses the tradeoffs between model knowledge, design complexity, and control performance. The presented decentralized designs require only SISO parametric models, and provide robustness to interaction through 1) independent designs, including the use of the structured singular value, and 2) sequential design. The developed framework is experimentally validated on an industrial flatbed printing system, operating with continuous media flow. The results demonstrate a large potential for

- This performance improvement is achieved by systematically designing for interaction. Through independent and sequential decentralized designs, see Table I, robustness is addressed separately per loop. Particularly in loops 1 and 3, less robustness is required, whereas the conservative robust SISO design applies the same filter $q$ to each loop.
- Using sequential design, the peak error during printing is improved by a factor 9 in $y$-direction (from 45 to $5 \mu m$), 37 in $x$-direction (from 150 to $4 \mu m$), and 70 in $\varphi$-direction (from 180 to $2.5 \mu rad$), compared to the case without RC.
- The power spectra of the converged errors corresponding to the sequential design and the case without RC are depicted in Fig. 12. It is observed that performance improvements are achieved up to the cutoff frequencies of the low-pass filters $q_i(z)$, see Table I.

For the considered printing application, especially the error reduction in $x$-direction is important. Errors in $y$-direction, i.e., approximately parallel to the print-pass direction, can be compensated by adjusting the inkjet timing.
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REFERENCES


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